Chapter V

Micro polar fluid model of steady blood flow in narrow tube with slip at the wall.

3.1 Abstract -

Poiseville flow of micropolar fluid has been considered as a model of blood flow in narrow tube. At the boundary, a slip in the axial velocity and a particle rotation depending upon wall effect parameters $s$ ($0 < S < 1$) and fluid vorticity have been introduced.

From the analysis we have shown that the slip helps inverse Fahracus Lindqvist effect (IFLE) to exist. Introduction of $s$ reduces the apparent viscosity of blood. Comparing the result with the experimental value, we have estimated the value of $\tilde{s}$ (a parameter depending on $s$ and suspension concentration $0 < \tilde{s} < 1$).

5.2 Introduction :

The components of blood are plasma, platelets. erythrocytes and leukocytes. Since RBC occupy 40-45 % fraction
of total blood volume. Therefore most of the rheological properties of blood depend on RBC. It is seen that the flow processes do not obey the same rule everywhere as they vary from small to large diameter tube. In narrow tubes (20-500 µm) we may propose the micropolar fluid theory as a model to explain the flow parameters characterizing the blood behaviour.

Several authors like Klime (1968), Cowin (1972), Ariman (1974) Brunn (1975), Erdogan (1980) and Chaturani (1984) have proposed micropolar fluid theory as a suitable model to explain the blood rheology in these vessels. They have used different boundary conditions at the wall.

In the present analysis we have studied the Poiseuille flow of micropolar fluid as a model of blood flow in 40 µm diameter tube and for 40% RBC concentration.

At the boundary we have introduced a slip in the axial velocity and a partial rotation depending upon the wall effect parameter s and fluid vorticity.

5.3 Mathematical Analysis:

Equations of motion for the Poiseuille flow of micropolar
fluid through a rigid circular tube of radius $R$ are -

\[(K_1 + K_2) \left( \frac{d^2 v}{d r^2} + \frac{1}{r} \frac{d v}{d r} \right) + \frac{2 L^2}{r^4} \frac{d (v \omega)}{d r} - \frac{d p}{d \gamma} = 0 \quad \ldots \quad (1)\]

\[K_3 \left( \frac{d^2 \omega}{d r^2} + \frac{1}{r} \frac{d \omega}{d r} - \frac{\omega}{r^2} \right) - 2 L^2 \frac{d v}{d r} - \gamma k_2 \omega = 0 \quad \ldots \quad (2)\]

Where $v$, $w$ are the axial and angular velocities respectively of the particle; $r$ and $z$ are the radial and axial coordinates; $p$ - the pressure; $k_1$ the shear viscosity of the suspension; $k_2$ the rotational viscosity; $k_3$ the viscosity of the gradient of particle's angular velocity.

The boundary conditions are

$v$ and $w$ remain finite at $r=0$

and \[v = -L \frac{d v}{d r}, \quad \omega = -\frac{3}{2} \frac{d v}{d r}, \quad \gamma = R \quad \ldots \quad (3)\]

Where $L$ is the slip parameter, $s$ the wall effect parameter.
The solutions of equations (1) and (2) with boundary conditions (3) are

\[ v = \frac{KR}{4K_1} \left[ 1 - \frac{R}{r} - 2 \varepsilon \alpha \left( \frac{I_0(Rt) - I_0(r\varepsilon)}{I_1(Rt)} \right) + 2 \psi (1 - e\delta) \right] \quad \ldots \quad (4) \]

\[ \omega = \frac{KR}{4K_1} \left[ \frac{R}{r} - \delta \frac{I_1(r\varepsilon)}{I_1(Rt)} \right] \quad \ldots \quad (5) \]

where, \( c = \frac{K_2}{K_1 + K_2} \), \( \varepsilon^2 = \frac{2K_1 c}{K_3} \), \( \psi = \frac{L}{R} \)

\[ K = -\frac{d\psi}{\delta^3} \quad , \quad \delta = \frac{1 - \varepsilon}{1 - e\delta} \]

\( I_n \) is the modified Bessel function of order \( n \).

From equation (4) the volumetric flow rate \( Q \) and the apparent viscosity \( K_a \) are obtained as -

\[ Q = \frac{\pi R^4 K}{8 K_1} \left[ 1 - \frac{4 \varepsilon \delta (I_0(Rt) - \frac{2}{(Rt)^2}) + 4 \psi (1 - e\delta)}{I_1(Rt)} \right] \quad \ldots \quad (6) \]

\[ K_a = \frac{K_1}{[1 - 4 \varepsilon \delta (I_0(Rt) - \frac{2}{(Rt)^2}) + 4 \psi (1 - e\delta)]} \quad \ldots \quad (7) \]
The velocity at the axis of the tube is

$$V_a = \frac{K_R^2}{4K_i} \left[ 1 - 2e^\bar{s} \left( \frac{I_0(R^2) - 1}{(R^2)I_1(R^2)} \right) + 2\psi (1-e^\bar{s}) \right]$$ \quad (8)

5.4 Discussion:

From equations (4) and (5) we observe that slip at the boundary affects the axial velocity whereas the particles rotation is unaffected.

We also observe that the introduction of slip reduces the apparent viscosity of the suspension and increases its axial velocity. But it has no effect on particles rotation.

Effect of $\bar{s}$ ($0 \leq s \leq 1$, a parameter depending on $s$ and concentration) on different flow parameters are seen and by comparing the results with the experimental value we have tried to estimate the value of $\bar{s}$.

For different values of $\bar{s}$ and $\psi$, the values of $K_a$, $V_a$, $-\frac{db}{dz}$ are shown in tables I and II. For $\psi = 0.02$, when $s$ varies from 0 to 1, $K_a$ varies from 2.02 to 2.88.
Variation of pressure gradient and axial velocity with respect to \( \psi \) and \( \overline{\mathcal{S}} \) are reported in table II.

Variations of velocity and rotational profiles for different values of \( \mathcal{S} \) are shown in table III & IV.

From these tables we observe that as \( \mathcal{S} \) increases, particles axial and rotational velocities decrease. \( \mathcal{J}_f \overline{\mathcal{S}} \) is assumed to be a measure of concentration, then it is seen that for dilute suspension, particle's rotation increases with tube radius. But for sufficiently large values of \( \mathcal{S} \) ( \( \overline{\mathcal{S}} > 0.85 \) ) the rotation at first increases, attains its maximum value and then decreases near the wall. For maximum concentration ( \( \overline{\mathcal{S}} = 1 \) ) the particle's rotation is maximum for \( \zeta = 0.75 \) which has been experimentally observed.
### Table - 1

**Variation of \( K_a \) and \( V_a \) vs \( \psi \) & \( \bar{s} \) for 40 % concentration**

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>( \bar{s} )</th>
<th>( K_a )</th>
<th>( V_a )</th>
<th>( V_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.00</td>
<td>2.02</td>
<td>3.11</td>
<td>0.12</td>
</tr>
<tr>
<td>0.02</td>
<td>0.26</td>
<td>2.19</td>
<td>2.85</td>
<td>0.11</td>
</tr>
<tr>
<td>0.02</td>
<td>0.35</td>
<td>2.25</td>
<td>2.79</td>
<td>0.11</td>
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<tr>
<td>0.02</td>
<td>0.49</td>
<td>2.36</td>
<td>2.66</td>
<td>0.10</td>
</tr>
<tr>
<td>0.02</td>
<td>0.66</td>
<td>2.52</td>
<td>2.50</td>
<td>0.10</td>
</tr>
<tr>
<td>0.02</td>
<td>1.00</td>
<td>2.88</td>
<td>2.20</td>
<td>0.08</td>
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</table>
Table - II

Variation of $-\frac{\partial b}{\partial Z}$ Va and Vs with respect to $\psi$ and $\bar{s}$.

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\bar{s}$</th>
<th>$-\frac{\partial b}{\partial Z}$</th>
<th>$V_\alpha$</th>
<th>$V_\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.591</td>
<td>65.31</td>
<td>2.47</td>
<td>0.048</td>
</tr>
<tr>
<td>0.02</td>
<td>0.591</td>
<td>64.27</td>
<td>2.53</td>
<td>0.097</td>
</tr>
<tr>
<td>0.03</td>
<td>0.591</td>
<td>64.27</td>
<td>2.57</td>
<td>0.097</td>
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<tr>
<td>0.04</td>
<td>0.591</td>
<td>61.37</td>
<td>2.66</td>
<td>0.101</td>
</tr>
<tr>
<td>0.05</td>
<td>0.591</td>
<td>60.32</td>
<td>2.67</td>
<td>0.244</td>
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Table - III

Variation of velocity Profile with respect to $\bar{s}$ at

constant $\psi = 0.02$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\bar{s}=0.00$</th>
<th>$\bar{s}=0.35$</th>
<th>$\bar{s}=0.50$</th>
<th>$\bar{s}=0.66$</th>
<th>$\bar{s}=1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>3.112</td>
<td>2.795</td>
<td>2.657</td>
<td>2.504</td>
<td>2.199</td>
</tr>
<tr>
<td>0.2</td>
<td>2.996</td>
<td>2.687</td>
<td>2.554</td>
<td>2.407</td>
<td>2.112</td>
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<tr>
<td>0.4</td>
<td>2.637</td>
<td>2.363</td>
<td>2.245</td>
<td>2.115</td>
<td>1.854</td>
</tr>
<tr>
<td>0.6</td>
<td>2.037</td>
<td>1.823</td>
<td>1.731</td>
<td>1.629</td>
<td>1.425</td>
</tr>
<tr>
<td>0.8</td>
<td>1.199</td>
<td>1.070</td>
<td>1.015</td>
<td>0.955</td>
<td>0.833</td>
</tr>
<tr>
<td>1.0</td>
<td>0.119</td>
<td>0.106</td>
<td>0.101</td>
<td>0.094</td>
<td>0.082</td>
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</tbody>
</table>
### Table - IV

**Variation of particle's rotational velocity with \( s \).**

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \bar{s}=0.00 )</th>
<th>( \bar{s}=0.35 )</th>
<th>( \bar{s}=0.50 )</th>
<th>( \bar{s}=0.66 )</th>
<th>( \bar{s}=0.75 )</th>
<th>( \bar{s}=1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.20</td>
<td>0.20</td>
<td>0.1372</td>
<td>0.1102</td>
<td>0.0804</td>
<td>0.0652</td>
<td>0.0203</td>
</tr>
<tr>
<td>0.40</td>
<td>0.40</td>
<td>0.2725</td>
<td>0.2178</td>
<td>0.1573</td>
<td>0.1266</td>
<td>0.0351</td>
</tr>
<tr>
<td>0.60</td>
<td>0.60</td>
<td>0.4043</td>
<td>0.3204</td>
<td>0.2276</td>
<td>0.1806</td>
<td>0.0408</td>
</tr>
<tr>
<td>0.80</td>
<td>0.80</td>
<td>0.5309</td>
<td>0.4159</td>
<td>0.2878</td>
<td>0.2232</td>
<td>0.0309</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>0.6500</td>
<td>0.5000</td>
<td>0.3400</td>
<td>0.2500</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table - V

Variation of apparent viscosity and axial velocity for different values of $s$ at constant $\Psi = 0.02$

<table>
<thead>
<tr>
<th>$\bar{s}$</th>
<th>$K_{\alpha}$</th>
<th>$V_{\alpha}$</th>
<th>$V_{\delta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>1.11</td>
<td>5.66</td>
<td>0.217</td>
</tr>
<tr>
<td>0.350</td>
<td>1.31</td>
<td>5.35</td>
<td>0.183</td>
</tr>
<tr>
<td>0.500</td>
<td>1.42</td>
<td>4.44</td>
<td>0.152</td>
</tr>
<tr>
<td>0.666</td>
<td>1.56</td>
<td>4.03</td>
<td>0.152</td>
</tr>
<tr>
<td>1.000</td>
<td>1.97</td>
<td>3.22</td>
<td>0.119</td>
</tr>
</tbody>
</table>
Fig (1) Variation of velocity profiles with $\bar{s}$
Fig(2) Variation of rotational velocity with $\bar{b}$
Fig (3) Variation of Maximum Axial Velocity
Fig(4). Variation of apparent viscosity
Fig. (5). Variation of $\psi$ with $\overline{S}$. 