Chapter 1

Introduction

The study of thermodynamics of strongly interacting matter has been an area of contemporary research for quite sometime now. Such studies are usually made in the extreme conditions of temperature and/or density where perturbative calculations might become meaningful. This is due to the fact that at such high density, the hadrons are expected to dissolve into their more fundamental constituents viz. the quarks and gluons, forming a new state of matter called Quark Gluon Plasma (QGP) [1]. We shall discuss this further in the context of asymptotic freedom.

In recent years, significant progress has been made to understand the behavior of QGP phase of matter leading to major advancement in the theoretical front addressing some of the subtle issues of the quasiparticles excitations in such an environment. Particularly important are the issues related to the infrared divergences which appear already in the vacuum field theory in dealing with the massless gluons. In a thermal bath, such infrared divergences get only worse because of the additional infrared singularity associated with the Bose distribution function involved in the calculation of various physical quantities like quasiparticle damping rate, quark mean free path etc [2-4].

One of the major developments, in this context, has been the Hard Thermal Loop (HTL) approximated perturbation theory where these issues are handled in a systematic manner and meaningful results are obtained after performing suitable resum-mations [4]. These apart, many calculations have been performed to study the high temperature transport phenomenon of QGP including calculations of various transport coefficients like viscosity, conductivity etc. [4-6]. Calculations have also been
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performed to estimate photon and dilepton rates in hot Quantum Chromodynamic (QCD) plasma [7-12]. Furthermore, the drag and momentum diffusion constants of the quarks in hot and dense QCD medium have also been estimated to study the equilibration phenomena for such a system [13-24]. In our case i.e. in dealing with cold dense plasma we use Hard Density Loop (HDL) corrected gluon propagator, which, in spirit, is very similar to the HTL resummation techniques as mentioned above.

Experimentally the heavy ion collisions provide opportunities to verify some of the theoretical results against measurements. In fact, the possibility of creating high temperature QGP by colliding heavy ions in the laboratory mimicking the conditions of the microsecond old universe has been a matter of intense research activities in the past decades. The Relativistic Heavy Ion Collider (RHIC) experiments at Brookhaven National Laboratory (BNL), and Large Hadron Collider (LHC) experiments at CERN have provided further impetus to these studies. Both of these experiments are concerned with high temperature plasma.

In contrast, the proposed experiments at GSI will focus on the compressed baryonic matter (CBM) i.e. matter with high baryonic chemical potential compared to temperature. It is to be noted that the high density quark matter can also exist in the core of neutron star with which we are presently concerned. Such studies are important to understand the properties of the astrophysical compact objects like neutron stars, pulsars, quasars etc. For example, experiments at Einstein laboratory, ROentgen SATellite (ROSAT), CHANDRA and X-ray Multi-Mirror Mission (XMM) have been performing measurements to understand the properties of neutron star [25-27]. The equation of state (EOS) for compact stars can also be explored in the forthcoming terrestrial accelerators such as Rare Isotope Beam (RIB) machines, Facility for Antiproton and Ion Research (FAIR) at Germany, Nuclotron-based Ion Collider fAcility (NICA) at Russia etc.

1.1 QCD Lagrangian

The basic theory that describes the strong interaction in terms of quarks and gluons is now known to be the Quantum Chromodynamics. These fundamental objects i.e. the quarks and gluons carry 'color' charges via which they couple with each other,
although in nature, quarks are always confined in colorless composites like mesons and baryons which are collectively called hadrons. In this thesis, however, we deal with only dense quark matter for which the dynamics are assumed to be controlled by the QCD alone. The calculations at such high densities can be performed perturbatively because of the existence of asymptotic freedom which we discuss in the next section [28].

QCD is a gauge theory with gauge group $SU(3)_c$. Here the index $c$ stands for color. The quarks belong to the fundamental representation of this group, and the gauge bosons i.e. gluons belong to the adjoint representation. Denoting the quark field with $\Psi$ and the gluon field with $A_\mu$, one writes the Lagrangian of QCD as [29]

$$L_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=1}^{N_f} \overline{\Psi}_{i,f} \left( i D_\mu - m_f \delta_{ij} \right) \Psi_{i,f}, \quad (1.1)$$

with $D_\mu = \gamma^\mu D_\mu$, $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$, and $D_{\mu ij} = \partial_\mu \delta_{ij} + ig A_\mu^a T^a_{ij}$. Apart from the invariance under $SU(3)_c$ gauge transformations, the Lagrangian is also invariant under global unitary transformations in flavor space as long as the differences of the quark masses can be neglected. In Eq.(1.1) we have written explicitly the sum over quark flavors ($f$). In the standard model, there are six flavors ($f = u, d, s, ...$) and for the light quarks $u, d$ one often assumes $m_f = 0$. Under this assumption the Lagrangian shows additional symmetry known as Chiral symmetry. Moreover, the Lagrangian is invariant under global $U(1)$ phase transformations of the quark fields [29].

1.2 Asymptotic freedom and QCD phase diagram

It is known that QCD is asymptotically free i.e. strong interaction becomes weak for processes involving high momentum transfers. Similarly, the QCD coupling constant also becomes weak at high temperature and/or density allowing the formation of QCD matter by which we mean phases of matter whose properties are governed not by the hadrons but by more fundamental constituents of matter viz. the quarks and gluons [29-32].

A basic property of QCD is the confinement of color charges i.e. at low density and temperature, the quarks and gluons remain confined in color singlet hadrons that
Constitute hadronic or nuclear matter. However, when the density or the temperature is high enough, the quarks and gluons start to play dominant roles in determining the thermodynamic properties of the system. At extreme temperature and/or density one expects a transition of hadronic matter into a phase dominated by quarks and gluons, where color is deconfined and interaction becomes screened [32, 33]. Such a phase, where any length scale of interest is greater than the screening length of the interaction, is known as Quark Gluon Plasma [1]. The QCD phase diagram is shown in Fig. (1.1). In general, this diagram can be depicted in terms of the temperature $T$ and baryon chemical potential $\mu$, as shown in Fig. (1.1) where the two extreme regions can be identified easily close to the $T$ and $\mu$ axis. The low temperature and high chemical potential region of the phase space corresponds to the phases which may be found at the core of the neutron star. Depending upon the density, it could be described in terms of hadronic or quark degrees of freedom including the possibility of superconducting phase as indicated in the diagram (1.1) [34–36]. The other extreme limit is the region of low chemical potential and high temperature. This part of the phase diagram corresponds to conditions very similar to the deconfined phase that might have existed in the early Universe before hadronization [37–39].

![Figure 1.1: Schematic QCD phase diagram of strongly interacting matter.](image)

It might be noted here, that in the high temperature with zero or low chemical potential two kinds of phase transitions may take place which has been labelled as quark-gluon plasma in Fig. (1.1). One possibility is the chiral phase transition which
might take place at high temperature where the broken chiral symmetry can be re­stored [40-42]. At vanishing chemical potential ($\mu = 0$), lattice QCD methods have been used extensively to study such chiral transitions [43-46]. At non-zero chemical potential, predictions on the order and exact location of the phase transition differ among the various lattice groups [47-51]. The other possibility is the deconfinement transition from hadronic state to a QGP state [52]. It is to be mentioned that the deconfinement phase transition is expected to be of first order if the quark mass is infinitely heavy [53-61], while the chiral phase transition could be of second order with two massless flavors [62-65]. We are particularly interested in the region where QCD matter corresponds to low temperature and high chemical potential which permits us to use the Fermi liquid theory (FLT) [66,67] and its extension to the case where non-Fermi liquid (NFL) corrections also become important. As an example of such corrections, we shall see how the neutrino mean free path or neutrino emissivity in dense quark system changes with NFL effects [68].

1.3 Ferromagnetic phase in dense quark matter

The main focus of the present investigation has been the exploration of the properties of dense quark matter (DQM) as mentioned above in the context of astrophysics. In particular, we have studied the possibility of ferromagnetism in DQM. This type of phase transition has been suggested recently in [69,70], which is also referred to as para-ferro phase transition. Part of the motivation to study the ferromagnetic phase transition in dense quark matter is provided by the discovery of 'magnetars' where an extraordinarily high magnetic field $\sim 10^{15} G$ might exist [70,71]. In [70], it is argued, that the origin of such a high magnetic field can be attributed to the existence of spin polarized quark matter [72]. The underlying mechanism of such a phase transition is analogous to what was originally proposed for the degenerate electron gas [73]. It is shown that for Coulomb interaction, the exchange correction to the energy is attractive, which, at low density, wins over the kinetic energy giving rise to a ferromagnetic state [73]. However, there are similarities and differences between quark matter and electron gas as discussed in ref. [70]. For slow moving massive quarks the dynamics is very similar to what happens in electron gas, while in the relativistic case, a completely different mechanism works when spin dependent lower component of the
Dirac spinor becomes important. In a series of works \([70,74,75]\) including the present author \([76]\), it has been shown that the strange quark matter can indeed exhibit such a phase transition below a critical density, while for light quark it never happens. Unlike ref. \([70]\), the model adopted in \([69]\) shows that both the light and heavy flavor systems can exhibit such phase transitions although the critical density for the strange matter is higher than the light quark systems. Subsequently various other calculations have been performed to address this issue \([36,71,72,75,77]\). For example, in \([36]\) it is shown that there is no contradiction between color superconductivity and ferromagnetism and both of these phases can co-exist. The same problem has been studied in \([75]\) in the large \(N_c\) and \(N_f\) limit while keeping \(N_c/N_f\) fixed. There, it is shown that spin polarized state can exist in dense quark system provided one considers only the exchange of electric (longitudinal) gluons. Inclusion of magnetic interaction, on the other hand, shows that the chiral density wave states as well as the color superconductivity disappear due to the presence of non-perturbative magnetic screening \([75]\). It might be mentioned, that such screening is now supported by the lattice calculation \([75,78]\). In \([36]\), it has been analytically shown that, if quarks are massless, ferromagnetism does not appear which is consistent with the conclusion drawn in \([75]\). According to ref. \([77]\), the ferromagnetism might appear in quark matter with a Goldstone boson current, where the magnetization is shown to be related to triangle anomalies.

We revisit the problem and invoke Relativistic Fermi Liquid Theory (RFLT) to examine the possibility of para-ferro phase transition in degenerate quark matter \([76]\). In particular, this has been accomplished by calculating the chemical potential \((\mu)\) and ground state energy (GSE) of degenerate quark matter in terms of the Landau parameters (LPs). The RFLT was first developed by Baym and Chin \([67,79]\) to study the properties of high density nuclear matter \([80]\). However, the formalism developed in ref. \([79]\) is valid for unpolarized matter and LPs calculated there are spin averaged. On the other hand, here, we deal with polarized quark matter which requires evaluation of the LPs with explicit spin dependencies.

For the computation of GSE of degenerate quark matter, we require the terms beyond the exchange diagram commonly known as the correlation energy \([81]\). Without such corrections, however, the calculations are known to remain incomplete as the higher order terms are plagued with infrared divergences arising out of the exchange
of massless gluons, indicating the failure of the naive perturbation series. This problem can be cured by reorganizing the perturbation theory where a particular class of diagrams, viz. the bubbles are resummed in order to obtain a finite result. In the case of degenerate electron matter this pioneering work was done by Gell-Mann and Brueckner (GB) commonly known as GB theory where the 'correlation energy ($E_{corr}$)' of electron gas at high density was calculated [82].

The knowledge of the GSE of spin polarized matter with the inclusion of bubble diagrams, is an important first step to include the corrections due to correlations to the spin susceptibility. Here, we present the spin susceptibility with corrections up to $O(g^4 \ln g^2)$ [83]. This work is very similar to that of Brueckner and Swada [84] and those of [85,86] where susceptibility was calculated for degenerate electron gas which we modify for the QCD matter. Unlike, degenerate electron gas, however, we have both the electric and magnetic interactions and the calculation is performed relativistically, while the known non-relativistic results appear when appropriate limits are taken.

Recently, in [87,88] the authors have studied the magnetic properties of degenerate quark matter in presence of weak uniform external magnetic field $B$. Similar investigation was also made in ref. [69] by evaluating the effective potential and employing quark magnetic moment as an order parameter. These calculations were, however, restricted to the case of unpolarized matter. On the contrary, we study the magnetic properties of polarized quark system. The expressions for magnetization and magnetic susceptibility have been presented in terms of the spin polarization parameter $\xi$ as we shall see later [89].

1.4 Magnetic interaction and NFL corrections

It is to be mentioned that at high density, for the quarks with momentum close to the Fermi momentum, relativistic effects become important, therefore, in this regime, the magnetic interaction can no longer be neglected. It is now known that with the inclusion of the transverse interaction the normal Fermi liquid description breaks down due to the vanishing of the quark propagator near the Fermi surface. This has been elaborated in [90] where the authors calculate Fermionic dispersion relations in ultradegenerate relativistic plasmas and show how such non-Fermi liquid (NFL)
behavior emerges from the vanishing of the Fermion propagator near the Fermi surface by calculating the group velocity of the corresponding quasiparticle excitations. This can be attributed to the absence of static screening of the magnetic gluon [90,91].

Historically, such a deviation from the normal Fermi liquid behavior for the first time was exposed in [92]. There, the specific heat of a degenerate gas at temperature $T$ due to the current-current interactions was calculated and the result was shown to contain the $T\ln T^{-1}$ term. Actually such an anomalous term emanates from the unscreened magnetic interactions, the origin of which can be understood from the medium modification of the fermion single particle spectra related to its self-energy in the plasma [90,91].

The fermion self-energy close to the Fermi surface receives a logarithmic enhancement due to the exchange of magnetic gluons [90,93]. A more rigorous discussion on how and why the dynamics change near the Fermi surface leading to the break down of Fermi liquid behavior or vanishing of the step discontinuity can be found in [91]. Departure from the Fermi liquid behavior have implications in determining the thermodynamic and transport properties of the quark component of neutron or proto neutron stars, viz. entropy, pressure, specific heat, viscosity etc. [29,31,91,94–96]. For example, ref. [93] examined NFL effects in the normal phase of high density QCD matter both using the Dyson-Schwinger equation and the renormalization group theory. Similar non-Fermi liquid terms, for ungapped quark matter, also appear in the calculation of neutrino emissivity [97]. Like emissivity the magnetic susceptibility shows similar non-Fermi liquid behavior as shown in [87]. In the present work we estimate and see the consequences of such effects in the case of neutrino mean free path (MFP) [68].

1.5 Outline of the thesis

In the present thesis, chapter 2 focuses on the formalism of relativistic Fermi liquid theory for the description of dense QCD matter. The Fermi liquid theory, originally was developed by Landau, is a semi-phenomenological approach which can be applied to study the thermodynamic properties of interacting fermionic system. We present here the extended version of the same by incorporating the relativistic effects [76,80].

Chapter 3 discusses the possibility of ferromagnetic phase transition in degenerate
quark matter within the framework of the Landau theory of relativistic Fermi liquids. This was accomplished by evaluating energy density ($E$) and chemical potential ($\mu$) of quark matter in terms of spin dependent LPs. Ground state energy density of the polarized quark matter is calculated here including the correlation corrections, without which, the perturbative evaluation remains incomplete [76,81].

Chapter 4 deals with the topic of spin susceptibility and magnetic susceptibility. Determination of spin susceptibility is necessary to know the possible para-ferro phase transition in DQM. On the other hand, the results for the magnetic susceptibility are important to understand the stability of the ferromagnetic phase [83,89].

In chapter 5, the non-Fermi liquid behavior of dense and warm QCD matter has been revealed. The exchange of dynamically screened magnetic gluons leads to infrared singularities in the fermion propagator for excitations near the Fermi surface causing the breakdown of the Fermi liquid description. Furthermore, here we examine the cooling rate of compact stars with non-Fermi liquid corrections which may enter through the specific heat ($C_v$) and neutrino emissivity ($\epsilon$). The latter is related directly to the neutrino mean free path which we evaluate in this chapter [68].

Lastly, in chapter 6 we summarize and relegate all the detailed calculations to the appendix.

Conventions

Throughout this thesis we shall use the following conventions. We use natural units, $\hbar = c = k_B = 1$. This convention has the added benefit that it establishes a simple relation between mass and length scale, through the relation $\hbar c = 197.33$ MeV fm. In natural units the relativistic energy-mass relationship reduces to $E^2 = p^2 + m^2$, and the units of mass and energy are the same. The Minkowski metric is $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Four momenta are denoted as $P_\mu = (p_0, p)$. The absolute value of the three momentum is denoted as $|p|$. A unit three vector is denoted as $\hat{p} = p/|p|$. 
Bibliography


