CHAPTER III

MULTIPLE FERRROMAGNETIC RESONANCE MODES IN A SEMI-INFINITE CYLINDRICAL BOUNDARY OF FERRITE MATERIAL

3.1: Introduction:

The magnetostatic solutions of multiple ferrromagnetic resonance in a ferrite semi-infinite cylinder are derived considering the inhomogeneous field distribution inside the material. The resonance modes for values of \( n = 0, 1, 2, 3, 4, 5 \ldots \) and for different values of \( \mathcal{H} \) and \( -\mathcal{L} \) are calculated. The dispersion curves are drawn for the medium. The values of \( \alpha/\mu \) for different values of \( \mathcal{H} \) and \( -\mathcal{L} \) are tabulated.

Under applied non-uniform radio frequency fields, non-uniform distributions of magnetisation can be excited in the ferrite material. These distributions can be calculated to a very good approximation by using the magnetostatic condition, \( \nabla \times \mathbf{H} = 0 \), and hence they are called magnetostatic modes. Ferrite materials of different boundaries are in great use in technology and in various scientific experiments. It is, therefore, very useful to have a tabulation of the characteristic modes that are most likely to be encountered in ferrite materials of different boundaries. For example, Fletcher and Bell (1959) have solved the magnetostatic problem of a sphere in a non-uniform radiofrequency field and have given the possible expressions for the resonant fields, potential functions and radio-frequency magnetic moment. Walker (1957) has given the magnetostatic solutions of ferrromagnetic resonance for the case of spheriodal and
elliptic boundaries. Paldas (1971) has solved the magnetostatic problem of ferrite material having a large conical boundary. The necessary assumptions made in solving the problems are:

(i) propagation effects are neglected i.e. $\lambda \gg a$ , where $\lambda$ = wavelength of the incident radiation and $a$ = radius of the sphere

(ii) The wavelength is long enough so that exchange effects are small, i.e. $\lambda \gg H_{ex} \frac{\mu}{M}$ , where $H_{ex}$ is the equivalent exchange field and $\mu$ is the magnetic lattice constant. (iii) Non-linear effects are neglected i.e. radio frequency fields are small compared with saturation i.e. $\lambda n f \ll 4\pi M$. The fact that some experimental observations agree well with these calculations, justifies, to some extent, these assumptions. For different geometry, solutions for ferrite media have also been obtained by Pilshchikov, Dunayev and Sedletskaya (1962), Monosov (1961) and others. The phenomenon of ferromagnetic resonance has been used widely as a tool for the investigation of the magnetic properties of ferrite materials. The variation with crystalline orientation of the field required for resonance yields the crystalline magnetic anisotropy constants. The field for resonance corrected for anisotropy effects gives the magnetic 'g' factor. Perhaps most important, the half-width of the resonance in $\omega$ or in $H$ may be interpreted according to the theory of Bloembergen (1950) or of Landau and Lifshitz (1935) to give the characteristic relaxation time of the magnetic system. It is an experimental fact that the resonant absorption is often far from symmetric in shape and may exhibit clearly defined secondary maxima, especially for larger sample sizes. The relaxation times inferred from the line widths
FIG - 8

FERRITE MATERIAL OF SEMI-INFINITE CYLINDRICAL BOUNDARY
are generally very short. Higher modes of ferrimagnetic resonance, in which the precessional phase varies from point to point within the sample, are shown to exist and to have fields for resonance in general different from the Kittel value (c.f. Kittel (1948)). The 'distorted' line shapes and "spurious" secondary maxima may be explained as caused by the accidental excitation of the higher modes of resonance. Coupling of the ferrimagnetic resonance mode of uniform prescessional phase to nearly degenerate modes of much higher order may be of great significance in the relaxation process. The experimental apparatus used for the resonant absorption measurements is similar in many respects to the recording system described by Tinkham, Solt, Davis and Strandbery (1956). The system consists of two basic units, the magnetic field sweep system and the microwave and detection circuits. The experimental arrangements to measure the resonant absorption in ferrite materials of different boundaries is discussed in Chapter I and the detection apparatus is explained by figures (c.f. Fig. 2.). The resonance observed in these experiments can be excited separately or in subgroups by choosing the appropriate orientation of $H_2$ and proper symmetry of the exciting microwave field. The resonance condition is also expected to depend on the shape of the specimen, and, in case of a single crystal, also on the orientation of the crystal.

3.2: Statement of the problem

We have considered the case of a ferrite specimen of semi-infinite cylindrical boundary. A semi-infinite cylinder can be considered as a non-symmetric body. In a semi-infinite cylinder,
due to its lack of symmetry, however, it can be expected that the internal field will be markedly inhomogeneous, namely when the internal fields smaller than $4\pi \mathcal{M}$ are considered. Consequently, it will be no more possible to describe the relation between the magnetization and the fields by equation (3.4) which is appropriate for a symmetrical body. The main interest of the paper lies in the fact that we have considered the inhomogeneity of the field distribution along the Z-axis along which the external field is applied. By considering the Z-component of magnetization, a function of x and y and not equal to constant, we have introduced the inhomogeneity inside the material. Some approximations are made.

3.3: Basic equations and the solution procedure:

The $\mathcal{A}$-axis is taken along the axis of the cylinder and form a right handed system. When the field distribution is homogeneous the differential equation for magnetostatic potential is given by

$$
(1+k) \left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) + \frac{\partial \psi}{\partial z} = 0.
$$

(3.1)

The corresponding differential equation for the field outside is

$$
\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial z} = 0.
$$

(3.2)

This differential equation for $\psi$ is valid when we consider the magnetic moment variation $m_z = 0$ which implies that $H_z$ is constant along the Z-axis. In order to introduce inhomogeneity we choose
Z-component of magnetisation $M_z$ as consisting of a constant part $M$ and a small perturbing part $m_2$ which is a function of $x$ and $y$ i.e.

$$M_z = M + m_2(x, y) \quad \quad \quad \quad (3.3)$$

We have the gyromagnetic equation

$$\frac{1}{\gamma} \frac{\partial \vec{M}}{\partial t} = (\vec{M} \times \vec{H}) \quad \quad (3.4)$$

which when written in terms of the components are

$$i \omega m_\alpha = \nabla \left[ m_y H - (m_z + M) h_\alpha \right], \quad \quad (3.5)$$

$$i \omega m_\gamma = \nabla \left[ (M + m_2) h_\alpha - m_\alpha H \right], \quad \quad (3.5)$$

$$i \omega m_z = \nabla \left[ m_x h_y - h_x m_y \right]. \quad \quad \quad \quad \quad (3.5)$$

where $h_z \approx H$. From these we arrive approximately,

$$m_\alpha = \frac{i}{4 \pi i \omega} \left[ k \frac{\partial \psi}{\partial x} - iv \frac{\partial \psi}{\partial y} \right], \quad \quad (3.6)$$

$$m_\gamma = \frac{i}{4 \pi i \omega} \left[ iv \frac{\partial \psi}{\partial x} + k \frac{\partial \psi}{\partial y} \right], \quad \quad \quad \quad (3.7)$$

$$m_2 = \frac{i}{4 \pi i \omega} \left[ \left\{ k \frac{\partial \psi}{\partial x} - iv \frac{\partial \psi}{\partial y} \right\} \frac{\partial \psi}{\partial y} - \left\{ iv \frac{\partial \psi}{\partial x} + k \frac{\partial \psi}{\partial y} \right\} \frac{\partial \psi}{\partial x} \right]. \quad \quad \quad \quad \quad (3.8)$$
Putting the value of $m_2$ from (3.8) in the eqn. (3.5) we have the final expressions for $m_x$, $m_y$, and $m_z$ as,

$$m_x = \frac{\gamma}{i\omega} \left[ \frac{1}{4\pi} \left\{ \frac{1}{\nu} \frac{\partial \psi}{\partial x} + ik \frac{\partial \psi}{\partial y} \right\} \cdot H - \frac{M}{4\pi\omega} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \right],$$

$$m_y = \frac{\gamma}{i\omega} \left[ M - \frac{\nu}{4\pi\omega} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \right],$$

$$m_z = -\frac{\nu}{4\pi\omega} \left[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right].$$

We have the field equations as $\text{div} \, \overrightarrow{H} = -4\pi \text{div} \, \overrightarrow{M}$, .. (3.11)

when expressed in terms of $\psi$, it becomes

$$\left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) = -4\pi \left[ \frac{\partial}{\partial x} (m_x) + \frac{\partial}{\partial y} (m_y) + \frac{\partial}{\partial z} (m_z) \right].$$

Substituting the values of $m_x$, $m_y$, and $m_z$ from (3.8) we have the differential equation for $\psi$ inside the material as

$$(1+k) \left[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right] + \frac{1}{\beta} \frac{\partial^2 \psi}{\partial z^2} + \beta \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) = 0.$$
where

$$k = \frac{\nu \gamma H}{\omega}, \quad \beta = \frac{\nu \gamma}{\omega},$$

(3.14)

so that the equation (3.1) is modified for the inhomogeneity as given by (3.13). When equations (3.2) and (3.13) are transformed in cylindrical co-ordinates

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial \psi}{\partial z^2} = 0,$$

(for exterior region) ..(3.15)

$$\left(1 + k\right) \left[ \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} \right] + \frac{\partial \psi}{\partial z^2} + \beta \frac{\partial}{\partial z} \left[ \frac{\partial \psi}{\partial r} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial \psi}{\partial \theta^2} \right] = 0,$$

(for interior region) .. (3.16)

The general solution for the eqn. (3.15) and (3.16) may be written in the forms

$$\Psi_{\text{int}} = \int_{0}^{\infty} A(\alpha') I_n(\alpha r) \cdot e^{i\phi} (z, \alpha') \, d\alpha'$$

$$\Psi_{\text{ext}} = \int_{0}^{\infty} C(\alpha) K_n(\alpha r) \cdot e^{i\phi} (z, \alpha) \, d\alpha$$

(3.17)

where

$$\Psi_{\text{int}} = \int_{0}^{\infty} A(\alpha') I_n(\alpha r) \cdot e^{i\phi} (z, \alpha') \, d\alpha'$$

$$\Psi_{\text{ext}} = \int_{0}^{\infty} C(\alpha) K_n(\alpha r) \cdot e^{i\phi} (z, \alpha) \, d\alpha$$

(3.17)
where
\[ \varphi(z, \alpha') = C_0 \alpha \sqrt{1 - \frac{\beta z^4}{4(1+k)^2}} \cdot z, \]
and
\[ \alpha = \alpha' \sqrt{1 - \frac{\beta z^4}{4(1+k)^2}}. \]

We have assumed here that \( \beta \) is small such that
\[ \sqrt{1 - \frac{\beta z^4}{4(1+k)^2}} \approx C_0 \alpha' z \sqrt{1 - \frac{\beta z^4}{4(1+k)^2}}. \]

Obviously the solutions (3.17) satisfy the condition
\[ \frac{\partial \psi}{\partial z} = 0 \quad \text{on} \quad z = 0. \]

There are two additional boundary conditions corresponding to the (H) and (B) normal conditions. Translated in terms of \( \psi \) these become

\[ \psi_{\text{int}} \bigg|_{r=a} = \psi_{\text{ext}} \bigg|_{r=a} \quad \text{.. (3.19)} \]

\[ (1+k) \frac{\partial \psi_{\text{int}}}{\partial r} \bigg|_{r=a} = \frac{\partial \psi_{\text{ext}}}{\partial r} \bigg|_{r=a} \quad \text{.. (3.20)} \]

From the first boundary condition, we get
\[ \alpha_k \left(k_0^2 \right) \int_0^\pi \int_0^{2\pi} A(x') I_n (\alpha z') \left( C_0 \alpha' z' \sqrt{1 - \frac{\beta z'^4}{4(1+k)^2}} \right) d\alpha' \quad \text{.. (3.21)} \]
Taking cosine integral inversion,
\[
\mathcal{C}(\chi) \mathcal{K}_n(\alpha') = \frac{2}{\pi} \int_0^\infty \cos z \, d\chi \int_0^\infty A(\chi') I_n(\alpha') \cos \left( \alpha' \sqrt{1 - \frac{\beta \chi'^2}{4(1+k)^2}} \right) d\chi'.
\]
\[
= \int_0^\infty A(\chi') I_n(\alpha') \delta \left( \alpha - \alpha' \sqrt{1 - \frac{\beta \chi'^2}{4(1+k)^2}} \right) d\chi'.
\]
\[
\mathcal{C}(\chi) \mathcal{K}_n(\alpha') = A(\alpha') I_n(\alpha')
\]  \hspace{1cm} (3.22)

where
\[
\alpha = \alpha' \sqrt{1 - \frac{\beta \chi'^2}{4(1+k)^2}}
\]  \hspace{1cm} (3.23)

where \(\delta\) is the Dirac delta function. From second boundary condition
\[
\left[ \mathcal{C}(\chi) \right] \mathcal{K}'_n(\alpha') = \int_0^\infty \alpha'(1+k) I_n'(\alpha') A(\alpha') \times \delta \left( \alpha - \alpha' \sqrt{1 - \frac{\beta \chi'^2}{4(1+k)^2}} \right) d\chi'.
\]  \hspace{1cm} (3.24)

From (3.22) and (3.24) we get as
\[
\frac{\mathcal{C}(\chi)}{A(\alpha')} = \frac{I_n(\alpha')}{\mathcal{K}_n(\alpha')}
\]  \hspace{1cm} (3.25)
and
\[
\frac{\alpha' C(\alpha')}{\alpha' A(\alpha')} = \frac{(1+k) I'_n(\alpha')}{K'_n(\alpha')}
\] .. (3.26)

From (3.15) and (3.16) we have the frequency equation as
\[
(1+k) \alpha' I_n(\alpha'\alpha) K_n(\alpha\alpha) - \alpha' I_n(\alpha'\alpha) K'_n(\alpha\alpha) = 0,
\] .. (3.27)

where
\[
\alpha = \alpha' \sqrt{1 - \frac{\beta'^2 \alpha'^2}{4(1+k)^2}}
\] .. (3.28)

The equation (3.17) can be written in terms of \(\alpha'\) as
\[
(1+k) \alpha' I_n(\alpha'\alpha) K_n \left\{ \alpha' \sqrt{1 - \frac{\beta'^2 \alpha'^2}{4(1+k)^2}}, \alpha \right\} - \alpha' \sqrt{1 - \frac{\beta'^2 \alpha'^2}{4(1+k)^2}} \times I_n(\alpha'\alpha) \times
K'_n \left\{ \alpha' \sqrt{1 - \frac{\beta'^2 \alpha'^2}{4(1+k)^2}}, \alpha \right\} = 0,
\] .. (3.29)

\[
K = \frac{2\sqrt{2}now}{\omega}, \quad \beta = \frac{\sqrt{2}now}{\omega}
\] .. (3.30)
Hence, $k$ and $\beta$ can be written in terms of $v, \gamma, H$ and $\omega$.

Eqn. (3.29) simplifies to

$$+ \frac{v \gamma H}{\omega} \alpha' \Gamma_n(\alpha') \ln \left\{ \alpha' \left(1 - \frac{v^2 \gamma^2 \alpha'^2}{4(\omega + v \gamma H)^2} \right)^{\frac{1}{2}} \right\} - \alpha' \left(1 - \frac{v^2 \gamma^2 \alpha'^2}{4(\omega + v \gamma H)^2} \right)^{\frac{1}{2}} \times$$

$$\Gamma_n(\alpha') \ln \left\{ \alpha' \left(1 - \frac{v^2 \gamma^2 \alpha'^2}{4(\omega + v \gamma H)^2} \right)^{\frac{1}{2}} \right\} = 0.$$  

**.. (3.31)**

$$\left(\frac{v \gamma H}{\omega}\right)$$ is written as

$$\frac{v \gamma H}{\omega} = \frac{H/4\pi M}{\omega/\gamma},$$

where

$$\omega = \frac{\omega}{4\pi M \gamma},$$

and

$$\frac{H_i}{4\pi M} = \frac{H_i}{4\pi M}$$

where $H_i = H_0 - \left(\frac{4\pi M}{3}\right).$  

**.. (3.32)**

When eqn. (3.19) is written in terms of $\omega$ and $\omega/\gamma$, it is

$$\frac{(\omega_H + \frac{1}{3})}{(\omega_H - \omega)} \times \Gamma_n(\alpha') \ln \left\{ \alpha' \left(1 - \frac{\alpha'^2}{64 \pi^2 H^2 \left[ (\omega_H - \omega) + (\omega_H + \frac{1}{3}) \right]^2} \times a \right\} -$$

$$\frac{\alpha'^2}{64 \pi^2 H^2 \left[ (\omega_H - \omega) + (\omega_H + \frac{1}{3}) \right]^2} \times \Gamma_n(\alpha') \ln \left\{ \alpha' \left(1 - \frac{\alpha'^2}{64 \pi^2 H^2 \left[ (\omega_H - \omega) + (\omega_H + \frac{1}{3}) \right]^2} \times a \right\} = 0.$$  

**.. (3.33)**
If we write \( \alpha' \) in the form \( \frac{\alpha'}{\mathcal{M}} \) and put \( \mathcal{M}=1 \) the expression (3.33) is

\[
\frac{\left(\alpha_{\mathcal{M}}^{\mathcal{N} + \frac{1}{2}}\right)}{\left(\alpha_{\mathcal{M}}^{\mathcal{N}} - \alpha_{\mathcal{N}}\right)} \times \int_{n}^{(d' / m)} \kappa_{n} \left\{ \frac{\alpha'}{\mathcal{M}} \left(1 - \frac{\alpha'^{2}}{64 \pi^{2} \left\{(\alpha_{\mathcal{M}}^{\mathcal{N}} - \alpha_{\mathcal{N}}) + (\alpha_{\mathcal{M}}^{\mathcal{N} + \frac{1}{2}})\right\}^{2}} \right) \right\} v_{n} - \\
\frac{\alpha'^{2}}{64 \pi^{2} \left\{(\alpha_{\mathcal{M}}^{\mathcal{N}} - \alpha_{\mathcal{N}}) + (\alpha_{\mathcal{M}}^{\mathcal{N} + \frac{1}{2}})\right\}^{2}} \times \int_{n}^{(d' / m)} \kappa_{n} \left\{ \frac{\alpha'}{\mathcal{M}} \left(1 - \frac{\alpha'^{2}}{64 \pi^{2} \left\{(\alpha_{\mathcal{M}}^{\mathcal{N}} - \alpha_{\mathcal{N}}) + (\alpha_{\mathcal{M}}^{\mathcal{N} + \frac{1}{2}})\right\}^{2}} \right) \right\} v_{n}
\]

(3.34)

Since \( \alpha' \) corresponds to the wave number and \( \alpha' \) is related to \( \alpha' \) by the relation \( \alpha = \alpha' \sqrt{1 - \beta' \alpha'^{2}} \), we have in (3.33) the characteristic equation. Expression (3.34) is the frequency equation for \( \mathcal{M}=1 \), for different magnetostatic modes excited inside a ferrimagnetic semi-infinite cylinder when an external magnetic field is applied along the \( Z \)-axis of the cylinder and the inhomogeneity of the internal field is taken into consideration.

We have numerically evaluated the values of \( \frac{\alpha'}{\mathcal{M}} \) for different values of \( \alpha_{\mathcal{M}}, \mathcal{N} \), and for the different azimuthal modes \( n = 0, 1, 2 \ldots 5, 6 \) from the expression (3.34). For the numerical evaluation we have used the recurrence relations \( \text{c.f.} \) Magnus and Oberhettinger (1951).
\[ K_n'(z) = -K_{n+1}(z) \]

\[ I_n'(z) = I_{n+1}(z) \]

and the expansions

\[ \eta(z) = \sum_{m=0}^{\infty} \frac{(\frac{1}{2}z)^{n+2m}}{m! [-\eta(n+m+1)]} \]

\[ U(z) = \frac{1}{2} \sum_{m=0}^{(n-1)} \frac{(-1)^m (\eta-m-1)!}{m! (\frac{1}{2}z)^{n-2m}} \]

\[ (-1)^m \sum_{m=0}^{n+1} \frac{(\frac{1}{2}z)^{n+2m} \left[ \log\left(\frac{1}{2}z\right) - \frac{1}{2} \psi(m+1) - \frac{1}{2} \psi(n+m+1) \right]}{m! (n+m)!} \]

The expression (3.34) has given several number of roots of \( \alpha'/M \) corresponding to different magnetostatic modes for every azimuthal modes. The curves \( \alpha'/M \) against \( \eta_H \) are plotted for different modes which represent the dispersion curves for the medium.

The different numerical values of \( \alpha'/M \) for a few values of \( \eta_H \) and \( \eta \) are given in table II.
FIRST MODES CORRESPONDING TO THE FIRST ROOTS OF THE FREQUENCY EQUATION AT DIFFERENT AZIMUTHAL HARMONICS (h) i.e. for $n = 0, 1, 2, \ldots, 6$ IN A SEMI-INFINITE CYLINDRICAL FERRITE
Table II

Different magnetostatic modes corresponding to the different roots of the frequency equation (3.34) in a semi-infinite cylindrical ferrite material.

<table>
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<th>$\eta_\perp$</th>
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SECOND MODES CORRESPONDING TO THE SECOND ROOTS OF THE FREQUENCY EQUATION AT DIFFERENT AZIMUTHAL HARMONICS (n) i.e. FOR n = 0, TO 6 IN A SEMI-INFINITE CYLINDRICAL FERRITE MATERIAL.
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<th>$\eta_\nu$</th>
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Third modes corresponding to the third roots of the frequency equation at different azimuthal harmonics i.e., for $\eta = 0, 1, 2, \ldots 6$ in a semi-infinite cylindrical ferrite material.
3.4: Results of numerical computation and discussion:

Numerical evaluations are made of the frequency equation (3.34) with the help of an IBM 1132 Computer. Different modes corresponding to the different roots of the frequency equation i.e. \( \alpha'/M \) at different azimuthal harmonies i.e. \( n = 0, 1, 2, 3, 4, 5, 6, \ldots \) etc. are found out at different values of \( \eta_H \) and \( \eta_L \). In our case \( \alpha' \) corresponds to wave number and \( \alpha' \) is related to \( \alpha \) by the relation

\[
\alpha = \alpha' \sqrt{1 - \frac{\beta^2 \omega^2}{4(M^2 + \kappa^2)^2}},
\]

where \( \beta = \frac{\nu Y}{\omega} \) and \( \kappa = \frac{\nu Y H}{\omega} \).

Equation (3.34) is the frequency equation for \( aM = 1 \) for different magnetostatic modes excited in a ferrimagnetic semi-infinite cylinder. Resonance curves are drawn by a plot of \( \alpha'/M \) Vs. \( \eta_H \). The resonance curves are shown in figures 9, 10, 11 which are far from symmetric in shape. The figures at the same time also represent the dispersion curves inside the semi-infinite ferrimagnetic medium. The necessary conditions and physical interpretations for the excitation of these non-symmetric multiple absorption peaks inside a ferrimagnetic medium is already discussed in sections (3.1) and (2.1) and chapter I of the thesis. The different resonant frequencies corresponding to the different roots of the frequency equation (3.34) for different values of \( \eta_H \) and \( \eta_L \) are calculated and is tabulated in Table II of this chapter.