Chapter-2

$S^*_g$-Continuous Functions in Topological Spaces

2.1 Introduction

The concept of semi-continuity in topological spaces was introduced by Norman Levine[36] in the year 1963. Also semi-generalized continuous functions and semi-generalized irresolute functions were introduced by P.Sundaram[68] in 1991. In this direction we introduce a new function called $S^*_g$-continuous function. Further we investigated some of its characterizations and placed $S^*_g$-continuous function between continuous function and semi-continuous function. Continuing this work, some new functions called $S^*_g$-irresolute map, strongly $S^*_g$-continuous function, perfectly $S^*_g$-continuous function, slightly $S^*_g$-continuous functions and totally $S^*_g$-continuous functions are also introduced and its properties and compositions are discussed here. The relationship between all these functions are also studied.

2.2. Preliminaries

**Definition 2.2.1:** A function $f: X \rightarrow Y$ is called a *semi-continuous* [36] if $f^{-1}(U)$ is a semi-open set in $(X, \tau)$ for every open set $U$ of $(Y, \sigma)$.

**Definition 2.2.2:** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a *sg-continuous* [68] if $f^{-1}(F)$ is a $sg$ -closed set in $(X, \tau)$ for every closed set $F$ of $(Y, \sigma)$.

**Definition 2.2.3:** A mapping $f:X \rightarrow Y$ is said to be *strongly-continuous* [37] if the inverse image of every subset in $Y$ is both open and closed in $X$. 
Definition 2.2.4: A mapping $f: X \rightarrow Y$ is said to be perfectly continuous\cite{[55]} if the inverse image of every open set in $Y$ is open and closed in $X$.

Definition 2.2.5: A function $f: X \rightarrow Y$ is called slightly continuous\cite{[32]} if the inverse image of every clopen set in $Y$ is open in $X$.

### 2.3 $S^*_g$-Continuous Function

The concept of $S^*_g$-continuous functions and a new space called $S^*_g$-$T_{1/2}$ space in topological spaces are introduced in this section and their relations with other continuous maps are studied.

Definition 2.3.1: A mapping $f: X \rightarrow Y$ is said to be $S^*_g$-continuous if the inverse image of every open set in $Y$ is $S^*_g$-open in $X$.

Theorem 2.3.2: Every continuous function is $S^*_g$-continuous.

Proof: Let $f: X \rightarrow Y$ be a continuous function. Then for every open set $U$ in $Y$, $f^{-1}(U)$ is open in $X$. Since every open set is $S^*_g$-open, $f^{-1}(U)$ is $S^*_g$-open in $X$. Hence $f: X \rightarrow Y$ is $S^*_g$-continuous.

Remark 2.3.3: The converse of the above theorem need not be true as can be seen from the following example.

Example 2.3.4: Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}\}$ and $S^*_gO(X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $Y = \{d, e\}$ with $\sigma = \{\emptyset, Y, \{d\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = f(b) = d$ and $f(c) = e$. Here $f^{-1}\{d\} = \{a, b\}$ which is $S^*_gO$ but not open. Hence $f$ is $S^*_g$-continuous but not continuous.
Theorem 2.3.5: Every $S_g^*$-continuous function is semi-continuous.

**Proof:** Let $f: (X, \tau) \to (Y, \sigma)$ be a $S_g^*$-continuous function. Let $V$ be an open set in $Y$. Since $f$ is $S_g^*$-continuous, $f^{-1}(V)$ is $S_g^*$-open in $X$. But every $S_g^*$-open is semi-open. Therefore $f^{-1}(V)$ is semi-open in $X$. Hence $f$ is semi-continuous.

Remark 2.3.6: The converse of the above theorem need not be true as can be seen from the following example.

Example 2.3.7: Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. In this space $S_g^*O(X, \tau) = \tau$. $SO(X, \tau) = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$ and $Y = \{e, f, g\}$ with $\sigma = \{\emptyset, Y, \{e, f\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = e, f(b) = f$ and $f(c) = g$. Here $f^{-1}\{e, f\} = \{a, b\}$ which is semi-open but not $S_g^*$-open. Hence the remark.

Theorem 2.3.8: Every $S_g^*$-continuous function is semi-generalized continuous.

**Proof:** Let $f: (X, \tau) \to (Y, \sigma)$ be a $S_g^*$-continuous function. Let $G$ be an open set in $Y$. Since $f$ is $S_g^*$-continuous, $f^{-1}(G)$ is $S_g^*$-open in $X$. Since every $S_g^*$-open is semi-generalized open, $f^{-1}(G)$ is semi-generalized open in $X$. Hence $f$ is semi-generalized continuous.

Theorem 2.3.9: If $f: X \to Y$ is $S_g^*$-continuous and $g: Y \to Z$ is continuous, then $gof: X \to Z$ is $S_g^*$-continuous.

**Proof:** Let $V$ be an open set in $Z$. Since $g$ is continuous, $g^{-1}(V)$ is open in $Y$. Also since $f$ is $S_g^*$-continuous, $f^{-1}(g^{-1}(V))$ is $S_g^*$-open in $X$. Therefore $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $S_g^*$-open in $X$. Hence $gof$ is $S_g^*$-continuous.

Definition 2.3.10: A topological space $(X, \tau)$ is said to be $S_g^*-T_{1/2}$ space if every $S_g^*$-open set of $X$ is open in $X$. 
**Remark 2.3.11:** In $S^*_g-T_{1/2}$ space, the concept of continuous and $S^*_g$-continuous functions coincides.

**Theorem 2.3.12:** Let $f: X \to Y$ be a mapping from a topological space $X$ into a topological space $Y$. Then $f$ is $S^*_g$-continuous iff the inverse image of every closed set in $Y$ is $S^*_g$-closed in $X$.

**Proof:** **Necessity.** Let $A$ be any closed set in $Y$. Then $Y \setminus A$ is open in $X$. Since $f$ is $S^*_g$-continuous, $f^{-1}(Y \setminus A)$ is $S^*_g$-open in $X$. Then $f^{-1}(Y \setminus A) = X \setminus f^{-1}(A)$ is $S^*_g$-open in $X$. Hence $f^{-1}(A)$ is $S^*_g$-closed in $X$.

**Sufficiency.** Let $O$ be an open set in $Y$ then $Y \setminus O$ is closed in $X$. By the assumption $f^{-1}(Y \setminus O)$ is $S^*_g$-closed in $X$. Therefore $f^{-1}(Y \setminus O) = X \setminus f^{-1}(O)$ is $S^*_g$-closed in $X$ which implies $f^{-1}(O)$ is $S^*_g$-open in $X$. Hence $f$ is $S^*_g$-continuous.

**Theorem 2.3.13:** If $f: X \to Y$ is $S^*_g$-continuous, then $f(S^*_g Cl(A)) \subseteq Cl(f(A))$.

**Proof:** Since $f(A) \subseteq Cl(f(A))$, $A \subseteq f^{-1}(Cl(f(A)))$. Since $f$ is $S^*_g$-continuous and $Cl(f(A))$ is a closed set in $Y$, $f^{-1}(Cl(f(A)))$ is $S^*_g$-closed set in $X$. Therefore $S^*_g Cl(A) \subseteq f^{-1}(Cl(f(A)))$. This implies $f(S^*_g Cl(A)) \subseteq Cl(f(A))$.

**Theorem 2.3.14:** Let $X$ and $Y$ be any topological spaces. If $f: X \to Y$ be a mapping then the following statements are equivalent.

(i) $f$ is $S^*_g$-continuous.

(ii) The inverse image of every closed set in $Y$ is $S^*_g$-closed in $X$.

(iii) $S^*_g Cl(f^{-1}(A)) \subseteq f^{-1}(Cl(A))$ for every set $A$ in $Y$.

(iv) $f(S^*_g Cl(A)) \subseteq Cl(f(A))$ for every set $A$ in $X$.

(v) $f^{-1}(Int(U)) \subseteq S^*_g Int(f^{-1}(U))$ for every set $U$ in $Y$. 

25
Proof: (i)⇒(ii) Follows from theorem 2.3.12

(ii)⇒(iii) Let A be any subset of Y. Then Cl(A) is closed in Y. Therefore by (ii),
$f^{-1}(Cl(A))$ is $S^*_g$-closed in $X$. Therefore $f^{-1}(Cl(A)) = S^*_g Cl(f^{-1}(Cl(A)) \supseteq S^*_g Cl(f^{-1}(A))$.

(iii)⇒(iv) Let A be any open subset of X. By (iii), $f^{-1}(Cl(A)) \supseteq S^*_g Cl(f^{-1}(A)) \supseteq S^*_g Cl(A)$. Hence $f(S^*_g Cl(A)) \subseteq Cl(f(A))$

(iv)⇒(v) Suppose $f(S^*_g Cl(A)) \subseteq Cl(f(A))$ for every set A in X. Then $S^*_g Cl(A)) \subseteq f^{-1}(Cl(f(A)))$ which implies $X - S^*_g Cl(A) \supseteq X - f^{-1}(Cl(f(A)))$ then $S^*_g Int(X - A) \supseteq f^{-1}(Int(Y - f(A)))$. Therefore $S^*_g Int(f^{-1}(U)) \supseteq f^{-1}(Int(U))$ for every set U=Y − f(A) in Y.

(v)⇒(i) Let A be an open set in Y. Then $f^{-1}(Int(A)) \subseteq S^*_g Int(f^{-1}(A))$ which implies $f^{-1}(A) \subseteq S^*_g Int(f^{-1}(A))$. Also since $f^{-1}(A) \supseteq S^*_g Int(f^{-1}(A))$. Hence $f^{-1}(A) = S^*_g Int(f^{-1}(A))$. Therefore $f^{-1}(A)$ is $S^*_g$-open in X. So (i).
2.4 $S^*_g$-irresolute Map

$S^*_g$-irresolute maps are defined and it is proved that the composition of two $S^*_g$-irresolute maps is again a $S^*_g$-irresolute map. Some characterization of $S^*_g$-irresolute maps are also elucidated.

**Definition 2.4.1:** A map $f: X \to Y$ is said to be $S^*_g$-irresolute if the inverse image of every $S^*_g$-open set in $Y$ is $S^*_g$-open in $X$.

**Remark 2.4.2:** A map $f: X \to Y$ is $S^*_g$-irresolute if the inverse image of every $S^*_g$-closed set in $Y$ is $S^*_g$-closed in $X$.

**Theorem 2.4.3:** If $f: X \to Y$ is a $S^*_g$-irresolute map then $f$ is $S^*_g$-continuous.

**Proof:** Let $O$ be an open set in $Y$. Since every open set is $S^*_g$-open, $O$ is $S^*_g$-open in $Y$. Since $f$ is $S^*_g$-irresolute, $f^{-1}(O)$ is $S^*_g$-open in $X$. Hence $f$ is $S^*_g$-continuous.

**Remark 2.4.4:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 2.4.5:** Let $X = Y = \{a, b, c\}$ with $\tau = \emptyset, X, \{a\}, \{c\}, \{a, c\}$ and $\sigma = \emptyset, Y, \{a\}, \{a, b\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b, f(b) = c$ and $f(c) = a$. Here $\tau = S^*_gO(X, \tau)$ and $S^*_gO(Y, \sigma) = \emptyset, X, \{a\}, \{a, b\}, \{a, c\}$.

Since $f^{-1}\{a, c\} = \{b, c\}$ is not $S^*_g$-open in $X$, $f$ is not $S^*_g$-irresolute.

**Theorem 2.4.6:** Let $f: X \to Y$ be a $S^*_g$-continuous map from $X$ into $Y$ and $Y$ is $S^*_g$-$T_{1/2}$ space. Then $f$ is $S^*_g$-irresolute.

**Proof:** Let $A$ be a $S^*_g$-open set in $Y$. Since $Y$ is $S^*_g$-$T_{1/2}$, $A$ is an open set in $Y$. Since $f$ is $S^*_g$-continuous, $f^{-1}(A)$ is $S^*_g$-open in $X$. Hence $f$ is $S^*_g$-irresolute.
**Theorem 2.4.7:** Let $X$, $Y$ and $Z$ be any three topological spaces. If $f : X \rightarrow Y$ is $S^*_g$-irresolute and $g : Y \rightarrow Z$ is $S^*_g$-irresolute, then $gof : X \rightarrow Z$ is $S^*_g$-irresolute.

**Proof:** Let $U$ be an $S^*_g$-open set in $Z$. Since $g$ is $S^*_g$-irresolute, $g^{-1}(U)$ is $S^*_g$-open in $Y$. Also since $f$ is $S^*_g$-irresolute, $f^{-1}(g^{-1}(U))$ is $S^*_g$-open in $X$. Therefore $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is $S^*_g$-open in $X$. Hence $gof$ is $S^*_g$-irresolute.

**Theorem 2.4.8:** If $f : X \rightarrow Y$ is $S^*_g$-irresolute and $g : Y \rightarrow Z$ is $S^*_g$-continuous, then $gof : X \rightarrow Z$ is $S^*_g$-continuous.

**Proof:** Let $F$ be an open set in $Z$. Since $g$ is $S^*_g$-continuous, $g^{-1}(F)$ is $S^*_g$-open in $Y$. Also since $f$ is $S^*_g$-irresolute, $f^{-1}(g^{-1}(F))$ is $S^*_g$-open in $X$. Therefore $(gof)^{-1}(F)$ is $S^*_g$-open in $X$. Hence $gof$ is $S^*_g$-continuous.
2.5 Strongly $S_g^*$-continuous function

Levine[37] introduced the concept of strongly continuous maps. In this section, we introduce the concept of strongly $S_g^*$-continuous function and discuss some results using this function.

**Definition 2.5.1:** A mapping $f: X \to Y$ is said to be *strongly $S_g^*$-continuous* if the inverse image of every $S_g^*$-open set in $Y$ is open in $X$.

**Theorem 2.5.2:** If $f: X \to Y$ is strongly $S_g^*$-continuous then $f$ is a continuous function.

**Proof:** Let $G$ be any open set in $Y$. Since every open set is $S_g^*$-open, $G$ is $S_g^*$-open in $Y$. Since $f: X \to Y$ is strongly $S_g^*$-continuous, $f^{-1}(G)$ is open in $X$. Hence $f$ is continuous.

**Remark 2.5.3:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 2.5.4:** Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = a$. Here $\tau = S_g^*O(X, \tau)$ and $S_g^*O(Y, \sigma) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$. Here $f$ is continuous but not strongly $S_g^*$-continuous, since $f^{-1}\{a, c\} = \{b, c\}$ is not open in $X$.

**Theorem 2.5.5:** A map $f: X \to Y$ is strongly $S_g^*$-continuous if and only if the inverse image of every $S_g^*$-closed set in $Y$ is closed in $X$.

**Proof:** Suppose that $f$ is strongly $S_g^*$-continuous. Let $B$ be any $S_g^*$-closed set in $Y$. Then $B^c$ is $S_g^*$-open in $Y$. Since $f$ is strongly $S_g^*$-continuous, $f^{-1}(B^c)$ is open in $X$. But $f^{-1}(B^c) = f^{-1}(Y \setminus B) = X \setminus f^{-1}(B)$. Hence $f^{-1}(B)$ is closed in $X$.

Conversely suppose that the inverse image of every $S_g^*$-closed set in $Y$ is closed in $X$. Let $G$ be any $S_g^*$-open set in $Y$. Then $G^c$ is $S_g^*$-closed set in $Y$. By assumption, $f^{-1}(G^c)$ is
closed in $X$. But $f^{-1}(G^*) = X \setminus f^{-1}(G)$. Hence $f^{-1}(G)$ is open in $X$. Therefore $f$ is strongly $S^*_g$-continuous.

**Theorem 2.5.6:** If $f: X \to Y$ is strongly $S^*_g$-continuous and $g: Y \to Z$ is $S^*_g$-continuous, then $gof: X \to Z$ is continuous.

**Proof:** Let $V$ be an open set in $Z$. Since $g: Y \to Z$ is $S^*_g$-continuous, $g^{-1}(V)$ is $S^*_g$-open in $Y$. Also since $f$ is strongly $S^*_g$-continuous, $f^{-1}(g^{-1}(V))$ is open in $X$. Therefore $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is open in $X$. Hence $gof$ is continuous.

**Theorem 2.5.7:** If $f: X \to Y$ is strongly $S^*_g$-continuous and $g: Y \to Z$ is $S^*_g$-irresolute, then $gof: X \to Z$ is strongly $S^*_g$-continuous.

**Proof:** Let $G$ be an $S^*_g$-open set in $Z$. Since $g: Y \to Z$ is $S^*_g$-irresolute, $g^{-1}(G)$ is $S^*_g$-open in $Y$. Also since $f$ is strongly $S^*_g$-continuous, $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is open in $X$. Hence $gof: X \to Z$ is strongly $S^*_g$-continuous.

**Theorem 2.5.8:** If $f: X \to Y$ is $S^*_g$-continuous and $g: Y \to Z$ is strongly $S^*_g$-continuous, then $gof: X \to Z$ is $S^*_g$-irresolute.

**Proof:** Let $U$ be an $S^*_g$-open set in $Z$. Since $g$ is strongly $S^*_g$-continuous, $g^{-1}(U)$ is open in $Y$. Also since $f$ is $S^*_g$-continuous, $f^{-1}(g^{-1}(U))$ is $S^*_g$-open in $X$. But $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$. Hence $gof: X \to Z$ is $S^*_g$-irresolute.

**Theorem 2.5.9:** Every strongly $S^*_g$-continuous function is $S^*_g$-continuous function.

**Proof:** Let $f: X \to Y$ be strongly-$S^*_g$-continuous. Let $O$ be any open set in $Y$. Since every open set is $S^*_g$-open, $O$ is $S^*_g$-open in $Y$. Therefore $f^{-1}(O)$ is open in $X$ which implies $f^{-1}(O)$ is $S^*_g$-open in $X$. Hence $f$ is $S^*_g$-continuous.

**Remark 2.5.10:** The converse of the above theorem need not be true as can be seen from the following example.
Example 2.5.11: Let \( X = Y = \{a, b, c\} \) with \( \tau = \emptyset, X, \{a\}, \{c\}, \{a, c\} \) and \( \sigma = \emptyset, Y, \{a\}, \{a, b\} \). Define \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a \), \( f(b) = c \) and \( f(c) = b \). Then \( \tau = S^*_g X, \tau \) and \( S^*_g \sigma = \emptyset, Y, \{a\}, \{a, b\}, \{a, c\} \). Here \( \{a, c\} \) is \( S^*_g \)-open in \( Y \), but \( f^{-1}\{a, c\} = \{a, b\} \) is not open in \( X \). So \( f \) is not strongly \( S^*_g \)-continuous.

Theorem 2.5.12: Every strongly continuous function is strongly \( S^*_g \)-continuous function.

Proof: Let \( f:X \rightarrow Y \) be strongly continuous. Let \( O \) be any \( S^*_g \)-open set in \( Y \). Since \( f \) is strongly continuous, \( f^{-1}(O) \) is open and closed in \( X \). Hence \( f \) is strongly \( S^*_g \)-continuous.

Remark 2.5.13: The converse of the above theorem need not be true as can be seen from the following example.

Example 2.5.14: Let \( X = Y = \{a, b, c\} \) with \( \tau = \emptyset, X, \{a\}, \{b\}, \{a, b\} \) and \( \sigma = \emptyset, Y, \{a, b\} \). The identity map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is strongly \( S^*_g \)-continuous but \( f \) is not strongly continuous. For the subset \( \{a, b\} \) of \( (Y, \sigma) \), \( f^{-1}\{a, b\} = \{a, b\} \) is open in \( (X, \tau) \) but not closed in \( (X, \tau) \).

Theorem 2.5.15: If \( f:X \rightarrow Y \) is strongly \( S^*_g \)-continuous and \( g:Y \rightarrow Z \) is strongly \( S^*_g \)-continuous, then \( gof: X \rightarrow Z \) is strongly \( S^*_g \)-continuous.

Proof: Let \( O \) be any \( S^*_g \)-open set in \( Z \). Since \( g \) is strongly \( S^*_g \)-continuous, \( g^{-1}(O) \) is open in \( Y \). By Theorem 1.3.3, \( g^{-1}(O) \) is \( S^*_g \)-open in \( Y \). Since \( f \) is strongly \( S^*_g \)-continuous, \( f^{-1}(g^{-1}(O)) \) is open in \( X \). Hence \( gof \) is strongly \( S^*_g \)-continuous.

Theorem 2.5.16: If \( f:X \rightarrow Y \) is continuous and \( g:Y \rightarrow Z \) is strongly \( S^*_g \)-continuous, then \( gof: X \rightarrow Z \) is strongly \( S^*_g \)-continuous.

Proof: Let \( O \) be any \( S^*_g \)-open set in \( Z \). Since \( g:Y \rightarrow Z \) is strongly \( S^*_g \)-continuous, \( g^{-1}(O) \) is open in \( Y \). Also since \( f \) is continuous, \( f^{-1}(g^{-1}(O)) \) is open in \( X \). Hence \( gof \) is strongly \( S^*_g \)-continuous.
**Theorem 2.5.17:** Let \((X,\tau)\) be any topological spaces and \(Y\) be a \(S^*_g-T_{1/2}\) space and \(f:X \to Y\) be a map. Then the following are equivalent:

(i) \(f\) is strongly \(S^*_g\)-continuous

(ii) \(f\) is continuous.

**Proof:** (i)⇒(ii) Let \(U\) be any open set in \(Y\). By Theorem 1.3.3, \(U\) is \(S^*_g\)-open in \(Y\). Then by (i), \(f^{-1}(U)\) is open in \(X\). Hence \(f\) is continuous.

(ii)⇒(i) Let \(O\) be any \(S^*_g\) open set in \(Y\). Since \(Y\) is a \(S^*_g-T_{1/2}\) space, \(O\) is open in \(Y\). Then by (ii), \(f^{-1}(U)\) is open in \(X\). Hence \(f\) is strongly \(S^*_g\)-continuous.

**Theorem 2.5.18:** Let \((X,\tau)\) be any topological spaces and \(Y\) be a \(S^*_g-T_{1/2}\) space and \(f:X \to Y\) be a map. Then the following are equivalent:

(i) \(f\) is \(S^*_g\)-irresolute

(ii) \(f\) is strongly \(S^*_g\)-continuous.

(iii) \(f\) is continuous

(iv) \(f\) is \(S^*_g\)-continuous.

**Proof:** The proof is straightforward.
2.6 Perfectly $S^*_g$-continuous function

In this section, we introduce the concept of perfectly $S^*_g$-continuous function and discuss some results using this function.

**Definition 2.6.1:** A mapping $f: X \to Y$ is said to be **perfectly $S^*_g$-continuous** if the inverse image of every $S^*_g$-open set in $Y$ is open and closed in $X$.

**Theorem 2.6.2:** If a map $f: X \to Y$ is perfectly $S^*_g$-continuous then it is strongly $S^*_g$-continuous.

**Proof:** Let $G$ be any $S^*_g$-open set in $Y$. Since $f: X \to Y$ is perfectly $S^*_g$-continuous, $f^{-1}(G)$ is open in $X$. Hence $f$ is strongly $S^*_g$-continuous.

**Remark 2.6.3:** The converse of the above theorem is not true as seen from the following example.

**Example 2.6.4:** Let $X = Y = \{a, b, c\}$ with $\tau=\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$. Here $S^*_g O(X, \tau) = \tau$. Define $f: (X, \tau) \to (X, \tau)$ as an identity map. Then $f$ is strongly $S^*_g$-continuous but not perfectly $S^*_g$-continuous, since $f^{-1}\{a\} = \{a\}$ is open in $X$ but not closed in $X$.

**Theorem 2.6.5:** A map $f: X \to Y$ is perfectly $S^*_g$-continuous if and only if $f^{-1}(G)$ is both open and closed in $X$ for every $S^*_g$-closed set $G$ in $Y$.

**Proof:** Assume that $f$ is perfectly $S^*_g$-continuous. Let $F$ be any $S^*_g$-closed set in $Y$. Then $F^c$ is $S^*_g$-open set in $Y$. Since $f$ is perfectly $S^*_g$-continuous, $f^{-1}(F^c)$ is both open and closed in $X$. But $f^{-1}(F^c) = X \setminus f^{-1}(F)$. Hence $f^{-1}(F)$ is both open and closed in $X$.

Conversely assume that the inverse image of every $S^*_g$-closed set in $Y$ is both open and closed in $X$. Let $G$ be any $S^*_g$-open set in $Y$. Then $G^c$ is $S^*_g$-closed set in $Y$. By assumption
$f^{-1}(G^c)$ is both open and closed in $X$. But $f^{-1}(G^c) = X \setminus f^{-1}(G)$ and so $f^{-1}(G)$ is both open and closed in $X$. Therefore $f$ is perfectly $S_g^*$-continuous.

**Theorem 2.6.6:** Every perfectly $S_g^*$-continuous function is perfectly continuous.

**Proof:** Let $f: X \to Y$ be perfectly $S_g^*$-continuous and $O$ be any open set in $Y$. Since every open set is $S_g^*$-open, $O$ is $S_g^*$-open in $Y$. Therefore $f^{-1}(O)$ is both open and closed in $X$. Hence $f$ is perfectly continuous.

**Remark 2.6.7:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 2.6.8:** Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ and $Y = \{d, e, f\}$ with $\sigma = \{\emptyset, Y, \{d\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = d, f(b) = f$ and $f(c) = e$. Then $\tau = S_g^*O(X, \tau)$ and $S_g^*O(Y, \sigma) = \{\emptyset, Y, \{d\}, \{d, e\}, \{d, f\}\}$. Here $\{d, e\}$ and $\{d, f\}$ are $S_g^*$-open in $Y$, but $f^{-1}\{d, e\} = \{a, c\}$ and $f^{-1}\{d, f\} = \{a, b\}$ are not open as well as not closed in $X$. So $f$ is not perfectly $S_g^*$-continuous.

**Theorem 2.6.9:** Let $f: (X, \tau) \to (Y, \sigma)$ be strongly $S_g^*$-continuous. Then $f$ is perfectly $S_g^*$-continuous if $(X, \tau)$ is a discrete topology.

**Proof:** Let $U$ be any $S_g^*$-open set in $(Y, \sigma)$. By hypothesis, $f^{-1}(U)$ is open in $(X, \tau)$. Since $(X, \tau)$ is a discrete topology, $f^{-1}(U)$ is closed in $(X, \tau)$. That is $f^{-1}(U)$ is both open and closed in $(X, \tau)$. Hence $f$ is perfectly $S_g^*$-continuous.

**Theorem 2.6.10:** If $f: X \to Y$ is perfectly $S_g^*$-continuous and $g: Y \to Z$ is perfectly $S_g^*$-continuous, then $g \circ f: X \to Z$ is perfectly $S_g^*$-continuous.

**Proof:** Let $O$ be any $S_g^*$-open set in $Z$. Since $g$ is perfectly $S_g^*$-continuous, $g^{-1}(O)$ is both open and closed in $Y$. By Theorem 1.3.3, $g^{-1}(O)$ is both $S_g^*$-open and $S_g^*$-closed in $Y,$
Since $f$ is perfectly-$S^*_g$-continuous, $f^{-1}(g^{-1}(O))$ is open and closed in $X$. Hence $gof$ is perfectly-$S^*_g$-continuous.

**Theorem 2.6.11:** If $f:X \to Y$ is continuous and $g:Y \to Z$ is perfectly $S^*_g$-continuous, then $gof:X \to Z$ is perfectly $S^*_g$-continuous.

**Proof:** Let $O$ be any $S^*_g$-open set in $Z$. Since $g$ is perfectly $S^*_g$-continuous, $g^{-1}(O)$ is both open and closed in $Y$. Since $f$ is continuous, $f^{-1}(g^{-1}(O))$ is open and closed in $X$. Hence $gof$ is perfectly $S^*_g$-continuous.

**Theorem 2.6.12:** If $f:X \to Y$ is perfectly $S^*_g$-continuous and $g:Y \to Z$ is $S^*_g$-irresolute, then $gof:X \to Z$ is perfectly $S^*_g$-continuous.

**Proof:** Let $O$ be any $S^*_g$-open set in $Z$. Since $g$ is $S^*_g$-irresolute, $g^{-1}(O)$ is $S^*_g$-open in $Y$. Since $f$ is perfectly $S^*_g$-continuous, $f^{-1}(g^{-1}(O))$ is both open and closed in $X$. Hence $gof$ is perfectly $S^*_g$-continuous.

**Theorem 2.6.13:** If $f:X \to Y$ is contra-continuous and $g:Y \to Z$ is perfectly $S^*_g$-continuous, then $gof:X \to Z$ is perfectly $S^*_g$-continuous.

**Proof:** Let $O$ be any $S^*_g$-open set in $Z$. Since $g$ is perfectly $S^*_g$-continuous, $g^{-1}(O)$ is both open and closed in $Y$. Since $f$ is contra-continuous, $f^{-1}(g^{-1}(O))$ is closed and open in $X$. Hence $gof$ is perfectly $S^*_g$-continuous.

**Theorem 2.6.14:** If $f:X \to Y$ is perfectly $S^*_g$-continuous and $g:Y \to Z$ is $S^*_g$-continuous, then $gof:X \to Z$ is perfectly continuous.

**Proof:** Let $O$ be any open set in $Z$. Since $g$ is $S^*_g$-continuous, $g^{-1}(O)$ is $S^*_g$-open in $Y$. Since $f$ is perfectly $S^*_g$-continuous, $f^{-1}(g^{-1}(O))$ is both open and closed in $X$. Hence $gof$ is perfectly continuous.
**Theorem 2.6.15:** If \( f: X \to Y \) is perfectly-continuous and \( g: Y \to Z \) is strongly \( S_g^* \)-continuous, then \( gof: X \to Z \) is perfectly \( S_g^* \)-continuous.

**Proof:** Let \( O \) be any \( S_g^* \)-open set in \( Z \). Since \( g \) is strongly-\( S_g^* \)-continuous, \( g^{-1}(O) \) is open in \( Y \). Since \( f \) is perfectly-continuous, \( f^{-1}(g^{-1}(O)) \) is both open and closed in \( X \). Hence \( gof \) is perfectly \( S_g^* \)-continuous.
2.7 Slightly $S_g^*$-continuous function

In 1997, Slightly continuity was introduced by Jain[32] and has been applied for semi-open and pre open sets by Nour[59] and Baker[5] respectively. In this section, slightly $S_g^*$-continuous has been introduced for $S_g^*$-open sets and various properties are discussed. Also the relationship between other functions are established.

**Definition 2.7.1:** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be *slightly $S_g^*$-continuous* at a point $x \in X$ if for each subset $V$ of $Y$ containing $f(x)$, there exists a $S_g^*$-open subset $U$ in $X$ containing $x$ such that $f(U) \subseteq V$. The function $f$ is said to be slightly $S_g^*$-continuous if $f$ is slightly $S_g^*$-continuous at each of its points.

**Example 2.7.2:** Let $X = \{a, b, c\} = Y$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$. $S_g^* O(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. The function $f: (X, \tau) \rightarrow (Y, \sigma)$ is defined as $f(a) = c, f(b) = a, f(c) = b$ is slightly $S_g^*$-continuous.

**Proposition 2.7.3:** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be slightly $S_g^*$-continuous iff the inverse image of every clopen set in $Y$ is $S_g^*$-open in $X$.

**Proof:** Suppose $f$ is slightly $S_g^*$-continuous. Let $V$ be a clopen set in $Y$ and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists a $S_g^*$-open set $U_x$ such that $x \in U_x$ and $f(U_x) \subseteq V$. Now $x \in U_x \subseteq f^{-1}(V)$. And $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Since arbitrary union of $S_g^*$-open sets is $S_g^*$-open, $f^{-1}(V)$ is $S_g^*$-open in $X$.

Conversely, let $f(x) \in V$ where $V$ is a clopen set in $Y$. Since $f$ is slightly $S_g^*$-continuous, $x \in f^{-1}(V)$ where $f^{-1}(V)$ is $S_g^*$-open in $X$. Let $U = f^{-1}(V)$. Then $U$ is $S_g^*$-open in $X$, $x \in U$ and $f(U) \subseteq V$. Then $f$ is slightly $S_g^*$-continuous.
**Theorem 2.7.4:** Let \( f: (X, \tau) \to (Y, \sigma) \) be a function then the following are equivalent.

1. \( f \) is slightly \( S^*_g \)-continuous.
2. The inverse image of every clopen set \( V \) of \( Y \) is \( S^*_g \)-open in \( X \).
3. The inverse image of every clopen set \( V \) of \( Y \) is \( S^*_g \)-closed in \( X \).
4. The inverse image of every clopen set \( V \) of \( Y \) is \( S^*_g \)-clopen in \( X \).

**Proof:**

(1) \( \Rightarrow \) (2): Follows from Proposition 2.7.3.

(2) \( \Rightarrow \) (3): Let \( V \) be a clopen set in \( Y \) which implies \( V^c \) is clopen in \( Y \). By (2), \( f^{-1}(V^c) = (f^{-1}(V))^c \) is \( S^*_g \)-open in \( X \). Therefore \( f^{-1}(V) \) is \( S^*_g \)-closed in \( X \).

(3) \( \Rightarrow \) (4): By (2) and (3) \( f^{-1}(V) \) is \( S^*_g \)-clopen in \( X \).

(4) \( \Rightarrow \) (1): Let \( V \) be a clopen subset of \( Y \) containing \( f(x) \). By (4) \( f^{-1}(V) \) is \( S^*_g \)-clopen in \( X \). Put \( U = f^{-1}(V) \) then \( f(U) \subseteq V \). Hence \( f \) is slightly \( S^*_g \)-continuous.

**Theorem 2.7.5:** Every slightly continuous function is slightly \( S^*_g \)-continuous.

**Proof:** Let \( f: X \to Y \) be slightly continuous. Let \( U \) be a clopen set in \( Y \). Then \( f^{-1}(U) \) is open in \( X \). Since every open set is \( S^*_g \)-open, \( f^{-1}(U) \) is \( S^*_g \)-open. Hence \( f \) is slightly \( S^*_g \)-continuous.

**Remark 2.7.6:** The converse of the above theorem need not be true as can be seen from the following example

**Example 2.7.7:** Let \( X = \{a, b, c, d\} \) with \( \tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\} \). Then \( S^*_gO(X, \tau) = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\} \). Let \( Y = \{p, q, r\} \) with \( \sigma = \{Y, \emptyset, \{p\}, \{q, r\}\} \). Define \( f: (X, \tau) \to (Y, \sigma) \) by \( f(a) = p \), \( f(b) = q \) and \( f(c) = f(d) = r \). Hence \( f^{-1}\{q, r\} = \{b, c, d\} \) is \( S^*_g \)-open but not open in \( X \). Thus \( f \) is slightly \( S^*_g \)-continuous but not slightly continuous.
**Theorem 2.7.8:** Every $S^*_g$-continuous function is slightly $S^*_g$-continuous.

**Proof:** Let $f : X \rightarrow Y$ be a $S^*_g$-continuous function. Let $U$ be a clopen set in $Y$. Then $f^{-1}(U)$ is $S^*_g$-open in $X$ and $S^*_g$-closed in $X$. Hence $f$ is slightly $S^*_g$-continuous.

**Remark 2.7.9:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 2.7.10:** Let $X = \{a, b, c\}, Y = \{p, q\}$. $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\} = S^*_gO(X, \tau)$. $\sigma = \{Y, \emptyset, \{p\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = q, f(b) = f(c) = p$. The function $f$ is slightly $S^*_g$-continuous but not $S^*_g$-continuous since $f^{-1}\{p\} = \{b, c\}$ is not $S^*_g$-open in $X$.

**Theorem 2.7.11:** If the function $f : (X, \tau) \rightarrow (Y, \sigma)$ is slightly $S^*_g$-continuous and $(Y, \sigma)$ is a locally indiscrete space then $f$ is $S^*_g$-continuous.

**Proof:** Let $U$ be an open subset of $Y$. Since $Y$ is locally indiscrete, $U$ is closed in $Y$. Since $f$ is slightly $S^*_g$-continuous, $f^{-1}(U)$ is $S^*_g$-open in $X$. Hence $f$ is $S^*_g$-continuous.

**Theorem 2.7.12:** If the function $f : (X, \tau) \rightarrow (Y, \sigma)$ is slightly $S^*_g$-continuous and $(X, \tau)$ is a $S^*_g-T_{1/2}$ space then $f$ is slightly continuous.

**Proof:** Let $U$ be a clopen subset of $Y$. Since $f$ is slightly $S^*_g$-continuous, $f^{-1}(U)$ is $S^*_g$-open in $X$. Since $X$ is a $S^*_g-T_{1/2}$ space, $f^{-1}(U)$ is open in $X$. Hence $f$ is slightly continuous.

**Theorem 2.7.13:** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be function

(i) If $f$ is $S^*_g$-irresolute and $g$ is slightly $S^*_g$-continuous then $gof : (X, \tau) \rightarrow (Z, \eta)$ is slightly $S^*_g$-continuous.

(ii) If $f$ is $S^*_g$-irresolute and $g$ is $S^*_g$-continuous then $gof$ is slightly $S^*_g$-continuous.
(iii) If \( f \) is \( S^*_g \)-irresolute and \( g \) is slightly continuous then \( gof \) is slightly \( S^*_g \)-continuous.

(iv) If \( f \) is \( S^*_g \)-continuous and \( g \) is slightly continuous then \( gof \) is slightly \( S^*_g \)-continuous.

(v) If \( f \) is strongly \( S^*_g \)-continuous and \( g \) is slightly \( S^*_g \)-continuous then \( gof \) is slightly continuous.

(vi) If \( f \) is slightly \( S^*_g \)-continuous and \( g \) is perfectly \( S^*_g \)-continuous then \( gof \) is \( S^*_g \)-irresolute.

(vii) If \( f \) is slightly \( S^*_g \)-continuous and \( g \) is contra-continuous then \( gof \) is slightly \( S^*_g \)-continuous.

**Proof:**

(i) Let \( U \) be a clopen set in \( Z \). Since \( g \) is slightly \( S^*_g \)-continuous, \( g^{-1}(U) \) is \( S^*_g \)-open in \( Y \). Since \( f \) is \( S^*_g \)-irresolute, \( f^{-1}(g^{-1}(U)) \) is \( S^*_g \)-open in \( X \). Since \( (gof)^{-1}(U) = f^{-1}(g^{-1}(U)) \), \( gof \) is slightly \( S^*_g \)-continuous.

(ii) Let \( U \) be a clopen set in \( Z \). Since \( g \) is \( S^*_g \)-continuous, \( g^{-1}(U) \) is \( S^*_g \)-open in \( Y \). Also since \( f \) is \( S^*_g \)-irresolute, \( f^{-1}(g^{-1}(U)) \) is \( S^*_g \)-open in \( X \). Hence \( gof \) is slightly \( S^*_g \)-continuous.

(iii) Let \( U \) be a clopen set in \( Z \). Then \( g^{-1}(U) \) is \( S^*_g \)-open in \( Y \). Therefore \( f^{-1}(g^{-1}(U)) \) is \( S^*_g \)-open in \( X \), since \( f \) is \( S^*_g \)-irresolute. Hence \( gof \) is slightly \( S^*_g \)-continuous.
(iv) Let $U$ be a clopen set in $Z$. Then $g^{-1}(U)$ is open in $Y$, since $g$ is slightly continuous. Also since $f$ is $S_g^*$-continuous. Also since $f$ is $S_g^*$-continuous, $f^{-1}(g^{-1}(U))$ is $S_g^*$-open in $X$. Hence $gof$ is slightly $S_g^*$-continuous.

(v) Let $U$ be a clopen set in $Z$. Then $g^{-1}(U)$ is $S_g^*$-open in $Y$, since $g$ is slightly $S_g^*$-continuous. Also since $f$ is strongly $S_g^*$-continuous, $f^{-1}(g^{-1}(U))$ is open in $X$. Therefore $gof$ is slightly continuous.

(vi) Let $U$ be a $S_g^*$-open in $Z$. Since $g$ is perfectly $S_g^*$-continuous, $g^{-1}(U)$ is open and closed in $Y$. Since $f$ is slightly $S_g^*$-continuous, $f^{-1}(g^{-1}(U))$ is $S_g^*$-open in $X$. Hence $gof$ is $S_g^*$-irresolute.

(vii) Let $U$ be a closed and open set in $Z$. Since $g$ is contra-continuous, $g^{-1}(U)$ is open and closed in $Y$. Since $f$ is slightly $S_g^*$-continuous, $f^{-1}(g^{-1}(U))$ is $S_g^*$-open in $X$. Therefore $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is $S_g^*$-open in $X$. Hence $gof$ is slightly $S_g^*$-continuous.

**Theorem 2.7.14:** If $f: (X, \tau) \to (Y, \sigma)$ is slightly $S_g^*$-continuous and $A$ is an open subset of $X$ then the restriction $f_{|A}: (A, \tau_A) \to (Y, \sigma)$ is slightly $S_g^*$-continuous.

**Proof:** Let $V$ be a clopen subset of $Y$. Then $(f_{|A})^{-1}(V) = f^{-1}(V) \cap A$ Since $f^{-1}(V)$ is $S_g^*$-open and $A$ is open, $(f_{|A})^{-1}(V)$ is $S_g^*$-open in the relative topology of $A$. Hence $f_{|A}$ is slightly $S_g^*$-continuous.
2.8 Totally \( S^*_g \)-continuous function

In this section, the concept of totally \( S^*_g \)-continuous functions are introduced and some of their basic properties are studied.

Definition 2.8.1: A map \( f: (X, \tau) \to (Y, \sigma) \) is said to be **totally \( S^*_g \)-continuous** if the inverse image of every open set in \((Y, \sigma)\) is \( S^*_g \)-clopen in \((X, \tau)\).

Example 2.8.2: Let \( X = Y = \{a, b, c\} \) with \( \tau = \{X, \emptyset, \{a\}, \{b, c\}\} = S^*_g O(X, \tau) \) and \( \sigma = \{Y, \emptyset, \{a\}\} \). Define \( f: (X, \tau) \to (Y, \sigma) \) by \( f(a) = a, \ f(b) = b, \ f(c) = c. \) Then \( f \) is totally \( S^*_g \)-continuous.

Theorem 2.8.3: Every perfectly \( S^*_g \)-continuous function is totally \( S^*_g \)-continuous.

Proof: Let \( f: (X, \tau) \to (Y, \sigma) \) be a perfectly \( S^*_g \)-continuous function. Let \( U \) be an open set in \((Y, \sigma)\). Then \( U \) is \( S^*_g \)-open in \((Y, \sigma)\). Since \( f \) is perfectly \( S^*_g \)-continuous, \( f^{-1}(U) \) is both open and closed in \((X, \tau)\) which implies \( f^{-1}(U) \) is both open and closed in \((X, \tau)\) which implies \( f^{-1}(U) \) is both \( S^*_g \)-open and \( S^*_g \)-closed in \((X, \tau)\). Hence \( f \) is totally \( S^*_g \)-continuous.

Remark 2.8.4: The converse of the above theorem need not be true as can be seen from the following example.

Example 2.8.5: Let \( X = \{a, b, c\} \) with \( \tau = \{\emptyset, X, \{a\}, \{b, c\}\} = \tau^c \) and \( Y = \{d, e, f\} \) with \( \sigma = \{\emptyset, Y, \{d\}\} \). Define \( f: (X, \tau) \to (Y, \sigma) \) by \( f(a) = d, \ f(b) = f, \ f(c) = e. \) Then \( S^*_g O(X, \tau) = \tau \) and \( S^*_g O(Y, \sigma) = \emptyset, Y, \{d\}, \{d, e\}, \{d, f\} \). Here \( \{d, e\} \) and \( \{d, f\} \) are \( S^*_g \)-open in \( Y \), but \( f^{-1}\{d, e\} = \{a, c\} \) and \( f^{-1}\{d, f\} = \{a, b\} \) are not open as well as not closed in \( X \). So \( f \) is not perfectly \( S^*_g \)-continuous but totally \( S^*_g \)-continuous.
**Theorem 2.8.6:** Every totally $S_g^*$-continuous function is $S_g^*$-continuous

**Proof:** Suppose $f: (X, \tau) \to (Y, \sigma)$ is totally $S_g^*$-continuous and $A$ is any open set in $(Y, \sigma)$. Since $f$ is totally $S_g^*$-continuous, $f^{-1}(A)$ is $S_g^*$-clopren in $(X, \tau)$. Hence $f$ is a $S_g^*$-continuous function.

**Remark 2.8.7:** The converse of the above theorem need not be true as can be seen from the following example

**Example 2.8.8:** Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Then $S_g^*O(X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$. The identity map $f: (X, \tau) \to (Y, \sigma)$ is $S_g^*$-continuous but not totally $S_g^*$-continuous since $f^{-1}\{a\} = \{a\}$ is $S_g^*$-open in $(X, \tau)$ but not $S_g^*$-closed in $(X, \tau)$.

**Remark 2.8.9:** The following two examples shows that totally $S_g^*$-continuous and strongly $S_g^*$-continuous are independent.

**Example 2.8.10:** Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\} = \tau^c$ and $Y = \{d, e, f\}$ with $\sigma = \{\emptyset, Y, \{d\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = d$, $f(b) = f$ and $f(c) = e$. Then $S_g^*O(X, \tau) = \tau$ and $S_g^*O(Y, \sigma) = \{\emptyset, Y, \{d\}, \{d, e\}, \{d, f\}\}$. Here $\{d, e\}$ and $\{d, f\}$ are $S_g^*$-open in $Y$, but $f^{-1}\{d, e\} = \{a, c\}$ and $f^{-1}\{d, f\} = \{a, b\}$ are not open in $X$. Hence $f$ is not strongly $S_g^*$-continuous but totally $S_g^*$-continuous.

**Example 2.8.11:** Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\} = S_g^*O(X, \tau)$ and $\sigma = \{\emptyset, Y, \{a, b\}\} = S_g^*O(Y, \sigma)$. The identity map $f: (X, \tau) \to (Y, \sigma)$ is strongly $S_g^*$-continuous but not totally $S_g^*$-continuous. For, the subset $\{a, b\}$ of $(Y, \sigma)$, $f^{-1}\{a, b\} = \{a, b\}$ is $S_g^*$-open in $(X, \tau)$ but not $S_g^*$-closed in $(X, \tau)$. 

43
**Theorem 2.8.12:** Let \( f: (X, \tau) \to (Y, \sigma) \) and \( g: (Y, \sigma) \to (Z, \eta) \) be any two functions.

(i) If \( f \) is \( S^*_g \)-irresolute and \( g \) is totally \( S^*_g \)-continuous then \( g \circ f \) is totally \( S^*_g \)-continuous.

(ii) If \( f \) is totally \( S^*_g \)-continuous and \( g \) is continuous then \( g \circ f \) is totally \( S^*_g \)-continuous.

**Proof:**

(i) Let \( U \) be an open set in \( Z \). Since \( g \) is totally \( S^*_g \)-continuous, \( g^{-1}(U) \) is \( S^*_g \)-clopen in \( Y \). Since \( f \) is \( S^*_g \)-irresolute, \( f^{-1}(g^{-1}(U)) \) is \( S^*_g \)-open and \( S^*_g \)-closed in \( X \). Therefore \( (g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) \) is \( S^*_g \)-clopen in \( X \). Hence \( g \circ f \) is totally \( S^*_g \)-continuous.

(ii) Let \( U \) be an open set in \( Z \). Since \( g \) is continuous, \( g^{-1}(U) \) is open in \( Y \). Also since \( f \) is totally \( S^*_g \)-continuous, \( f^{-1}(g^{-1}(U)) \) is \( S^*_g \)-clopen in \( X \). Hence \( g \circ f \) is totally \( S^*_g \)-continuous.

**Remark 2.8.13:** The following diagram gives pictorial representation of the discussion in this chapter.