

## INTRODUCTION

The concept of continuity is one of the most important and useful notions of analysis and topology. In recent years a considerable amount of research has been devoted to questions involving many different types of generalized continuity of functions. Prominent among those which have been studied are quasi [48], semi [50], almost [62] [90], somewhat [33] [59], nearly [72] [90], somewhat nearly [72], pseudo [61], simple [4], and Baire or B-continuity [55]. In this connection we should also mention cliquish functions [85], which is very closely related to quasi-continuity.

It is found that simply-continuity, cliquishness, B-continuity are all generalizations of quasi-continuity.

All the above-discussed non-continuous functions involve open set or generalized open sets like semi-open, simple-open or others. Certain of these non-continuous functions have properties similar to those of continuous functions and they hold, in many part, parallel to the theory of continuous functions. The analogy in their definitions and results suggests the need of formulating a unified theory in the setting of functions. Our aim is to introduce a new type of generalized continuous function in the category sense without any generalized open set and suggest to name such functions as  $B^*$ -continuous functions.

Multifunctions or multi-valued mappings are still something exotic in the heads of mathematicians despite the fact that it can be very naturally obtained. The inverse of a function that is not injective is nothing but a multifunction. But that is not all. Multifunctions have many applications in mathematical programming, probability, statistics, different inclusions and fixed-point theory. More natural and interesting aspects of multifunctions arise from our observations of life, thinking, from nature as well as from problems, which are directly linked to such abstract areas.

In topology, there has been recently significant interest in characterizing and investigating the properties of several weak forms of continuity of multifunctions. The development of such a theory is in fact very well motivated. In economics, in the so-called parameterised maximization problem [88] the major interest is how this multifunction changes; in particular how and when it changes 'continuously'. That is why, extending and studying different forms of generalized continuity for multifunctions is a real problem also beyond the field of topology. The famous theorem of maximum [88] for example, involved several topological concepts. In brief, it states that every continuous function with compact range and non compact-valued continuous constraints has continuous maximum and the solution of the parameterised maximization problem is an upper semi-continuous multifunction.

An attempt has been made to discuss about  $B^*$ -continuous multifunctions along with some weaker forms of multifunctions and try to give some characterization and study some properties as well as the graph of these multifunctions.

Another important concept in analysis is the infinite matrix transformation theory, especially the theory of summability. In the later part of our thesis we have studied some properties of  $A$ -continuous function, introduced by Antoni and Salat.  $A$ -continuous function may be treated as a bridge between generalized continuity and infinite matrix transformation theory.

With the evolution of the concept of continuous function the property of continuity that came into the mathematicians' mind was the intermediate value property, better known as the Darboux property. A continuous function has the Darboux property. But a function having Darboux property may not be continuous at all. In the concluding part of our thesis we have studied some results relating the functions with intermediate value-type properties.

This thesis consists of eight chapters, which are believed to contain original results. We give below a brief summary of each chapter.

In **CHAPTER – 1**, we have introduced the notions of  $B^*$ -set and  $B^*$ -continuous function and studied the inter-relation of this type of function with other generalized continuous functions. Some properties of this type of function are obtained and some non-trivial examples are given.

In **CHAPTER – 2**, the concept of new type of oscillation of a function is introduced and we have studied some properties of  $B^*$ -continuous functions and characterize  $B^*$ -continuous function by this new oscillation-type function.

**CHAPTER – 3** deals with some preserving properties of  $B^*$ -continuous functions under pointwise convergence of transfinite sequence of  $B^*$ -continuous functions and uniform convergence of sequence of  $B^*$ -continuous functions in metric spaces. Also it is established that the uniform convergence of a sequence of functions is transmitted to the sequence of corresponding oscillation-type function.

In **CHAPTER – 4** we have introduced new notions of upper and lower  $B^*$ -continuous multifunctions as well as upper and lower  $B^*$ -cluster continuous multifunctions defined on a topological space  $X$ , and obtained some characterizations and some properties of such functions in connection with  $B^*$ -closed or  $B^*$ -open sets.

In **CHAPTER – 5** we have introduced the concepts of upper and lower weakly  $B^*$ -continuous multifunctions, and obtained some characterizations and several properties concerning upper and lower  $B^*$ -continuous as well as upper and lower weakly  $B^*$ -continuous multifunctions. The relationship between these multifunctions and their graphs are investigated.

Infinite matrix transformation theory plays an important role in analysis, especially on the theory of summability. In **CHAPTER – 6** we have established some results relating to a real-valued  $A$ -continuous function satisfying some conditions of regular infinite matrix  $A$  and the nature of set of points of  $A$ -discontinuity of the function is investigated.

In **CHAPTER – 7** we have studied some convergence properties of a sequence of  $A$ -continuous functions and defined  $A$ -oscillation of a function and tried to characterize the  $A$ -continuous function with the help of  $A$ -oscillation.

**CHAPTER – 8** deals with the behaviour of functions with intermediate-value-type properties in some restricted form.