6.1 **Introduction**

In this chapter we shall study the DMR in quantum well wires of tetragonal semiconductors, II-VI semiconductors and Bi in section 6.2, 6.3 and 6.4 respectively. The well-known expressions of wide gap materials have also been obtained in all the cases.

6.2 **Formulation of DMR in quantum well wires of tetragonal semiconductors:**

The electron energy spectrum in quantum well wires (QWWs) of tetragonal semiconductors can be written following equation (2.1) as

\[
C(E) = A(E) \left( \frac{\hbar v}{d_y} \right)^2 + B(E) \left( \frac{l_\pi}{d_o} \right)^2 + A(E) k_x^2. \tag{6.1}
\]

The density-of-states function can be expressed using equation (6.1) as

\[
N(E) = \frac{2}{\pi} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} \frac{1}{A(E) \left( C(E) - A(E) \left( \frac{\hbar v}{d_y} \right)^2 - B(E) \left( \frac{l_\pi}{d_o} \right)^2 \right)^{\frac{1}{2}}}
\]

\[
\left[ \frac{1}{A(E) \left( C(E) - B(E) \left( \frac{l_\pi}{d_o} \right)^2 - A(E) \left( \frac{\hbar v}{d_y} \right)^2 \right)} - \frac{A(E)}{\left( A(E) \right)^2} \right] \left[ C(E) - A(E) \left( \frac{\hbar v}{d_y} \right)^2 - B(E) \left( \frac{l_\pi}{d_o} \right)^2 \right] H(E - E_f) \right] \tag{6.2}
\]

where \( E_m \) can be obtained by putting \( E_m \) and \( k_x = 0 \) in equation (6.1). Combining equation (6.2) with the Fermi-Dirac occupation probability factor and using the generalized Sommerfeld's lemma, the expression for electron concentration per unit length can be written as...
For Lax model, the dispersion relation in the present case is given by
\[
\left[ E(1 + \alpha E) + \epsilon E_0 d_0 (1 + 2\alpha E) - \frac{\hbar^2 k_x^2}{2m_1} - \frac{\hbar^2 k_y^2}{2m_2} \right]^{\frac{1}{2}} + \epsilon E_0 d_0 (1 + 2\alpha E) - \frac{\hbar^2 k_x^2}{2m_1} - \frac{\hbar^2 k_y^2}{2m_2} \right]^{\frac{1}{2}} = \frac{3}{2} \left( \epsilon \frac{E_0 \hbar (1 + 2\alpha E)}{\sqrt{2m_3}} \right) l \pi
\]

For ellipsoidal model, the equation (5.13) gets simplified into the form
\[
\left[ E + \epsilon E_0 d_0 - \frac{\hbar^2 k_x^2}{2m_1} - \frac{\hbar^2 k_y^2}{2m_2} \right]^{\frac{1}{2}} - \left[ E - \epsilon E_0 d_0 - \frac{\hbar^2 k_y^2}{2m_2} \right]^{\frac{1}{2}} = \frac{3}{2} \left( \epsilon \frac{E_0 \hbar}{\sqrt{2m_3}} \right) l \pi
\]
\[ n_0 = \frac{4}{\pi} \sum_{n_y=1}^{n_\text{max}} \sum_{l=1}^{L_\text{max}} \left[ \delta_{11}(E_F) + \delta_{13}(E_F) \right] \]

where \( \delta_{11}(E_F) = \frac{1}{A(E_F)} \left[ C(E_F) - A(E_F) \left( \frac{m^*}{d_y} \right)^2 - B(E_F) \left( \frac{1}{d_0} \right)^2 \right] \)

\[ \delta_{12}(E_F) = \sum_{r=1}^{S} \left[ \delta_{11}(E_F) \right] \]

and \( E_F \) is the Fermi energy in QWW as measured from the band-edge. Thus using equations (2.11) and (6.3), the DMR in QWWs of tetragonal semiconductors can be expressed as

\[ \frac{D}{\mu} \left| \begin{array}{c} \delta_{11}(E_F) + \delta_{13}(E_F) \end{array} \right| = \frac{1}{\mu} \left| \begin{array}{c} \sum_{n_y=1}^{n_\text{max}} \sum_{l=1}^{L_\text{max}} \right| \delta_{11}(E_F) + \delta_{13}(E_F) \]

**Special cases**

(a) Under the conditions \( \Delta_y = \Delta_\perp = \Delta, \ \delta = 0 \) and \( m^*_y = m^*_\perp = m^* \), equation (6.1) assumes the form

\[ \gamma(E) = \frac{h^2 k^2}{2 m^*} + \frac{\hbar^2 \pi^2}{2 m^*} \left[ \left( \frac{n_y}{d_y} \right)^2 + \left( \frac{l}{d_0} \right)^2 \right] \]

where the function \( \gamma(E) \) has been defined in equation (2.23). Therefore equation (6.5) represents the electron dispersion law in QWWs of non-parabolic materials whose energy band structures are defined by three-band Kane model. Thus the density-of-states function, electron concentration and the DMR can, respectively, be expressed as

\[ N(E) = \frac{2 \sqrt{2 m^*}}{h} \sum_{n_y=1}^{n_\text{max}} \sum_{l=1}^{L_\text{max}} \gamma(E) H \left( E - E_{\text{Dirac}} \right) \]

\[ n_0 = \frac{2 \sqrt{2 m^*}}{\hbar} \sum_{n_y=1}^{n_\text{max}} \sum_{l=1}^{L_\text{max}} \left[ \delta_{13}(E_F) + \delta_{14}(E_F) \right] \]
where $E_{\text{in}}$ can be obtained from the equation $\gamma(E_{\text{in}}) = \Delta_{13}$. Then $\Delta_{13} = \left(\frac{\hbar^2 \pi^2}{2m^*}\right)^{\frac{1}{2}} \left[\left(\frac{n_y}{d_y}\right)^2 + \left(\frac{l}{d_0}\right)^2\right]$.

\[
\delta_{13}(E_p) = \sqrt{\gamma(E_p) - \Delta_{13}} \quad \text{and} \quad \delta_{16}(E_p) = \sum_{r=1}^{g} L(r,0) \left[\delta_{13}(E_p)\right]
\]

(b) For $\Delta >> E_g$ and $\Delta << E_g$, equation (6.5) assumes the form

\[
E(1 + \alpha E) = \frac{\hbar^2 k^2}{2m^*} + \Delta_{13} \tag{6.9}
\]

which is one dimensional dispersion relation of the carriers whose energy band structures are obtained by two-band Kane model. Thus the density-of-states function, the electron concentration and the DMR can, respectively, be expressed as

\[
N(E) = \frac{2\sqrt{2m^*}}{h} \sum_{n_y=1}^{n_{\text{in}}} \sum_{l=1}^{l_{\text{um}}} \left(1 + 2 \alpha E\right) \frac{H(E - E_{\text{in}})}{(E(1 + \alpha E) - \Delta_{13})^{\frac{1}{2}}} \tag{6.10}
\]

\[
n_0 = \frac{2\sqrt{2m^*}}{\pi h} \sum_{n_y=1}^{n_{\text{in}}} \sum_{l=1}^{l_{\text{um}}} \left[\delta_{13}(E_p) + \delta_{16}(E_p)\right] \tag{6.11}
\]

and

\[
D = \frac{1}{\mu} \sum_{n_y=1}^{n_{\text{in}}} \sum_{l=1}^{l_{\text{um}}} \left[\delta_{13}(E_p) + \delta_{16}(E_p)\right] \tag{6.12}
\]

\[
E_{\text{in}} \left(1 + \alpha E_{\text{in}}\right) - \Delta_{13} = 0 , \quad \delta_{13}(E_p) = \sqrt{\left(E_p \left(1 + \alpha E_p\right) - \Delta_{13}\right)} \quad \text{and} \quad \delta_{16}(E_p) = \sum_{r=1}^{g} L(r,0) \left[\delta_{13}(E_p)\right]
\]

Under the condition $\frac{E_p}{E_g} << 1$, the expressions of the electron statistics and the DMR in QWWs of non-parabolic materials whose electron energy spectra obey the two-band Kane model can, respectively, be expressed as
\[ n_0 = \frac{2\sqrt{2m^*\hbar k_B T}}{\hbar} \sum_{n_x=1}^{n_{\text{max}}} \sum_{l=1}^{l_{\text{max}}} \left[ \frac{\left(1 + \frac{3}{2} \alpha_{18}\right) F_1(\delta_{19})}{2} + \frac{3}{4} \alpha k_B T F_1(\delta_{19}) \right] (\delta_{17})^{-\frac{1}{2}} \]

and

\[ D = \frac{k_B T}{\mu} \frac{\sum_{n_x=1}^{n_{\text{max}}} \sum_{l=1}^{l_{\text{max}}} \left[ \frac{1}{2} (1 + \frac{3}{2} \alpha_{18}) F_3(\delta_{19}) + \frac{3}{4} \alpha k_B T F_3(\delta_{19}) \right]} {\sqrt{\delta_{17}}} \]

where \( \delta_{18} = \Delta_{13}(1 + \alpha_{13})^{-1}, \delta_{17} = 1 + \alpha_{13}, \delta_{19} = \frac{[E_F - \Delta_{13}]}{(k_B T)} \)

(c) For wide gap materials \( \alpha \to 0 \) and the equations (6.9), (6.10), (6.13) and (6.14) assume the following forms [6.7]

\[ E = \hbar^2 k^2 \frac{2m^*}{2m^*} + \Delta_{13} \]

\[ N(E) = \frac{2\sqrt{2m^*}}{\hbar} \sum_{n_x=1}^{n_{\text{max}}} \sum_{l=1}^{l_{\text{max}}} \frac{H(E - E_{1\text{pp}})}{(E - \Delta_{13})^{\frac{1}{2}}} \]

\[ n_0 = \frac{2\sqrt{(2m^*\hbar k_B T)}}{\hbar} \sum_{n_x=1}^{n_{\text{max}}} \sum_{l=1}^{l_{\text{max}}} F_1(\delta_{19}) \]

and

\[ D = \frac{k_B T}{\mu} \frac{\sum_{n_x=1}^{n_{\text{max}}} \sum_{l=1}^{l_{\text{max}}} F_3(\delta_{19})}{\sqrt{\delta_{17}}} \]

where \( \delta_{19} = \frac{[E_F - \Delta_{13}]}{(k_B T)} \)

For bulk specimens of wide gap materials converting the summations over \( l \) and \( n_x \) to the integrations over the same variables, the equations (6.17) and (6.18) assume the well-known forms of equations (3.12) and (3.13) respectively.
6.3 Formulation of DMR in quantum well wires of II-VI semiconductors

The modified carrier energy spectrum in QWWs of II-VI semiconductors can be written following equation (2.3) as

\[ k_z^2 = \frac{1}{4a_0^2} \left[ -\lambda c_0 \pm \sqrt{c_0^2 + 4a_0 \left( E - b_0 \left( \frac{l\pi}{d_y^2} \right) \right)} \right]^2 - \left( \frac{mn_y}{d_y} \right)^2 \]  

(6.19)

Combining equation (6.19) with the Fermi-Dirac occupation probability factor, the electron concentration per unit length can be expressed as

\[ n_0 = \frac{4}{\pi} \left[ \sum_{n_y=1}^{n_{ym}} \sum_{l=1}^{l_{xm}} \left( \delta_{20}(E_F) + \delta_{21}(E_F) \right) \right] \]  

(6.20)

where \( \delta_{20}(E_F) = \frac{1}{(4a_0^2)} \left[ -\lambda c_0 \pm \sqrt{c_0^2 + 4a_0 \left( E_F - b_0 \left( \frac{l\pi}{d_y^2} \right) \right)} \right]^2 - \left( \frac{mn_y}{d_y} \right)^2 \)

and \( \delta_{21}(E_F) = \sum_{r=1}^{s} I(r,0) \delta_{20}(E_F) \)

Using equations (6.20) and (2.21) the DMR in QWWs of II-VI semiconductor can be written as

\[ D = \frac{1}{\mu} \left| \sum_{n_y=1}^{n_{ym}} \sum_{l=1}^{l_{xm}} \left[ \delta_{20}(E_F) + \delta_{21}(E_F) \right] \right| \]  

(6.21)

\[ \mu = \left| e \sum_{n_y=1}^{n_{ym}} \sum_{l=1}^{l_{xm}} \left[ \delta_{20}(E_F) + \delta_{21}(E_F) \right] \right| \]

Under the limiting conditions \( c_0 \to 0 \) and \( a_0 = b_0 \) the equation (6.21) gets simplified to equation (6.18).
6.4 Formulation of DMR in quantum wires of Bi in accordance with various band models

6.4.1 McClure and Choi model

The dispersion relation of the carriers in QWWs of Bi can be written following equation (2.42) as

\[ a_4(E, n_y) = k_r^2 \alpha_4(n_y) + \left( \frac{l\pi}{d_0} \right) a_6(n_y) \]  \hspace{1cm} (6.22)

The use of equation (6.22) leads to the expression of the density-of-states function as

\[ N(E) = \frac{g_v}{\pi} \sum_{n_y=1}^{\infty} \frac{1}{\sqrt{a_4(n_y)}} \left[ 1 + 2\alpha E - \frac{\alpha}{2m_y} \left( \frac{\hbar m_y}{d_0} \right)^2 \left( 1 - \frac{m_z^2}{m_y^2} \right) \right] a_4(E, n_y) - \left( \frac{l\pi}{d_0} \right) a_6(n_y) \right)^{1/2} \frac{H(E - E_{in})}{V} \] \hspace{1cm} (6.23)

where \( E_{in} \) can be obtained through the equation

\[ a_4(E_{in}, n_y) = \left( \frac{l\pi}{d_0} \right) a_6(n_y) \] \hspace{1cm} (6.24)

Combining equation (6.23) with the Fermi-Dirac occupation probability factor the electron concentration can be written as

\[ n_0 = \left( \frac{g_v}{\pi} \right) \sum_{n_y=1}^{\infty} \sum_{l=1}^{\infty} \left[ \delta_{22}(E_p) + \delta_{23}(E_p) \right] \] \hspace{1cm} (6.25)

where

\[ \delta_{22}(E_p) = \left[ \left\{ a_4(E_p, n_y) - \left( \frac{l\pi}{d_0} \right) a_6(n_y) \right\} \right]^{1/2} \]

and

\[ \delta_{23}(E_p) = \sum_{r=1}^{s} L(r, 0) [\delta_{22}(E_p)] \]

Combining equation (6.25) and (2.11), the expression of DMR can be written as
\[ D = \frac{1}{\mu} \sum_{n_{s,1}} \sum_{l=1}^{\text{num}} \left[ \delta_{23}(E_F) + \delta_{23}(E'_{F}) \right] \]

\[ \frac{\mu}{e} = \frac{\sum_{n_{s,1}} \sum_{l=1}^{\text{num}} \left[ \delta_{23}(E_F) + \delta_{23}(E'_{F}) \right]}{e} \]  \hspace{1cm} (6.26)

### 6.4.2 Hybrid model

The one-dimensional energy spectrum for this model can be written from equation (2.48) as

\[ E(1+\alpha E) = \frac{\hbar^2 k_F^2}{2m_1} + \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 + \left( \frac{\hbar n_y}{2d_y} \right)^2 + \frac{\alpha \hbar^4 \pi^4 (n_y)}{4M_2^2 (d_y)^4} \]  \hspace{1cm} (6.27)

The use of equation (6.27) leads to the expression of the density-of-state function as

\[ N(E) = \frac{2g_y}{\hbar} \sum_{n_{s,1}} \sum_{l=1}^{\text{num}} \frac{1}{\sqrt{a_y(n_y)}} \left[ 1 + 2\alpha E - \alpha (1-\gamma_0) \left( \frac{\hbar n_y}{2d_y} \right)^2 \right] \]

\[ \left[ E(1+\alpha E) - \frac{l^2 \hbar^2 \pi^2}{2m_3 d_0^2} - \frac{\pi^2 n_y^2 \hbar^2 \beta(E_{m_y})}{2M_2 d_y^2} - \frac{\hbar^4 \pi^4}{4M_2^2 d_y^4} \right]^{1/2} H(E-E_{m_y}) \]  \hspace{1cm} (6.28)

where \( E_{m_y} \) can be expressed through the equation

\[ E_{m_y} (1+\alpha E_{m_y}) = \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 + \left( \frac{\hbar n_y}{2d_y} \right)^2 + \frac{\alpha \hbar^4 \pi^4 (n_y)}{4M_2^2 (d_y)^4} \]  \hspace{1cm} (6.29)

Combining equation (6.28) with the Fermi-Dirac occupation probability factor, the electron concentration per unit length can be written as

\[ n_0 = \left( \frac{2g_y}{\pi} \right) \sum_{n_{s,1}} \sum_{l=1}^{\text{num}} \left[ \delta_{24}(E_F) + \delta_{25}(E_F) \right] \]  \hspace{1cm} (6.30)

\[ \delta_{24}(E_F) = \left( \frac{\sqrt{2m_1}}{\hbar} \right) E_F (1+\alpha E_F) - \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 - \frac{\beta(E_F)}{2M_2} \left( \frac{\hbar n_y}{2d_y} \right)^2 - \frac{\alpha \hbar^4 \pi^4 (n_y)}{4M_2^2 (d_y)^4} \]  \hspace{1cm} (6.30)

and \( \delta_{25}(E_F) = \sum_{r=1}^{s} L(r,0) \left[ \delta_{24}(E_F) \right] \)
Combining equations (2.11) and (6.30), the DMR in QWWs of Bi in accordance with hydrid model can be written as

$$ D = \frac{1}{\mu} \sum_{n_y=1}^{n_y_{max}} \sum_{l=1}^{l_{max}} \left[ \delta_{2n}^n(E_p) + \delta_{2n}^{n+1}(E_p) \right] $$

(6.31)

6.4.3 **Cohen model**

The carrier energy spectrum in QWWs of Bi for Cohen model can be expressed following equation (2.4) as

$$ E(1 + \alpha E) = \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{1}{d_0} \right)^2 + \frac{\alpha E}{2m_2} \left( \frac{\hbar m_{\gamma}}{d_y} \right)^2 + \frac{\hbar^2 \pi^2}{2m_2} \left( 1 + \alpha E \right) \left( \frac{n_y}{d_y} \right)^2 + \frac{\alpha h^4 \pi^4}{4m_2 m_2} \left( \frac{n_y}{d_y} \right)^4 $$

(6.32)

The density-of-states function can be written as

$$ N(E) = \frac{2g_{\pi}}{h} \sqrt{2m_1} \sum_{n_y=1}^{n_y_{max}} \sum_{l=1}^{l_{max}} \left[ 1 + 2\alpha E + \frac{\alpha E}{2m_2} \left( \frac{\hbar m_{\gamma}}{d_y} \right)^2 - \frac{\alpha h^2 \pi^2}{2m_2} \left( \frac{n_y}{d_y} \right)^2 \right] $$

$$ \left[ E(1 + \alpha E) - \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{1}{d_0} \right)^2 + \frac{\alpha E}{2m_2} \left( \frac{\hbar m_{\gamma}}{d_y} \right)^2 - \frac{\hbar^2 \pi^2}{2m_2} \left( 1 + \alpha E \right) \left( \frac{n_y}{d_y} \right)^2 - \frac{\alpha h^4 \pi^4}{4m_2 m_2} \left( \frac{n_y}{d_y} \right)^4 \right]^{1/2} H(E - E_{\omega_\gamma}) $$

(6.33)

where $E_{\omega_\gamma}$ can be expressed through the equation

$$ E_{\omega_\gamma}(1 + \alpha E_{\omega_\gamma}) = \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{1}{d_0} \right)^2 + \frac{\alpha(E_{\omega_\gamma})}{2m_2} \left( \frac{\hbar m_{\gamma}}{d_y} \right)^2 + \frac{\hbar^2 \pi^2}{2m_2} \left( 1 + \alpha E_{\omega_\gamma} \right) \left( \frac{n_y}{d_y} \right)^2 + \frac{\alpha h^4 \pi^4}{4m_2 m_2} \left( \frac{n_y}{d_y} \right)^4 $$

(6.34)

The electron concentration can be expressed as

$$ n_0 = \left( \frac{2g_{\pi}}{\pi} \right) \sum_{n_y=1}^{n_y_{max}} \sum_{l=1}^{l_{max}} \left[ \delta_{2n}^n(E_p) + \delta_{2n}^{n+1}(E_p) \right] $$

(6.35)
where \( \delta_{26}(E_p) = \)
\[
\left( \frac{2m_1}{\hbar} \right)^2 \left[ E_p \left( 1 + \alpha E_p \right) - \frac{\hbar^2 \pi^2}{2m_3} \frac{l}{d_0} \right]^2 - \frac{\hbar^2 \pi^2}{2m_2} \left( \frac{n_y}{d_y} \right)^2 - \frac{\hbar^2 \pi^2}{2m_2} \left( 1 + \alpha E_p \right) - \left( \frac{\alpha \hbar^4 \pi^4}{4m_1 m_2} \frac{n_y}{d_y} \right)^{\frac{1}{2}}
\]
and \( \delta_{27}(E_p) = \sum_{r=0}^{n_x} L(r,0) [ \delta_{26}(E_p) ] \)

Combining equations (2.11) and (6.35), the DMR can be expressed as
\[
\frac{D}{\mu} = \frac{1}{|e|} \left[ \sum_{n_x=1}^{n_x} \sum_{l=1}^{l_{max}} [ \delta_{26}(E_p) + \delta_{27}(E_p) ] \right]
\] (6.36)

6.4.4 Lax model

The one dimensional carrier dispersion law for this model can be written following equation (2.48) as
\[
E \left( 1 + \alpha E \right) = \frac{\hbar^2 k^2}{2m_1} + \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 + \frac{\hbar^2 \pi^2}{2m_2} \left( \frac{n_y}{d_y} \right)^2
\] (6.37)

The density-of-states function can be written as
\[
N(E) = \frac{2g_x}{\hbar} \sqrt{2m_1} \sum_{n_y=1}^{n_y} \sum_{l=1}^{l_{max}} \left[ 1 + 2\alpha E \left[ E \left( 1 + \alpha E \right) - \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 - \frac{\hbar^2 \pi^2}{2m_2} \left( \frac{n_y}{d_y} \right)^2 \right] \right]^{\frac{1}{2}} H(E - E_{br})
\] (6.38)

where \( E_{br}, (1 + \alpha E_{br}) = \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 + \frac{\hbar^2 \pi^2}{2m_2} \left( \frac{n_y}{d_y} \right)^2 \)

The electron concentration assumes the form
\[
n_0 = \left( \frac{2g_x}{\pi} \right)^{\frac{1}{2}} \sum_{n_y=1}^{n_y} \sum_{l=1}^{l_{max}} [ \delta_{24}(E_p) + \delta_{25}(E_p) ]
\] (6.39)

where \( \delta_{24}(E_p) = \left( \frac{\sqrt{2m_1}}{\hbar} \right)^2 \left[ E_p \left( 1 + \alpha E_p \right) - \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 - \frac{\hbar^2 \pi^2}{2m_2} \left( \frac{n_y}{d_y} \right)^2 \right]^{\frac{1}{2}}
\)
and \( \delta_{2\nu}(E_F) = \sum_{r=1}^{s} L(r, 0) \left[ \delta_{\chi}(E_F) \right] \)

Combining equations (6.39) and (2.11), the DMR can be expressed as

\[
\frac{D}{\mu} = \frac{1}{|e|} \sum_{n_{\pi}=1}^{n_{\pi_{\text{max}}}} \sum_{l=1}^{l_{\text{max}}} \left[ \frac{\delta_{\chi}(E_F) + \delta_{2\nu}(E_F)}{2} \right]
\]

(6.40)

6.4.5 Ellipsoidal model

For ellipsoidal model the carrier energy spectrum, the carrier concentration and the DMR in QWWs of Bi can, respectively, be expressed as

\[
E = \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 + \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{n_y}{d_y} \right)^2
\]

(6.41)

\[
n_0 = \left( \frac{2g_Y (2\pi m_0 k_B T)^{1/2}}{h} \right)^2 \sum_{n_{\pi}=1}^{n_{\pi_{\text{max}}}} \sum_{l=1}^{l_{\text{max}}} F_{1/2} (\delta_{\pi})
\]

(6.42)

and

\[
D = \frac{k_B T}{|e|} \sum_{n_{\pi}=1}^{n_{\pi_{\text{max}}}} \sum_{l=1}^{l_{\text{max}}} \frac{F_{1/2} (\delta_{\pi})}{2}
\]

(6.43)

where \( \delta_{\pi} = \frac{1}{k_B T} \left[ E_F - \frac{\hbar^2 \pi^2}{2m_2} \left( \frac{n_y}{d_y} \right)^2 - \frac{\hbar^2 \pi^2}{2m_3} \left( \frac{l}{d_0} \right)^2 \right] \)