2.1 Introduction

The Einstein relation for the diffusivity-mobility ratio of the carriers in semiconductors (DMR) is known to be very useful [2.1] since the diffusion constant can be obtained from the ratio by knowing the experimental values of the mobility. In addition, the DMR is more accurate than any of the individual relations for the diffusivity or the mobility which are considered to be the two most widely used properties of carrier transport in semiconductors. In electronic devices since the performance of semiconductor devices at the device terminals and the speed of operation of modern switching devices are significantly influenced by the degree of carrier degeneracy, the simplest way of analysing them would be to use the expression for the DMR which, in turn, enables us to express the above features of the devices made of degenerate materials in terms of carrier concentration [2.2-2.3]. The connection of the DMR with the velocity auto-correlation function [2.4] and the relation of the same ratio with the screening length [2.5] have been studied. The classical value of the DMR is equal to \( (k_B T/e) \), \((k_B, T \text{ and } e \) are Boltzmann constant, temperature and the magnitude of the carrier charge respectively) and this relation is the well-known Einstein relation [2.1]. The relation is valid both for electrons and holes. In this conventional form, the relation holds only for non-degenerate materials although its validity has been suggested erroneously for degenerate compounds [2.6]. It is well-known from the fundamental works of Landsberg [2.7] that the Einstein relation, in degenerate materials is essentially determined by the energy band structures. It has therefore, different values in various degenerate materials and varies significantly with electron concentration, with the magnitude of the quantizing magnetic field, with quantizing electric field as in inversion layers, with size quantization as in ultrathin films, with magneto-size quantization, with quantum wires etc. The nature of some of these variations have been studied in the literature [2.8-2.40]. Nevertheless, there still remain scopes in the investigations made while the interest for further researches of the
DMR in non-parabolic semiconductors having different band structures under various quantization of band states is becoming increasingly important. With a view to exploring some of these aspects, an attempt is made to study the DMR in non-parabolic materials under certain external conditions in this present chapter.

In Section 2.1, the basic formulation of the DMR for multiband degenerate materials has been presented. Besides, a suggested experimental method of determining the DMR in semiconductors having degenerate carrier concentration having arbitrary dispersion laws has also been given in the same chapter. In Section 2.2, the DMR has been formulated in tetragonal semiconductors by deriving the generalized electron energy spectrum incorporating the anisotropies of the energy band constants within the frame work of \( k.p \) formalism. The DMRs' in II-VI semiconductors and also in Bi have also been discussed by using the respective energy band spectra. It is worth remarking that the effects of quantizing magnetic field on the band structures of non-parabolic materials are more striking that of the parabolic one and are easily observed in experiments. Under magnetic quantization, the general characteristics of the band structure remain same, but in each band the energy of the electron corresponding to the velocity transverse to the magnetic field becomes discrete due to the quantization of the area of the \( k \)-space in the direction perpendicular to the direction of application of the quantizing magnetic field. This quantum nature of Landau levels leads to a host of interesting transport phenomena. In Section 2.3, the magneto DMRs' in the tetragonal semiconductors, II-VI materials and Bi have been discussed. It should be noted that, the cross-field configuration is fundamental in studying the transport phenomena in solids and investigations of the electrons in non-parabolic materials offer interesting physical possibilities [2.41]. Optical investigations of bulk semiconductors started with the theoretical work of Hansel and Peter [2.42] who indicated that an influence of the electric field on the Landau levels should lead to observable effects in cyclotron resonance transition in semiconductors. Aranov [2.43] pointed out that the electric field effects should be visible in interband magneto-optical transitions. In Section 4, the discussions have been made on the DMRs' of the said materials under cross-field configurations.

The remarkable developments of the fine line lithography (FLL), molecular beam epitaxy (MBE), metal organic chemical vapour deposition (MOCVD) and other experimental techniques, have introduced an interesting era of fabricating new artificial materials known as quantum wells.
formed between two planer heterojunctions [2.44]. Apart from the obvious technological importance, the thinfilm technology offers the possibilities of observing many new effects that might provide more careful and significant informations regarding two dimensional degenerate electron gases in semiconductors. The quantum size effects (QSE) is observed in ultrathin films when the film dimension becomes comparable to the de-Broglie wave length of the carriers and produces significant changes in the electron transport of nano-devices. The restriction of the motion of the carriers in the direction normal to the film may be viewed as carrier confinement in a one dimensional infinitely deep potential well, leading to the quantization of the wave vector of the carriers in a direction perpendicular to the surface of the film, producing thereby a discrete energy spectrum and a density-of-states function that is step-like in energy [2.45]. Heterostructures based on various materials are currently widely investigated because of the enhancement of carrier mobility [2.46]. These properties make them suitable for applications in quantum well lasers [2.47], FET's [2.48], high speed digital networks [2.49], high frequency microwave circuits [2.50], optical modulators [2.51], optical switching systems [2.52] and other devices. Two dimensional carrier gases in quantum well have been studied by using magneto-resistive measurement [2.53], transistor devices [2.54] and photo-luminescence spectroscopy [2.55].

In Section 2.5, the DMR's in ultrathin films of II-VI semiconductors, tetragonal semiconductors and Bi have respectively presented. The calculations have been extended to study the DMRs' under magnetosize quantization for the said types of ultrathin materials in the same chapter. We have plotted the DMR's as functions of various physical variables taking few specimens of the aforementioned materials as examples.

2.2 Theoretical Background

Section 2.2.1

2.2.1.1 Generalized formulations of DMR for multiband degenerate semiconductors

The dispersion law of the carriers in i-th band in multiband semiconductors can, in general, be expressed as
\[
E = \left( \frac{\hbar^2 K^2}{2m^*_e(E)} \right) + E_i = E_u + E_i
\]  
(2.1)

where, \( \hbar = h/2\pi \), \( h \) is the Planck's constant, \( m^*_e(E) \) is the effective carrier mass and equilibrium distribution function can be written as

\[
f_{oi} = \left[ 1 + \exp(K_B T)^{-1}(E_x - E_i) \right]^{-1}
\]  
(2.2)

where \( E_F \) is the Fermi energy. The carrier concentration in \( n_i \) in the \( i \)-th band is given by

\[
n_i = (4\pi^3)^{-1} \int f_{oi} d\mathbf{k} = n_t(E_{pi}), \quad E_{pi} = E_F - E_i
\]  
(2.3)

The solution of Boltzmann transport equation leads to the expression of the conduction current \( j_d \) contributed by the carriers in the \( i \)-th band due to an electric field \( \zeta \) in the \( z \)-direction as

\[
j_d = -(4\pi^3)^{-1} \left( \zeta^2 / \hbar^2 \right) \left[ (\nabla_k \cdot E)^2 \right] \tau_i (\partial f_{oi} / \partial E_i) d\mathbf{k} \quad \text{\( \phi(n_i, \mu_i, \zeta) \)}
\]  
(2.4)

where \( \mu_i \) and \( \tau_i \) are the mobility and the relaxation time respectively for the carrier in \( i \)-th subband. The same equation (2.4) is valid for scattering mechanism for which the relaxation time is applicable where

\[
f_i = f_{oi} - \left[ (\nabla_k \cdot E) \left( \frac{\partial f_{oi}}{\partial E_i} \right) \phi_i(E) \right] \left( \frac{e\zeta}{\hbar} \right)
\]

The diffusion current contributed by the carriers in the \( i \)-th band can be written as

\[
j_{di} = -(4\pi^3)^{-1} \left( e / \hbar^2 \right) \left[ (\nabla_k \cdot E)^2 \right] \tau_i \left( \frac{\partial f_{oi}}{\partial E_i} \right) d\mathbf{k}
\]  
(2.5)

The term \( \left( \frac{\partial f_{oi}}{\partial z} \right) \) can be expressed as

\[
\left( \frac{\partial f_{oi}}{\partial z} \right) = \left[ \left( \frac{\partial f_{oi}}{\partial E_{pi}} \right) \left( \frac{\partial f_{oi}}{\partial z} \right) \right] = -\left( \frac{\partial f_{oi}}{\partial E_u} \right) \left( \frac{\partial f_{oi}}{\partial z} \right)
\]  
(2.6)

Besides \( \left( \frac{\partial n}{\partial z} \right) = \left( \frac{\partial}{\partial z} \right) \sum_i n_i(E_{pi}) = \left( \frac{\partial f_{oi}}{\partial z} \right) \phi_i \)
where \( \bar{\phi}_i = \sum_j \left( \frac{\partial n_j(E_{Fj})}{\partial E_{Fj}} \right) \left( \frac{\partial E_{Fj}}{\partial E_{Fj}} \right) \)

Replacing \( \frac{df_{mj}}{dz} \) on equation (2.5) by using equation (2.5) and (2.6) we get

\[
J_{Di} = -\left( \frac{e}{\hbar^2} \right)^2 \left( \frac{e}{\hbar} \right) (N_z E)^2 \tau (\frac{df_{m}}{dz}) (\frac{\partial n}{\partial z})^{-1} dk
\]

\[
= -\left( \frac{e}{|e|} \right) n_i/\mu_i (\bar{\phi}_i)^{-1} \left( \frac{\partial n}{\partial z} \right)
\]

(2.8)

Thus the total diffusion current can be written as

\[
J_0 = \sum_i J_{Di} = -\left( \frac{e}{|e|} \right) \left( \frac{e}{\hbar} \right) n_i/\mu_i (\bar{\phi}_i)^{-1} = -De \left( \frac{\partial n}{\partial z} \right)
\]

(2.9)

Thus we can write

\[
\frac{D}{\mu} = \frac{n}{|e|} \left[ \sum_i n_i/\mu_i (\bar{\phi}_i)^{-1} \right] \left[ \sum_i n_i/\mu_i \right]^{-1}
\]

(2.10)

For \( E_i \)'s are independent of \( z \), equation (2.10) gets simplified to the well-known form as

\[
\frac{D}{\mu} = \frac{n}{|e|} \left( \frac{\partial n}{\partial E_p} \right)
\]

(2.11)

In the electric quantum limit in inversion layers \( i=j=0 \) and equation (2.10) transforms as

\[
\frac{D}{\mu} = \frac{n_e}{|e|} \left( \frac{\partial n_e}{\partial (E_{ps} - E_o)} \right)
\]

(2.12)

where \( n_e, E_{ps} \) and \( E_o \) and the electron concentration, the energy of the electric subband and the Fermi energy in the electric quantum limit respectively.

It can be concluded that equation (2.11) is valid for different kinds of multiband semiconductors except for inversion layers and thin films of heterostructures with very large charge densities. For these exceptional cases the equation (2.10) should be used for the evaluation of DMR. For inversion layers under electric quantum limit, the equation (2.10) should be used.

Equation (2.11) is also valid for heterostructures if the contribution of the charge density to the internal potential is negligible.
2.2.1.2 Suggestion for experimental determination of DMR for degenerate semiconductors having arbitrary dispersion laws

(a) It is well-known that the thermoelectric power of the electrons in semiconductors in the presence of a very large magnetic field is independent of scattering mechanism and can be written [2.56] as

\[ G = \frac{S}{|e|n} \]  

(2.13)

where \( S \) is the entropy. Following Tsidilkovski [2.57], the equation (2.13) can be written under the condition of carrier degeneracy

\[ G = \left( \frac{\pi^2 K_B^2 T}{3n} \right) \left( \frac{\partial n}{\partial E_F} \right) \]  

(2.14)

Using equations (2.11) and (2.14) we get

\[ \frac{D}{\mu} = \left( \frac{\pi^2 K_B^2 T}{3G} \right) \left( |e|^2 \right) \]  

(2.15)

Thus we can determine the DMR by knowing \( G \) which can be experimentally measured [2.58].

(b) For inversion layers, under electric quantum limit, the equation (2.14) assumes the form

\[ G = \left( \frac{\pi^2 K_B^2 T}{3n_0} \right) \left[ \frac{dn_0}{d(E_{Fg} - E_0)} \right] \]  

(2.16)

Using equation (2.16) and (2.12) we get the same equation (2.15) or quantum wires and heterostructures with small charge densities, the relation between \( D/\mu \) and \( G \) is again by given by equation (2.15). The equation (2.15) is also valid in the presence of quantizing magnetic field and also under cross-field configuration. Thus the equation (2.15) is independent of the dimensions of quantum confinement. Obviously our analysis is not valid for totally k-space quantized system. It may be noted that in magneto-size quantization and also in magneto inversion-layers, the electron motion is possible due to the motion of the electrons in the broadened levels. It may also be noted in this context that for non-degenerate semiconductors,
Nag and Chakravarti [2.59] have shown that

\[ \frac{D}{\mu} = P_n |e| b \]  

(2.17)

where \( P_n \) is the available noise power in the band width \( b \). We wish to remark that equation (2.17) is valid only under non-degenerate electron concentration and the experimental results on \( P_n \) have yet to be reported in the literature to the best of the knowledge of this author.

Section 2.3

2.3.1 Formulation of DMR in bulk specimens of tetragonal semiconductors

\( A_{2g}^m B_{2u}^s \) and ternary chalcopyrite semiconductors are called tetragonal semiconductors since they have the tetragonal crystal structure [2.60]. These materials are being increasingly used as nonlinear optical elements [2.61] and light emitting diodes [2.62]. Rowe and Shay [2.63] have demonstrated that the quasi-cubic model can be used to explain the observed splitting and symmetry properties of the conduction and valence bands at the zone center of \( k \) space of the aforementioned semiconductors. The s-like conduction band is singly degenerate and the p-like valence band is triply degenerate. The latter splits into three subbands because of spin-orbit and crystal field interactions. The largest contribution of the crystal field parameter occurs from the non-cubic potential [2.65]. Incorporating the crystal potential to the Hamiltonian, Bodner [2.66] proposed a dispersion relation of the conduction electrons in the same semiconductor by using the assumption of an isotropic spin-orbit splitting parameter. It would, therefore, be of much interest to investigate the DMR in these materials by generalizing the above model within the framework of \( k.p \) formalism. This is done, in what follows, by taking \( n-Cd_3As_2 \) as an example of tetragonal semiconductors, which is being increasingly used in Hall pick-ups and thermal detectors.

The form of \( k.p \) matrix for tetragonal semiconductors can be expressed as

\[ H = \begin{bmatrix} H_1 & H_2 \\ H_2^* & H_1 \end{bmatrix} \]

(2.18)

where
\[
H_1 = \begin{bmatrix}
E_g & P_{\mu} K_x & 0 & 0 \\
P_{\mu} K_x & \left(\delta + \frac{1}{3}\Delta_{\mu}\right) & \left(\frac{\sqrt{2}}{3}\Delta_1\right) & 0 \\
0 & \left(\frac{\sqrt{2}}{3}\Delta_1\right) & \left(-\frac{2}{3}\Delta_1\right) & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
H_2 = \begin{bmatrix}
0 & 0 & \left(\frac{P_{\perp}}{\sqrt{2}}\right)(k_x - ik_y) & \left(\frac{P_{\perp}}{\sqrt{2}}\right)(k_x - ik_y) \\
0 & 0 & 0 & 0 \\
-f_{-1} & 0 & 0 & 0 \\
f_{+1} & 0 & 0 & 0
\end{bmatrix}
\]

in which \(E_g\) is the energy band gap, \(P_{\mu}\) and \(P_{\perp}\) are the momentum matrix elements parallel and perpendicular to the crystal axis respectively, \(\delta\) is the crystal field splitting parameter, \(\Delta_{\mu}\) and \(\Delta_1\) are the spin-orbit splitting parameters along and perpendicular to the C-axis respectively and \(i = \sqrt{-1}\). Thus, neglecting the contribution of the higher bands and the free electron energy, the diagonalisation of the above matrix leads to the dispersion relation of the conduction electron in bulk specimens of tetragonal semiconductors as

\[
C(E) = A(E)k_x^2 + B(E)k_z^2
\]

(2.19)

where

\[
C(E) = E(E + E_g)\left[(E + E_g)(E + E_g + \Delta_{\mu}) + \delta\left(E + E_g + \frac{2}{3}\Delta_{\mu}\right) + \frac{2}{9}\left(\Delta_{\mu}^2 - \Delta_1^2\right)\right], \quad k_z^2 = k_x^2 + k_y^2,
\]

\(E\) is the energy as counted from the edge of the conduction band in the vertically upward direction in the absence of any quantization.

\[
A(E) = \frac{\hbar^2 E_g (E_g + \Delta_{\mu})}{2m_+^*\left(E_g + \frac{2}{3}\Delta_{\mu}\right)}\left[\delta\left(E + E_g + \frac{2}{3}\Delta_{\mu}\right) + \left(E + E_g\right)\left(E + E_g + \frac{2}{3}\Delta_{\mu}\right) + \frac{1}{9}\left(\Delta_{\mu}^2 - \Delta_1^2\right)\right],
\]

\[
B(E) = \frac{\hbar^2 E_g (E_g + \Delta_{\mu})}{2m_-^*\left(E_g + \frac{2}{3}\Delta_{\mu}\right)}\left[\left(E + E_g\right)\left(E + E_g + \frac{2}{3}\Delta_{\mu}\right)\right], \quad \hbar = \frac{\hbar}{2\pi}, \quad \hbar\text{ is Planck's constant and}
\]

\(m_+^*\) and \(m_-^*\) are the longitudinal and transverse effective electron masses at the edge of the conduction band respectively.
The use of equation (2.2) leads to the expression of the density-of-states function as

\[ D_0(E) = \frac{2}{(2\pi)^3} \left( \frac{d}{dE} \right)[V(E)] \]

\[ = (3\pi^2)^{-1} p(E) \quad (2.20) \]

where \( V(E) \) is the volume of \( k \) space as formed by equation (2.2)

\[
P(E) = \left[ \frac{3/2 \left( \frac{C_1(E)\sqrt{C(E)}}{A(E)\sqrt{B(E)}} \right) - A_1(E)C(E)}{A^2(E)\sqrt{B(E)}} \right]^{3/2}
\]

\[ C_1(E) = \left[ (2E + E_g)C(E)[E(E + E_g)]^{-1} + E(E + E_g)\left(2E + 2E_g + \delta + \Delta_H\right) \right]^{-1} \]

\[ A_1(E) = \left[ 2m_\perp \left( E_g + \frac{2}{3}\Delta_\perp \right) \right]^{-1} \left[ \hbar^2 E_g \left( E_g + \Delta_\perp \right) \right] \left[ \delta + 2E + 2E_g + \frac{2}{3}\Delta_H \right] \quad \text{and} \]

\[ B_1(E) = \left[ 2m_\parallel \left( E_g + \frac{2}{3}\Delta_\parallel \right) \right]^{-1} \left[ \hbar^2 E_g \left( E_g + \Delta_\parallel \right) \right] \left[ 2E + 2E_g + \frac{2}{3}\Delta_H \right] \]

Combining equation (2.20) with the Fermi-Dirac occupation probability factor and using the generalized Sommerfeld's lema [2.64], the electron concentration can be written as

\[ n_0 = (3\pi^2)^{-1} [M(E_F) + N(E_F)] \quad (2.21) \]

where

\[ M(E_F) = \left[ \frac{C(E_F)^3}{A(E_F)\sqrt{B(E_F)}} \right], \quad N(E_F) = \sum_{r=1}^{n} L(r,0)M(E_F), \]

\[ L(r,J) = \frac{2(K_B T)^{2r} \zeta(2r) \zeta(2r) (d^{2r}\cdot J)}{(dE_F)^{2r+J}} \]

\( r \) and \( J \) are the sets of real positive integers and \( \zeta(2r) \) is the Zeta function of order \( 2r \) [2.65].

Thus using equations (2.21) and (2.11) we get
\[
\frac{D}{\mu} = \frac{1}{|e|} \frac{[M(E_f) + N(E_f)]}{[p(E_p) + q(E_p)]} \tag{2.22}
\]

where \(q(E_p) = \sum_{r=1}^{4} L(r,1)[M(E_p)]\)

Special cases

(a)

Under the substitutions \(\delta = 0, \Delta_H = \Delta_\parallel = \Delta\) (the isotropic spin-orbit splitting parameter) and \(m_{\perp}^* = m_{\parallel}^* = m^*\) (the isotropic effective electron mass at the edge of the conduction band), equation (2.19) assumes the form

\[
\frac{\hbar^2 k^2}{2m^*} = \gamma(E), \quad \gamma(E) = \frac{E(E + E_g)(E + E_g + \Delta)(E_x + \frac{2}{3} \Delta)}{E_x(E_x + \Delta)(E + E_x + \frac{2}{3} \Delta)} \tag{2.23}
\]

which is the standard dispersion relation of the conduction electrons of III-V semiconductors and is known as three band model [2.67] of Kane. The III-V materials find extensive applications in Bragg reflectors, distributed feedback lasers [2.68], passive filter devices [2.69], high electron mobility transistors [2.70] and also in integrated optoelectronics. Under the aforementioned limiting conditions, the basic forms of equations (2.21) and (2.22) remain unchanged where

\[
M(E_p) = \left[ \frac{2m^* \gamma(E_p)}{\hbar^2} \right]^{3/2}
\]

and

\[
p(E_p) = \frac{2}{3} \sqrt{\gamma(E_p) \gamma(E_p)} \left[ \frac{2m^*}{\hbar^2} \right]^{3/2}
\]

in which \(\gamma(E_p) = \gamma(E_p) \left[ (E_p + E_g)^{-1} + (E_p + E_g + \Delta)^{-1} - \left( E_p + E_g + \frac{2}{3} \Delta \right)^{-1} \right]\)
Under the inequalities $\Delta \gg E_g$ (e.g. InSb) or $\Delta \ll E_g$ (i.e. n-GaAs), the equation (2.23) assumes the form
\[
\frac{\hbar^2 k^2}{2m} = E(1 + \alpha E), \quad \alpha = \frac{1}{E_g}
\] (2.24)
which is known as two-band Kane model. Under the above conditions, the expressions for the electron statistics and the DMR can, respectively, be expressed through the equations (2.21) and (2.22)

where
\[
m(E_{\text{p}}) = \left[ E_{\text{p}}(1 + \alpha E_{\text{p}}) \frac{2m^*}{\hbar^2} \right]^2 \quad \text{and} \quad p(E_{\text{p}}) = \left[ \frac{2m^*}{\hbar^2} \right]^2 \left( 1 + 2\alpha E_{\text{p}} \right) \left[ E_{\text{p}}(1 + \alpha E_{\text{p}}) \right]^{1/2}
\]

(c) Under the conditions $\Delta \gg E_g$ or $\Delta \ll E_g$ together with the condition $\frac{E_{\text{p}}}{E_g} \ll 1$, we can write
\[
n_0 = N_c \left[ F_{y/2}(\eta) + \left( \frac{15\alpha k_B T}{4} \right) F_{y/2}(\eta) \right]
\] (2.25)

and
\[
\frac{D}{\mu} = \left[ \frac{k_B T}{|e|} \right] \left[ \frac{F_{y/2}(\eta) + \left( \frac{15\alpha k_B T}{4} \right) F_{y/2}(\eta)}{F_{y/2}(\eta) + \left( \frac{15\alpha k_B T}{4} \right) F_{y/2}(\eta)} \right]
\] (2.26)

where
\[
N_c = 2 \left( \frac{2m^* k_B T}{\hbar^2} \right)^{1/2}, \quad \eta = \frac{E_{\text{p}}}{k_B T} \quad \text{and} \quad F_i(\eta) \text{ is the one parameter Fermi-Dirac integral of order } t \text{ which can be written as},
\]
\[
F_i(\eta) = \left( \frac{1}{\Gamma_{y,t}} \right) \int_0^\infty y^t (1 + \exp(y - \eta))^{-1} dy, \quad y > -1
\] (2.27)

where $\Gamma_{y,t}$ is the complete Gamma function or for all $t$, analytically continued as a complex contour integral around the negative axis.
\[ F_t(\eta) = A_t \int_{-\infty}^{0+} y' \left( 1 + \exp(-y - \eta) \right)^{-1} dy \] (2.28)

in which \( A_t = \frac{\Gamma_t}{2\pi \sqrt{-1}} \)

(d) For wide band-gap materials \( E_g \to \infty \) and equations (2.25) and (2.26) assumes the forms

\[ n_0 = N_c F_{y_1}(\eta) \] (2.29)

\[ \frac{D}{\mu} = \left( \frac{k_B T}{e} \right) \frac{F_{y_2}(\eta)}{F_{y_1}(\eta)} \] (2.30)

The equations (2.29) and (2.30) were derived for the first time by Landsberg [2.7].

(e) Combining equations (2.25) and (2.26) and using the formula \( \frac{d}{dt} [F_t(\eta)] = F_{t-1}(\eta) \) easily derived from equation (2.27) and (2.28) together with the fact that under extreme degeneracy

\[ F_{y_2}(\eta) = \left[ \frac{4}{3\sqrt{\pi}} \right] (\eta)^{3/2} \]

and we can write

\[ n_0 = \frac{1}{3\pi^2} \left[ \frac{2m^* E_F (1 + \alpha E_F)}{\hbar^2} \right]^{3/2} \] (2.31)

\[ \frac{D}{\mu} = \left( \frac{2}{3} \right) \frac{E_F (1 - \alpha E_F)}{\epsilon_0^2} \] (2.32)

For \( \alpha \to 0 \), the equations (2.31) and (2.32) get simplified as

\[ n_0 = \frac{1}{3\pi^2} \left[ \frac{2m^* E_F}{\hbar^2} \right]^{3/2} \] (2.33)

and \[ \frac{D}{\mu} = \frac{2E_F}{3|\epsilon_0^2|} \] (2.34)
Under the condition of non-degenerate electron concentration $\eta \ll 0$ and $F_\eta(t) = \exp(\eta)$ for all $t$. Therefore the equations (2.25), (2.26), (2.29) and (2.30) assume the well known forms as

$$n_0 = N_c \exp(\eta) \quad (2.35)$$

and

$$\frac{D}{\mu} = \frac{k_B T}{|e|} \quad (2.36)$$

### 2.3.2 Formulation of DMR in bulk specimens of II-VI semiconductors

The group theoretical analysis shows that based on the symmetry properties of the conduction and valence band wave functions, both the energy bands of II-VI semiconductors should have general form [2.75, 2.76]

$$E = a_k^2 + b_0 k_z^2 + C_0 k_z \quad (2.37)$$

where $a_0 = \frac{\hbar^2}{2m_1}$, $b_0 = \frac{\hbar^2}{2m_2}$, $C_0 = \pm 1$ represents the splitting of the two spin-states by the spin-orbit coupling and crystalline field. We wish to note that the II-VI semiconductors possess the appropriate direct band gap to producing light emitting diodes and lasers from blue to red wavelength.

Using (2.37), the density-of-states function can be written as

$$D_n(E) = \frac{1}{4\pi^2 a_0^2} [p_1(E)] \quad (2.38)$$

where

$$p_1(E) = \left[ \frac{C_0^2}{4 E b_0} + a_c (E/b_0)^{\nu^2} \pm \frac{a_c C_0}{2a_0 b_0} \left\{ \sin^{-1}(Q(E)) + \sqrt{1 - Q^2(E)} \right\} \pm \frac{P(E) Q(E)}{\sqrt{Q^2(E)}} \right] [1 - Q_1(E)],$$

$$Q(E) = \left[ \frac{2a_0 E}{\sqrt{C_0^2 + 4 a_0 E}} \right], \quad Q_1(E) = \frac{a_c \left[ 1 - 2(Q(E))^2 \right]}{Q(E) [C_0^2 + 4a_0 E]}.$$
and \( P(E) = \left( \frac{C_0}{\sqrt{2a_0b_0}} \right) \left( a_0 E + \frac{C_0}{4} \right) \)

Combining equation (2.38) with the Fermi-Dirac occupation probability factor, the carrier concentration can be expressed as

\[
n_o = \frac{M_1(E_F)N_1(E_F)}{4\pi^2a_0^2}
\]

where \( M_1(E_F) = -2\frac{C_0^2}{E_F} + \frac{2}{3} \left( \frac{a_0(E_F)}{\sqrt{b_0}} \right) \pm \frac{p(E_F)}{\sqrt{b_0}} \left( 1 - \frac{Q^2(E)}{Q(E)} \right) \)

and \( N_1(E_F) = \sum_{r=1}^{4} L(r,0)M_1(E_F) \)

Combining equations (2.39) and (2.11), the DMR in bulk specimens of II-VI semiconductors can be written as

\[
D = \frac{1}{\mu} \left[ M_1(E_F) + N_1(E_F) \right]
\]

where \( q_1(E_F) = \sum_{r=1}^{4} L(r,1)M_1(E_F) \)

Under the condition \( C_0 \to 0 \), the equation (2.37) assumes the form

\[
E = ak_x^2 + b_x k_x^2
\]

Thus under the condition \( C_0 \to 0 \), the equation (2.40) reduces to the well-known form as given by equation (2.30).

2.3.3 Formulation of DMR in bulk specimens of Bi

It is well-known that the carrier energy spectrum in Bi differ considerably from simpler spherical surfaces of degenerate electron gas and several models have been developed to describe the energy band structure of Bi. Earlier works demonstrated that the electronic properties of Bi could be described by ellipsoidal parabolic model. Sohengberg indicated that the de Hass-Van
Alphen and Cyclotron resonance experiments supported the one band model, though the later work shows that Bi could be described by two-band model since the magnetic field dependence of any aspects of Bi supports the above model. Magneto-optical results, longitudinal magnetostriction and ultrasonic quantum oscillation data favour the Lax ellipsoidal model, whereas Kao, Dinger and Lawson and Koch and Zensen indicated that the Cohen model, in better agreement with the experimental results. In a work on magneto surface resonance, Takaoka et. al., concluded that neither the Lax nor the Cohen model is adequate. They proposed hybrid dispersion relation. In 1977, McClure and Choi presented a new model of Bi which is more greater than those currently in use. They showed that it can fit the data of a large number of magneto-oscillatory and resonance experiments. In what follows, we shall study the influence of above energy band models on the DMR in bulk specimens of Bi.

(a) **McClure and Choi model**

The dispersion relation of the carriers in Bi can be written, following McClure and Choi, as

\[
E(1 + \alpha E) = \frac{P_x^2}{2m_1} + \frac{P_y^2}{2m_2} + \frac{P_z^2}{2m_3} + \frac{P_x P_y}{m_1m_2} \left[ 1 - \left( \frac{m_2}{m m_1} \right) \right] + \frac{P_y^4}{4m_1m_2} - \frac{\alpha P_x^2 P_y^2}{4m_1m_2} - \frac{\alpha P_y^2 P_z^2}{4m_1m_3} \quad (2.42)
\]

where \( E \) is the carrier energy as measured from the band-edge, \( m_1, m_2 \) and \( m_3 \) are the effective carrier masses at the band-edge along x, y and z direction respectively and \( m'_z \) is the effective-mass tensor component at the top of the valence band (for electrons) or at the top of the conduction band (for holes).

The use of equation (2.42) density-of-states function as

\[
D_0(E) = \frac{2g_s g_v}{(2\pi\hbar)^2} p_s(E) \quad (2.43)
\]

where \( g_s \) is the spin degeneracy, \( g_v \) is the valley degeneracy,

\[
p_s(E) = \frac{d}{dE} [M_s(E)]
\]
\[ M_2(E_F) = \left[ C_0(E_F)(1 + \alpha(E_F)) - a_0^2 C_0 - a^4 C_0 C_2 \gamma \frac{1}{2a} \ln \left( \frac{a + S_0(E_F)}{a - S_0(E_F)} \right) \right] + C_0 C_2(2S_0(E_F) + \frac{1}{3} S_0^3(E_F)) \]

where
\[ C_0 = \sqrt{E_g A n_{m_0} \sqrt{E_g m_1 m_2}} \]
\[ C_1(E) = \left[ 1 + \frac{\alpha E}{2m_2} \left( 1 - \frac{m_1}{m_2} \right) \right] \]
\[ C_2 = \frac{\alpha}{4m_2 m_2}, \quad a^3 = 2m_1 E_g \]

Using equation (2.43), the carrier concentration can be expressed as
\[ n_0 = \frac{4E_g}{h^2} [M_2(E_F) + N_2(E_F)] \]

where
\[ N_2(E_F) = \sum_{r=1}^{4} L(r,0)[M_2(E_F)] \]

Thus combining equation (2.44) and (2.11), we can write the expression of DMR in Bi according with McClure and Choi model as
\[ \frac{D}{\mu} = \frac{1}{\varepsilon} \left[ M_2(E_F) + N_2(E_F) \right] \]

where \( q_2(E_F) = \sum_{r=1}^{4} L(r,1)[p_2(E_F)] \)

(b) Hybrid model

The energy spectrum of the carriers of Bi in accordance with the Hybrid model can be expressed as
\[ E(1 + \alpha E) = \frac{P_x^2}{2m_1} + \frac{P_y^2}{2m_2} + \beta(E) \frac{P_x^4}{4m_2^2} - \gamma_0 P_y \alpha \]

where \( \beta(E) = [1 + \alpha E(1 - \gamma_0) + \delta_0] \gamma_0 = \frac{M_2}{M_2}, \delta_0 = \frac{M_2}{M_2}, M_2 \) is the effective mass tensor
component along the bisectrix axis due to the influence of the remote bands at the bottom of the conduction band (for electrons) or at the top of the valance band (for holes) and $M_2$ is the effective mass tensor component along the bisectrix axis due to the influence of the remote bands at the top of the valence band (for electrons) or at the bottom of the conduction band (for holes). The basic forms of equation (2.44) and (2.45) will be in this case

$$M_1(E_F) = \left[ E_F(1 + \alpha E_F) + \beta(E_F) \frac{S_1(E_F)}{6M_2} - \alpha \gamma_0 \frac{S_1^4(E_F)}{20M_2^2} \right],$$

$$S_1(E_F) = \sqrt{\frac{M_2}{h \gamma_0}} \left[ \beta(E_F) + \sqrt{\beta^2(E_F) + 4\alpha \gamma_0 E_F(1 + \alpha E_F)} \right],$$

$$P_1(E_F) = \left[ (1 + 2\alpha E_F) + \frac{S_1(E_F) \alpha(1 - \gamma_0)}{6M_2} - \beta(E_F) \frac{U_1(E_F)}{6M_2} - \alpha \gamma_0 \frac{S_1^4(E_F) U_1(E_F)}{4M_2^4} \right],$$

$$t_1(E_F) = \left[ \frac{M_2}{\alpha \gamma_0 S_1(E_F)} \right] \left[ \alpha(\gamma_0 - 1) + \beta(E_F) \alpha(1 - \gamma_0) + 2\alpha \gamma_0 (1 + \alpha E_F) \right] \sqrt{\beta^2(E_F) + 4\alpha \gamma_0 E_F(1 + \alpha E_F)}.$$ 

(c) Cohen model

The dispersion law in Bi in accordance with this model is given by

$$E(1 + \alpha E) = \frac{p_x^2}{2m_1} + \frac{p_y^2}{2m_2} + \frac{\alpha E_F^2}{2m_2} + \frac{p_y^2(1 + \alpha E_F)}{2m_2} + \frac{\alpha \gamma_F^4}{4m_2 m_2}, \quad (2.47)$$

The basic forms of equation (2.44) and (2.45) will not change in this case where,

$$M_1(E_F) = 2\pi \sqrt{m_1 m_3} \left[ E_F(1 + \alpha E_F) S_1(E_F) - \frac{t_2(E_F) S_1^3(E_F)}{3} - \frac{C_1}{5} S_1^4(E_F) \right],$$

$$S_2(E_F) = \frac{1}{\sqrt{2C_2}} \left[ \sqrt{t_2^2(E_F) + 4C_2 E_F(1 + \alpha E_F)} - \sqrt{t_2(E_F)} \right],$$

$$t_2(E_F) = \left[ \frac{1 + \alpha E_F}{2m_2} - \frac{\alpha E_F}{2m_2} \right],$$

$$P_2(E_F) = 2\pi \sqrt{m_1 m_3} \left[ (1 + 2\alpha E_F) S_2(E_F) + E_F(1 + \alpha E_F) S_2^2(E_F) + S_3^2(E_F) - t_3 S_2^2(E_F) S_3(E_F) - C_2 S_2^4(E_F) S_3(E_F) \right].$$
The carrier degeneracy spectrum of Bi in accordance with the Lax model is given by

\[ S_3(E_F) = \frac{1}{4C_2 S_2^4(E_F)} \left[ t_3(E_F) + \frac{2C_2(1 + 2\alpha E_F)}{t_4(E_F)} \right] - t_3 \]

The basic forms of equations (2.44) and (2.43) will also not change for this model.

Where

\[ M_2(E_F) = \left( \frac{2\pi}{3} \right) \sqrt{(2m_1m_2m_3)^2 \frac{1}{2}(1 + \alpha E_F)\sqrt{E_F(1 + \alpha E_F)}} \]

Under the conditions \( \alpha E_F \ll 1 \), the expression for \( n_0 \) and DMR in Bi in accordance with Lax non-parabolic and ellipsoidal model get simplified to equations (2.25) and (2.26) where

\[ N_2 = \left[ 2\pi(m_1m_2m_3)^{\frac{1}{3}} \frac{k_BT}{\hbar^2} \right]^3 \]

(e) Ellipsoidal parabolic model

The band model is given by

\[ E(1 + \alpha E) = \frac{P_x^2}{2m_1} + \frac{P_y^2}{2m_2} + \frac{P_z^2}{2m_3} \]

For this model, the expressions of carrier concentration and DMR are respectively given by equations (2.29) and (2.30) where \( N_2 \) has been defined as above.
2.4 Results and Discussions

Using \( n-Cd_3As_2 \) as an example of tetragonal semiconductors together with the parameters

\[ m_{11}^* = 0.03 m_0, \quad m_{-}^* = 0.04 m_0, \quad E_g = 0.095 \text{eV}, \delta = 0.085 \text{eV}, \Delta_H = 0.24 \text{eV}, \Delta_L = 0.29 \text{eV} \text{, and } T = 2.42 \text{K}, \]

we have plotted the DMR as a function of electron concentration as shown in [2.37] plot a of figure [2.1] in which the circular plot b exhibits the same dependence by using equation (2.15) and taking the experimental values of the thermoelectric power of the electrons in \( n-Cd_3As_2 \) in the presence of a classically large magnetic field as given in reference [2.50]. The plot c corresponds to the three-band Kane model where we have taken \( m^* = 0.035 m_0 \) and \( \Delta = 0.245 \text{eV} \) for the purpose of numerical computations. The plots of d and e corresponds to two-band Kane model and that parabolic energy band respectively. The dotted line corresponds to \( \delta = 0 \) so that we can study the influence of crystal field splitting of the DMR in tetragonal semiconductors.

It appears from figure 2.1 that, the DMR in tetragonal materials increases with increasing carrier degeneracy as expected for degenerate semiconductors and is in good agreement with the suggested experimental method of determining the same ratio for arbitrary dispersion law. It is also seen that the tetragonal crystal field effects the DMR of the electrons quite significantly in degenerate tetragonal semiconductors for the whole range of the concentration considered here. For a fixed value of electron concentration, the DMR of the electrons is smaller as compared to that in the absence of \( \delta \). Although the DMR increases non-linearly with electron concentration in other limiting cases, the rates of increase are different from that in the proposed band model since the variations of the DMRs' with electron concentration are totally band structure dependent. The well-known classically of the DMR is \( D/\mu = k_B T/|e| \) and is equal to 0.36meV at 4.2K. This is therefore not shown in the figure 2.1, as it would be senseless in such a figure. In recent years, the mobility of electrons in non-parabolic semiconductor has extensively investigated, but the diffusion constant (a very important device parameter which can not be easily determined) of such materials has relatively been less investigated. Thus the theoretical results of our paper will be useful in determining the diffusion constants even for parabolic energy bands. We wish to note that in formulating the basic dispersion relation we have considered the crystal field-splitting parameters, the anisotropies in the momentum-matrix
elements and the spin-orbit splitting parameters, respectively, since these are the significant
physical features of the tetragonal semiconductors. In the absence of crystal-field splitting
together with the assumptions of isotropic effective electron mass and isotropic spin-orbit
splitting parameters respectively, our basic equation (2.19) converts in to the well-known form of
three-band Kane model as given by equation (2.23). The three-band Kane model must be used as
such in studying the electronic properties of n-InAs where the spin-orbit splitting parameter is of
the order of the band gap $E_g$. However, for non-parabolic materials $\Delta \gg E_g$ (eg. InSb etc) or
$\Delta \ll E_g$ (eg. GaAs, InP etc). Under this conditions, the equations (2.23) gets simplified into the
form $E(1 + E/E_g) = \frac{\hbar^2 k^2}{2m^*}$, which is equation (2.24) and is known as two-band Kane model
as written above. Finally, for $E_g \rightarrow \infty$, as for parabolic energy bands the above equation
transforms into the well-known form $E = \frac{\hbar^2 k^2}{2m^*}$. Thus our basic relation (2.19) covers
various semiconductors having different band structures. Finally we wish to note that, although
the many body effects and the formation of band tails in degenerate semiconductors have not
been considered in the theoretical formulation, this simplified analysis exhibits the basic features
of the DMR in tetragonal materials and the agreement between the theory and the suggested
experimental method of determining the same ratio becomes rather significant inspite of the
aforementioned approximations.

Using equations (2.39) and (2.40) we have plotted the DMR [2.11] versus hole
concentration in p-CdS together with the parameters $m_1^* = 0.7m_0, m_\mu^* = 0.5m_0, C_0 = 1.2 \times 10^2$,
even at $T = 4.2K$ as shown in figure 2.2 in which the plot for $C_0 = 0$ has also been given for the
purpose assessing the influence of the splitting of the two spin states the spin-orbit coupling and
the crystalline field on DMR.
Figure 2.1: Plot of the DMR versus electron concentration in Bulk specimens of (a) n-Cd$_3$As$_2$, (b) $\delta = 0$, (c) Three band model of Kane, (d) Two band model of Kane and (e) parabolic energy bands. The circular points have been obtained by using equation 2.15 and taking the experimental values of the thermoelectric power of the electrons in n-Cd$_3$As$_2$ in the presence of a classically large magnetic field as given in the reference.
Figure 2.2 Plot of the DMR for p-CdS versus $n_o$ for (a) $C_o=0$ and (b) $C_o \neq 0$
Figure 2.3 Plot of the DMR as a function of electron concentration in Bi in accordance with (a) McClure and Choi model, (b) Hybrid model, (c) Cohen model, (d) Lax model and (e) Ellipsoidal parabolic model.
It appears from figure 2.2 that the DMR increases with increasing hole concentration at a rate greater than that corresponding to zero value of $C_0$. Moreover, for relatively low values of $p_0$, the effect of $C_0$ decreases whereas the same parameter affects the DMR quite significantly for relatively higher values for carrier degeneracy. $C_0$ lowers the values of DMR in the whole range of concentration considered as compared with that corresponding to $C_0 = 0$.

Using equations (2.43) and (2.44) and taking the parameters $g_3 = 2$, $g_v = 3$, $m_1 = m_0/172$, $m_2 = m_0/0.78$, $m_3 = m_0/88.5$, $T = 4.2K$, $E_F = \left[13.6 + (2.1 \times 10^{-3})^T + (2.5 \times 10^4 T^2)\right]$ MeV, $M_2 = 1.28m_0$ and $M_1 = 0.8m_0$, we have plotted in curve a of figure 2.3, the DMR in Bi as a function of electron concentration in accordance with McClure and Choi model. The curves b, c, d and e exhibit the same dependence in accordance with hybrid, Cohen, Lax and ellipsoidal models respectively by using the appropriate equations.

It appears from figure 2.3 that the DMR's in bulk specimens of Bi increase with increasing $n_o$ for all the models. Since $E_F$ is an increasing function of $n_o$ and the DMR increases monotonically with $E_F$, therefore DMR increases with increasing carrier degeneracy. For a single value of $n_o$, there is a single value of DMR in accordance with McClure and Choi model exhibits least numerical values as compared to other models. Since the DMR's are monotonic function of $E_F$, the natures of variations of DMR's will follow the general trend of variations of the Fermi energy with respect to a perpendicular physical variable. For various models of value of the DMR with respect to $n_o$ are different since these are quantities. The relative assessment with all types of band models of Bi with respect to $n_o$ becomes apparent from figure 2.2.

We wish to note that under the condition $\alpha \to 0$, the McClure and Choi, the Cohen and Lax models as given by equation (2.42), (2.47) and (2.48) respectively reduce to equations (2.49). Also under the limiting conditions $\alpha \to 0$ $\delta >> 1$, the hybrid as given by equation (2.48) gets simplified to equation (2.49). Thus, under certain limiting conditions, all the four models reduce to ellipsoidal parabolic energy bands and the expressions for $n_o$ and DMR under the same conditions reduce to the well-known expression of Landsberg [2.7]. The Cohen model is used to describe the dispersion relation of the carriers of lead chalgenide materials. The Lax model under
the condition of the isotropic effective mass of the carriers of the band edge (i.e. \( m_1 = m_2 = m_3 = m^* \)) reduces the two-band Kane model, which is often used to study the physical features of III-V compound semiconductors, in general, excluding n-InAs. Thus the analysis is valid not only for Bi but also for all types of lead chalcogenides, II-V semiconductors excluding n-InAs, and wide-gap materials respectively. Besides, the influence of the energy band models on the DMR of Bi has also been studied. Finally it can be, noted that the present analysis is also valid for holes of Bi with the proper change in the band parameters.

Section 2.5

DMR in non-parabolic materials under magnetic quantization

In this section an attempt has been made to investigate the DMR under magnetic quantization in tetragonal semiconductors, II-VI semiconductors and Bi in Section 2.5.1, 2.5.2 and 2.5.3 respectively. The well-known expressions of parabolic energy bands have also been obtained in all the cases.

2.5.1 Formulation of DMR in tetragonal semiconductors under quantization

The modified electron energy spectra in tetragonal semiconductors under arbitrary magnetic quantization can be written, extending the method given by Wallace, as

\[
C(E) = Z_\pm(n, E, \theta) + \alpha(E, \theta)(k_F')^2
\]

where

\[
Z_\pm(n, E, \theta) = \left( \frac{2eB}{h} \right) \left( n + \frac{1}{2} \right) \left[ A(E) \left\{ A(E) \cos^2 \theta + B(E) \sin^2 \theta \right\} \right]^{\frac{1}{2}} \pm \left[ \frac{\left| E B^2 \epsilon \right|}{6} \left( \frac{E_g + \Delta_1}{m^* \left( E_g + \frac{2}{3} \Delta_1 \right)} \right) \right]^{\frac{1}{2}}
\]

\[
\left\{ E + E_g + \delta + \frac{\Delta_2^2 - \Delta_1^2}{3 \Delta_2} \right\} \left( \frac{\Delta_2^2 \cos^2 \theta (E_g + \Delta_1)}{m^* \left( E_g + \frac{2}{3} \Delta_1 \right)} \right) \left( \frac{E + E_g \Delta_2 \sin^2 \theta (E_g + \Delta_2)}{m^* \left( E_g + \frac{2}{3} \Delta_1 \right)} \right)^{\frac{1}{2}}
\]

2.5.2 Formulation of DMR in II-VI semiconductors under quantization

The modified electron energy spectra in II-VI semiconductors under arbitrary magnetic quantization can be written, extending the method given by Wallace, as

\[
C(E) = Z_\pm(n, E, \theta) + \alpha(E, \theta)(k_F')^2
\]

where

\[
Z_\pm(n, E, \theta) = \left( \frac{2eB}{h} \right) \left( n + \frac{1}{2} \right) \left[ A(E) \left\{ A(E) \cos^2 \theta + B(E) \sin^2 \theta \right\} \right]^{\frac{1}{2}} \pm \left[ \frac{\left| E B^2 \epsilon \right|}{6} \left( \frac{E_g + \Delta_1}{m^* \left( E_g + \frac{2}{3} \Delta_1 \right)} \right) \right]^{\frac{1}{2}}
\]

\[
\left\{ E + E_g + \delta + \frac{\Delta_2^2 - \Delta_1^2}{3 \Delta_2} \right\} \left( \frac{\Delta_2^2 \cos^2 \theta (E_g + \Delta_1)}{m^* \left( E_g + \frac{2}{3} \Delta_1 \right)} \right) \left( \frac{E + E_g \Delta_2 \sin^2 \theta (E_g + \Delta_2)}{m^* \left( E_g + \frac{2}{3} \Delta_1 \right)} \right)^{\frac{1}{2}}
\]
\[ a(E, \theta) = \frac{A(E)B(E)}{\{A(E)\cos^2\theta + B(E)\sin^2\theta\}} \]

\( n(=0,1,2,\ldots) \) is the Landau quantum number, \( k_z(=k_x\cos\theta + k_y\sin\theta) \) is the direction of application of the quantizing magnetic field \( B \) which makes an angle \( \theta \) with \( k_z \) axis and lies into \( K_x, k_y \) plane and the function \( A(E), B(E) \) and \( C(E) \) have already been defined in connection with equation (2.19) of Section 2.3.

The general formula of the magneto-density-of-states function including spin and broadening can be written as

\[ N_B(E) = \frac{|B|}{2\pi^2\hbar} \sum_{n=0}^{\infty} \left[ \frac{\partial \phi}{\partial E} \right] \]  \hspace{1cm} (2.50)

where \( \phi = \text{Real part of } \left[ k_z(E_n) \right] \) \( E_0 = E_F + i\Gamma, \quad i = \sqrt{-1}, \quad \Gamma = \pi k_B T_D \) is broadening parameter [2.3] and \( T_D \) is Dingle temperature. Thus the electron concentration can be written as

\[ n_0 = \frac{|B|}{2\pi^2\hbar} \sum_{n=0}^{\infty} \int_0^\infty \left[ \frac{\partial \phi}{\partial E} \right] dE \]  \hspace{1cm} (2.51)

where \( f_0 \) is the Fermi-Dirac occupation probability factor. Thus combining the appropriate equations and using the generalized Sommefeld's lemma [82], the electron concentration in tetragonal semiconductors can be expressed as

\[ n_0 = \left( \frac{|B|}{2\pi^2\hbar} \right)^2 \sum_{n=0}^{\infty} \left[ g_1(n, E_{FB}, \theta) + g_2(n, E_{FB}, \theta) \right] \]  \hspace{1cm} (2.52)

where \( E_{FB} \) is the Fermi energy in the presence of magnetic quantization as measured from the edge of the conduction band in the absence of any quantization and the functions \( g_1(n, E_{FB}, \theta) \) and \( g_2(n, E_{FB}, \theta) \) are defined in Appendix 4. Combining equations (2.11) and (2.52), the magneto-DMR in tetragonal semiconductors can be written as

\[ \frac{D}{\mu} = \left[ \sum_{n=0}^{\infty} \left\{ g_1(n, E_{FB}, \theta) + g_2(n, E_{FB}, \theta) \right\} \right] \left[ \sum_{n=0}^{\infty} \left\{ g_1(n, E_{FB}, \theta) + g_2(n, E_{FB}, \theta) \right\} \right]^{-1} \]  \hspace{1cm} (2.53)

where the primes denote the differentiation with respect to energy and in this case \( E_{FB} \).
Special cases

(a) Under the substitutions $\delta = 0$, $\Delta = \Delta_1 = \Delta$ and $m^* = m_1^* = m^*$, equation (2.49) assumes the form

$$\gamma(E) = \left(n + \frac{1}{2}\right) \hbar \omega_0 + \frac{\hbar^2 k^2}{2 m^*} \pm \frac{eBh\Delta}{6m^*} \left(E + E_g + \frac{2}{3} \Delta \right)$$

(2.54)

where $\omega_0 = \frac{|eB|}{m}$, the function $\gamma(E)$ has been defined in equation (2.23) and the equation (2.54) is the well-known magneto three-band Kane model [2.5]. For this model the equation (2.52) and (2.53) get simplified as

$$n_0 = \left(\frac{|eB|}{2\hbar^2 m^*}\right)^2 \sum_{n=0}^{\infty} \left[G_1(n, E_{FB}) + G_2(n, E_{FB})\right]$$

(2.55)

and

$$D = \frac{1}{\mu} \left[ \sum_{n=0}^{\infty} \left(G_1(n, E_{FB}) + G_2(n, E_{FB})\right) \right]$$

(2.56)

where

$$G_1(n, E_{FB}) = \left[\hbar^2(E, n) + \hbar^2(E, E_{FB}) + h_1(E, n)\right]^2$$

$$h_1(E, n) = \left\{E + \frac{2}{3} \Delta \right\} \left[\hbar^2(E, E_{FB}) + \hbar^2(E, n)\right] \left\{E + E_g + \frac{2}{3} \Delta \right\}^{-2} + \Gamma^{-2}$$

$$\left\{E + E_g + \frac{2}{3} \Delta \right\} + \Gamma \left[\hbar^2(E, E_{FB}) - \Gamma^{-2} - 2\Delta - \hbar^2(E, E_g) + \Delta \hbar^2(E, E_g) - \Delta \Gamma^{-2} \right]$$

$$\left[\hbar^2(E, n) - \Gamma^{-2} \hbar^2(E, n) - 2E \Delta \hbar^2(E, E_g) + \Delta \Gamma + \Gamma \hbar^2(E, E_g)\right]$$

$$- \left(n + \frac{1}{2}\right) \hbar \omega_0 \pm \frac{eBh\Delta}{6m^*} \left(E + E_g + \frac{2}{3} \Delta \right)$$

(2.56)
\[ h_s(E) = \left( E + \frac{2 \Delta}{3} \right)^2 \left( E + \frac{2 \Delta}{3} \right)^2 \left( E + \frac{2 \Delta}{3} \right)^2 \ \	frac{\Gamma}{E + \frac{2 \Delta}{3}} \left[ \left( E + \frac{2 \Delta}{3} \right)^2 - \Gamma^3 + 2 \Gamma E (E + \frac{2 \Delta}{3}) + \Delta \Gamma + \Gamma \Delta (E + \frac{2 \Delta}{3}) \right] \ \	frac{\Gamma e \hbar \Delta}{6m^*} \left( E + \frac{2 \Delta}{3} \right)^2 + \Gamma^2 \right] \]

and

\[ G_s(n, E_{FB}) = \sum_{r=1}^S G_{s}(n, E_{FB}) \]

(b) Under the approximation \( \Delta \gg E_g \), the equation (2.23) assumes the form

\[ E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{\hbar^2 k^2}{2m^*} \pm \frac{1}{2} g^* \mu_B B \] (2.57)

where \( \alpha = \frac{1}{E_g}, g^* = \frac{m_0}{m}, m_0 \) is the free electron mass and \( g^* = \frac{e \hbar}{2m_0} \). Equation (2.57) is the well-known magneto-dispersion relation in accordance with two band Kane model, the basic forms of equation (2.55) and (2.56) will be unchanged where

\[ h_{s}(n, E_{FB}) = \left[ E_{FB} \left( 1 + \alpha E_{FB} \right) - \alpha \Gamma - \left( n + \frac{1}{2} \right) \hbar \omega_0 \pm \frac{1}{2} \mu_B B g^* \right] \]

and

\[ h_{s}(E_{FB}) = \Gamma(1 + 2\alpha E_{FB}) \]

(c) For relatively wide band gap semiconductors, \( E_g \to \infty \) and the equation (2.57) assumes the well-known form

\[ E = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{\hbar^2 k^2}{2m^*} \pm \frac{1}{2} g^* \mu_B B \] (2.58)

The basic form of \( n_0 \) and DMR as given by equation (2.55) and (2.56) will be unaltered for parabolic bands where
\[ h_1(n,E_{ph}) = \left[ E_{ph} - \left( \eta + \frac{1}{2} \hbar \omega_0 \pm \frac{1}{2} \mu_B Bg^* \right) \right] \] and \[ h_2(n,E_{ph}) = \Gamma \]

(d) In the absence of spin and broadening, the expression of \( n_0 \) and DMR of two band Kane model can be expressed under the assumption \( \alpha E_{ph} \ll 1 \) as

\[ n_0 = N_e \theta \sum_{n=0}^{\infty} \frac{1}{\sqrt{a}} \left[ \left( 1 + \frac{3}{2} \alpha a b \right) F_{1/2} \left( \frac{\eta}{4} \right) + \left( \frac{3}{4} \alpha k_B T \right) F_{3/2} \left( \frac{\eta}{4} \right) \right] \] (2.59)

\[ \left( \frac{D}{\mu} \right)_B = \frac{k_B T}{\left| e \right|} \sqrt{\sum_{n=0}^{\infty} \frac{1}{\sqrt{a}} \left[ \left( 1 + \frac{3}{2} \alpha a b \right) F_{1/2} \left( \frac{\eta}{4} \right) + \left( \frac{3}{4} \alpha k_B T \right) F_{3/2} \left( \frac{\eta}{4} \right) \right]} \] (2.60)

where \( \theta = \frac{\hbar \omega_0}{k_B T} \), \( \alpha = 1 + \alpha \left( \eta + \frac{1}{2} \hbar \omega_0 \right) \), \( b = \left( \eta + \frac{1}{2} \hbar \omega_0 / a \right) \) and \( \eta' = \frac{E_{ph} - b}{k_B T} \)

(e) For parabolic energy bands \( \alpha \to 0 \) and the equations (2.59) and (2.60) get simplified as

\[ n_0 = \sum_{n=0}^{\infty} F_{1/2} \left( \frac{\eta}{4} \right) \] (2.61)

\[ \left( \frac{D}{\mu} \right)_B = \frac{k_B T}{\left| e \right|} \sqrt{\sum_{n=0}^{\infty} F_{1/2} \left( \frac{\eta}{4} \right)} \] (2.62a)

where \( \eta' = \frac{1}{k_B T} \left[ E_{ph} - \left( n + \frac{1}{2} \right) \hbar \omega_0 \right] \)

Using equations (2.61), (2.62a), (2.29) and (2.30) together with the facts as used in obtaining equations (2.31) and (2.32) from equations (2.25) and (2.26) we can write

\[ \sum_{n=0}^{\infty} \left( \frac{\eta'}{e} - n - \frac{1}{2} \right)^2 = \left( \frac{2}{3} \right) \left( \frac{\eta'}{e} \right)^{3/2} \] (2.62b)

and

\[ \frac{(D/\mu)_B}{(D/\mu)_0} = 2 \left( \frac{\eta}{e} \right) \left( \sum_{n=0}^{\infty} \left( \frac{\eta'}{e} - n - \frac{1}{2} \right)^2 \right) \] (2.62c)
where $\left( \frac{D}{\mu} \right)_o$ indicates the DMR in the absence of quantizing magnetic field. It may be noted that the equations (2.62b) and (2.62c) were derived for the first time by Butcher et al.

### 2.5.2 Formulation of DMR in II-VI semiconductor under magnetic quantization

The magneto-energy spectrum in II-VI semiconductors can be expressed using equation (2.20) as

$$E = b_0 k^2 + \overline{\Psi}_\pm(n)$$  \hspace{1cm} (2.63a)

where

$$\overline{\Psi}_\pm(n) = \frac{2|e|B}{\hbar} \left( n + \frac{1}{2} \right) a_0 \pm \frac{1}{2} g^* \mu_B B + \lambda C_0 \left( \frac{n + 1}{2} \right)^{1/2}$$

The use of equation (2.63a) leads to the expression of electron concentration as

$$n_0 = \left( \frac{|e|B}{2\pi^2 \hbar} \right) \sum_{n=0}^\infty \left[ G_3(n,E_{PB}) + G_4(n,E_{PB}) \right]$$  \hspace{1cm} (2.63b)

where

$$G_4(n,E_{PB}) = \left[ E_{PB} - \overline{\Psi}_\pm(n) \right] \left[ E_{PB} - \overline{\Psi}_\pm(n) \right]^2 + \Gamma^2$$

and

$$G_4(n,E_{PB}) = \sum_{r=1}^4 L(r,0) \left[ G_3(n,E_{PB}) \right]$$

combining equations (2.11) and (2.63b) we get

$$\left( \frac{D}{\mu} \right)_B = \frac{1}{|e|} \left[ \sum_{n=0}^\infty \left\{ G_3(n,E_{PB}) + G_4(n,E_{PB}) \right\} \right]$$  \hspace{1cm} (2.64)

Neglecting spin and broadening together with isotropic effective mass of the carriers, the equations (2.63b) and (2.64) get simplified to the form as given by the equations (2.61) and (2.62a) respectively.
2.5.3 Formulation of DMR in Bi under magnetic quantization in accordance with various band models

(a) McClure and Choi Model

The energy eigen value equation in the presence of a direct quantizing magnetic field B along bisectrix axis can be written following equation (2.42) as

\[
[H_0(1+\alpha H_0)]\psi_B = H_B\psi_B = \frac{E(1+\alpha E)}{4m_2}\psi_B
\]  \hspace{1cm} (2.65)

where \(H_B\) is the Hamiltonian in the absence of magnetic quantization and

\[
H_B = \left[\frac{\hat{p}_x^2 - e|B|^2}{2m_1} + \frac{\hat{p}_z^2}{2m_2} + \frac{\hat{p}_y^2}{2m_3} + \left(\frac{\hat{p}_y^2}{2m_2}\right)\alpha E\left(1 - \frac{m_2}{m_1}\right) + \alpha - \frac{\hat{p}_y^2}{4m_2m_1} - \alpha \frac{\hat{p}_y^2}{4m_2m_2} - \left(\frac{\alpha}{4m_2m_2}\right)\hat{p}_y^2\right]
\]

Therefore the modified magneto-dispersion relation of the carriers in Bi in accordance with McClure and Choi model can be written including spin-splitting as

\[
E(1+\alpha E) = \left(n + \frac{1}{2}\right)\hbar \omega_0 + \frac{p_y^2}{2m_2} + \left(\frac{p_y^2}{2m_2}\right)\alpha E\left(1 - \frac{m_2}{m_1}\right) + \frac{\alpha p_y^4}{4m_2m_2} - \left(\frac{\alpha p_y^4}{4m_2m_2}\right)\left(n + \frac{1}{2}\right)\hbar \omega_0 \pm \frac{1}{2} g^* \mu_0 B
\]  \hspace{1cm} (2.66)

where \(\omega_0 = \frac{e|B|}{\sqrt{m_1m_2}}\)

Therefore the carrier concentration can be written including spin and broadening effects, as

\[
n_0 = \frac{g_s e^2}{\hbar^2} (C_2)^2 \sum_{\alpha=0}^{1} (G_2 + G_3)
\]

where \(G_2 = \frac{\alpha}{4m_2m_1}\), \(G_3 = \left(T_1 + \sqrt{T_1 + T_2}\right)^{1}\)

\[
T_1 = \sqrt{Q_1 - g_1 - g_2 E}, \quad T_2 = g_1 + \sqrt{\frac{g_1^2 + g_2^2}{2}}
\]

\[
Q_1 = \left(g_1 - g_2 E\right)^2 - g_2 E^2 + 4C_4 \left[\alpha E^2 - \alpha T^2 + E - \left(n + \frac{1}{2}\right)\hbar \omega_0 \pm \frac{1}{2} g^* \mu_0 B\right]
\]

\[
g_1 = \frac{1}{2m_2} \left[1 - \alpha \left(n + \frac{1}{2}\right)\hbar \omega_0\right], \quad g_2 = \left(\frac{\alpha}{2m_2}\right)\left(1 - \frac{m_2}{m_1}\right)
\]
Combining equations (2.11) and (2.66) we get the magneto-DMR in Bi in accordance with McClure and Choi model as

\[
\left( \frac{D}{\mu} \right)_{B} = \frac{1}{2} \sum_{\alpha=0}^{\infty} \left[ G_\alpha + G_{e\alpha} \right] \left[ G_\alpha' + G_{e\alpha}' \right]^{-1} 
\]  
(2.68)

(b) Hybrid model

The magneto dispersion relation of the carriers in Bi in accordance with Hybrid model can be expressed as

\[
E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \beta(E) \frac{p_r^2}{2m_2} + \alpha \gamma_0 \frac{p_r^4}{4m_2^2} \pm \frac{1}{2} g^* \mu_0 B
\]  
(2.69)

Thus for Hybrid model, the forms of equations (2.67) and (2.68) remain unchanged where

\[
C_2 = \frac{\alpha \gamma_0}{4m_2^2}, \quad g_1 = 1 + \delta_0 \quad \text{and} \quad g_2 = \frac{(1 - \gamma_0)}{2m_2}
\]

(c) Cohen model

\[
E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \alpha(\beta) \frac{p_r^2}{2m_2} + \alpha \gamma_0 \frac{p_r^4}{4m_2^2} \pm \frac{1}{2} g^* \mu_0 B + \left( \frac{p_r^2}{2m_2^2} \right) (1 + \alpha E)
\]  
(2.70)

In this case, the basic forms of the equations (2.67) and (2.68) will not change where

\[
C_2 = \frac{\alpha}{4m_2m_2}, \quad G_1 = \frac{1}{2m_2} \quad \text{and} \quad g_2 = \alpha \left[ \frac{1}{m_2} - \frac{1}{m_2} \right]
\]
(d) **Lax model**

The magneto energy spectrum in accordance with the Lax model of Bi can be expressed as

\[
E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{p_y^2}{2m_2} \pm \frac{1}{2} g^* \mu_B B
\]

(2.71)

In this case, the basic forms of equations (2.55) and (2.56) will not change only the term \( eB\sqrt{m^*}\sqrt{2\pi^2\hbar^4} \) of equation (2.55) assumes the form \( eB\sqrt{\frac{m^*}{2\pi^2\hbar^2}} \) and the symbol \( \omega_0 \) assumes the form \( eB\sqrt{\frac{m^*}{m_2}} \).

(e) **Ellipsoidal parabolic model**

In this case, the modified magneto dispersion law of the carrier can be written as

\[
E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{1}{2} g^* \mu_B B + \frac{\hbar^2 k^2}{2m_2
\]

(2.72)

In this case, the basic forms of the equations (2.55) and (2.56) will unaltered where \( eB\sqrt{\frac{m^*}{2\pi^2\hbar^2}} \to eB\sqrt{\frac{m^*}{2\pi^2\hbar^2}} \) and \( \omega_0 = eB\sqrt{\frac{m^*}{2\pi^2\hbar^2}} \).

In the absence of broadening we get

\[
n_0 = \left[ g^* N_e e / 2 \sum_{n=0}^{\infty} \frac{F_1(\eta^n)}{n^2} \right]
\]

(2.73)

and

\[
\left( \frac{D}{\mu} \right)_B = \left( \frac{\sum_{n=0}^{\infty} \frac{F_3(\eta^n)}{n^2}}{\sum_{n=0}^{\infty} \frac{F_1(\eta^n)}{n^2}} \right)
\]

(2.74)

where \( \eta = \left[ \frac{1}{k_B T} \left( E_{FB} - \left( \eta + \frac{1}{2} \right) \hbar \omega_0 \pm \frac{1}{2} g^* \mu_B B \right) \right] \), \( N_c = 2 \left( \frac{2\pi m^* k_B T}{\hbar^2} \right)^{\frac{3}{2}} \),

\[
e = \frac{\hbar \omega_0}{k_B T}, \quad \omega_0 = \frac{eB}{\sqrt{m^* m_2}}, \quad m^* = (m_1, m_2, m_3)^{\frac{1}{3}}
\]
2.6 Results and Discussions

Using equations (2.52), (2.53), (2.21) and (2.22) and taking the parameters as used in obtaining figure 2.1 together with $T_D = 3 K$, $T= 4.2 K$, $B = 1$ Tesla and $n_o = 2.2 \times 10^{22}$ $m^{-3}$, the plots of the normalized DMR versus $e$ (neglecting magnetic freeze out) have been in figure 2.4. Besides, using the equations (2.7), (2.8) and (2.4), (2.5) (with the proper definitions of $M(E_F)$ and $P(E_F)$ as valid for three-band Kane model) for three-band Kane model and also taking the appropriate equations for two-band Kane model the plots of the DMR according to both the models, respectively, have been shown in the same figure for the purpose of comparison. Using the same parameters as used in obtaining figure 2.4 the plots of the normalized DMR have been shown as functions of $1/B$ and $n_o$ in figures 2.5 and 2.6 respectively.

It appears from figure 2.4 that the DMR in $n$-Cd$_3$As$_2$ is a periodic function of the orientation of the quantizing magnetic field. It is also apparent from same figure that he DMR exhibit the spiky variation corresponding to specified values of $e$ when the magnetic quantum number changes from one fixed value to the other. The ratio increases at a high rate in a monotonous manner with increasing angle of orientation of the quantizing magnetic field. up to $e = 60^\circ$, beyond which the ratio suddenly drops to a lower value due to the change in magnetic quantum number. Moreover $a = 70^\circ$ to $90^\circ$ the ratio increases with $e$ and from $e = \pi/2$, the whole process starts again. In the absence of anisotropies, the DMR turns out to be independent of a since the basic dispersion relation becomes spherical instead of being ellipsoidal. Thus the DMR for both the three and two-band Kane's model becomes independent of $e$ as shown in figure 2.4.

It appears from figure 2.5 that the DMR oscillates with $1/B$ due to SdH effect. The band anisotropies enhance the numerical value of the DMR in $n$-Cd$_3$As$_2$ as compared to other types of band models. At extremely large values of the quantizing magnetic field, the condition for quantum limit ($n = 0$) will be reached when the DMR will found to decrease with increasing magnetic field. The DMR increases in an oscillatory way with electron concentration for all the models. Our calculation is only valid under the conditions of carrier degeneracy since under non-degenerate conditions, the DMR varies with only temperature in a linear manner. The DMR will in general, be anisotropic in the presence of magnetic quantization. It appears that for
Figure 2.4 The plot of the normalized DMR versus $\theta$ in n-Cd$_3$As$_2$ in accordance with (a) Our generalized band model, (b) Three band model of Kane and (c) Two band model of Kane.
Figure 2.5: Plot of the normalized DMR versus electron concentration in accordance with (a) Our generalized band model, (b) Three band model of Kane, (c) Two band model of Kane and (d) Parabolic energy bands.
Figure 2.6  Plot of the normalized DMR under magnetic quantization as a function of hole concentration in p-CdS for (a) \( C_0 = 0 \) and (b) \( C_0 \neq 0 \)
Figure 2.7: Plot of the normalized DMR in Bi under quantizing magnetic field as a function of electron concentration in (a) McClure and Choi model, (b) Hybrid model, (c) Cohen model, (d) Lax model and (e) Ellipsoidal parabolic model.
investigating the DMR under magnetic quantization, we have determined the magnetic field
directional element of the corresponding tensor of DMR as a function B. Thus finally note that
the DMR defined here refers to the direction of the application of the magnetic field.

Using the same parameters as used in obtaining figure 2.2 together with $g^* = 2.4$, $T_d = 3K$,
$T = 2.2K$, $B = 3$ Tesla, we have plotted the normalized DMR as a function of $p_0$ by using the
equations (2.63b), (2.64), (2.39) and (2.40) in p-CdS where the dashed plot corresponds to
parabolic energy bands as shown in figure 2.6.

Using the same parameters as used in obtaining figure 2.3 together with $g^* = 55$, $T = 2.2K$,
$T_d = 3K$, $B = 1$ Tesla and using the appropriate equations, we have plotted the normalized DMR
versus $n_0$ in Bi in accordance with McClure and Choi, Hybrid, Cohen, Lax and parabolic
ellipsoidal models of Bi as shown in figure 2.7.

It appears that the DMR oscillates both with $n_0$ and $1/B$ due to the SdH effect. The
quantum oscillation in McClure and Choi model show up much more significantly with respect
to other models. The relative comparison among the band models of Bi is also apparent from the
figures. The exact numerical values of the DMR with respect to $n_0$ and $B$ are totally band
structure dependent as noted already. It may also be stated that under arbitrary oriented
quantizing magnetic field, the forms of the equations in accordance with various band models
will change although the basic qualitative features will be unaltered.
The functions $g_1(n, E_{PB}, e)$ and $g_1(n, E_{PB}, e)$ are defined as follows:

$$g_1(n, E_{PB}, e) = \left[ S_1^2(n, E_{PB}, e) + S_2^2(n, E_{PB}, e) \right]^{\frac{1}{2}}$$

and

$$g_2(n, E_{PB}, e) = \sum_{r=1}^{s} 2(k_B T)^r (1 - 2^{-2r}) \gamma(2r) \left[ \frac{d^{2r}}{dE_{PB}^{2r}} g_1(n, E_{PB}, e) \right]$$

where

$$S_1(n, E_{PB}, e) = \frac{1}{[C_1^2(E, e) + C_2^2(E, e)]} \left[ Y_1(n, E_{PB}, e) C_1(E, e) + Y_2(n, E_{PB}, e) C_2(E, e) \right],$$

$$C_1(E, \theta) = \sqrt{\left[\{A(E) \cos^2 \theta + b_1(E) \sin^2 \theta\}^2 + \{a_1(E)b_1(E) + A_1(E)b_2(E)\}\right]}$$

$$b_1(E) = h^2 E_x (E_x + \frac{2}{3} \Delta_0) \left(2m_1 E_x + \frac{2}{3} \Delta_0\right)^{-1} \left(E + E_x + \frac{2}{3} \Delta_0 (E + E_x)\right),$$

$$a_1(E) = h^2 E_x (E_x + \frac{2}{3} \Delta_0) \left(2m_1 E_x + \frac{2}{3} \Delta_0\right)^{-1} \Gamma,$$

$$b_2(E) = h^2 E_x (E_x + \frac{2}{3} \Delta_0) \left(2m_1 E_x + \frac{2}{3} \Delta_0\right)^{-1} 2\Gamma (E + E_x) + \frac{2}{3} \Delta_0 \Gamma\Delta_0,$$

$$C_2(E, \theta) = \sqrt{\left[\{A(E) \cos^2 \theta + b_2(E) \sin^2 \theta\}^2 + \{a_1(E)b_1(E) + A_1(E)b_2(E)\}\right]}$$

$$a_1(E)b_1(E) + \{A(E)b_2(E)\}\left\{A(E) \cos^2 \theta + b_1(E) \sin^2 \theta\right\} - \{b_1(E)A(E) - a_1(E)b_2(E)\}$$

$$\{a_1(E) \cos^2 \theta + b_2(E) \sin^2 \theta\}$$

$$Y_1(n, E, e) = e_1(E) - \omega_1(n, E, e),$$
\[
e_1(E) = \left( (E^2 - \Gamma^2)^2 - 4E^2\Gamma^2 + (3E_g + \Delta_u + \delta)\Gamma(E^2 - \Gamma^2) - 2E\Gamma^2 \right) \\
+ \left( (E^2 - \Gamma^2)^2 - 2E^2\Gamma_E + 2\delta\Delta_u + \frac{2}{3}\delta\Delta_u \right) \\
+ \left( E(E_g) \left( E_g + \Delta_u + \delta E_g + \frac{2}{3}\delta\Delta_u - \frac{2}{9}(\Delta_u^2 - \Delta_1^2) \right) \right) \\
\omega_1(n, E_g, \theta) = \left[ \frac{2eB}{h} \left( n + \frac{1}{2} \right) \rho_1(E, \theta) \pm I_1(E, \theta) \right], \\
\rho_1(E, \theta) = \left( \frac{1}{\sqrt{2}} \right)^2 \left[ \left( \rho_2(E, \theta) + \rho_1(E, \theta) \right)^2 + \rho_3(E, \theta) \right]^{\frac{1}{2}}, \\
\rho_2(E, \theta) = \left[ a^2(E)\sin^2\theta - a_2^2(E)\cos^2\theta + A(E)b_1(E)\sin^2\theta - a_1(E)b_2(E)\sin^2\theta \right], \\
\rho_3(E, \theta) = \left[ 2A(E)a_1(E)\cos^2\theta + a_1(E)b_1(E)\sin^2\theta + A(E)b_1(E)\sin^2\theta \right], \\
I_1(E, \theta) = \left( \frac{1}{\sqrt{2}} \right)^2 \left[ I_1^2(E, \theta) + I_2^2(E, \theta) \right]^{\frac{1}{2}}, \\
I_2(E, \theta) = \left[ \frac{eBhE_g}{6} \left( \frac{E_g + \Delta_1}{m_1(E_g + \Delta_1)} \right) \right]^{\frac{1}{2}} \left[ \frac{A^2_C\cos^2\theta + (E_g + \Delta_1)}{m_1(E_g + \Delta_1)} \right] \\
\left\{ E + E_g + \delta + \frac{(\Delta_u^2 - \Delta_1^2)^2}{3\Delta_u^2} \right\}^{\frac{1}{2}} \\
\frac{\{\Delta_1^2 - \sin^2\theta(E_g + \Delta_u)\}}{\left( m_1 \left( E_g + \frac{2}{3}\Delta_1 \right) \right)} \left\{ E + E_g \right\}^{\frac{1}{2}} \left( E^2 - \Gamma^2 \right) \\,
\[ h(E,0) = eBhE^{\frac{e}{fe+Ax}} - 2fC\cos^2 \theta + A^2 \]

\[ m, \ast \cdot 3A, J \]

\[ \left[ 2\Gamma \left( E + E_g + \Gamma + \frac{\Delta_{II}^2 - \Delta_{I}^2}{3\Delta_{II}} \right) \right] \left[ \begin{array}{c} \Delta_{II}^2 \sin^2 \theta (E_g + \Delta_{II}) \\ 3\Delta_{II} \end{array} \right] \left[ \begin{array}{c} 1 \\ \frac{1}{2} \end{array} \right] \]

\[ \bar{Y}_2(n,E,\theta) = [e_2(E) - \omega_2(n,E,\theta)] \]

\[ e_2(E) = \frac{4\Gamma \left( E^2 - \Gamma^2 \right) + 2\Gamma E^2 + \Gamma \left( E^2 - \Gamma^2 \right)}{3E_g + \Delta_{II} + \delta} \]

\[ + 2\Gamma \left( 3E_g^2 + 2\delta_{II} E_g + 2\delta E + \frac{2}{3} \delta_{II} + \frac{2}{9} \Delta_{II}^2 + \frac{2}{9} \Delta_{I}^2 \right) \]

\[ + 2\Gamma E_g \left( E_g^2 + E_g \Delta_{II} + \delta E_g + \frac{2}{3} \Delta_{II} + \frac{2}{9} \left( \Delta_{II}^2 - \Delta_{I}^2 \right) \right) \]

\[ \beta_1(E,\theta) = \left( \frac{1}{\sqrt{2}} \right) \left[ \rho_2^2(E,\theta) + \rho_3^2(E,\theta) \right]^{\frac{1}{2}} \]

\[ \beta_2(E,\theta) = \left( \frac{1}{\sqrt{2}} \right) \left[ I_2^1(E,\theta) + I_2^2(E,\theta) \right]^{\frac{1}{2}} \]

\[ S_2(n,E,\theta) = \frac{1}{C_1^2(E,\theta) + C_2^2(E,\theta)} \left[ \bar{Y}_2(n,E,\theta)C_1(E,\theta) - \bar{Y}_1(n,E,\theta)C_2(E,\theta) \right] \]

\[ \Gamma(=nk_BT_D) \] is the broadening parameter in which \( T_D \) is Dingle temperature and \( \zeta(2\tau) \) is the zeta function of order 2r.

**Section 2.7**

**2.7.1 DMR in non-parabolic semiconductors under cross-field configuration**

In this Section we shall discuss the DMR in tetragonal semiconductors, II-VI semiconductors and Bi in the presence of crossed electric and quantizing magnetic fields in Section 2.7.2, 2.7.3 and 2.7.4 respectively.
2.7.2 Analysis of the DMR in tetragonal semiconductors under cross-field configuration

The equation 2.19 can be written as

\[ U(E) = \frac{p^2}{2M_\perp} + \left[ \frac{p^2 V(E)}{2M_\parallel} \right] \] (2.75)

where \( U(E) = \left[ E(1+\alpha E) \left( \left( E + E_g \right) \left( E + E_g + \Delta_\parallel \right) + \delta \left( E + E_g + \frac{2}{3} \Delta_\parallel \right) \right) + \frac{2}{9} \left( \Delta_\parallel^2 - \Delta_\perp^2 \right) \right] \left[ U_1(E) \right]^{-1}, \]

\[ \alpha = \frac{1}{E_g}, \quad U_1(E) = \left[ \left( E + E_g \right) \left( E + E_g + \frac{2}{3} \Delta_\parallel \right) + \delta \left( E + E_g + \frac{1}{3} \Delta_\parallel \right) + \frac{1}{9} \left( \Delta_\parallel^2 - \Delta_\perp^2 \right) \right], \]

\[ p_x = \hbar k_x, \quad M_\perp = m^\star \left( \frac{E_g + \frac{2}{3} \Delta_\parallel}{E_g + \Delta_\perp} \right), \quad M_\parallel = m^\star \left( \frac{E_g + \frac{2}{3} \Delta_\parallel}{E_g + \Delta_\parallel} \right), \]

and \( V(E) = \left( E + E_g \right) \left( E + E_g + \frac{2}{3} \Delta_\parallel \right) \left/ U_1(E) \right. \).

Therefore, extending the method as given by Zawadzki and Lax the modified electron spectrum in the presence of a quantizing magnetic field \( B \) along \( z \)-axis and the electric field \( \varepsilon_0 \) along \( x \)-axis can be expressed as

\[ U(E) = \left( n + \frac{1}{2} \right) \hbar \omega_\perp + \frac{\hbar^2 k_x^2}{2\alpha(E)} - \left( \frac{\varepsilon_0}{B} \right) \hbar k_x \rho(E) - \frac{M_\parallel^2}{2B^2} \rho^2(E) \] (2.76)

where

\[ \omega_\perp = \frac{eB}{M_\perp}, \quad \alpha(E) = \frac{M_\parallel}{V(E)}, \]

\[ \rho(E) = U(E) \left[ \frac{1 + 2\alpha E}{E(1+\alpha E)} \frac{J(E)}{I(E)} + \frac{1}{E + E_g + \Delta_\parallel} \frac{1}{E + E_g + \frac{2}{3} \Delta_\parallel} \right], \]

\[ J(E) = \left[ I(E) \left( \frac{C_\parallel(E)}{1 + A_\parallel(E)} \right) - \frac{H_\parallel(E)}{I(E)} \right], \quad I(E) = \frac{1 + A_\parallel(E)}{1 + G_\parallel(E)}. \]
The electron concentration can be expressed as

\[ n_0 = \frac{1}{L_x} \sum_{n=0}^{\infty} \int_{x_1}^{x_2} \frac{\partial f_0}{\partial E} dE \]  

(2.77)

where \( L_x \) is the simple dimension along x-axis, \( x_1 \) is the lower limit of \( k_y \), \( x_2 \) is the upper limit of \( k_y \) and \( E_l \) is the Landau energy in the presence of cross-field configuration. Thus using equation (2.76) and (2.77) we get

\[ n_0 = K_i \sum_{n=0}^{\infty} [I_1(E_p) + I_2(E_p)] \]  

(2.78)

where

\[ K_i = \left[ \frac{2\sqrt{2}}{3L_x^2\hbar^2e_0} \right] \]

\[ I_1(E_p) = \text{Real part of} \]

\[ \left[ \left\{ U(E_o) - \left( n + \frac{1}{2} \right) \hbar \omega_c + e_0 L_0 \rho(E_o) - \frac{(M_0 \omega_c^2 \rho^2(E_o))}{2\hbar^2} \right\} \frac{1}{\rho(E_o)} - \left( n + \frac{1}{2} \right) \hbar \omega_c - \frac{(M_0 \omega_c^2 \rho^2(E_o))}{2\hbar^2} \right] \left[ \frac{\sqrt{\rho(E_o)}}{\rho(E_o)} \right] \]

\[ E_o = E_p + \Gamma, \text{ } E_p \text{ is the Fermi energy in this case, } i = \sqrt{-1} \text{ and } I_1(E_{pg}) = \sum_{r=1}^{i} L(r,0) [I_1(E_{pg})] \]

Using equations (2.11) and (2.78), the DMR can be expressed as
Special cases

(a) Under the substitutions \( \Delta_H = \Delta_L = \Delta \), \( m^{*}_n = m^*_L = m^* \) and \( \delta = 0 \), equation (2.76) assumes the form

\[
\gamma(E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{\hbar^2 k^2}{2m^*} \left( \frac{\varepsilon_0}{B} \right) \left[ \hbar k_j L_j(E) \right] - \frac{m^* \varepsilon_0^2 L^2}{2B^2}
\]  

where

\[
\gamma(E) = \left[ E \left( 1 + \alpha E \right) \left( E + 2 \Delta_H \right) \left( E + 2 \Delta_L \right) \right] \left( \frac{E + E_g + \Delta}{E + \Delta} \right)
\]

and

\[
\omega_0 = \frac{eB}{m}
\]

The equation (2.80) is the modified dispersion law under cross-field configuration in accordance with three-band Kane model. The expression of the electron concentration can be written as

\[
n_n = \frac{2\sqrt{2m^*B}}{3\varepsilon_0^2 \hbar^2 \pi^2 L^2_x} \sum_{m=0}^{\infty} \left[ I_3(E_F) + I_4(E_F) \right]
\]  

where

\[
I_3(E_F) = \text{Real part of} \left[ \left\{ \frac{1}{L_1(E_F)} \right\} \left\{ \gamma(E) - \left( n + \frac{1}{2} \right) \hbar \omega_1 + e \varepsilon_0 L_1 \left( E_0 \right) - \frac{m^* \varepsilon_0^2 L^2(E_0)}{2B^2} \right\} \right]
\]

\[
- \left\{ \gamma(E) - \left( n + \frac{1}{2} \right) \hbar \omega_1 - \frac{m^* \varepsilon_0^2 L^2(E_0)}{2B^2} \right\}^{\frac{3}{2}}
\]
and $I_4(E_F) = \sum_{n=0}^{\infty} L(r,0)I_3(E_F)$

Thus using equation (2.11) and (2.77) the DMR under cross-field configuration in accordance with three-band Kane model can be expressed as

$$\frac{D}{\mu} = \frac{1}{\mu} \left[ \sum_{n=0}^{\infty} \left[ I_3(E_{FB}) + I_4(E_{FB}) \right] \right]$$

(2.82)

(b) For semiconductors having parabolic energy bands $\alpha=0$ and the equation (2.80) gets simplified as

$$E = \left( n + \frac{1}{2} \right) \hbar \omega_o + \frac{\hbar^2 k^2}{2m} - \frac{e \varepsilon_0 k_x}{m \omega_o} - \frac{e^2 \varepsilon_0^2}{m^* \omega_o^2}$$

(2.84)

Thus neglecting broadening, the expressions for $n_0$ and DMR for parabolic energy bands in the presence of crossed electric and magnetic field can, respectively, be written as

$$n_0 = C_\frac{1}{2} \sum_{n=0}^{\infty} \left[ F_1(\eta_1) - F_1(\eta_2) \right]$$

(2.85)

and

$$D = \frac{k B T}{\mu} \left[ \sum_{n=0}^{\infty} \left[ F_1(\eta_1) - F_1(\eta_2) \right] \right]$$

(2.86)

where

$$\eta_1 = \frac{1}{k B T} (E_F - E_2), \quad E_2 = \left( n + \frac{1}{2} \right) \hbar \omega_o - e \varepsilon_0 L_x + \frac{m^* \varepsilon_0^2}{2B^2}$$

$$\eta_2 = \frac{1}{k B T} (E_F - E_3), \quad E_2 = E_3 + e \varepsilon_0 L_x, \quad C_\frac{1}{2} = \frac{2B \sqrt{2m^* \pi (k_B T)^3}}{L_\frac{1}{2} \hbar^2 \varepsilon_0}$$

Under the condition $\varepsilon_o \to 0$, the equation (2.85) and (2.86) get simplified to the equations (2.61) and (2.62a) respectively.
2.7.3 Analysis of the DMR in II-VI semiconductors under cross-field configuration

The modified carrier energy spectrum for II-VI semiconductors can be expressed in the present case, using equation (2.37), as

\[ E = \left( n + \frac{1}{2} \right) \hbar \omega_2 + \frac{\hbar^2 k_1^2}{2m_{\text{it}}} - \left( \varepsilon_0^2 m_{\text{i}}^* \right) \frac{1}{2B^2} + D_0 \sqrt{\left( n + \frac{1}{2} \right) \hbar \omega_2 + \frac{\varepsilon_0^2 m_{\text{i}}^*}{2B^2}} \]  

(2.87)

where \( \omega_2 = \frac{eB}{m_{\text{i}}} \) and \( D_0 = \left[ \frac{\varepsilon_0^2 m_{\text{i}}^*}{h} J_0 \right]^{\frac{1}{2}} \)

The electron concentration can be expressed using equations (2.87) and (2.77) as

\[ n_0 = \frac{1}{3} \frac{\varepsilon_0^2 m_{\text{i}}^*}{\hbar^2 \varepsilon_{\text{f}} x} \sum_{n=0} I_5(E_F) + I_6(E_F) \]  

(2.88)

where \( I_5 = \text{Real part of} \left[ E_0 + L_3 \left( \frac{\varepsilon_0^2}{B} \right) \hbar x_1 \right]^{\frac{3}{2}} - \left[ E_0 + L_3 \left( \frac{\varepsilon_0^2}{B} \right) \hbar x_2 \right]^{\frac{3}{2}} \),

\[ E_0 = E_F + i\Gamma, \quad L_3 = \left[ \frac{\varepsilon_0^2 m_{\text{i}}^*}{2B^2} + \left( n + \frac{1}{2} \right) \hbar \omega_2 - D_0 \sqrt{\left( n + \frac{1}{2} \right) \hbar \omega_2 + \frac{\varepsilon_0^2 m_{\text{i}}^*}{2B^2}} \right] \]

\[ x_1 = \frac{eBL_{\text{e}}}{h} + x_2, \quad x_2 = \left( \frac{2D_0}{\hbar} \right) \left( \frac{\varepsilon_0^2 m_{\text{i}}^*}{\sqrt{2}} \right) \left( \frac{\varepsilon_0^2 m_{\text{i}}^*}{Bh} \right) \]

and \( I_6(E_F) = \sum_{n=0} I_5(L(r,0)) \left[ I_5(E_F) \right] \)

Thus using equations (2.11) and (2.88) we can write the expression of DMR in II-VI materials under cross-field configuration as

\[ D = \frac{1}{\mu} \frac{\sum_{n=0} I_5(E_F) + I_6(E_F)}{\sum_{n=0} I_5'(E_F) + I_6'(E_F)} \]  

(2.89)

Under the condition \( C \rightarrow 0, \quad A_0 = B_0 = \frac{\hbar^2}{2m^*} \) and neglecting broadening, the equations (2.88) and (2.89) get simplified to the equations (2.85) and (2.86) respectively.
2.7.4 Analysis of the DMR under cross-field configuration in Bi in accordance with various band models

(a) McClure and Choi model

The dispersion relations of the carriers in Bi under cross electric field $\varepsilon_0$ along trigonal axis and quantizing magnetic field $B$ along bisectrix axis can be expressed using equation (2.42) as

$$E(l + \alpha\varepsilon) = \left(n + \frac{1}{2}\right)\hbar\omega_o + \frac{p_y^2}{2m_y} - \left(\frac{\varepsilon_0}{B}\right)(1 + 2\alpha\varepsilon)\hbar k_x - \frac{1}{2}m\left(\frac{\varepsilon_0}{B}\right)^2(1 + 2\alpha\varepsilon)^2$$

$$+ \left(\frac{p_y^2}{2m_z}\right)\alpha\varepsilon\left(1 - \frac{m_z}{m_y}\right) + \frac{\alpha p^2}{4m_z m}\alpha^2 + \left(\frac{\alpha p_y^2}{2m_z}\right)\left(n + \frac{1}{2}\right)\hbar\omega_o \pm \frac{1}{2}\frac{g^*\mu_o B}{m^2}$$

(2.90)

where $\omega_o = \frac{eB}{\sqrt{m,m_3}}$

Using equations (2.77) and (2.90), the electron concentration is given

$$n_o = \frac{(4B)(C_o)}{2\pi^2 h^2 L_0 \varepsilon_0} \sum_{m=0}^{\infty} \left[I_{\gamma}(E_\gamma) + I_{\delta}(E_\delta)\right]$$

(2.91)

where

$$I_{\gamma}(E_\gamma) = \text{Real part of}$$

$$\left[\frac{1}{(1 + 2\alpha\varepsilon_0)}\left[\frac{1}{2}\left(b_5(E_0) - b_3(E_0)^2\right) - \frac{1}{3}b_3(E_0)\right]^\frac{1}{2} - \frac{1}{3}b_3(E_3)\right]$$

$$I_{\delta}(E_\delta) = \sqrt{b_4(E_0) + b_3(E_3)B_1(E_0)}$$

$$b_4(E_0) = \frac{1}{2m_z^2} + \frac{\alpha\varepsilon_0}{2m_z}\left[\left(1 - \frac{m_z}{m_y}\right) - \alpha\left(\frac{1}{2}n + \frac{1}{2}\right)\hbar\omega_o\right]$$

$$C_2 = \frac{\alpha}{4m_2m}, \quad a_2(E) = E_0(1 + \alpha\varepsilon_0) - \left(n + \frac{1}{2}\right)\hbar\omega_o + \frac{1}{2}m_2(\varepsilon_0 B)^2(1 + 2\alpha\varepsilon)^2 \pm \frac{1}{2}g^*\mu_o B$$

$$b_3(E_3) = \frac{\varepsilon_0 h}{BC_2}(1 + 2\alpha\varepsilon_3), \quad b_5(E_3) = \frac{b_3(E_3)}{2C_2}, \quad b_3(E_0) = \left[b_4(E_0) + b_3(E_3)B_1(E_0)\right]^\frac{1}{2}$$
and \( I_x(E_p) = \sum_{n=0}^{\infty} L(r,0) [I_x(E_p)] \)

Thus using equations (2.91) and (2.11) we get

\[
\frac{D}{\mu} = \frac{1}{\epsilon} \sum_{n=0}^{\infty} I_x(E_p) + I_x'(E_p)
\]

(2.92)

(b) Hybrid model

The modified dispersion law for Hybrid model in the present case assume the form

\[
E_o(1 + \alpha E_0) = \left(n + \frac{1}{2}\right) \hbar \omega_0 \left(\frac{\varepsilon_0}{B}\right) \hbar k_x \left(1 + 2\alpha E\right) - m_1 \frac{\varepsilon_0^2}{2B^2} \left(1 + 2\alpha E\right)^3 + \beta(E) \frac{p_v^2}{2M_2} + \alpha(\epsilon) \frac{p_v^2}{4M_2} \pm \frac{1}{2} g^* \mu_0 B
\]

... (2.93)

The basic form of the equations (2.91) and (2.92) as valid for McClure and Choi model will be unaltered for Hybrid model where \( C_2 = \frac{\alpha \gamma_0}{4M_2} \) and \( b_2(E_0) = \beta(E) \frac{2M_2}{2m_2} \)

(c) Cohen model

The modified electron energy spectrum in accordance with Cohen model assumes the form

\[
E_o(1 + \alpha E_0) = \left(n + \frac{1}{2}\right) \hbar \omega_0 \left(\frac{\varepsilon_0}{B}\right) \hbar k_x \left(1 + 2\alpha E\right) - m_1 \frac{\varepsilon_0^2}{2B^2} \left(1 + 2\alpha E\right)^3 + \alpha(\epsilon) \frac{p_v^2}{2m_2}
\]

\[
\pm \frac{1}{2} g^* \mu_0 B + \left(\frac{p_v^2}{2m_2}\right) \left(1 + \alpha E\right) \alpha\gamma_0 \frac{p_v^2}{4M_2}
\]

(2.94)

The basic form of the equations (2.91) and (2.92) as valid for McClure and Choi model will also be valid for Cohen where \( b_2(E_0) = \left[\frac{(1 + \alpha E_0)}{2m_2} - \frac{\alpha(E_0)}{2m_2}\right] \)
(d) **Lax model**

Lax model is the anisotropic two-band Kane model. For this model, the modified electron energy spectrum assumes the form

\[ E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{\hbar^2 k^2}{2m_i} \pm \frac{1}{2} g^* \mu_B - \left( \frac{\varepsilon_0}{B} \right) \hbar k_x (1 + 2\alpha E) - \frac{\varepsilon_0^2}{2B^2} (1 + 2\alpha E)^2 \]  

(2.95)

For the Lax model the expression of \( n_0 \) can be written as

\[ n_0 = \frac{B\sqrt{2m_i}}{3\pi^2 \hbar^2 \varepsilon_0 I_s} \sum_{n=0}^\infty \left[ I_s(E_F) + I_{10}(E_F) \right] \]  

(2.96)

where

\[ I_s(E_F) = \text{Real part of} \left[ \frac{1}{(1 + 2\alpha E_F)} \left( a_s(E_0) + \left( \frac{\varepsilon_0}{B} \right) \hbar B_s(E_F) (1 + 2\alpha E_0) \right) \right]^{\frac{3}{2}} - \left( a_s(E_0) + \left( \frac{\varepsilon_0}{B} \right) \hbar B_s(E_0) (1 + 2\alpha E_0) \right)^{\frac{3}{2}} \]

and \( I_{10}(E_F) = \sum_{n=0}^\infty L(r,0)[I_s(E_F)] \)

Thus using equations (2.49) and (2.11) we get

\[ \frac{D}{\mu} = \frac{1}{|e|} \sum_{n=0}^{\infty} I_s(E_F) + I_{10}'(E_F) \]  

(2.97)
For ellipsoidal parabolic model the $E - K$ relation for this case can be expressed as

$$E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \left( \frac{\varepsilon_0}{B} \right) \hbar k_x - m_1 \frac{\varepsilon_0^2}{B} + \frac{\hbar^2 k_x^2}{2m_1} + \frac{1}{2} g \cdot \mu_0 B$$

(2.98)

In this case the forms of the equations (2.96) and (2.97) will be unaltered where

$$I_{\nu}(E_{\nu}) = \text{Real part of} \left[ \left\{ a_2(E_0) + \left( \frac{\varepsilon_0}{B} \right) \hbar B_0(E_0) \right\}^3 - \left\{ a_2(E_0) + \left( \frac{\varepsilon_0}{B} \right) \hbar B_2(E_0) \right\}^3 \right],$$

$$a_2(E_0) = \left[ E_0 - \left( n + \frac{1}{2} \right) \hbar \omega_0 \pm \frac{1}{2} g \cdot \mu_0 B - \frac{m_1 \varepsilon_0^2}{2B^2} \right],$$

$$B_2(E_0) = -\frac{m_1 \varepsilon_0}{Bh} \quad \text{and} \quad B_1(E_0) = \frac{eBL_1}{h} + B_2(E_0).$$
Figure: 2.8 Plot of the normalized DMR under cross-field configuration as a function of electron concentration in n-Cd$_3$As$_2$ in accordance with
(a) Our generalized band model, (b) Three-band model of Kane
(c) Two band model of Kane and (d) Parabolic energy bands
Figure 2.9 Plot of normalized DMR under cross-field configuration as a function of magnetic quantum limit for all the cases of figure 2.8.
2.8 Results and Discussion

Using equation (2.78) and (2.79) and taking the same parameters as used in obtaining figure 2.4 we have plotted the DMR in $n-Cd_{3}As_{2}$ as a function of electron concentration as shown in plot a of figure 2.8 in which the same dependences has been plotted for three-band Kane model, two-band Kane model and that of parabolic energy bands for the purpose of relative assessment among the band models. The graph $\delta = 0$ has been shown for the purpose of evaluating the influence of cross-field splitting on the DMR in tetragonal semiconductors under cross-field configuration. The presence of crystal-field splitting parameter decreases the DMR in the whole ranges of the concentration considered here. The DMR's oscillate with $n_{0}$, which are totally band structure dependent. In figure 2.9, we have drawn the above cases versus magnetic field in magnetic quantum limit. The DMR decreases with increasing magnetic field in the magnetic quantum limit as expected for degenerate semiconductors.

In the presence of magnetic quantization, the electrons can move only in direction of magnetic field where as under cross-field configuration the motion along the two directions are possible. This important properties of cross-field configuration the motion along the two directions are possible. This important properties of cross-field is the removal of degeneracy which in turns introduces the same anisotropy for even parabolic energy bands and this anisotropy depends on the electron energy in addition to Landau quantum number. Besides all the transport coefficients becomes sample dimension dependent. The above facts influence the Fermi energy strongly which, strongly effect the DMR.