A Simple Theoretical Analysis of the Effective Electron Mass in Bismuth under Different Physical Conditions

12.1. Introduction:

The importance of studying the effective mass in the field of semiconductor science is well known in the literature. Nevertheless, it appears that the influence of quantum confinement on the effective mass of the electron at the Fermi level (EMM) in semimetals has yet to be investigated. We shall take Bi as example of semimetals that has been the subject of a large number of experimental and theoretical investigations at low temperatures since it is easy to observe the influence of quantizations of band states in it [12.1-12.3]. Bi is considered to be a semimetal because its electronic properties are between those of metal and semiconductor. The dispersion relation of the carriers in Bi could be described by Lax Ellipsoidal non-parabolic model [12.5], whereas Dinger and Lawson [12.6] indicated that the Cohen model [12.7] is in better agreement with the experimental cyclotron resonance results. In a work on magnetic surface resonance Takaoka et al. [12.8] have proposed a hybrid band model of Bi. In addition, McClure and Choi [12.9] presented a new model of Bi, which was more general than those in use. They showed that it can fit the data for a large number of oscillatory and resonance experiments. Therefore it would be interest to study the EMM in Bi in accordance with all the aforementioned models under various quantizations of band states.

In this chapter, in section 12.2.1 we shall formulate the EMM in Bi in accordance McClure and Choi, hybrid and Cohen models in the presence of crossed electric and magnetic fields since the cross field configuration is fundamental for classical and quantum transport phenomena in solids [12.10]. In section 12.2.2, we have studied the EMM in quantum wells of Bi in accordance with various band models. In quantum wells, the carrier motion along the direction
normal to the film may be viewed as carrier confinement in an infinitely deep square potential well leading to the quantization of the wave vector of the carriers along the direction normal to the film [12.11]. In sections 12.2.3 and 12.2.4 we have extended the calculation of the EMM in electric field aided ultrathin films of Bi and also in the presence of crossed electric and magnetic fields respectively.

Low dimensional structures [12.12, 12.13] having quantum confinements in two dimensions such as quantum well wires (QWWs) have in the last few years attracted much attention not only for their potential in uncovering new phenomena in nano-structured electronics but also their interesting device applications [12.14, 12.15]. We have studied the EMM in quantum well wires of Bi in accordance with various models in section 12.2.5. Finally in sections 12.2.6 and 12.2.7 we investigated the EMM in electric field added QWWs of Bi and also in 1D bismuth under cross field configuration respectively.

12.2. Theoretical Background:

12.2.1 Formulation of EMM in Bi under cross field configuration in accordance with various band models

(a) McClure and Choi model:

The carrier energy spectrum in accordance with the McClure and Choi model can be written as [12.9]

\[
E(1+\alpha E) = \frac{p_x^2}{2m_1} + \frac{p_y^2}{2m_2} + \frac{p_z^2}{2m_3} + \frac{\alpha \bar{p}_x^2}{4m_1m_2} + \frac{\alpha \bar{p}_y^2}{4m_1m_3} + \frac{\alpha \bar{p}_z^2}{4m_2m_3}
\]  

(12.1a)

where \( E \) is the total energy of the carriers as measured from the edge of the band, \( \alpha = \frac{1}{E_g} \), \( E_g \) is band gap, \( \bar{p} = \hbar \tilde{k}, \hbar = h/2\pi \), \( h \) is the Plank constant, \( \tilde{k} \) is the carrier wave vector,
$m_i (i=1,2,3)$ are the effective carrier masses at the band edge $p_x, p_y$ and $p_z$ directions respectively and $m_i$ is the longitudinal effective mass of the holes for the valance band for the case of electrons or the longitudinal mass of the electrons for the conduction bands in the case of holes.

The modified carrier dispersion law in Bi in the presence of an electric field $\varepsilon_0$ along trigonal axis and quantizing magnetic field $B$ along bisetrix can be written as

$$E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 + \left( \frac{p_x^2}{2m_2} \right) - \left( \frac{\varepsilon_0}{B} \right) (1 + 2\alpha E) \hbar k_x - \left( \frac{m_1}{2} \right) \left( \frac{\varepsilon_0}{B} \right) (1 + 2\alpha E)^2 + \left( \frac{\alpha E p_z^2}{2m_2} \right) \left( 1 - \frac{m_2}{m_i} \right) - \left( \frac{\alpha p_z^2}{4m_2 m_i} \right) \left( n + \frac{1}{2} \right) \hbar \omega_0 \pm \frac{1}{2} g^* \mu_0 B$$

where $n(=1,2,...)$ is the Landau number, $C_0 = \frac{eB}{\sqrt{m_1 m_2}}$, $\mu_0$ is the Bohr magnetron and $g^*$ is the effective g factor at the band edge. The use of equation (12.1b) leads to the expression of EMM along $y$ and $x$ directions, respectively, as

$$m_{y,z}^*(E_{PB}) = \frac{1}{4C_2} \left[ -\left( \frac{\alpha}{2m_2} \right) + \left( \alpha_1 (\alpha_1 + \alpha_2 E_{PB}) + 2C_2 (1 + 2\alpha E_{PB}) \right) \right]$$

and

$$m_{x,z}^*(n, E_{PB}) = \left( \frac{B}{\varepsilon_0} \right)^2 - 2\alpha (1 + 2\alpha E_{PB}) \left[ E_{PB} (1 + 2\alpha E_{PB}) - \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{m_1}{2} \left( \frac{B}{\varepsilon_0} \right)^4 (1 + 2\alpha E_{PB}) \right]$$

$$\pm \frac{1}{2} g^* \mu_0 B \left[ 1 + 2\alpha E_{PB} \right] \left[ (1 + 2\alpha E_{PB}) \alpha m_1 + \left( \frac{\varepsilon_0}{B} \right)^2 (1 + 2\alpha E_{PB}) \right]$$

where $\alpha_1 = \left[ \frac{1}{2m_2} \left( 1 - \frac{m_2}{m_i} \right) - \alpha \left( n + \frac{1}{2} \right) \hbar \omega_0 \right]$, $\alpha_2 = \frac{\alpha}{2m_2}$, $C_2 = \frac{\alpha}{4m_2 m_i}$ and $E_{PB}$ is the Fermi energy in the present case. It appears then that, the evaluation of $m_{y,z}^*(E_{PB})$ and $m_{x,z}^*(E_{PB})$ from equations (12.2) and (12.3) as function of doping requires an expression of carrier statistics that can, in turn, be expressed including spin and broadening as
\[ n_0 = h_0 \sum_{n=0}^{n} [U_1(E_{pB}) + U_2(E_{pB})] \]  

(12.4)

where \( h_0 = \left( \frac{2BC}{n^2 \hbar^2 e_c L_z} \right) \), \( L_z \) is the sample length along the Z direction.

\[ U_1(E_{pB}) = \text{Real part of} \left[ \left( \frac{1}{1 + 2\alpha E_0} \right) \frac{1}{5} \left( b_3(E_0) - b_4(E_0) \right)^{\frac{2}{5}} - \frac{1}{5} \left( b_4(E_0) - b_3(E_0) \right)^{\frac{2}{5}} + \frac{1}{3} b_5(E_0) \left( b_3(E_0) - b_4(E_0) \right)^{\frac{3}{5}} - \frac{1}{3} b_3(E_0) \left( b_5(E_0) - b_4(E_0) \right)^{\frac{3}{5}} \right] \]

\( E_0 = E_{pB} + i\Gamma, E_p \) is the Fermi energy, \( i = \sqrt{-1}, \Gamma(\alpha K_B T_D) \) is the broadening parameter, \( k_B \) is the Boltzmann constant, \( T_D \) is the Dingle temperature.

\[ b_4(E_0) = \sqrt{b_4(E_0) + b_2(E_0)B_1(E_0)}, \]

\[ b_2(E_0) = \left[ \frac{2m_2}{2m_2} \left( 1 - \frac{m_2}{2m_2} \right) - \frac{1}{2m_2} \left( n + \frac{1}{2} \right) \hbar \omega_0 \right], \]

\[ b_3(E_0) = \left( \frac{e_s \hbar}{B} \right) (1 + 2\alpha E_0) B_1(E_0) = \left[ \frac{eB L_2}{h} + b_3(E_0) \right], \]

\[ B_3(E_0) = \left( \frac{m_2 e_s}{B h} \right) (1 + 2\alpha E_0) B_3(E_0) = \frac{b_4(E_0)}{2c_2}, \]

\[ b_4(E_0) = \sqrt{b_4(E_0) + b_2(E_0)B_1(E_0)}, \]

\[ U_2(E_{pB}) = \sum_{r=1}^{s} \nabla_r [U_2(E_{pB})], \text{ } r \text{ is the set of real positive integers.} \]

\[ \nabla_r = 2(k_B T)^{2r} (1 - 2^{1-2r}) \xi(2r) \frac{d^{2r}}{dE_p^{2r}} \text{ } T \text{ is temperature and } \xi(2r) \text{ is the Zeta function of order } 2r. \]

(b) **Hybrid model:**

The dispersion law of the carriers for the hybrid model under cross-field configuration is given


\[ E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 - \left( \frac{e \hbar k_x}{B} \right)(1 + 2\alpha E) - m_1 e^2 \left( \frac{1 + 2\alpha E}{B} \right)^2 + \beta(E) p_y^2 + \left( \frac{\alpha p_y^4}{4m_2^2} \right) \pm \frac{1}{2} \cdot \mu_4^* B \]

(12.5)

where \( \beta(E) = [1 + \alpha E(1 - \gamma) + \delta] \), \( \gamma = \frac{M_2}{M_1} \), \( \delta = \frac{M_2}{m_2} \) and the other notations are defined in [12.16]. Using equation (12.5) \( m^*_{x,\pm}(n, E_{FB}) \) and \( m^*_{y,\pm}(n, E_{FB}) \) can be expressed as

\[
m^*_{x,\pm}(n, E_{FB}) = \left( \frac{B}{\varepsilon_0} \right)^2 \left[ \frac{2\alpha}{(1 + 2\alpha E_{FB})^2} \left( E_{FB} \left( 1 + 2\alpha E_{FB} \right) - \left( n + \frac{1}{2} \right) \hbar \omega_0 - \left( \frac{e_0^2 m_1}{2B^2} \right) \left( 1 + 2\alpha E_{FB} \right) \right) \pm \frac{1}{2} \cdot \mu_0^* B \right] + \frac{1}{(1 + 2\alpha E_{FB})} \left( 1 + 2\alpha E_{FB} + 2m_1 \alpha \left( \frac{e_0}{B} \right)^2 \left( 1 + 2\alpha E_{FB} \right) \right) \]

(12.6)

and

\[
m^*_{y,\pm}(E_{FB}) = \frac{1}{4C_2} \left[ -g_3 + \frac{\{ \beta^2(E_{FB}) \} \left( 2m_2 \right)^2 - 4C_2}{\left( n + \frac{1}{2} \right) \hbar \omega_0 \pm \frac{1}{2} \cdot \mu_4 B - E_{FB}(1 + \alpha E_{FB})} \right] \left( \frac{\beta(E_{FB}) g_3^2}{2m_2} + 2C_1(2\alpha E_{FB}) \right) \]

(12.7)

where \( C_2 = \frac{\alpha \gamma}{2M_2^2} \) and \( g_3 = \frac{\alpha(1 - \gamma)}{2M_2} \). The basic form of equation (12.4) will be unchanged in this case where \( b_2(E_0) = \frac{\beta(E_0)}{2M_2} \).

(c) Cohen model:

The modified carrier dispersion law is given by

\[
E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_0 - \left( \frac{e \hbar k_x}{B} \right)(1 + 2\alpha E) - m_1 e^2 \left( \frac{1 + 2\alpha E}{B} \right)^2 - \frac{\alpha p_y^4}{2m_2^2} \left( 1 + \alpha E \right) + \frac{\alpha p_y^4}{4m_2 m_2} \left( 1 + \alpha E \right) \]

(12.8)

The EMM along x and y directions are, respectively, given by
The EMM along $x$ and $y$ directions are, respectively, given by

$$m^*_{x,\pm}(n, E_{FB}) = \left(\frac{B}{\varepsilon_0}\right)^2 \left[ \frac{2\alpha}{(1 + 2\alpha E_{FB})^2} \left( E_{FB} \left(1 + 2\alpha E_{FB}\right) - \left(n + \frac{1}{2}\right)\hbar \omega_0 + \left(\frac{\varepsilon_0^2 m_1}{2B^2}\right)^{\pm 1/2} \mu B \right) \right] +$$

$$\frac{1}{1 + 2\alpha E_{FB}} \left[ \left(1 + 2\alpha E_{FB} + 2m_1\alpha \left(\frac{\varepsilon_0}{B}\right)^2 \left(1 + 2\alpha E_{FB}\right) \right) \right]$$

and

$$m^*_{y,\pm}(E_{FB}) = \frac{1}{4C_2} \left[ \frac{g_1 + g_2 (g_1 + g_2 (E_{FB}) + 2C_2 (1 + \alpha E_{FB}))}{(g_1 + g_2 (E_{FB}))^2 - 4C_2 \left(n + \frac{1}{2}\right)\hbar \omega_0 \pm \frac{1}{2} g \mu B - E_{FB} (1 + \alpha E_{FB})} \right]$$

where $C_2 = \frac{\alpha}{2m_1 m_2}$, $g_1 = \frac{1}{2m_2}$ and $g_2 = \frac{\alpha}{2} \left(\frac{1}{m_2} - \frac{1}{m_1}\right)$.

The basic form of $n_0$ as given by equation (12.4) will not change in this where

$$b_0(E_0) = \left[ \frac{1}{2m_2} (1 + \alpha E_0) - \frac{1}{2m_2} \alpha E_0 \right].$$

Besides, under the condition $E \to 0$, all the expressions of $m^*_{x,\pm}(n, E_{FB}) \to \infty$ as they should. In the absence of electric field $m^*_{y,\pm}(n, E_{FB})$ for all models will exist.

12.2.2. **Formulation of EMM in ultrathin films of Bi in accordance with various band models**
(a) McClure and Choi model:

In the presence of size quantization along $k_y$ direction, the modified carrier energy spectrum in ultrathin films of Bi in accordance with the McClure and Choi model can be written as

$$E(1+\alpha E) = p^2 \left[ \frac{1}{2m_1} \left( \frac{\alpha \hbar^2 \pi^2}{4m_2 m_3} \right) \left( \frac{n_{y}}{2d_y} \right)^2 \right] + p^2 \left[ \frac{1}{2m_3} \left( \frac{\alpha \hbar^2 \pi^2}{4m_2 m_3} \right) \left( \frac{n_{y}}{2d_y} \right)^2 \right] +$$

$$\left( \frac{n_{y}}{2d_y} \right) \left[ \frac{1}{2m_2} \left( \frac{\alpha E}{2m_2} \right) \left( 1 - \frac{m_1}{m_2} \right) + \left( \frac{\alpha \hbar^2 \pi^2}{4m_2 m_3} \right) \left( \frac{n_{y}}{2d_y} \right)^2 \right]$$

Where $n_{y}$ and $2d_y$ are the size quantum number and the thickness along the $y$ direction respectively.

The EMM is given by

$$m^*(n_{y}, E_{ps}) = \frac{1}{\sqrt{m_1 m_3}} \left[ \frac{1+2\alpha E_{ps}}{\alpha E_{ps}} \left( \frac{n_{y}}{2d_y} \right)^2 \left( 1 - \frac{m_2}{m_2} \right) \right]$$

$$\left[ 1 - \left( \frac{\alpha \hbar^2 \pi^2}{2m_2} \right) \left( \frac{n_{y}}{2d_y} \right)^2 \right]$$

(12.12)

where $E_{ps}$ is the Fermi energy in the presence of size quantization measured from the band edge in the absence of any quantization.

The surface carrier concentration is given by

$$n_s = \left( \frac{g_s \sqrt{m_1 m_3}}{\pi \hbar^2} \right)^{\frac{N_{y_{\text{max}}}}{m_s}} \sum_{n_{y}=1}^{n_{y_{\text{max}}}} \left[ J_1 + J_2 \right]$$

(12.13)

where

$$J_1 = \left[ E_{ps} \left( 1 + \alpha E_{ps} \right) - \left( \frac{\pi n_{y}}{2d_y} \right)^2 \right] \left[ \frac{1}{2m_1} + \frac{\alpha E_{ps}}{2m_1} \left( 1 - \frac{m_1}{m_2} \right) - \frac{\alpha \hbar^2 \pi^2}{4m_2 m_3} \left( \frac{n_{y}}{2d_y} \right)^2 \right]$$

$$\left[ 1 - \left( \frac{\alpha \hbar^2 \pi^2}{2m_2} \right) \left( \frac{n_{y}}{2d_y} \right)^2 \right]$$

$E_{ps}$ is the Fermi energy in the present case and $J_2 = \sum_{r=1}^{s} \nabla_r [J_1]$. 

(b) **Hybrid model:**

The modified dispersion relation in ultrathin films of Bi in accordance with the hybrid model can be expressed as

\[
E(1 + \alpha E) = \left( \frac{p_x^2}{2m_1} + \frac{p_z^2}{2m_2} + \frac{\hbar n_y}{2d_y} \right) \left[ 1 + 2 \frac{\hbar n_y}{2d_y} \left( \frac{\beta(E)}{2M_2} + \frac{\alpha \gamma}{4M_2^2} \left( \frac{\hbar n_y}{2d_y} \right)^2 \right) \right] \]

(12.14)

The EMM is given by

\[
m^* (n, E_{gs}) = \sqrt{m_1 m_3} \left[ 1 + 2 \alpha E_{gs} + \alpha (\gamma - 1) \left( \frac{\hbar n_y}{2d_y} \right)^2 \right] \]

(12.15)

The basic form of surface carrier concentration for hybrid model is given by equation (12.13) where

\[
J_s = \left[ E_{gs} (1 + \alpha E_{gs}) - \left( \frac{\pi \hbar n_y}{2d_y} \right)^2 \left( \frac{\beta E_{gs}}{2\gamma} + \frac{\alpha \gamma}{4M_2^2} \left( \frac{\pi \hbar n_y}{2d_y} \right)^2 \right) \right] \]

(c) **Cohen model:**

The modified carrier energy spectrum in quantum wells of Bi can be written in accordance with the Cohen model as

\[
E(1 + \alpha E) = \left( \frac{p_x^2}{2m_1} + \frac{p_z^2}{2m_2} \right) + \alpha \frac{\hbar n_y}{2d_y} \left( \frac{\hbar n_y}{2d_y} \right)^2 + \left( \frac{\hbar n_y}{2d_y} \right)^2 \left[ \frac{2M_2}{2m_2} \left( \frac{\hbar n_y}{2d_y} \right)^4 \right] \]

(12.16)

The EMM can be expressed as

\[
m^* (n, E_{gs}) = \sqrt{m_1 m_3} \left[ 1 + 2 \alpha E_{gs} - \frac{\hbar n_y}{2d_y} \left( \frac{\beta}{2M_2} + \frac{\alpha \gamma}{4M_2^2} \right) \right] \]

(12.17)

The form of equation (12.13) will be unchanged in this case where
12.2.3. **Formulation of EMM in electric field aided ultrathin films of Bi in accordance with various band models**

(a) **McClure and Choi model:**

In the presence of an electric field $E_0$ along x-axis, the modified carrier energy spectrum in ultrathin films of Bi in accordance with the McClure and Choi model can be expressed as

\[
(p_x + p_y d_x^2 - (p_x - p_y d_x^2)^2 = \left( \frac{3n_p p_x^2}{2} \right)
\]

(12.18)

where

\[
p_x = \left( \frac{2m_i}{1 + \alpha E} \right)^{-1} E(1 + \alpha E) - \left( \frac{P_x^2}{2m_i} \right) - \left( \frac{\alpha E_p^2}{2m_i} \right) \left( \frac{1}{m_i} \right) + \left( \frac{\alpha E_p^2}{4m_i^2} \right) + \left( \frac{\alpha E_p^2}{4m_i^2} \right)
\]

The expression of EMM and $n_y$ can, respectively, be written as

\[
m_y^* = \left( p_y \frac{dp_y}{dE} \right)_{p_x=0, E=E_{FS}}
\]

(12.19)

\[
m_y^* = \left( p_y \frac{dp_y}{dE} \right)_{p_x=0, E=E_{FS}}
\]

(12.20)

and

\[
n_x = \left( \frac{2g_v}{h} \sum_{n=1}^{\infty} \left( A_n \frac{df_n}{dE} \right) dE \right)
\]

(12.21)
where $A_0$ is the area of $p_y p_z$ surface given by equation (12.18), $f_0$ is the Fermi Dirac function and $E$ is the root of equation (12.18) when $p_x = p_z = 0$. Since the nature of the equation (12.18) is transcendental, the analytical solutions of EMM and $n_x$ are not possible.

(b) **Hybrid model:**

For this model, the form of equation (12.18) remains unchanged

$$p_1 = 2m_1 \left[ E(1 + \alpha E) - \left( \frac{p_y^2}{2m_1} \right) - \left( \frac{p_z^2}{2m_1} \right) - \left( \frac{\beta(E)p_y}{2m_2} \right) - \left( \frac{\alpha p_y^4}{4M_y^2} \right) \right]$$

and

$$p_2 = 2m_1 \left[ e\varepsilon_0(1 - \gamma)p_y^2 \right]$$

The basic forms of equation (12.19) to (12.21) will be unaltered in this case.

(c) **Cohen model:**

For this model the basic forms of equations (12.18) to (12.21) will not change where

$$p_1 = 2m_1 \left[ E(1 + \alpha E) - \left( \frac{p_x^2}{2m_1} \right) - \left( \frac{p_y^2}{2m_1} \right) - \left( \frac{\alpha p_x^2}{2m_2} \right) - \left( \frac{\alpha p_y^4}{4m_2 m_1^2} \right) \right]$$

and

$$p_2 = 2m_1 \left[ e\varepsilon_0(1 + 2\alpha E)\alpha \varepsilon_0 - \frac{\alpha e\varepsilon_0 p_x^2}{2m_2} \right]$$

12.2.3.1. **Formulation of EMM in ultrathin films of Bi under cross field configuration in accordance with various band models**

(a) **McClure and Choi model:**
For an electric field $e_0$ along the trigonal axis and the magnetic field $B$ along bisectrix axis, the modified carrier energy spectra in the present case can be written as

$$L_s \sqrt{\left(\frac{\zeta_2^2 - 1}{\xi_1^2} - L_s \sqrt{\left(\frac{\zeta_2^2 - 1}{\xi_1^2} + \xi_2^2 \right)}\right)} + \xi_2^2 \left[ \sin^{-1} \left( \frac{L_s}{\zeta_1} \right) - \sin^{-1} \left( \frac{L_s}{\zeta_1} \right) \right] = 2 \zeta_2 \hbar n_x$$

(12.22)

where

$$L_s = \left( \frac{\zeta_3 \pm \xi_2 d_2}{2 \xi_2} \right)$$

$$\zeta_3 = \frac{\varepsilon e_0 - 2 \alpha e e_0 + 2 \hbar k e B \left( \frac{1}{2m_4} - \frac{\alpha \hbar^2 k_x^2}{4m_2 m_3} \right) + \alpha e e_0 \hbar^2 k_y^2 \left( \frac{m_2}{m_2} - \frac{m_3}{m_3} \right)}{\frac{1}{2m_2} - \frac{\alpha \hbar^2 k_y^2}{4m_2 m_3}}$$

$$\xi_2^2 = \frac{\varepsilon e_0^2 + \varepsilon B^2 \left( \frac{1}{2m_1} - \frac{\alpha \hbar^2 k_y^2}{4m_2 m_3} \right)}{\frac{1}{2m_3} - \frac{\alpha \hbar^2 k_y^2}{4m_3 m_3}}$$

$$\zeta_1^2 = \zeta_4 + \frac{\xi_3^2}{4 \xi_2^2}$$

and

$$\zeta_4 = \frac{E(1 + \alpha E) - \hbar^2 k_x^2 \left( \frac{2m_4}{2m_2} \right) - \hbar^2 k_y^2 \left( \frac{1}{2m_2} + \frac{\alpha E}{2m_2} \left( \frac{m_2}{m_2} \right) + \frac{\alpha \hbar^2 k_x^2}{4m_2 m_3} \right)}{\frac{1}{2m_3} - \frac{\alpha \hbar^2 k_y^2}{4m_3 m_3}}$$

The expression of EMM and $n_0$ can respectively, be written as
\[ m^*_x = \left( \hbar^2 k_x \frac{d k_x}{dE} \right)_{k_y=0, E=E_{FS}} \]  
(12.23)

\[ m^*_y = \left( \hbar^2 k_y \frac{d k_y}{dE} \right)_{k_x=0, E=E_{FS}} \]  
(12.24)

\[ n_s = \left( -\frac{q_v}{2\pi^2} \right) n^2 \max_{\nu} \sum_{m_z=1}^{\infty} \left\{ p_0 \frac{df}{dE} \right\} dE \]  
(12.25)

where \( p_0 \) is the enclosed by equation (12.22), \( t' \) is obtained from equation 912.22) by putting \( E = t' \) and \( k_x = k_y = 0 \).

(b) Hybrid model:

For hybrid model the basic forms of equations (12.22) and (12.25) will not change where

\[ \xi_3 = 2m_3 \left[ \frac{p_x eB}{m_1} - ee_0 - 2\alpha E e e_0 \right] \]

\[ \xi_2 = 2m_2 \left[ \frac{e^2 B^2}{2m_1} - a e^2 e_0^2 \right] \]

and

\[ \xi_4 = 2m_3 \left[ E(1 + \alpha E) - \left( \frac{p_x^2}{2m_1} \right) - \left( \frac{\alpha e p_y^2}{2m_2} \right) - (1 + \alpha E) \left( \frac{p_y^2}{2m_2} \right) \left( \frac{\alpha e p_y^4}{4m_2 m_1} \right) \right] \]

12.2.4. Formulation of EMM in quantum well wires (QWWs) of Bi in accordance with various band models

(a) McClure and Choi model:

In this case carrier dispersion law is given by

\[ E(1 + \alpha E) = a_4 k_z^2 + a_5 \]  
(12.26)
where
\[ a_4 = h^2 \left[ \frac{1}{2m_3} - \frac{1}{4m_3m_2} \left( \frac{\hbar n_y}{2d_y} \right)^2 + \left( \frac{\alpha E}{2m_2} \right) \left( 1 - \frac{m_2}{m_3} \right) \left( \frac{\hbar n_y}{2d_y} \right)^2 \right] \]
\[ + \left( \frac{\alpha}{4m_3m_2} \right) \left( \frac{\hbar n_y}{2d_y} \right)^4 - \left( \frac{\alpha h^4 \pi^4}{4m_3m_1} \right) \left( \frac{n_y^2}{4d_y^2 \gamma^2} \right) \]

The use of equation (12.26) leads to the expression of the EMM as
\[ m_i^*(n_y, E_{F_1}) = \left( \frac{\hbar}{2a_4} \right) \left[ 1 + 2\alpha E_{F_1} - \left( \frac{\alpha}{2m_2} \right) \left( 1 - \frac{m_2}{m_3} \right) \left( \frac{\hbar n_y}{2d_y} \right)^2 \right] \]  
\[ (12.27) \]

where \( E_{F_1} \) is the Fermi energy in the present case.

The carrier concentration per unit length is given by
\[ n_{id} = \left( \frac{2e}{\sqrt{m_1} m_3} \right) \sum_{n_y} [J_3 + J_4] \]  
\[ (12.28) \]

where
\[ J_3 = \frac{1}{a_4} \sqrt{[E_{F_1}(1 + E_{F_1}) - \alpha_3(E_{F_1})]} \]

and
\[ J_4 = \sum_{r=1}^{5} 2k_0 T^2 (1 - 2r^2) \left[ 2r \frac{d^2}{dE_{F_1}} dE_{F_1} \right] \]

(b) Hybrid model:

The carrier dispersion law for the hybrid model is given by
\[ E(1 + \alpha E) = \left( \frac{\hbar^2 \pi^2}{2m_3} \right) \left( \frac{n_y}{2d_y} \right)^2 + \left( \frac{p_4^2}{2m_3} \right) + \beta(E) \left( \frac{\hbar n_y}{2d_y} \right)^2 + \left( \frac{\alpha Y}{4M_2^2} \right) \left( \frac{\hbar n_y}{2d_y} \right)^4 \]  
\[ (12.29) \]

The EMM is given by
\[ m_i^*(n_y, E_{F_1}) = m_i \left[ 1 + 2\alpha E_{F_1} - \alpha (1 - \gamma) \left( \frac{\hbar n_y}{2d_y} \right)^4 \right] \]  
\[ (12.30) \]
The basic form of \( n_{o} \) as given by equation (12.28) will not change in this case where \( a_{4} = \frac{\hbar^{2}}{2m_{s}} \)

and \( \alpha_{s} = \left( \frac{\hbar^{2}\pi^{2}}{2m_{s}} \right) \left( \frac{n_{s}}{2d_{s}} \right)^{2} + \beta (E_{E_{v_{1}}}) \left( \frac{\hbar m_{v}}{2M_{2}} \right)^{2} + \left( \frac{\alpha_{r}}{4M_{2}^{2}} \right) \left( \frac{\hbar m_{v}}{2d_{r}} \right)^{4} \)

(c) **Cohen model:**

The modified dispersion law for Cohen model can be written as

\[
E(1 + \alpha E) = \left( \frac{\hbar^{2}\pi^{2}}{2m_{s}} \right) \left( \frac{n_{s}}{2d_{s}} \right)^{2} + \frac{\hbar^{2}k_{s}^{2}}{2m_{s}} \left( \frac{\hbar m_{v}}{2d_{r}} \right)^{2} + \left[ -\frac{\alpha E}{2m_{s}} + \frac{1 + \alpha E}{2m_{s}} + \frac{\alpha h^{2}\pi^{2}}{4m_{s}m_{2}} \left( \frac{n_{s}}{2d_{r}} \right)^{2} \right] .
\]

(12.31)

12.2.6 **Formulation of EMM in Electric Field aided QWWs of Bi in accordance with various band models**

(a) **McClure and Choi model:**

In this case, the modified dispersion law can be expressed as

\[
(p_{3} + p_{d_{s}}^{3} - (p_{3} - p_{d_{s}})^{3} = -\frac{3}{2} n_{o} \hbar p_{s}
\]

where

\[
p_{3} = \frac{\left[ E(1 + \alpha E) - \frac{p_{s}^{2}}{2m_{s}} - \left( \frac{\hbar^{2}}{2m_{s}} \right) \left( \frac{n_{s}}{2d_{s}} \right)^{2} - \left( \frac{\alpha \hbar^{2}}{2m_{s}} \right) \left( 1 - \frac{m_{s}}{m_{s}} \right) \left( \frac{n_{s}}{2d_{s}} \right)^{2} + \left( \frac{\alpha \hbar^{4}}{4m_{s}m_{2}} \right) \left( \frac{n_{s}}{2d_{s}} \right)^{4} + \left( \frac{\alpha \hbar^{2}k_{s}^{2}}{4m_{s}m_{2}} \right) \left( \frac{n_{s}}{2d_{s}} \right)^{2} \right]}{\left[ \frac{1}{2m_{s}} - \frac{\alpha h^{2}}{4m_{s}m_{2}} \left( \frac{n_{s}}{2d_{r}} \right)^{2} \right]}
\]
The expression for EMM and carrier concentration per unit length, can, respectively be expressed for 1D motion along z-direction as

\[
m_{z}^* = \left( \frac{\hbar^2 k_z}{dE} \right) \bigg|_{E=E_{F1}}
\]

(12.33)

and

\[
n_{1D} = \left( \frac{g_y}{2\pi} \right) \sum_{k_z} \int k_z \left( \frac{df}{dE} \right) dE
\]

(12.34)

where \( Q_0 \) is obtained by substituting \( E = Q_0 \) and \( k_z = 0 \) in equation (12.32). Again for conduction along \( p_y \) direction, the EMM and \( n_{1D} \) in the present case can be written as

\[
m_{y}^* = \left( \frac{\hbar^2 k_y}{dE} \right) \bigg|_{E=E_{F1}}
\]

(12.35)

and

\[
n_{1D} = \left( \frac{g_y}{\pi} \right) \sum_{k_y} \int k_y \left( \frac{df}{dE} \right) dE
\]

(12.36)

where \( Q_0 \) is obtained by putting \( E = Q_0 \) and \( p_y = 0 \) in equation (12.32) where

\[
P_3 = \frac{E(1 + \alpha E) - \frac{p_y^2}{2m_2} + \frac{\hbar^2}{2m_3} \left( \frac{n_z\pi}{2d_y} \right)^2 - \frac{p_y^2}{2m_2} \alpha E \left( 1 - \frac{m_2}{m_1} \right)}{\frac{\alpha p_y^4}{4m_2 m_3} + \frac{\alpha h n_{zz}^2}{2d_z}}
\]

and

\[
P_4 = \frac{\varepsilon \varepsilon_0 (1 + 2\alpha E) - \frac{\alpha \varepsilon_0 \hbar^2}{2m_2} \left( \frac{n_y\pi}{2d_y} \right)^2 \left( 1 - \frac{m_2}{m_1} \right)}{\frac{1}{2m_1} - \frac{\alpha \hbar^2}{4m_1 m_2} \left( \frac{n_y\pi}{2d_y} \right)^2}
\]
\[
P_4 = \left[ e\varepsilon_0 - \frac{p_y^2}{2m_2} - \alpha e\varepsilon_0 \left(1 - \frac{m_2}{m_1}\right) + 2\alpha e\varepsilon_0 E \right] \frac{1}{2m_1 - \left(\frac{\alpha p_y^2}{4m_1m_2}\right)}
\]

(b) **Hybrid model:**

For the hybrid model the forms of equations (12.33) and (12.34) remain unchanged where

\[
p_3 = 2m_1 \left[ E(1 + \alpha E) - \frac{\hbar^2}{2m_2} \left(\frac{n_y\pi}{2d_y}\right)^2 - \frac{p_y^2}{2m_3} - \frac{\beta(E)p_y}{2M_z}\left(\frac{hn_y\pi}{2d_y}\right)^2 - \left(\frac{\alpha\hbar^4}{4M_2^2}\right)\left(\frac{n_y\pi}{2d_y}\right)^2\right]
\]

and

\[
p_4 = 2m_1 \left[ e\varepsilon_0 (1 + 2\alpha E) - \alpha e\varepsilon_0 (1 - \gamma) \frac{\hbar^2}{2m_2} \left(\frac{n_y\pi}{2d_y}\right)^2\right]
\]

Besides for the motion of the carriers along \( p_y \) direction, the expression of the EMM and \( n_{lo} \) as given by equations (12.35) and (12.36) remain same where

\[
p_3 = 2m_1 \left[ E(1 + \alpha E) - \frac{p_y^2}{2m_2} - \frac{\hbar^2}{2m_3} \left(\frac{n_y\pi}{2d_y}\right)^2 - \frac{\beta(E)p_y^2}{2M_z} - \left(\frac{\alpha\hbar^4}{4M_2^2}\right)\right]
\]

and

\[
p_4 = 2m_1 \left[ e\varepsilon_0 (1 + 2\alpha E) - \alpha e\varepsilon_0 \frac{p_y^2}{2m_2} (1 - \gamma)\right]
\]

(c) **Cohen model:**

For this model we have to use
\[ p_3 = 2m_1 \left[ E(1 + \alpha E) - \frac{p_3^2}{2m_3} + \alpha E \left( \frac{\hbar n_y \pi}{2d y} \right)^2 - 1 + \alpha E \left( \frac{\alpha n_y \hbar}{2d y} \right)^2 - \left( \frac{\alpha h^4}{4m_2 m_2} \right) \left( \frac{n_y \pi}{2d y} \right)^4 \right] \]

and \[ p_4 = 2m_1 \left[ e \varepsilon_0 (1 + 2\alpha E) + \left( \frac{\alpha e \varepsilon_0}{2m_2} \right) \left( \frac{\hbar n_y \pi}{2d y} \right)^2 \right] \] for equations (12.33) and (12.34) and

\[ p_3 = 2m_1 \left[ E(1 + \alpha E) - \left( \frac{\hbar^2}{2m_3} \right) \left( \frac{n_y \pi}{2d y} \right)^2 + \left( \frac{\alpha E p_y^2}{2m_2} \right) + \left( \frac{p_y^2}{2m_2} \right) (1 + \alpha E) - \left( \frac{\alpha \hbar^4}{4m_2 m_2} \right) \right] \]

and \[ p_4 = 2m_1 \left[ e \varepsilon_0 (1 + 2\alpha E) - \left( \frac{p_y^2}{2m_2} \right) \alpha E \varepsilon_0 \right] \] for equations (12.35) and (12.36) respectively.

### 12.2.7 Formulation of EMM in QWWs under cross field configuration in accordance with various band models

(a) **McClure and Choi model:**

In the presence of a magnetic field \( B \) along bisectrix axis and electric field \( \varepsilon_0 \) along the trigonal axis the modified carrier dispersion law in the present case can be expressed as

\[ (\omega_{k+} - \omega_{k-}) \sqrt{a_z^2 - \omega_{k+}^2} - (\omega_{k-} - \omega_{k+}) \sqrt{a_z^2 - \omega_{k-}^2} + a_z^2 \left[ \sin^{-1} \left( \frac{\omega_{k+}}{a_z} \right) - \sin^{-1} \left( \frac{\omega_{k-}}{a_z} \right) \right] = 2 \hbar n_r \zeta_6 \]  \hspace{1cm} (12.37)

where
\[ \omega_{1,1} = \left[ \frac{\eta_2}{2 \zeta_6} \right]^{\pm} \zeta_1 \alpha \]

\[ \eta_2 = \frac{\left( \frac{1}{2m_1} + \frac{\alpha \hbar^2}{4m_2} \left( \frac{mn_y}{2d_x} \right)^2 \right)}{\left( \frac{1}{2m_3} + \frac{\alpha \hbar^2}{4m_3} \frac{mn_y}{2d_x} \right)} \]

\[ \zeta_6^2 = \left[ \frac{1}{2m_2} + \frac{\alpha \hbar^2}{4m_2} \left( \frac{mn_y}{2d_x} \right)^2 \right] \left( -\alpha^2 \left( \epsilon_0^2 \right) + e^2 B^2 \left( \frac{1}{2m_1} + \frac{\alpha \hbar^2}{4m_2} \frac{mn_y}{2d_x} \right) \right), \]

\[ a_i^2 = \zeta_7 + \frac{\eta_2^2}{4 \zeta_6^2}, \]

\[ \zeta_7^2 = \left[ \left( E + \alpha E \right) - \frac{h^2 k_x^2}{2m_1} + \frac{\alpha \hbar^4 k_x^4}{4m_2} \left( \frac{mn_y}{2d_x} \right)^2 \right] \left( \frac{1}{2m_2} + \frac{\alpha \hbar}{2m_2} \left( 1 - \frac{m_2}{m_1} \right) + \frac{\alpha \hbar^2}{4m_2 m_3} \left( \frac{mn_y}{2d_x} \right)^2 \right) \]

The equation \(12.37\) is valid for the motion along \(x\)-direction. The EMM can be written as

\[ m_x^* = \left( \frac{h^2 k_x}{dE} \right)_{E=E_{F_1}} \]  \(12.38\)

where \( E_{F_1} \) is the Fermi energy in this case.

The carrier concentration can be expressed as

\[ n_{ID} = \left( \frac{g_v}{2\pi} \right) \sum_{n_x,n_y}^{\infty} \int k_z \left[ \frac{d f_0}{dE} \right] dE \]  \(12.39\)

where \( K_0 \) is the subband energy. For motion along \( k_y \) direction, the basic form of equation \(12.37\) remains same where...
\[ \eta_1 = \left[ -e\varepsilon_\mathrm{g} - 2\alpha e\varepsilon_\mathrm{g} + 2\hbar e \left( \frac{m_x}{2a_x} \right)^2 \left( \frac{1}{2m_1} + \frac{\alpha \hbar^2 k_y^2}{4m_2m_3} \right) + \left( \alpha e\varepsilon_\mathrm{g} p_y^2 \right) \left( 1 - \frac{m_1}{m_2} \right) \right] \]

\[ \xi_\delta = \left[ \frac{1}{2m_3} + \frac{\alpha p_y^2}{4m_2m_3} \right]^{-1} \left[ e^2 B^2 \left( \frac{1}{2m_1} + \frac{\alpha p_y^2}{4m_2m_3} \right) - \alpha e^2 (\varepsilon_\mathrm{g})^2 \right] \]

\[ \xi_\gamma = \left[ (E + \alpha E) - \left( \frac{\hbar m_x}{2d_x} \right) \left( \frac{1}{2m_1} - \frac{\alpha p_y^2}{4m_2m_3} \right) - p_y^2 \left( \frac{1}{2m_2} + \frac{\alpha E}{2m_3} \left( 1 - \frac{m_1}{m_2} \right) + \frac{\alpha p_y^2}{4m_2m_3} \right) \right] \]

Therefore the EMM and carrier concentration can, respectively, be expressed as

\[ m_y^* = \left( \frac{\hbar^2 k_y}{m_2} + \frac{dk_y}{dE} \right) \big|_{E=E_F} \]  \hspace{1cm} (12.40)

and

\[ n_0 = \left( \frac{g_y}{2\pi} \right) \sum_{\gamma} K_y \left( \frac{df_0}{dE} \right) dE \]  \hspace{1cm} (12.41)

K_\gamma is the subband energy in this case.

(b) **Hybrid model:**

For the hybrid model, basic forms of equations (12.37) to (12.39) will not change where

\[ \eta_2 = \left[ \frac{heBk_x}{m_2} + \frac{\hbar^2}{2m_2} e\varepsilon_\mathrm{g} \left( 1 - \gamma \right) \left( m_y \right) - e\varepsilon_\mathrm{g} - 2\alpha e\varepsilon_\mathrm{g} \right] (2m_3) \]
\[
\zeta^2 = 2m_1 \left( \frac{e^2 B}{2m} - \alpha e^2 (\varepsilon_0)^2 \right)
\]

and
\[
\zeta^2 = 2m_3 \left[ E(1 + \alpha E) - \frac{h^2 k_x^2}{2m_1} - \frac{h^2}{2M_2} \left( \frac{n_x}{2d} \right)^2 \left( 1 + \alpha E(1 - \gamma) \right) - \frac{\alpha \gamma h^4 k_y^4}{4M_2^2} \right]
\]

For motion along \( k_y \) direction, the basic forms of equations (12.40) and (12.41) will not change where

\[
\eta_2 = \left[ \frac{\varepsilon}{m_2} \frac{m_n}{2d_x} + \frac{\alpha}{2M_2} \gamma (1 - \gamma) - \varepsilon_0 - 2\alpha E^2 \varepsilon_0 \right] (2m_2)
\]

\[
\zeta^2 = 2m_3 \left( \frac{e^2 B^2}{2m} - \alpha e^2 (\varepsilon_0)^2 \right)
\]

and
\[
\zeta^2 = 2m_3 \left[ E(1 + \alpha E) - \frac{h^2 k_x^2}{2m_1} - \frac{h^2}{2M_2} \left( \frac{n_x}{2d} \right)^2 \left( 1 + \alpha E(1 - \gamma) \right) - \frac{\alpha \gamma h^4 k_y^4}{4M_2^2} \right]
\]

\((c)\) **Cohen model:**

For this model, the basic forms of equation (12.37) to (12.39) will not change where

\[
\eta_2 = 2m_1 \left[ \frac{p_x^2 eB}{m_1} - \varepsilon_0 - 2\alpha E^2 \varepsilon_0 \right]
\]

\[
\zeta^2 = 2m_3 \left( \frac{e^2 B^2}{2m} - \alpha e^2 (\varepsilon_0)^2 \right)
\]

and
\[
\zeta^2 = 2m_3 \left[ E(1 + \alpha E) - \frac{p_x^2}{2m_1} + \frac{\alpha E h^2}{2m_2} \left( \frac{n_x}{2d} \right)^2 - \frac{1 + \alpha}{2m_2} \left( \frac{h m_y}{2d} \right)^2 - \frac{\alpha}{4m^2} \left( \frac{h m_y}{2d} \right)^2 \right]
\]

For motion along \( k_y \) direction, the basic forms of equations (12.40) and (12.41) will not change where

\[
\eta_2 = \left[ \frac{h m_y eB}{2d^2 m} - \varepsilon_0 - 2\alpha E^2 \varepsilon_0 \right] (2m_2)
\]
\[ \zeta^2 = 2m_s \left( \frac{e^2 B^2}{2m_i} - \alpha e^3 \langle \varepsilon_n \rangle^2 \right) \]

and \[ \zeta = 2m_i \left[ E(1 + \alpha E) - \frac{\hbar^2}{2m_i} \left( \frac{n_{2x}}{2d_z} \right)^2 - \frac{\hbar^2 p_z^2}{2m_z} - \frac{(1 + \alpha E)p_y^2}{2m_y} - \frac{\alpha p_x^4}{4m_{1y}} \right] \]

12.3 Results and Discussions:

Using the appropriate equations together and the parameters [12.16] \( g_s = 2, \ g_e = 3, \ m_1 = m_o/172, \ m_2 = m_o/0.8, \ m'_3 = m_0/0.78, \ m_3 = m_o/0.885, \ T = 4.2K, \)

\( E_s = [13.6 + (2.1X10^{-3})T + (2.5X10^{-4}T^2)]neV. \)

\( M_2 = 1.28m_0 \ and \ M'_2 = 0.8m_0 \) we have plotted \( \frac{m^*_s(n, E_{PB})}{m} \) for the first subband under cross-field configuration versus \( n_0 \) in Bi for the McClure and Choi, Hybrid, Cohen, Lax and parabolic ellipsoidal energy band models as shown by curves a, b, c, d, and e respectively of figure 12.1. From figure 12.1 it appears that the EMM in the presence of crossed electric and quantizing magnetic fields depends on the Fermi energy and the magnetic quantum number. For a single value of \( n_0, \) EMM is double valued due to presence of spin-splitting term as given in the expression of \( m^*_s(n, E_{PB}) \) for all the models. Besides all the transport properties will be sample dispersion dependent. These properties are the characteristic features of the cross-field transport. This conclusion is also valid for isotropic parabolic energy bands and the cross-field introduces the mass anisotropy even for relatively wide gap materials which, in turn, is a function of Fermi energy, electron spin and Landau quantum number respectively. For the \( E \rightarrow 0, \ m^*_s(n, E_{PB}) \rightarrow \infty \) for all the band models of Bi as it should.

In figure 12.2 we have plotted the normalized EMM for the first two subbands in ultra thin films of Bi versus surface electron concentration by using all the band models excluding the parabolic energy bands. The mass depends on the size quantum number in all the models excluding Lax and parabolic energy bands, solely due to the presence of band non-parabolicity. We wish to state that the band non-parabolicity can also explain the dependence of the EMM on...
Figure 12.1 Plot of $\frac{m^{*}\left(n, E_{FB}\right)}{m_{i}}$ versus $n_{0}$ in Bi under cross-field configuration for the first Subbands in accordance with (a) McClure and Choi; (b) Hybrid; (c) Cohen; (d) Lax and (e) parabolic ellipsoidal energy band models respectively ($B = 1 \text{Tesla}, L_{x} = L_{y} = L_{z} = 1 \text{m}, E_{0} = 10^{3} \text{V/m and } \Gamma = 10^{-3} \text{eV}$).
Figure 12.2 Plot of $m^*(n_x, E_{FS})/\sqrt{m_1 m_3}$ versus $n_x$ in ultrathin films for the first subbands in accordance with the band models of figure 12.1 excluding the parabolic energy bands ($2d_x = 40nm$).
Figure 12.3 Plot $m^*_y/m_2$ versus $n_s$ in electric field aided ultrathin films of Bi for first four subbands in accordance with all the band models of figure 12.1 ($2d_z = 40nm, E^*_0 = 10^3 V/m$).
Figure 12.4 Plot $m^*/m_i$ versus $n_i$ in electric field aided ultrathin films of Bi for first two subbands in accordance with all the band models of figure 12.1 ($2d_x = 40\text{nm}, E_0 = 10^3 \text{V/m}$).
Figure 12.5 Plot \( m_x' / m_1 \) versus \( 2d_z \) in ultrathin films of Bi for first two subbands in accordance with all the band models of figure 12.1 \((n_0 = 10^4 \, m^{-3}, B = 1\, \text{Tesla} \, \text{and} \, E_0 = 10^4 \, V/m)\).
Figure 12.6 Plot $\frac{m_y}{m_2}$ versus $2d_z$ in ultrathin films of Bi for first two subbands under cross-field configuration in accordance with all the band models of figure 12.1 ($n_0 = 10^4 m^{-2}, B = 1 \text{Tesla and } E_0 = 10^4 V/m$).
Figure 12.7 Plot of $m_z (n_y, E_{F1})/m_3$ versus $n_{1D}$ in Bi under cross-field configuration for the first Subbands in accordance with (a) McClure and Choi; (b) Hybrid and (c) Cohen models respectively. The plot $d'$ refers to Lax model ($2d_x = 2d_y = 40 \text{nm}, E_0 = 10^9 V/m$)
Figure 12.8 Plot of $m^*/m_3$ versus $n_{1D}$ for the first Subbands in electric field aided quantum wires of Bi for 1D motion along z-direction in accordance with all the band models of figure 12.1 ($2d_x = 2d_y = 40\text{nm}, E_0 = 10^3 V/m$).
Figure 12.9 Plot of $m^*/m_2$ versus $n_{1D}$ for the first few subbands in electric field aided quantum wires of Bi for 1D motion along $k_y$-direction in accordance with all the band models of figure 12.1 ($2d_x = 2d_y = 40 nm$, $E_0 = 10^3 V/m$)
Figure 12.10 Plot of $m^*_x/m_1$ versus $1/B$ in quantum wires of Bi under cross-field configuration for few subbands in accordance with all the band models of figure 12.1 for the free motion along $k_x$ direction. ($2d_x = 2d_y = 40nm$, $E_0 = 10^4 V/m$ and $n_{1D} = 10^{10}/m$)
Figure 12.11 Plot of $m^*/m_2$ versus film thickness in quantum wires of Bi under cross-field configuration for the first few subbands in accordance with all the band models of figure 12.1 for the free motion along $k_z$ direction.

\[2d_x - 2d_y = 40 \text{ nm}, \quad E_0 = 10^4 \text{ V/m}, \quad B = 1 \text{ Tesla}, \quad \text{and } n_{1D} = 10^{10} / \text{m}^2\]
the Fermi energy as in the Lax model but cannot take into account its dependence on the size quantum number in addition to Fermi energy. In the McClure and Choi, Hybrid and Cohen models of Bi, the same non-parabolicity also explain the dependence of the EMM on the size quantum number which is the characteristic features of the aforementioned band models for two dimensional quantum confinement.

In figure 12.3 and 12.4 we have plotted \( \frac{m^*_x}{m_2} \) and \( \frac{m^*_y}{m_3} \) for a few subbands in the electric field aided films of Bi versus \( n_s \) in accordance with all the band models of Bi by using appropriate equations for the present system. It appears that \( m^*_x \) and \( m^*_y \) depend upon the electric field, the Fermi energy and the size quantum number for all the models. The EMM corresponding to the lowest subbands exhibits the highest numerical values and the nature of variations of the EMM are totally band structure dependent.

In figure 12.5 and 12.6 we have plotted the normalized \( m^*_x \) and \( m^*_y \) in ultrathin films of Bi under cross-field configuration for a few subbands, in accordance with all the band models as a function of \( 2d_z \). The EMM depends on the Fermi energy, size quantum number, electric field and magnetic field respectively for the present 2D confinement. The Landau quantization phenomena is not possible here. The EMM decrease with increasing film thickness in various manners and appearance of the humps are due to the redistribution of the electrons among the size quantized levels. In figure 12.7 we have plotted the normalized EMM of a few subbands in quantum wires of Bi in accordance with all the band models of figure 12.1 excluding parabolic energy bands. The EMM of all the models excluding the Lax model, depend on the Fermi energy and the size quantum numbers that is solely due to the influence of band non-parabolicity. The above feature is the characteristic property of the McClure and Choi, Cohen and Hybrid models of Bi with respect to 1D systems.

In figures 12.8 and 12.9 we have plotted the normalized \( m^*_x \) and \( m^*_y \) in electric field aided quantum wires of Bi versus \( n_{1D} \) for all the band models of Bi by using the appropriate equations.
In figures 12.10 and 12.11 we have plotted normalized $m_x^*$ and $m_y^*$ for all the models in quantum well wires under cross field configuration versus magnetic field and films thickness respectively. It appears from the figures 12.10 and 12.11 that EMM depend on the size quantum number, the Fermi energy and the electric field and the magnetic field for all types of band models of Bi due to the particular physical characters of 1D systems under cross-field configurations.

It appears from all the figures that the quantum oscillations in EMM in Bi in accordance with McClure and Choi model show up much more significantly as compared to models of Bi. We may note that the quantum oscillations which occur in degenerate quantum confined materials would further be influenced by the index dependent EMM in semimetals and contributions of the EMM to the oscillatory mobility would be important. The carrier energy spectra in Bi could be described by the McClure and Choi, Hybrid, Cohen, Lax and Ellipsoidal parabolic models respectively as they are often used by various authors to investigate the different physical features of the same material under various physical conditions. We have formulated the EMM in Bi in accordance with all types of band models. Under certain limiting conditions, the EMM and the carrier statistics lead to the well-known results of wide-gap materials.

We wish to note that the Cohen model is used to describe the dispersion relation of the carriers in lead chalcogenides. The Lax model under condition of isotropic effective mass of the carriers at the band edge (i.e., $m_1 = m_2 = m_3 = m^*$) reduce to the well-known two band Kane model which is often used to describe the electronic properties of III-V compounds excluding $n-InAs$ [12.10]. Furthermore for $E_g \to \infty$, together with the conditions of the isotropic effective mass of the carriers at the band edge, all the models get simplified to the well-known form of isotropic parabolic bands as $E = \hbar^2 k^2 / 2m^*$, which is being widely used for investigating the electronic properties of relatively wide-gap materials under different physical conditions. Our analysis is also valid for holes with the proper change in band parameters. Thus the formulation of the present chapter is not only valid for semimetals like Bi but also for all types of lead chalcogenides, ternary and quaternary.