A BRIEF HISTORICAL DEVELOPMENT:

The first idea that comes to our mind as we hear the word "continuous" is uninterrupted, remaining together, not broken or smooth flowing. There is no doubt that functions and of course continuous functions stand among the most important and most researched points in the whole of the mathematical science.

In the late eighteenth century at the time of Euler, d' Alembert and Daniel Bernoulli, 'continuity' referred to a constancy of the analytical expression of a function, rather than to connectedness of its graph, which is the modern idea of continuity. Indeed, essentially all functions treated in the eighteenth century analysis were continuous in the modern sense; 'discontinuity' then referred either to failure at isolated points, where the analytical expression changed, of a function to be smooth (in the modern sense), or to the lack of any analytical expression at all, as in the case of freehand curves.

Most of the fundamental and important properties of mathematics are satisfied by continuous functions. However, various studies have revealed that many important properties of mathematics are also satisfied by functions, which are not continuous. For example, every continuous function is Riemann integrable. But there are Riemann integrable functions, which have discontinuities even of second kind at some points.

These studies help to generalize the concept as well as the properties of continuous function. Many different forms of continuous functions have been introduced over the years. In 1922 H. Blumberg [7] first introduced the concept of almost continuity under the name 'densely approaching'. In 1932, S. Kempisty [48] introduced the notion of quasi-continuity for real functions as a generalization of the notion of continuity. In 1953 V. Ptak [78] intensively studied 'near continuity' which is another name of almost continuity. In 1959 J. Stalling [84] introduced the almost continuous function and later Z. Frolik (1961) [32], T. Husain (1966) [44] further carried out the research on almost continuous functions. In 1963 N. Levine [50] introduced the notion of semi-continuity with the help of semi-open set. A. Neubrunnova (1973) [61] showed that the semi-continuity in the sense of Levine is equivalent to the quasi-continuity. Studies had also revealed the invariance of almost continuity and semi-continuity.

Ever since its introduction quasi-continuity had been most intensively studied. There are various reasons for the interest of this study. One of the main reasons is the relatively good connection between the continuity and quasi-continuity in spite of the generality of the latter. Next is a deep connection of quasi-continuity with mathematical analysis, topology and even with probability. Quasi-continuous function is also closely associated with various other generalized continuous functions.

In 1952 W. Bledsoe [6] introduced the notion of neighbourliness of a function and in 1953 H. P. Thielman [85] introduced the notion of cliquishness of a function. In 1961 S. Marcus [52] established an interrelationship between the concepts of quasi continuity, neighbourliness and cliquishness of functions. Marcus established a number of results relating these concepts to measurability of functions. One typical result states that there exists a function which is quasi continuous at each point of an interval but is not Lebesgue integrable. Some of his results also have applications to differentiability of functions of several variables.

In 1961 N. Levine [49] introduced the notions of weakly continuous and w*-continuous functions and illustrated the various connections between these two weakened forms of continuity with sequential continuity and continuity.

In 1962 C. J. Neugebauer [64] established the connection between quasi continuity and Blumberg sets the existence of which was first demonstrated by Blumberg [7] in 1922. He showed that a function f on an interval I_0 is quasi-continuous if and only if f has a strongly Blumberg set.

In 1963 Eugen V. Dobrescu [23] studied about semi-continuity along with one of its extensions. In 1969 N. Biswas [5] generalized the notion of semi-continuity by simple continuity with the help of simple open sets.

In 1971, J. S. Lipinski and T. Salat [51] examined the structures of sets of points of quasi-continuity and cliquishness of functions. In 1973 V. Popa and Stan [75] introduced and studied the concepts of weakly quasi-continuous functions.

In his paper in 1973, A. Neubrunnova [61] studied the relations that exist between certain 'weak continuity' properties of functions, for example quasi-continuity, cliquishness, the Denjoy property, continuity almost everywhere and approximate continuity. He also showed that the set on which a function is cliquish but not continuous is necessarily an F_σ-set of first category.

T. Noiri (1981) investigated the interrelation among the weak continuity, semi-continuity and some weak forms of open functions. E. Grande [38], in 1982, showed that the set of points of discontinuity of the quasi-continuous planar real functions f: $\mathbb{R}^2 \rightarrow \mathbb{R}$ is exactly the F_σ sets of first category. The relationship between simple continuity and cliquishness was studied by J. Dobos [20] in 1987. In 1985 T. Noiri and B. Ahmed [67] defined semi-weak continuity. In 1986 J. Ewert defined some weaker forms of continuity, quasi-continuity and cliquishness for maps of a topological space into a bi-topological space and studied various properties of these classes.

In 1990 T. Noiri [65] gave several characterizations of almost quasi-continuous function and showed that the notion of almost quasi-continuity coincides with that of semi-weak continuity.

In 1993 T. Natkaniec [58] studied about the Cartesian products and diagonals of quasi-continuous and cliquish maps. In 1994-95, Z. Grande [39] studied about strong quasi-continuity of functions.

The sum of quasi-continuous functions need not be quasi-continuous. But several mathematicians like E. Stronska (1992), T. Natkaniec (1992), J. Borsik (1993), A. Maliszewski (1995), Z. Grande (1995-96) studied about decomposition of various generalized continuous functions into finite sum of quasi-continuous functions and some algebraic properties of quasi-continuous functions.

It is known that the pointwise limit of a sequence of functions does not carry many important properties of the members of the sequence onto the limit function. Consequently, various types of convergence have been introduced which are stronger than pointwise convergence.

In 1920 W. Sierpinski [83] initiated the investigation of pointwise convergence of transfinite sequence of functions. In 1974 A. Neubrunova showed that quasi-continuity and cliquishness of functions from separable metric spaces to metric spaces are preserved under the operation of taking pointwise limits of transfinite sequences of type Ω. In the same year T. Salat [82] proved that like quasi-continuous and cliquish functions somewhat continuous functions are closed under uniform convergence.

In 1976 B. Jonezyk [46] introduced a notion of 'upper quasi-uniform convergence' of a sequence of real-valued functions. He proved that if $\{f_n\}$ is a sequence of lower semi-continuous functions that is upper quasi-uniformly convergent to f, then f is lower semi-continuous. Also if X is countably compact and $\{f_n\}$ is a sequence of continuous functions convergent on X to f, then f is lower semi-continuous if and only if the convergence is upper quasi-uniform on X.

In 1981 J. Dobos and T. Nemce [21] investigated the nature of the quasi-uniform limit of sequences of quasi-continuous, somewhat continuous and cliquish functions. In 1982 Dobos [22] together with Salat further studied quasi-uniform convergence of sequence of cliquish functions.

In 1992 J. Borsik [9] proved that every real-valued real cliquish function is a quasi-uniform limit of a sequence of quasi-continuous function. In the same year A. Neubrunova and T. Salat [62] proved that if the range of the function is a metric space then almost quasi-continuity need not be preserved by pointwise convergence but is preserved by uniform convergence.

In 1994 J. Ewert [27] studied the uniform and pointwise limit of sequence of simply continuous functions and in 1994-95 S. Drahovsky, T. Salat and V. Toma [25] studied various types of local convergences of sequence of functions based on the concept of oscillation of sequence of functions.

Many types of generalised continuous functions have also been introduced for multifunctions and the multiplication of the results in this direction never stops.

T. Banzaru and N. Crivat [4] defined quasi-continuous multifunctions in 1975. Since then many properties of quasi-continuous and weakly quasi-continuous, irresolute and strongly irresolute multifunctions are studied by T. Noiri and V. Popa [68], [69].

In 1987 M. Matejdes [55] introduced the notions of B-continuity or Baire continuity of multifunctions. These functions connect the generalised continuous functions with the sets having Baire property.

In 1988 J. Ewert and T. Neubrunn [31] investigated some properties of quasi-continuous multifunctions. In 1990 Ewert [30] further studied quasi-continuity of multi-valued maps with respect to the qualitative topology.

In 1983 T. Banzaru and N. Crivat [18] surveyed convergence in multifunction spaces. In it they considered different forms of convergence for nets of multifunctions and investigated relationship between them. They also studied the continuity of the limits of such nets. They defined pointwise, uniform and quasi-uniform convergence and investigated the transfer of the properties of continuity and quasi-continuity from the net's terms to its limits. They also gave some conditions of characterisation for the continuity and quasi-continuity of the pointwise limit for multifunction nets.

In 1992 T. Noiri and V. Popa [69] studied about upper and lower quasi-continuous multifunctions. They obtained several characterisations and preservation theorems for such multifunctions. In the same year in another paper on multi-valued mapping Popa [68] showed that an almost continuous mapping with compact images in a regular space has a closed set of quasi-continuity points.

In 1993 Matejdes [54] introduced a definition of cliquishness for multifunction in his study about the selection of multifunctions. In 1993 M. Przemski [77] studied some relationship between various classes of generalised continuous and quasi-continuous multi-valued maps.

In 1993-94 Matejdes [57] studied the structure of the set of semi-continuity points, the Baire property of multifunctions, limits of multifunctions, separate and joint continuity for multifunctions.

Among the basic properties of continuous functions perhaps the intermediate value property or Darboux property is the most naturally arising one. Though Louis Arbogast first singled out the intermediate value property of a function in 1791, the property today is more widely known as Darboux property owing its name to the great mathematician Darboux, who intensively studied various properties of functions with intermediate value property. Darboux first showed that a function having intermediate value property might not be continuous.

There is a close connection between functions having intermediate value-type properties and connectivity function. Each connectivity function is a Darboux function. But in 1969, J. B. Brown [12] showed that there are Darboux functions, which are not connectivity functions. In 1982 R. G. Gibson and F. Roush [36] generalised the intermediate value property by weak Cantor intermediate value property (WCIVP) and Cantor intermediate value property (CIVP). They investigated the Cantor intermediate value property for functions f: [0,1] $\rightarrow$ [0,1]. It is obvious that any continuous function has CIVP. The authors constructed a connectivity function, which does not have CIVP, and also a CIVP function, which is not Darboux.

In 1991-92, Z. Grande [40] studied about the Darboux property of the sum of cliquish functions. In 1994-95 Grande alongwith A. Maliszewski and T. Natkaniec [41] showed that every almost continuous function possesses the Darboux property.

For the past forty years the matrix transformation theory has been flourishing area of research for many mathematicians. The theory of sequence spaces and infinite matrices in the last two decades of the nineteenth century were motivated by problems in Fourier series, Power series and systems of equations with infinitely many variables. Prior to this, particular infinite matrices have been of great interest to summability theorists for more than a century.

It was the celebrated German Mathematician O. Toeplitz [86], who in 1911, brought the methods of linear space theory to bear on problems connected with matrix transformations on the sequence space. The general theory of matrix transformations was motivated by special and classical results in summability, which were obtained independently by N. E. Norlund (1920) [70], M. Riesz (1924)

[79], Borel (1928) and others. Borel (1909) [8] proved a famous theorem that almost all sequences of 0's and 1's are (C, 1) summable to 1/2.

In 1948 R. C. Buck [14] gave the idea of Cesaro continuity or C-continuity of a real-valued function and proved that a real-valued function is linear if it is C-continuous atleast at one point.

In 1980, J. Antoni and T. Salat [2] generalised the notion of C-continuity by A-continuity of real functions. These can be treated as some generalisations of continuous function that bridge the two most important concepts of mathematical analysis, viz. continuity and infinite matrix.