CHAPTER I

General Introduction

1.1 Introduction:

To study blood flow in human circulatory system, it is necessary to know the basic principles of fluid dynamics as well as the special characteristics of the cardiovascular system. Accordingly, the basic equations of continuity and momentum for fluid flow in different coordinate systems are given. As a special case, the solution for the Hagen Poiseuille flow in a circular tube have been obtained. The inlet length flows, i.e. flows in the entrance regions of pipes before these flows become fully developed have been given. This is necessary since blood flow is mostly inlet length flow. Since the relation between stress and strain rate is not linear in blood, various types of non Newtonian fluid have been discussed.

The constitution of blood, viscosity and non-newtonian character of blood have been examined.
**Composition of blood :-**

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>RBC (erythrocyte)</td>
<td>45 %</td>
</tr>
<tr>
<td>other cellular components</td>
<td></td>
</tr>
<tr>
<td>[WBC (Leucocytes), Platelets etc.]</td>
<td>3 %</td>
</tr>
<tr>
<td>Plasma</td>
<td>52 %</td>
</tr>
</tbody>
</table>

**RBC :-**

They are deformable cells having complex aqueous solution of hemoglobin, most of the rheological properties are dominated by these particles.

**WBC :-**

1-2 % of blood volume.

A suspension of WBC shows higher viscosity than for the same volume fraction of RBC.
White cells increase blood viscosity in diseases where there is a large increase in leucocytes concentration.

**Plasma :-**

It is an aqueous solution of ions inorganic and organic molecules. Viscosity of plasma mainly depends on plasma proteins.

**Serum :-**

If the blood is allowed to clot and the solid material is removed, the remaining fluid is called serum. This has the same composition as plasma except that fibrinozen.

**Measurement of viscosity :-**

Cone and Plate viscometers are used for measuring the viscosity of blood.
Constitution of Blood -

Blood consists of a suspension of cells in an aqueous solution called plasma which is composed of about 90 percent water and 7% protein. There are about $5 \times 10^9$ cells in a millilitre (1 cc) of healthy human blood, of which about 95 percent are red cells or erythrocytes whose main function is to transport oxygen from the lungs to all the cells of the body and the removal of carbon dioxide formed by metabolic processes in the body to the lungs. About 45 percent of the blood volume in an average man is occupied by red cells. This fraction is known as the hematocrit, of the remaining, white cells or leucocytes constitute about one sixth or 1 percent of the total, and these play a role in the resistance of the body to infection; platelets form 5 percent of the total, and they perform a function related to blood clotting.

Shearing Stress -

Two types of forces act on a fluid element. One of them is body force and the other is surface force. The body force is proportional to the mass of the body on which it acts while the
surface force acts on the boundary of the body and so it is proportional to the surface area.

Suppose $F$ is a surface force acting on an elementary surface area $ds$ at the point $P$ of surface $S$. Let $F_1$ and $F_2$ be resolved parts of $F$ in the directions of tangent and normal at $P$. The normal force per unit area is called normal stress and is also called pressure. The tangential force per unit area is called shearing stress. Hence $F_1$ is a kind of shearing stress and $F_2$ is a normal stress.

**Viscosity**

Viscosity is that property of real fluid as a result of which they offer some resistance to shearing, i.e. sliding moment of one particle past or near another particle. Viscosity is also known as internal friction of fluid. All known fluids have this property in varying degrees. Viscosity of Glycerine and oil is large in comparison to viscosity of water or gases.

**1.5 Shear Tensor and Strain Rate Tensor**

The shearing stress between any thin sheets of a fluid is defined as

$$\tau = \frac{\text{Force}}{\text{Area}}$$

**now**

$$\tau = \frac{U}{y_0}$$

Where $y_0$ is the distance between two planes s.t. one is at rest while the other is moving with uniform velocity $U$ parallel to itself.
The fluid lies in between these two plates.

Hence \( \tau = \mu \frac{u}{y_0} \)

Where \( \mu \) is the constant of proportionality and is defined as viscosity.

Sometimes \( \frac{u}{y_0} \) is denoted by \( \frac{du}{dy} \) also it is denoted by \( \tau \) or by \( e \).

\( \frac{du}{dy} = \tau = e \) is called velocity gradient,

In the relation \( \tau = \mu \frac{du}{dy} \) when \( \mu \) is constant it is clear that shear stress varies with velocity gradient.

1.6 Non Viscous Fluid

A fluid is said to be non-viscous fluid if it is incapable of exerting shearing stress. The following have the same meaning: Perfect fluid, Ideal fluid, Inviscous fluid, Non viscous fluid, Frictionless Fluid. In this case \( \tau = 0 \), so that \( \mu = 0 \)
1.7 **Viscous Fluid** -

A fluid is said to be viscous if it exerts shearing stress. Thus in this case

\[ \tau \neq 0 \text{ so that } \mu \neq 0 \]

1.8 **Elastic Bodies** -

Elastic Body is one for which velocity gradient vanishes. In this case

\[ \frac{du}{dy} = 0 \text{ so that } \mu = \infty \]

1.9 **Newtonian Fluid** -

A fluid is said to be Newtonian if its viscosity does not change with the rate of deformation. In this case the equation

\[ \tau = \mu \left( \frac{du}{dy} \right) \]

is similar to the equation \( y = mx \), where \( y = \tau \), \( m = \mu \), \( x = \frac{du}{dy} \). Hence Newtonian fluid is represented by straight line.

1.10 **Non Newtonian Fluid**

A fluid is said to be Non-Newtonian if its viscosity varies with the rate of deformation, all are variables in the equation

\[ \tau = \mu \left( \frac{du}{dy} \right) \]
Therefore Non-Newtonian fluid is represented by curve. The main classes of Non-Newtonian fluids are Bingham plastics, Pseudo Plastics and Dialants.

or

Non Newtonian fluids are those where stress is Non linearly proportional to strain (velocity gradient)

1.11 Viscosity of Blood -

Blood is neither homogeneous nor Newtonian. Plasma in isolation may be considered Newtonian with a viscosity of about 1.2 times that of water. For whole blood, we can measure effective viscosity (apparent viscosity), and this is found to depend on shear rate, The constitutive equations proposed for whole blood are as follows:

i) \( \tau = \mu \dot{\gamma}^n \) (Power law equation)

ii) \( \tau = \mu \dot{\gamma} \) (Newtonian equation)

iii) \( \tau = \mu \dot{\gamma}^n + \tau_0 (\dot{\gamma} \geq \dot{\gamma}_0) \) (Hershel - Bulkley equation)

iv) \( \tau^2 = \mu_2 \dot{\gamma}^2 + \tau_0^2 \) (Casson equation)

v) \( \tau = \tau_y + \mu \dot{\gamma} \) where \( \mu \) is newtonian viscosity (Binghan equation)
1.12 Cardiovascular System -

The cardiovascular system consists of the following:

1) The heart (which acts as a pump, whose elastic, muscular walls contract periodically making possible the pulsatile flow of blood).

2) The distributory system (comprising arteries and arterioles for sending blood to the various organs of the body)

3) The diffusing system (made up of fine capillaries which are in contact with the cells of the body)

4) The collecting system of veins (which collects blood depleted of oxygen and full of products of metabolic processes of the system).

The organs which supplement the function of the cardiovascular system are (i) the lungs which provide a region of interphase transfer of \( O_2 \) to the blood and removal of \( CO_2 \) from it, and (ii) The kidney, liver, and spleen, which help in maintaining the chemical quality of blood under normal conditions and under conditions of extreme stress.
Deoxygenated blood enters the right atrium (RA) from where it goes to the right ventricle (RV) when the heart contracts, the tricusped valve between the RA and RV closes and blood is pushed out to the lungs through the pulmonary artery (PA) which branches to the right and left lungs where \(O_2\) is removed and blood is oxygenated. The blood returns from the lungs through the pulmonary vein (PV) to the left atrium (LA) and then it goes to the left ventricle (LV) and from there, due to contraction of the heart, it enters the aorta from which it travels to other arteries and the rest of the vascular system.

1.13 Fahraous Lindqvist Effect (F-L effect) -

In arteries, blood flows in two layers, a plasma layer near the walls consisting of only the plasma and almost no cells and a core layer consisting of red cells in plasma.

1.14 Blood as a Transport Medium for the Body -

Blood, the red fluid of the blood vessels with which we are all familiar, is the transport medium of the body. It is the medium by which all living tissues are related to their external environment; i.e. from the outside world. Blood is pumped to and fro between tissues and elementary canal to take up oxygen and nourishment respectively, and it helps in returning the products of oxidation and metabolism in the tissues to the outside world by the lungs, the skin and kidney. It is also the medium by which growth and repair substances are transported to the tissues and by which the various controlling glands of the body can distribute their chemical message.
Blood is somewhat viscid fluid. In man and in all other vertebrate animals with the exception of two (the amphioxus and leptocephalus), it is red in colour. It consists of a continuous yellowish aqueous phase, the plasma, in which formed elements are suspended. The formed elements consist of red blood cells (erythrocytes), white blood cells (leukocytes) and platelets (thrombocytes). Plasma is made up of water (92%) and contains traces of inorganic and organic salts. The inorganic and small organic molecules contribute little to plasma viscosity which is primarily dependent on the plasma proteins. If blood is allowed to clot and the solid material is removed, the remaining fluid is called serum. This has essentially the same composition as plasma except that fibrinogen and some of the clotting factors have been removed. About 7% by weight, of plasma is protein mainly albumin, globulin and fibrinogen; which have molecular weight ranging from 44,000 to 10,00000. About half of the protein mass is albumin. The significance of plasma protein is its multi natures mentioned below:

1. Plasma Proteins are responsible for osmotic pressure, the level of which is important for regulating water exchange between blood and tissues.

2. They possess buffer properties and maintain the acid-base equilibrium of blood.

3. They produce a definite viscosity of plasma which is important in maintaining blood pressure.

4. They promote stabilization of the blood by providing conditions that prevent sedimentation of erythrocytes.
5- They play an important role in coagulation.

6- Plasma proteins are important factors in immunity.

If blood to which an anticoagulant has been added, is poured into a rest tube and centrifuged, the corpuscles tend to settle at the bottom and the blood is divided into two layers, namely, a red lower layer consisting of the formed elements and a transparent colour-less or slightly yellowish upper layer consisting of plasma. The Leukocytes form of this white film between the echthrocytes and the plasma since their specific gravity is less than that of the erythrocytes.

The red cells surface carries a net negative charge but in stationary blood, cells interact with each other to form aggregates. The aggregates commonly consist of 6 to 10 cells stocked face to face and such a cluster of cells in called a rouleaux. Secondary aggregation of rouleaux also occurs building up a complex three dimensional structure. When blood is sheared, these rouleaux break up at sufficiently high shear rate the cell exists as an individual. Blood flows in the form of different laminar containing different types of cells. Considering these factors, the Rheological properties of blood might be expected to be rather complex.

In normal blood cellular components others than red cells compose only about 3% of the total cell volume and usually have a small effect on blood viscosity. White cells may significantly increase blood viscosity in diseases where there is large increase in Leukocytes concentration.

The red blood cells are about 97% of the total cell volume in the blood. Consequently, the removal of white blood cells and
platelets does not measurably modify the experimentally determined flow properties of these suspensions. The concentration of most red blood cells in suspension is generally reported as the suspension volume fraction occupied by the red blood cells, called hematocrit.

The hematocrit is normally about 42-45% of volume. The erythrocytes consist of a thin flexible unstrechable membrane with an interior filled with saturated hemoglobin solution (Viscosity 6.0 centipose) The membrane is highly deformable if the change in the surface area is small, but becomes very much stiffer if the deformation produces a large change in the area of the cell membrane.

The red blood cells have the shape of biconcave disc which can deform, however into a bullet shaped entity during passage through small capillaries. The cell has a large surface area relative to its volume.

Two properties of blood are essential for the preservation of life. The first is that blood remains fluid in the blood vessels throughout its life and second is that it rapidly becomes solid when shed. The maintenance of fluidity is necessary for the circulation of blood, while the solidification of the shed blood provides indispensable defence against excessive bleeding from wounds. The coagulation of the blood is due to formation of a jelly by the deposition of protein material called fibrin and it is the formation of this body that the fundamental change in blood clotting occurs.

To prevent this from occurring, various anticoagulents are added to the blood when it is drawn from the animal.

The effects of an anticoagulant on rheological properties of
blood are difficult to assertain because of the difficulty in making reliable rheological measurements in the short time available between the blood drawing and the blood clotting. The effect of anti coagulents on blood viscosity appears to be small.

1.15 Human Circulatory System -

In the human body circulatory system of blood consists of four functions Respiratory, Nutritive excretory, Protective and Regulatory. The heart that provides the energy for the circulation has four compartments (ie left and right atrium and left and right ventricles) interconnected to each other by one way valves. Blood coming from body tissues enters the right atrium. Through the venul caval contraction of right atrium forces blood pass the tricuspid valves into the right ventricle. From this point there begin two subdivisions the pulmonary and the systematic circulation. As the term shows, the former services the lung and latter the various systems of the body. The blood vessel have internal diameter in the range of 2.5 cm in aorta to about 4 microns in capillary.

Blood is pumped from the heart into the aorta from where it goes to the circulatory system consisting of about 40 large arteries, 1600 main artery branches, 1800 terminal branches, 4,00,00,000 arterioles, 1,20,00,00,000 capillaries, 8,00,00,000 valves, 1,800 terminal veins, 1600 main venous branches, 40 large venous branches, 40 large veins and then returns to the heart through the venule caval.

It is estimated that the heart muscles it self consume about 18% of the energy required to sustain life. only some energy
gose in to the mechanical work of pumping blood. The heart beats about 70 times in one minute in an average person at rest. Blood flow in smaller blood vessels approaches steady flow condition otherwise pulsatile. The pressure in the aorta rises rapidly to its maximum (systolic) value of about 120 mm/Hg. The requirement of the circulation is the supply of oxygen required for metabolic process. In an average man at rest, the $O_2$ required is 200 ml/minute for under physical stresses, the need may rise to 5 litre/min.

In an average man it is necessary for the heart to circulate 5 to 6 litres of blood per minute or about the 5 times the metabolic requirement of oxygen. The metabolic requirements are the energy requirements for biological functions.

In fluid mechanics the circulatory system is classified in to two parts viz. macrocirculation and microcirculation.

1.16 Macrocirculation -

The flow of blood in vessels of diameters greater than 500 μm (such flow occurs in aorta) is called macrocirculation. The flow is characterized by high Reynolds number, defined as the ratio of inertial to viscous forces. Turbulent flow can occur in blood vessels for the Reynolds number greater than 2300. The governing equations for analysing of such flow conditions should include inertial effects, effect of curvature of blood vessels, pulsatile flow and the distensibility of the vessel wall. Blood can be taken as homogeneous and continuous fluid when vessel diameter is large in comparison to the dimensions of red blood cells. Pulsatile flow effects and Pressure wave propagations have been studied in macrocirculation by a number of research workers, Skalek and Taylor. The flow disturbances at bifurcations, bends etc. and
their effects on Pathological states are studied by Patel.

1.17 Microcirculation -

When the diameter of blood vessel is less than 500 \( \mu \text{m} \) [i.e. arterioles, capillaries, venules] the circulation is called microcirculatory. This is responsible for 80\% pressure drop in circulatory system. In capillary bed the transfer of nutrients to and removal of wastes from the living cells of the body is a part of microcirculation. The flow is characterized by very low Reynolds numbers (\( \text{Re} < 1 \)). But we can not neglect the size of the red blood cell, compared to blood vessels size.

1.18 Boundary Layer Flow, The Cone Flow and Fully Developed Flow of Fluid -

When a fluid enters a tube from a large reservoir where the velocity is uniform and parallel to the axis of the tube, the velocity profile is a flat surface at the entry.
Immediately after entry, the velocity near the surface is affected by the friction of the surface, but the velocity profile near the axis still remains flat. As the fluid moves further in the tube, the flat portions decrease, and at the section corresponding to A, the paraboloidal velocity profile for the fully developed flow is reached. The flow in the region OA is called the entry region (or inlet) flow and the flow beyond A (in region III) is called the fully developed flow. The length OA is called the entry length. The flow in the entry length portion itself consists of two parts. The flow in region I near the surface is called the boundary layer flow, the flow in region II is called the core flow or the plug flow.

In plug region, \[ e = \frac{dv}{dr} = 0 \]
\& \( \tau \leq \tau_y \) where \( \tau_y \) is yield stress
\& \( v = \text{constant} \)

i.e. plug flow exists wherever the shear stress does not exceed the yield stress.

1.19 Rheology of Blood -

Blood behaves like a time-dependent non-Newtonian fluid and the basic rheological property of blood is its viscosity. The viscosity of whole blood is about 4.0 centipose and of plasma is about 1.2 centipose at 37°C. The specific gravity of whole blood is 1.05 to 1.06 and that of plasma is about 1.03. Thus the red blood cells tend to sediment slowly in plasma. The increased viscosity relative to water is produced mainly by the presence of plasma proteins. Both, the molecular shape and concentration of the protein are important. Fibrinogens which has an elongated molecule
has a marked influence; although forming less than 5% of the total plasma proteins. It is responsible for about 20% of the plasma viscosity elevations.

1.20 The Viscosity of Blood

It is an important factor in determining the local pressure variation through the cardio-vascular system which in turn influences the local flow rates through each section of the vascular network. The clinical importance of blood viscosity as a parameter lies in its sensitivity to small variations in composition. One can often diagnose pathological states by determining a change in blood viscosity.

There are several rheological parameters (e.g. plasma, blood cells, hematocrit etc.) which can effect the blood viscosity. The viscosity of plasma increases with its protein concentration. But some proteins have different influences on plasma viscosity depending on their shape and size. The influence of Fibrinogen on plasma viscosity can be seen in the difference between plasma and serum viscosities. Serum usually has a viscosity which is 20% less than that of plasma. Many relationships have been suggested to express blood viscosity as a function of cell concentration, plasma viscosity and shear rate. When temperature is increased, the viscosity of blood and plasma is fallen. Measurements should be made at constant temperature (37°C).

Platelets and white cells, in general have little influences on blood viscosity, because they are present in very minute quantity as compared to red cells.
The deformation of red blood cells allows the blood to remain fluid up to hematocrit of 98%; rigid cell without deformation will cease to flow at a rigid cell without deformation will cease to flow at a cell concentration of about 60%. The hematocrit also influence the deformation of the erythrocytes i.e raising of the cell concentration produces an increase in cell deformation and therefore fall in viscosity of blood occurs.

In stationary blood, rouleaux is formed which intracts to produce larger aggregates. At low flow rates the presence of these red cell structures strongly influence the viscosity of the blood. The size of the rouleaux and aggregates progressively decreases as the shear rate increases. In normal blood the disaggregation is probably complete at shear rate 50 sec.\(^{-1}\) approximately. The viscosity of blood increases when shear rate falls, but it is unceration as to what happens when the shear rate actually falls to zero. The build up of a 3-D of increasing aggregates suggests that blood may show a yield stress.

1.21 Blood as a Non-Newtonian Fluid -

Blood, from fluid mechanics point of view can be thought as a suspension of erythrocytes in a Newtonian liquid called plasma. In small vessels the dimensions of red blood cells are not negligible as compared to the size of tube. Therefore blood can not be considered as a homogeneous fluid in microcirculatory systems (diameter less than 500 m,) It is a specific property of blood that exhibits a yield stress. Hence if applied shear stress is below a critical value (\(\tau < \tau_y\)) the response will be elastic and on removal of stress the shape of blood film is unaltered. otherwise (\(\tau > \tau_y\))
flow takes place and blood behaves as a non Newtonian fluid.

It is found that suspended blood cells are responsible for the non Newtonian nature of blood rheology. At low shear rates the blood exhibits yield stress and behaves like casson fluid 
\[ \tau^2 = \gamma \tau^2 + \mu^2 \varepsilon^2 \] constitutive equations suggested by Herchel and Bulkley \( \tau = \gamma + \mu \varepsilon \) has also been used to describe the shear rate dependence of blood.

Blood rheology can be helpful in diagnosis of some blood problems.

1.22 Survey of literature -

The German form of the word (rheologic) and the description of small viscometer as a microrheometer are found in literature since early days. But in its modern sense, the term rheology was coined by prof. E.C. Bingilam and formally adopted and defined at the foundation meeting of the American society of rheology in December 1929 in Washington, as "The science of deformation and flow of matter"

However, the subject of hydrodynamics and aerodynamics are not included in rheology.

The first mathematical paper of blood flow was given by Leonhard Euler in 1862. He developed one dimensional equations of invisicid flow of an incompressible fluid in an elastic tube. Blood flow in capillaries is of great interest to Physiologists involved in microvascular research. The pressure drop relation in microcirculation was obtained by Posiseuille in 1940. The most remarkable result on blood flow was of Fahraeus and Lindquist, showing that the apparent viscosity (effective viscosity)
of blood decreases as tube diameter decreases from 500 μm. This effect has been confirmed by other investigators (Coklet and Dintenfass). Taylor studied dispersion process in Newtonian fluid flows through a circular tube.

1.23 Geometrical aspect of Vessels -

A striking characteristic of the circulatory system is its geometrical complexity. Blood must flow through many bends, bifurcations, stenosis and tapering during its journey. Most of the progress in understanding the mechanics of the circulations is based on the investigations of the flow in straight uniform tubes.

It is only within the last two decades that the effects of geometric transitions on blood flow have begun to be explained.

1.24 Stenosis -

Stenosis causes the narrowing of the blood vessels due to the development of abnormal tissues and gives way to serious circulatory disorder by reducing the blood supply. Hemodynamic characteristics may be changed due to this undesirable growth which could be injurious to normal health. In recent years many workers have investigated the flow characteristics of blood through artery in presence of mild stenosis.

Young gave a theoretical analysis of the effect of time dependent stenosis on flow characteristics of blood. Recently Sinha and Singh studied the effect of sténosis on blood flow through the couple stress.
1.25 Capillary blood flow:

The first microscoping observation and depiction of the capillaries is attributed to Malpighi who published Sketches of the capillaries of the lung.

Poiseuille studied the flow of water, alcohol and mercury in fine glass capillaries in the form:

\[ Q = K \frac{P D^4}{L} \]  

Where \( Q \) is the discharge due to pressure drop \( P \) over the length \( L \), through a capillary of diameter \( D \). The coefficient \( K \) was found to be dependent on the liquid flowing and the temperature \( T \):

\[ K = k(1 + AT + AT^2) \]  

Where \( k, A \) and \( A' \) are constants depending on the fluid flowing. Latter on the equation (1) was modified in the form:

\[ Q = \frac{128PD^4}{\pi L} \]  

The theoretical model of capillary blood flow was the axial-train model in which a solid cylindrical core represents a line of axisymmetric blood cells flowing in a cylindrical tube. In terms of the diameter ratio \( \lambda \) (core diameter divided by the tube diameter), the ratio of the core velocity \( U \) to the ratio of the mean velocity \( V \) of entire suspension is

\[ \frac{U}{V} = \frac{2}{1 + \lambda^2} \]  

Where the relative apparent Viscosity \( \eta \) of the suspension is

\[ \eta = \frac{1}{1 - \lambda^4} \]  

The main function of capillary is to maintain oxygenation in tissue.

1.26 Tapering:

Many blood vessels have the characteristic of taper cylinders rather than straight cylinders.

Block has given the idea that blood vessels which carry blood towards tissues should be treated as long slowly tapered cones rather than cylinders.
Table 3
Protein Concentration in human Plasma

<table>
<thead>
<tr>
<th>Protein</th>
<th>Conc. (g/L)</th>
<th>Molecular Weight (ml)</th>
<th>Mol.Sizes (Aum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albumin</td>
<td>45</td>
<td>69,000</td>
<td>15 x 4</td>
</tr>
<tr>
<td>Globulin</td>
<td>27</td>
<td>1,50,000</td>
<td>24 x 4.5</td>
</tr>
<tr>
<td>Fibrinogen</td>
<td>3</td>
<td>3,40,000</td>
<td>70 x 4</td>
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</table>
## Table 1
### Basic information

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<th>Vessel</th>
<th>Diameter (cm)</th>
<th>Length (cm)</th>
<th>Wall Thickness</th>
<th>Average Vescocity (cm/see)</th>
<th>Average Reynolds Number</th>
<th>Maximum Velocity (cm/see)</th>
<th>Maximum Reynolds Number</th>
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</thead>
<tbody>
<tr>
<td>Arteria</td>
<td>2.5</td>
<td>50</td>
<td>0.2</td>
<td>48</td>
<td>3400</td>
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<td>Arteries</td>
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<td>Arterioles</td>
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<td>Capillaries</td>
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<tr>
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<td>Veins</td>
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<td>Venacava</td>
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<td>38</td>
<td>3300</td>
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<tr>
<td>Arteria</td>
<td>Arteries</td>
<td>Arterioles</td>
<td>Capillaries</td>
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<tr>
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<td>5</td>
<td>10</td>
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</tr>
</tbody>
</table>

Table 2
Mean Pressure (mm Hg) in Blood Vessels
Bulk rheological behaviour of blood

Bulk rheological behaviour of blood is important to understand the mechanism of circulation in good health and in disease. This gives the direct clinical relevance of blood rheology to diagnosis and to the cause of disease. Viscosity is the main rheological parameter in blood flow.

Several research scholars have proposed following formulae -

Einstein (1906) proposed that

\[ \mu_r = 1 + c\phi, \ c \text{ being a constant and having a value of } 2.5 \text{ for rigid spheres where} \]

\[ \mu_r = \frac{\text{Viscosity of suspension } \mu_s}{\text{Viscosity of suspending medium } \mu_m} \]

\[ \phi = \text{Volume concentration of particles. He assumed that particles are small, spherical, rigid and non interacting and suspension is dilute. None of these assumptions are valid for blood.} \]
He modified Einstein equation for deformable spherical particles and suggested that -

\[ \mu_r = 1 + 2.5 \phi + 2.35 \phi^2 + \cdots \]

Brooks (1970),

He suggested that

\[ \mu_r = a_0 + a_1 H + a_2 H^2 + \cdots \]

where \( H \) denotes the Hematocrit fraction, \( a_0, a_1, a_2, \ldots \) are constants related to physical mechanism occurring during flow:

Dintenfass (1965):-

gave the relationship as

\[ \mu_r = (1 - KH)^{-2.5} \]

where \( K \) is a constant.

Quemada (1983):-

\[ \mu_r = e^{2.5H} \]
**Diffusion**: 

In biological sciences, the study of diffusivity of nutrients, metabolic products, drugs and other solutes is of utmost importance. Many life giving materials mixed in the blood reach to different parts of the body by the process of diffusion. Accordingly, we obtain diffusion equation and then give its solution in one dimension and in two dimensions in cartesian form and in cylindrical polar coordinates.

**The Diffusion Equation** :-

**Fick's Laws of Diffusion** :-

Let \( C(x,y,z,t) \) be the concentration of a solute or the amount of the solute per unit volume at the point \((x,y,z)\) at time \(t\). Due to the concentration gradient \( \text{grad} \, c \), there is a flow of solute given by the current density vector \( \mathbf{j} \), which, according to Fick’s first law of diffusion, is given by -

\[
\mathbf{j} = -D \, \text{grad} \, c
\]

\[
= -D \, \nabla c \quad \ldots \quad \ldots \quad (1)
\]
Here the quantities $J_x, J_y, J_z$ give respectively the amounts of the solute crossing the planes perpendicular to $x, y, z$ axes per unit area per unit time.

The negative signs in (1) & (2) indicate that the flow takes place in the direction of decreasing concentration. D. carry on with $x, y, z$, but we shall take it to be constant.

Let us consider a volume $V$ with surface $F$. The rate of change of the amount of the solute is given by

\[ \frac{\partial}{\partial t} \int_V c \, dV. \]
The amount of the solute which comes out of surface \( s \) per unit time is given by

\[ \int_s J \cdot nds \quad \ldots \quad \ldots \quad (4) \]

by Gauss - Divergence theorem

\[
\frac{\partial}{\partial t} \int_V C \cdot dV + \int_s \mathbf{J} \cdot nds = - \int_s J \cdot nds
\]

\[
= \int_s (D \text{ grad}) \cdot nds
\]

\[
= \int \text{Div} (D \text{ grad}) \, dy \, dz \quad \ldots \quad (5)
\]

\[
\therefore \int \left[ \frac{\partial C}{\partial t} - \text{Div} (D \text{ grad}) \right] \, dy \, dz = 0 \quad (6)
\]

Since (6) holds for all volumes, we get Fick's Second law of diffusion

\[
\frac{\partial C}{\partial t} = \text{div} \, (D \text{ grad}C) \quad \ldots \quad (7)
\]

Since \( D \) is assumed to be constant, we get the diffusion equ.

\[
\frac{\partial C}{\partial t} = D \cdot \text{div} \, (\text{grad}C) = D \nabla^2 C
\]

\[
\frac{\partial C}{\partial t} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \quad \ldots \quad (8)
\]
The solution of one dimensional equation:
\[ \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \]

is
\[ c = \frac{m}{(4\pi D t)^{3/2}} e^{-x^2/4Dt} \] ... (9)

For the axially symmetric case, the diffusion equation in cylindrical polar coordinates is:
\[ \frac{\partial c}{\partial t} = D \left[ \frac{\partial^2 c}{\partial r^2} + \frac{1}{r} \frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial z^2} \right] \] ... (10)

If the fluid is incompressible, \( \text{div} \mathbf{v} = 0 \) then diffusion equation is:
\[ \frac{\partial c}{\partial t} + (u \cdot \nabla) c = D \nabla^2 c \] ... (11)