CHAPTER - III

ANALYSIS OF OSCILLATOR NETWORKS

3.1 INTRODUCTION

Oscillator may be defined as an electronic device for generating a.c. voltage from d.c. power. In this electronic oscillator circuit there is no input signal voltage or current. In so far as the function of converting d.c. power into alternating power is concerned, an oscillator functions in a manner similar to electronic amplifier. Considering an electronic oscillator, d.c. power is supplied for biasing which results an a.c. power in the output terminals. The principal difference between oscillator and amplifier is that in case of oscillator the frequency, waveform and magnitude of the output a.c. voltage are determined by the circuit itself but in case of amplifier those are governed by the external source. In an oscillator, on the other hand, the frequency, waveform and magnitude of the output a.c. voltage depend only on the transistor or valve and the associated circuit and no external controlling voltage is required.
In oscillator, the characteristics of a network determine the frequency of oscillation whereas the combined characteristics of the transistor and circuit determine the condition of oscillation. Oscillator in which transistor operates on the linear portion of its characteristic and thereby produces sinusoidal voltage, is called a sinusoidal or harmonic oscillator. On the other hand, in a relaxation oscillator, transistor operates over non-linear region of its characteristic to produce voltage widely different in shape from sinusoidal one.

In the present analysis ordinary audio and radio frequency oscillators are considered on the assumption that the transistor parameters are all real at the frequency under consideration. The wave shape is assumed to be purely sinusoidal. Here in this chapter the frequency and the condition of oscillation of different oscillatory network will be derived by following tensor method of approach as developed by Kron.

Usually equivalent circuits are used for the analysis of oscillatory networks. Using the equivalent circuit the equations are established to determine the performance characteristics of oscillators with the help of well-known circuit theorems and Kirchhoff's laws. These equations may be solved either by the process of elimination of unknown variables or
by using Barkhausen criterion $K\beta = 1$, where $K$ is the nominal gain and $\beta$ is the feedback fraction.

The analysis of oscillatory network problems by conventional methods are given in the books of Fitchen\textsuperscript{16}, Hakim\textsuperscript{25,26}, Cattermole\textsuperscript{8}, Gartner\textsuperscript{17}, Mithal\textsuperscript{45} and others. Nichols\textsuperscript{48}, Getgen\textsuperscript{18}, Srikantaswamy\textsuperscript{59,60} and others have used matrix method in the analysis of oscillatory network problems in their different papers. Bacon\textsuperscript{1} established some formulae which are applicable to a network of any number of sections in a single stage phase shift oscillator. Ginzton and Hallingsworth\textsuperscript{21} calculated the performance equation and frequency of oscillation of a phase shift oscillator by the ordinary method. This analysis has been extended to four mesh phase shift oscillator by Sheer\textsuperscript{56}. Cherne\textsuperscript{9} has developed a general expression for condition of oscillation in an oscillator with an $n$ stage RC network. Dutta Roy\textsuperscript{15} has derived three possible transistor Wien bridge oscillator from analogy with corresponding vacuum tube circuits. He also derived approximate formulae for frequency of oscillation. Hooper and Jackets\textsuperscript{32} discussed the basic criteria of resistance capacitance oscillator using transistors. Brodie\textsuperscript{7} described a method for calculating the condition of oscillation in transistor feedback circuits. Cote\textsuperscript{10,11,12} made general classification of oscillators as $Z$-type, $Y$-type, $G$-type and $H$-type oscillatory
He developed one group of equations for each type by using matrix method. Using the same method of approach Zelinger determined the oscillatory conditions of Colpitts oscillator both in valve and transistor configurations. Hajek determined the oscillatory condition of four pole networks. A generalized form of RC oscillator circuit using negative impedances, of which the Wien bridge type of oscillators are shown to be special cases, is described by Pasupathy. Tensor method of approach has been employed, in the study of feedback valve oscillators, to determine the frequency of oscillations and corresponding starting conditions by Mitra and Bhattacharyya. The method is extended here to study the different types of oscillators where transistor is the active element.

In subsection 3.3a performance equation of generalized feedback harmonic oscillator is developed by tensor method of approach from which the performance equations of some practical oscillators are derived. Subsection 3.3b deals with the tensor analysis of single stage RC oscillators. In subsection 3.3c analysis of the Wien bridge oscillator is made by tensor method of approach. Frequency and condition for oscillations of this Wien bridge oscillator circuit are derived.

3.2 Method of Approach

A feedback oscillator may be considered as consisting of a two-port active element interconnected to a two-port
passive circuit. There are four possible ways in which a pair of two-port networks may be connected: (a) series connection, (b) parallel connection, (c) parallel-series connection and (d) series-parallel connection. These four possible connections are shown in Fig. 3.1. Thus there are four possible ways in which an active element is connected to a passive circuit. Therefore, to analyze any harmonic oscillator circuit, one need only express the circuit as a two-port network whose input and output ports are terminated in one of the four possible ways:

(a) Input and output short-circuited.
(b) Input and output open-circuited.
(c) Input open, output shorted.
(d) Input shorted, output open.

These four possibilities correspond respectively to Z, Y, G and H-oscillators as shown in Fig. 3.2. Analysis presented here is confined to only Z and Y type oscillators. Z-type oscillator is solved by using mesh method of approach and Y-type oscillator is solved by using junction-pair method of approach.

The behaviour of any mesh network is given by

$$\Delta E' = Z' \cdot \Delta I' \quad \ldots (3.1)$$
Fig. 3.1. Connection of two-part networks.
Fig. 3.2. Basic types of Oscillators.
where \( \Delta E' \) = impressed voltage tensor
\( \Delta I' \) = response current tensor
\( Z' \) = impedance tensor of the network.

For a stable network response quantity would be zero when impressed quantity is absent. For oscillatory networks, however, response quantity \( \Delta I' \) may exist in spite of the absence of impressed quantity \( \Delta E' \).

From equation (3.1)
\[
\Delta I' = Z'^{-1} \Delta E' \quad \ldots (3.2)
\]

\( Z'^{-1} \) has the form of a fraction whose numerator is a matrix \( Z_c \) containing all the cofactor of \( Z' \) and the denominator is the determinant \( D \) of \( Z' \), so the equation (3.2) may be written as
\[
\Delta I' = \frac{Z_c}{D} \Delta E' \quad \ldots (3.3)
\]

Under oscillatory condition the above equation (3.3) reduces to
\[
\Delta I' = \frac{Z_c}{D} \cdot 0
\]
or
\[
\Delta I' = \frac{0}{D} \quad \ldots (3.4)
\]
as the impressed quantity \( \Delta E' \) is absent.
But $\Delta I'$ does exist and is not zero as the expression (3.4) seems to indicate, therefore, the denominator must also be zero. Then $\Delta I' = \frac{0}{0}$, which may be an actual number, and consequently the network may oscillate. Hence $D = 0$ of a network indicates that the network is oscillatory. The elements of $D$ may have both real and imaginary parts, and thus $D$ may have two parts: one real part and another imaginary part. Both these real and imaginary parts must separately be zero. From these two equations the frequency and starting condition of oscillation can be determined.

In case of Y-type oscillator, the same method of reasoning is followed.

The behaviour of any junction network is given by

$$\Delta I' = Y' \cdot \Delta E' \quad \ldots (3.5)$$

where

$\Delta I' = \text{impressed current tensor}$

$\Delta E' = \text{response voltage tensor}$

$Y' = \text{admittance tensor of the network}.$

For oscillatory networks, response quantity $\Delta E'$ may exist in spite of the absence of impressed quantity $\Delta I'$.

From equation (3.5)

$$\Delta E' = Y'^{-1} \cdot \Delta I' \quad \ldots (3.6)$$
$Y^{-1}$ has the form of a fraction whose numerator is a matrix $Y_c$ containing all the cofactors of $Y'$ and the denominator is the determinant $D$ of $Y'$, so the equation (3.6) can be written as

$$
\Delta E' = \frac{Y_c}{D} \cdot \Delta I' \quad \ldots \quad (3.7)
$$

Under oscillatory condition the above equation (3.7) reduces to

$$
\Delta E' = \frac{Y_c}{D} \cdot 0
$$

or

$$
\Delta E' = \frac{0}{D} \quad \ldots \quad (3.8)
$$

But $\Delta E'$ does exist and is not zero as the expression (3.6) seems to indicate. Therefore, the denominator must also be zero. Then $\Delta E' = \frac{0}{D}$, which may be an actual number and consequently the network may oscillate. Hence $D = 0$ of the network indicates that the network is oscillatory. The elements of $D$ may have both real and imaginary parts, and thus $D$ may have two parts: one real part and another imaginary part. Both these real and imaginary parts must separately be zero. From these two equations the frequency and starting condition of oscillation can be determined.
3.3 ANALYSIS OF FEEDBACK OSCILLATOR NETWORKS

3.3a Single Stage LC Oscillators

The generalized transistor oscillator network is shown in Fig. 3.3 and its Y-equivalent is shown in Fig. 3.4. Fig. 3.5 shows the junction equivalent of the network.

The old impedance tensor of the passive feedback network is given by

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & z_1 & z_{12} & & & \\
2 & z_{12} & z_{22} & & & \\
3 & & z_3 & & & \\
4 & & & z_4 & & \\
5 & & & & z_5 & \\
6 & & & & & z_6 \\
\end{array}
\]

The old admittance tensor of the passive feedback network is given by
Fig. 3.3. The Generalized transistor Oscillator network.

Fig. 3.4. $Y$-equivalent of the Generalized transistor Oscillator network.
The admittance tensor of the common emitter transistor is given by

$$Y_1 = \begin{bmatrix} Y^1 & Y^{12} \\ Y^{12} & Y^2 \\ & & Y^3 \\ & & & Y^4 \\ & & & & Y^5 \\ & & & & & Y^6 \end{bmatrix}$$

where

$$Y^1 = \frac{z_2}{z_1 z_2 - (z_{12})^2}, \quad Y^{12} = -\frac{z_{12}}{z_1 z_2 - (z_{12})^2},$$

$$Y^2 = \frac{z_1}{z_1 z_2 - (z_{12})^2}, \quad Y^3 = \frac{1}{z_3}, \quad Y^4 = \frac{1}{z_4},$$

$$Y^5 = \frac{1}{z_5} \quad \text{and} \quad Y^6 = \frac{1}{z_6}.$$
Fig. 3.5. The junction equivalent of the Generalized Oscillator network.
The old admittance tensor $\mathbf{Y}$ of the interconnected network is given by $\mathbf{Y} = (\mathbf{Y}_1 + \mathbf{Y}_2)$ as

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & b & c \\
1 & y_1 & y_{12} & & & & & \\
2 & y_{12} & y_2 & & & & & \\
3 & & & y_3 & & & & \\
4 & & & & y_4 & & & \\
5 & & & & & y_5 & & \\
6 & & & & & & y_6 & \\
b & & & & & & y_{bb} & y_{bc} \\
c & & & & & & y_{cb} & y_{cc}
\end{array}
\]

... (3.9)

The assumed junction-pair voltage tensor ($\Delta \mathbf{E}'$) is given by

\[
\Delta \mathbf{E}' = \begin{bmatrix}
\Delta \mathbf{E}_{p'} \\
\Delta \mathbf{E}_{q'}
\end{bmatrix}
\]

... (3.10)

Fig. 3.5 shows the branch voltages and junction-pair response voltages. The corresponding equations expressing the
relationships between branch and junction-pair voltages give the junction-pair voltage transformation tensor \( A \) as

\[
\begin{align*}
\Delta E_1 &= \Delta E_{p'}, \\
\Delta E_2 &= \Delta E_{q'}, \\
\Delta E_3 &= \Delta E_{p'}, \\
\Delta E_4 &= \Delta E_{q'}, \\
\Delta E_5 &= -\Delta E_{p'} + \Delta E_{q'}, \\
\Delta E_6 &= -\Delta E_{p'} + \Delta E_{q'}, \\
\Delta E_b &= \Delta E_{p'}, \\
\Delta E_c &= \Delta E_{q'}
\end{align*}
\]

The transpose of \( A \), obtained by interchanging the columns and rows of the righthand member of equation (3.11), is

\[
A_t = \begin{bmatrix}
1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\]

The admittance tensor of the network as obtained from the relation \( Y' = A_t \cdot (Y \cdot A) \) is given by
When the circuit is oscillatory the determinant $D$ of $Y'$ is zero. Thus,

$$
D = \det(Y') = \left( Y_1 + Y_3 + Y_5 + Y_6 + y_{bb} \right) \left( Y_2 + Y_4 + Y_5 + Y_6 + y_{cc} \right) 
- \left( Y_{12} - Y - Y_6 + y_{bc} \right) \left( Y_{12} - Y - Y_6 + y_{cb} \right) = 0
$$

... (3.14)

If the internal feedback of the active element (i.e., transistor) is assumed to be negligible, then equation (3.14) reduces to

$$
(Y_1 + Y_3 + Y_5 + Y_6 + y_{bb}) (Y_2 + Y_4 + Y_5 + Y_6 + y_{cc}) 
- (Y_{12} - Y - Y_6 + y_{bc}) (Y_{12} - Y - Y_6 + y_{cb}) = 0
$$

... (3.15)

since $y_{bc}$ is negligibly small.

The performance equation of the single stage generalized LC oscillator is given by equation (3.15), which is a complex equation containing both real and imaginary parts. The performance equation (3.15) will now be used to determine the frequency and the condition of oscillations of some special types of oscillatory networks.
(i) **Colpitts Oscillator**:

Fig. 3.6 shows the circuit diagram of a transistor Colpitts oscillator, while 3.7 shows the corresponding Y-equivalent. It is being assumed that the operating frequency is sufficiently low to ensure that the transistor parameters are real, i.e., independent of frequency. The generalized oscillator circuit shown in Fig. 3.4 becomes a colpitts oscillator if each of the admittances \( Y_1, Y_2, Y_6, Y_1^2 \) be zero when its performance equation reduces to

\[
(Y + j\omega C)(Y + j\omega C^2) - (Y^2_{5})(-Y^2_{5} + Y_{cb}) = 0 \quad \ldots (3.16)
\]

where the values of the passive elements are:

\[
Y_3 = j\omega C, \quad Y_4 = j\omega C \quad \text{and} \quad Y_5 = -\frac{j}{\omega L}
\]

Hence the equation (3.16) becomes

\[
(j\omega C - \frac{j}{\omega L} + Y_{bb})(j\omega C - \frac{j}{\omega L} + Y_{cc}) - (Y^2_{5})(-Y^2_{5} + Y_{cb}) = 0
\]

or

\[
\left( -\frac{(n+1)c}{L} - n\omega^2 C^2 + Y_{bb} Y_{cc} \right) + j\omega \left( -nC Y_{bb} + C Y_{cc} \right) - \frac{Y_{bb} Y_{cb} + Y_{cc}}{\omega^2 L} = 0
\]

\[
\ldots (3.17)
\]

Equating real part of equation (3.17) to zero gives
Fig. 3.6. The Colpitts Oscillator.

Fig. 3.7. Y-equivalent of Colpitts Oscillator.
\[
\omega^2 = \frac{1}{LC} (1 + \frac{1}{n}) + \frac{y_{bb} y_{cc}}{nc^2} \quad \cdots \quad (3.18)
\]

If, however, the ratio \( \frac{C}{L} \) is sufficiently large such that

\[
\frac{C}{L} \gg \frac{y_{bb} y_{cc}}{1 + n} \quad \cdots \quad (3.19)
\]

then equation (3.18) reduces to

\[
\omega^2 = \frac{1}{LC} (1 + \frac{1}{n}) \quad \cdots \quad (3.20)
\]

Equation (3.20) shows that, if the inequality of equation (3.19) is satisfied, then the frequency of oscillation is determined by the tuned circuit and, therefore, practically independent of the transistor parameters.

Equating imaginary part of the equation (3.17) to zero gives

\[
\frac{y_{bb} y_{cb} + y_{cc}}{\omega^2 L} = nC_y y_{bb} + C_y y_{cc} \quad \cdots \quad (3.21)
\]

Substituting the value of \( \omega \) from equation (3.20) into equation (3.21) and after simplification, the expression for condition of oscillation becomes

\[
y_{cb} = ny_{bb} + \frac{y_{cc}}{n} \quad \cdots \quad (3.22)
\]
The Clapp oscillator is similar to Colpitts one except that the inductor of the latter circuit is replaced by series combination of an inductor and a capacitor as shown in Fig. 3.8 and its Y-equivalent is shown in Fig. 3.9. The generalized oscillator circuit shown in Fig. 3.4 becomes a Clapp oscillator if each of the admittances \( Y_1, Y_2, Y_6, Y_{12} \) be zero when its performance equation becomes

\[
(Y^3 + Y^5 + Y_{bb}) (Y^4 + Y + Y_{cc}) - (-Y^5) (-Y^5 + Y_{cb}) = 0 \quad \cdots (3.23)
\]

where the passive elements are given by

\[
Y_3 = j\omega C_1, \quad Y_4 = j\omega C_2 \quad \text{and} \quad Y_5 = \frac{j\omega C}{1 - \omega^2 LC}
\]

Hence the equation (3.23) becomes

\[
\left( j\omega C_1 + \frac{j\omega C}{1 - \omega^2 LC} + Y_{bb}\right) \left( j\omega C_2 + \frac{j\omega C}{1 - \omega^2 LC} + Y_{cc}\right)
\]

\[
\quad + \frac{j\omega C}{1 - \omega^2 LC} \left( - \frac{j\omega C}{1 - \omega^2 LC} + Y_{cb}\right) = 0
\]

or

\[
-\omega^2 C_1 C_2 + \frac{\omega^2 C(C_1 + C_2)}{1 - \omega^2 LC} - Y_{bb} Y_{cc} \quad \cdots (3.24)
\]

\[
+ j \frac{-\omega C_1 y_{bb} + \omega C y_{cb} + \omega C y_{cc}}{1 - \omega^2 LC} = 0 
\]

\[
\cdots (3.24)
\]
The Clapp Oscillator.

**Fig. 3.8. The clapp Oscillator.**

**Fig. 3.9. Y-equivalent of clapp Oscillator.**
Equating real part of equation (3.24) to zero gives

\[ \omega^2 c_1 c_2 + \frac{\omega^2 C (C_1 + C_2)}{1 - \omega^2 LC} - y_{bb} y_{cc} = 0 \quad \ldots (3.25) \]

If \( C_2 = nC_1 \), above equation (3.25) becomes

\[ \omega^2 \left[ nC_1 + (1+n)C \right] - y_{bb} y_{cc} = 0 \quad \ldots (3.26) \]

If it is assumed that

\[ \left[ nC_1 + (1+n)C \right] \gg \frac{LC}{C_1} \cdot y_{bb} y_{cc} \quad \ldots (3.27) \]

then it can be shown that equation (3.26) simplifies to

\[ \omega^2 = \frac{1}{L} \left( \frac{1}{C} + \frac{1}{C_1} + \frac{1}{nC_1} \right) \quad \ldots (3.28) \]

and the frequency of oscillation, therefore, becomes independent of the transistor parameters.

Equating imaginary part of equation (3.24) to zero gives

\[ \omega nC_{1y} y_{bb} + \omega C_{1y} y_{cc} + \left( \frac{\omega C_{y_{bb} + y_{cb} + y_{cc}}}{1 - \omega^2 LC} \right) = 0 \quad \ldots (3.29) \]

where \( C_2 \) is replaced by \( nC_1 \).

Substituting the value of \( \omega \) from expression (3.28) into equation (3.29) gives
Equation (3.30) gives the condition of oscillation of Clapp oscillator.

(iii) Hartley Oscillator:

Fig. 3.10 shows the circuit diagram of a transistor Hartley oscillator, while Fig. 3.11 shows the corresponding Y-equivalent circuit diagram. The generalized oscillator circuit shown in Fig. 3.4 becomes a Hartley oscillator if each of the admittances \( Y^3, Y^4, Y^5 \) be zero when its performance equation becomes

\[
(Y + jY')(Y + jY') - (Y_{12} Y^6)^2 (Y - jY') (Y - jY') = 0
\]

... (3.31)

where the passive elements are given by

\[
Y_1 = \frac{-jL_2}{\omega(L_1 L_2 - M^2)}, \quad Y_2 = \frac{-jL_2}{\omega(L_1 L_2 - M^2)}, \quad Y_{12} = \frac{-jN}{\omega(L_1 L_2 - M^2)}
\]

and \( Y_6 = j\omega \), since \( Z_1 = j\omega L_1 \), \( Z_2 = j\omega L_2 \), \( Z_6 = \frac{1}{j\omega} \)

and \( Z_{12} = -j\omega M \).
Fig. 3.10. The Hartley Oscillator.

Fig. 3.11. Y- equivalent of Hartley Oscillator.
Hence, the equation (3.31) becomes

\[
\left( -\frac{jL_2}{\omega(L_1L_2-M^2)} + \frac{j\omega}{\omega(L_1L_2-M^2)} \right) \left( -\frac{jL_1}{\omega(L_1L_2-M^2)} + \frac{j\omega}{\omega(L_1L_2-M^2)} \right)
\]

\[
-\left( -\frac{jM}{\omega(L_1L_2-M^2)} - \frac{j\omega}{\omega(L_1L_2-M^2)} - \frac{j\omega}{\omega(L_1L_2-M^2)} - \frac{j\omega}{\omega(L_1L_2-M^2)} \right) = 0
\]

or

\[
\frac{C(L_1+L_2)}{L_1L_2-M^2} + \frac{2MC}{L_1L_2-M^2} \frac{1}{\omega(L_1L_2-M^2)} + \frac{y_{bb}y_{cc}}{\omega(L_1L_2-M^2)}
\]

\[
+j \left( -\frac{C(y_{bb}+y_{cb}+y_{cc})}{\omega(L_1L_2-M^2)} + \frac{M y_{cb} - L_1 y_{bb} - L_2 y_{cc}}{\omega(L_1L_2-M^2)} \right) = 0
\]

... (3.32)

Equating real part of equation (3.32) to zero gives

\[
\frac{1}{\omega^2} = C(L_1+L_2+2M) + \frac{y_{bb}y_{cc}(L_1L_2-M^2)}{\omega(L_1L_2-M^2)}
\]

... (3.33)

If \( L_1 \) and \( L_2 \) are replaced by \( l \) and \( nL \) respectively, then above equation (3.33) becomes

\[
\frac{1}{\omega^2} = C(L+nL+2M) + \frac{y_{bb}y_{cc}(nL^2-M^2)}{\omega(nL^2-M^2)}
\]

... (3.34)

If the two coils of the feedback network are mutually coupled and have a coupling coefficient \( k \), which is less than unity, then their mutual inductance \( M \) is given by
Substituting the value of $M$ from equation (3.35) into equation (3.34) gives

$$\frac{1}{\omega^2} = C\left(1 + 2k\sqrt{n} + n\right) + nL^2 \left(1 - k^2\right)y_{bb}y_{cc} \quad \ldots (3.36)$$

If it is assumed that

$$\frac{C}{L} \gg \frac{n(1-k^2)}{1+2k\sqrt{n} + n} y_{bb}y_{cc} \quad \ldots (3.37)$$

then equation (3.36) reduces to

$$\omega^2 = \frac{1}{\omega C\left(1+2k\sqrt{n} + n\right)} \quad \ldots (3.38)$$

Equating imaginary part of equation (3.32) to zero gives

$$\omega^2 C \left(y_{bb} + y_{cb} + y_{cc}\right) \left(L_1L_2-M^2\right) + M y_{cb} - L_1y_{bb} - L_2y_{cc} = 0 \quad \ldots (3.39)$$

Replacing $L_1$, $L_2$, and $M$ by $L$, $nL$ and $kL\sqrt{n}$ respectively, equation (3.39) becomes

$$\omega^2 \left(nL^2 \left(y_{bb} + y_{cb} + y_{cc}\right) \left(1-k^2\right) + kL\sqrt{n}y_{cb} - L_1y_{bb} - L_2y_{cc} = 0 \quad \ldots (3.40)$$

Substituting the value of $\omega$ from equation (3.38) into equation (3.40) gives
Expression (3.41) gives the condition of oscillation of Hartley type oscillator.

(iv) Tuned Collector Oscillator:

The tuned collector oscillator network is shown in Fig. 3.12 and its Y-equivalent is shown in Fig. 3.13. The generalized oscillator circuit shown in Fig. 3.4 becomes a tuned collector oscillator if each of the admittances \( Y_3, Y_5 \) and \( Y_6 \) be zero when its performance equation becomes

\[
(Y_1 + Y_{bb})(Y_2 + Y_A + Y_{cc}) - Y_{12}(Y_{12} + Y_{cb}) = 0 \quad \ldots \quad (3.42)
\]

where the passive elements are given by

\[
Y_1 = \frac{-jL_2}{\omega(L_1L_2 - M^2)} , \quad Y_2 = \frac{-jL_1}{\omega(L_1L_2 - M^2)} , \quad Y_{12} = \frac{-jM}{\omega(L_1L_2 - M^2)} \quad \text{and} \quad Y_4 = j\omega C
\]

since \( Z_1 = j\omega L_1 \), \( Z_2 = j\omega L_2 \), \( Z_{12} = -j\omega M \) and \( Z_4 = \frac{1}{j\omega C} \).

Hence the equation (3.42) becomes
Fig. 3.12. The tuned collector Oscillator.

Fig. 3.13. Y-equivalent of tuned collector Oscillator.
\[
\begin{bmatrix}
\frac{-jL_2}{\omega(L_1L_2-M^2)} + y^{bb} \\
\frac{-jL_1}{\omega(L_1L_2-M^2)} + j\omega + y^{cc}
\end{bmatrix}
+ \frac{jM}{\omega(L_1L_2-M^2)} \begin{bmatrix}
\frac{-jM}{\omega(L_1L_2-M^2)} + y^{cb}
\end{bmatrix} = 0
\]

or
\[
\begin{bmatrix}
\frac{L_2C}{L_1L_2-M^2} - \frac{1}{\omega(L_1L_2-M^2)} + y^{bb}y^{cc}
\end{bmatrix}
+ j \begin{bmatrix}
\frac{M y^{cb}}{\omega(L_1L_2-M^2)} - \frac{L_2y^{cc}+L_1y^{bb}}{\omega(L_1L_2-M^2)} + \omega y^{bb}
\end{bmatrix} = 0
\]

\[\ldots (3.43)\]

Equating real part of equation (3.43) to zero gives

\[\frac{1}{\omega^2} = L_2C + (L_1L_2-M^2)y^{bb}y^{cc}\]

or

\[\omega^2 = \frac{1}{L_2C}\]  
\[\ldots (3.44)\]

Equating imaginary part of equation (3.43) to zero gives

\[M y^{cb} - L_2y^{cc} - L_1y^{bb} + \omega^2 C(L_1L_2-M^2)y^{bb} = 0\]  
\[\ldots (3.45)\]

Substituting the value of \(\omega\) from equation (3.44) into equation (3.45) gives the condition for oscillation as
(v) Tuned Base Oscillator:

The tuned base oscillator network is shown in Fig. 3.14. The Y-equivalent of the tuned base oscillator is shown in Fig. 3.15. The generalized oscillator network shown in Fig. 3.4 becomes a tuned base oscillator if each of the admittances $Y_1$, $Y_5$ and $Y_6$ be zero when its performance equation becomes

$$\left( \frac{1}{Y_1} + \frac{1}{Y_2} \right) \left( \frac{1}{Y_3} + \frac{1}{Y_4} \right) - \left( \frac{1}{Y_5} \right) \left( \frac{1}{Y_6} + \frac{1}{Y_7} \right) = 0 \quad \ldots \ (3.47)$$

where the passive elements are given by

$$Y_1 = \frac{-jL_2}{\omega (L_2 - M^2)} \quad Y_2 = \frac{-jL_1}{\omega (L_1 - M^2)} \quad Y_5 = \frac{-jM}{\omega (L_1 L_2 - M^2)} \quad \text{and} \quad Y_3 = j\omega C.$$ 

Hence the equation (3.47) becomes

$$\left[ \frac{-jL_2}{\omega (L_1 L_2 - M^2)} + j\omega C + \frac{Y_{bb}}{y} \right] \left[ \frac{-jL_1}{\omega (L_1 L_2 - M^2)} + \frac{Y_{cc}}{y} \right]$$

$$+ \frac{jM}{\omega (L_1 L_2 - M^2)} \left[ \frac{-jM}{\omega (L_1 L_2 - M^2)} + \frac{Y_{cb}}{y} \right] = 0$$
Fig. 3.14. The tuned base Oscillator.

Fig. 3.15. Y-equivalent of tuned base Oscillator.
Equating real part of equation (3.48) to zero gives

\[
\frac{L_1 C}{L_1 L_2 - M^2} \cdot \frac{1}{\omega^2 (L_1 L_2 - M^2)} + y_{bb} y_{cc} = 0 \quad \ldots (3.48)
\]

Equating imaginary part of equation (3.48) to zero gives

\[
-j \left[ \frac{L_1 y_{bb} + L_2 y_{cc} - M y_{cb}}{\omega (L_1 L_2 - M^2)} - \omega y_{cc} \right] = 0 \quad \ldots (3.50)
\]

Substituting the value of \( \omega \) from equation (3.49) into equation (3.50) gives the condition for oscillations as

\[
y_{cb} = \frac{L_1}{M} y_{bb} + \frac{M}{L_1} y_{cc} \quad \ldots (3.51)
\]
3.3b Single Stage RC Oscillators

(i) High-pass RC Phase Shift Oscillator

The High-pass RC oscillator circuit is shown in Fig.3.16 and its Z-equivalent circuit diagram is shown in Fig.3.17. Fig.3.18 shows the mesh equivalent of the network.

The old impedance tensor of the passive feedback network is given by

$$
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1 & z_1 & & & & & \\
2 & & z_2 & & & & \\
3 & & & z_3 & & & \\
4 & & & & z_4 & & \\
5 & & & & & z_5 & \\
6 & & & & & & z_6 \\
7 & & & & & & z_7
\end{array}
$$

The impedance tensor of common emitter transistor is given by

$$
\begin{bmatrix}
    b \\
    c
\end{bmatrix}
\begin{bmatrix}
    z_{bb} & z_{bc} \\
    z_{cb} & z_{cc}
\end{bmatrix}
$$
The high-pass R C phase shift Oscillator.

Fig. 3.16. The high-pass R C phase shift Oscillator.

Fig. 3.17. Z-equivalent of high-pass R C phase shift Oscillator.

Fig. 3.18. The mesh equivalent of high-pass R C phase shift Oscillator.
The old impedance tensor $Z$ of the interconnected network is given by $Z = (Z_1 + Z_2)$ as

$$Z = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & b & c \\
1 & Z_1 \\
2 & Z_2 \\
3 & Z_3 \\
4 & Z_4 \\
5 & Z_5 \\
6 & Z_6 \\
7 & Z_7 \\
b & Z_{bb} & Z_{bc} \\
c & Z_{cb} & Z_{cc}
\end{bmatrix} \quad \ldots (3.52)$$

The assumed mesh current tensor $(\Delta I')$ is given by

$$\Delta I' = \begin{bmatrix}
p' & \Delta I_p' \\
q' & \Delta I_q' \\
r' & \Delta I_r' \\
s' & \Delta I_s'
\end{bmatrix} \quad \ldots (3.53)$$

where $\Delta I_p'$, $\Delta I_q'$, $\Delta I_r'$, $\Delta I_s'$ are assumed new mesh currents.
Fig. 3.18 shows the branch currents and mesh response currents. The corresponding equations expressing the relationships between branch and mesh currents give the current transformation tensor \( C \) as

\[
\begin{align*}
\Delta I^1 &= \Delta I^q' - \Delta I^r', \\
\Delta I^2 &= \Delta I^r' - \Delta I^s', \\
\Delta I^3 &= -\Delta I^p' + \Delta I^s', \\
\Delta I^4 &= \Delta I^r', \\
\Delta I^5 &= \Delta I^s', \\
\Delta I^6 &= \Delta I^p', \\
\Delta I^7 &= \Delta I^p', \\
\Delta I^p &= \Delta I^p', \\
\Delta I^c &= -\Delta I^q'.
\end{align*}
\]

\[
C = \begin{pmatrix}
1 & 1 & -1 \\
2 & 1 & -1 \\
3 & -1 & 1 \\
4 & 1 & . \\
5 & 1 & . \\
6 & 1 & . \\
7 & 1 & . \\
b & 1 & . \\
c & -1 & .
\end{pmatrix}
\]  

\( \cdots \text{(3.54)} \)

The transpose of \( C \), obtained by interchanging the columns and rows of the right-hand member of equation (3.54), is

\[
C_t = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & b & c \\
1 & -1 & 1 & 1 & 1 \\
q' & 1 & . & . & . & . \\
r' & -1 & 1 & 1 \\
s' & -1 & 1 & 1 \\
\end{pmatrix}
\]  

\( \cdots \text{(3.55)} \)
The impedance tensor of the network as obtained from the relation $Z' = C_l \cdot (Z \cdot C)$ is given by

$$

tabular{cccc}
p' & q' & r' & s' \\
\hline
- \frac{Z_3}{Z_6} + \frac{Z_7}{Z_{bb}} & -Z_{bc} & -Z_3 \\
- \frac{Z_{cb}}{Z_1} & Z_1 + Z_{cc} & -Z_1 \\
- \frac{Z_2}{Z_1} & Z_1 + Z_2 + Z_4 & -Z_2 \\
- \frac{Z_3}{Z_2} & -Z_2 & Z_2 + Z_3 + Z_5 \\
\hline

(3.56)

In the actual high-pass R-C phase shift oscillator network (Fig.3.16) or in its Z-equivalent connection (Fig.3.17), the impedances $Z_1, Z_2, Z_3$ etc are given by

$$
Z_1 = \frac{R}{\omega L} \\
Z_2 = R \\
Z_3 = R \\
Z_4 = -\frac{j}{\omega C} \\
Z_5 = -\frac{j}{\omega C} \\
Z_6 = -\frac{j}{\omega C} \\
Z_7 = R - h_{bb}

(3.57) \quad (3.58) \quad (3.59) \quad (3.60) \quad (3.61) \quad (3.62) \quad (3.63)

Substituting from equations (2.26) to (2.29) and (3.57) to (3.63) into equation (3.56) gives
The equation of performance of the actual network under oscillatory condition is obtained by equating determinant $D$ of the impedance tensor $Z'$ to zero. The frequency as well as the condition of oscillations can be obtained first by equating real and imaginary parts to zero and then by proper simplification of these two equations.

Hence, under oscillatory condition

$D = \det (Z') =$

<table>
<thead>
<tr>
<th>$p'$</th>
<th>$q'$</th>
<th>$r'$</th>
<th>$s'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p'$</td>
<td>$(2R - \frac{j}{\omega C}) - \frac{h^* c}{h^* c} \cdot \frac{h^* b}{h^* c}$</td>
<td>$\frac{h^* c}{h^* c}$</td>
<td>$- \frac{h^* b}{h^* c}$</td>
</tr>
<tr>
<td>$q'$</td>
<td>$\frac{h^* c}{h^* c}$</td>
<td>$\frac{R}{n} + \frac{1}{h^* c}$</td>
<td>$- \frac{R}{n}$</td>
</tr>
<tr>
<td>$r'$</td>
<td>$- \frac{R}{n}$</td>
<td>$R(1 + \frac{1}{n}) - \frac{j}{\omega C}$</td>
<td>$-R$</td>
</tr>
<tr>
<td>$s'$</td>
<td>$-R$</td>
<td>$-R$</td>
<td>$2R - \frac{j}{\omega C}$</td>
</tr>
</tbody>
</table>

$\ldots$ (3.65)
If, now, it is assumed that $h^c_b \ll 1$ and $\frac{1}{h^c_c}$ is very large, then equation (3.65) reduces to:

$$-3R^3\left(1+\frac{1}{n}\right) - \frac{4R}{\omega^2C^2} - 2R^3 - \frac{R(1+\frac{1}{n})}{\omega^2C^2} + \frac{R^3\cdot h^c_c}{n\cdot h^c_b} \cdot j$$

$$+ \frac{j}{\omega C} \left( \frac{1}{\omega^2C^2} - 2R^2 - R^2(4+\frac{4}{n}) \right) = 0 \quad \ldots (3.66)$$

Equating imaginary part of equation (3.66) to zero gives the frequency of oscillation as:

$$\omega = \frac{1}{CR} \sqrt{\frac{1}{6n+4}} \quad \ldots (3.67)$$

Equating real part of equation (3.66) to zero gives:

$$R^3 + \frac{3R^3}{n} - \frac{R(5+\frac{1}{n})}{\omega^2C^2} + \frac{R^3\cdot h^c_c}{n\cdot h^c_b} = 0 \quad \ldots (3.68)$$

Substituting the value of $\omega$ from equation (3.67) into equation (3.66) gives the condition of oscillation as:

$$h^c_c \cdot h^c_b = 29n + 23 + \frac{4}{n} \quad \ldots (3.69)$$

(ii) **Low-pass RC Phase Shift Oscillator:**

The low-pass RC oscillator circuit is shown in Fig.3.19 and its Z-equivalent circuit diagram is shown in Fig.3.20. Fig.3.21 shows the mesh equivalent of the network.
Fig. 3.19. The low-pass R C phase shift oscillator.

Fig. 3.20. Z-equivalent of low-pass R C phase shift oscillator.

Fig. 3.21. The mesh equivalent of low-pass R C phase shift oscillator.
The old impedance tensor of the passive feedback network is given by

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1 & Z_1 & & & & & \\
2 & & Z_2 & & & & \\
3 & & & Z_3 & & & \\
4 & & & & Z_4 & & \\
5 & & & & & Z_5 & \\
6 & & & & & & Z_6 \\
7 & & & & & & Z_7 \\
\end{array}
\]

The impedance tensor of common emitter transistor is given by

\[
\tilde{Z}_2 = \begin{bmatrix}
  b & c \\
  z_{bb} & z_{bc} \\
  z_{cb} & z_{cc} \\
\end{bmatrix}
\]

The old impedance tensor of the interconnected network is given by \( Z = (Z_1 + Z_2) \) as
The assumed mesh current tensor \(\Delta I'\) is given by

\[
\Delta I' = \begin{bmatrix}
\Delta I^p' \\
\Delta I^q' \\
\Delta I^r' \\
\Delta I^s' \\
\Delta I^t'
\end{bmatrix}
\]

where \(\Delta I^p'\), \(\Delta I^q'\), \(\Delta I^r'\), \(\Delta I^s'\) and \(\Delta I^t'\) are the assumed new mesh currents.
Fig. 3.21 shows the branch currents and mesh response currents. The corresponding equations expressing the relationships between branch and mesh currents give the current transformation tensor $C$ as

$$\begin{align*}
\Delta I_1 &= \Delta I_{q'} - \Delta I_{r'} \\
\Delta I_2 &= \Delta I_{r'} - \Delta I_{s'} \\
\Delta I_3 &= \Delta I_{s'} - \Delta I_{t'} \\
\Delta I_4 &= -\Delta I_{p'} + \Delta I_{t'} \\
\Delta I_5 &= \Delta I_{s'} \\
\Delta I_6 &= \Delta I_{t'} \\
\Delta I_7 &= \Delta I_{p'} \\
\Delta I^b &= \Delta I_{p'} \\
\Delta I^c &= -\Delta I_{q'}
\end{align*}$$

$$C = \begin{array}{ccccccc}
p' & q' & r' & s' & t' \\
1 & 1 & -1 & & & & \\
2 & 1 & -1 & & & & \\
3 & & 1 & -1 & & & \\
4 & -1 & 1 & & & & \\
5 & & & & 1 & & \\
6 & & & & & 1 & \\
7 & 1 & & & & & \\
b & & & & & & 1 \\
c & & & & & -1 &
\end{array}$$

... (3.72)
The transpose of $C$, obtained by interchanging the columns and rows of the right hand member of equation (3.72), is

\[
\begin{array}{cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & b & c \\
p' & & & & & -1 & & 1 & 1 & \\
q' & 1 & & & & & & & -1 \\
C_t = r' & -1 & 1 & & & & & & \\
s' & & 1 & 1 & & & & & \\
t' & & -1 & 1 & 1 & & & & \\
\end{array}
\]

\[= \mathbf{111} =\]

... (3.73)

The impedance tensor $Z'$ of the network is obtained from the relation $Z' = C_t \cdot (Z \cdot C)$ as

\[
\begin{array}{cccccc}
 & p' & q' & r' & s' & t' \\
p' & Z_4 + Z_7 + Z_{bb} & -Z_{bc} & & & -Z_4 \\
q' & -Z_{cb} & Z_1 + Z_{cc} & -Z_1 & & \\
Z' = r' & & -Z_1 & Z_1 + Z_2 & -Z_2 & \\
s' & & & -Z_2 & Z_2 + Z_3 + Z_5 & -Z_3 \\
t' & & & & -Z_4 & Z_3 + Z_4 + Z_6 & \\
\end{array}
\]

... (3.74)
In the actual RC phase shift oscillator of low-pass type
the impedances \( z_1, z_2, z_3 \) etc are given by

\[
\begin{align*}
z_1 &= \frac{R}{n} \quad \ldots (3.75) \\
z_2 &= -\frac{j}{\omega C} \quad \ldots (3.76) \\
z_3 &= -\frac{j}{\omega C} \quad \ldots (3.77) \\
z_4 &= -\frac{j}{\omega C} \quad \ldots (3.78) \\
z_5 &= R \quad \ldots (3.79) \\
z_6 &= R \quad \ldots (3.80) \\
z_7 &= R - h_{bb} \quad \ldots (3.81)
\end{align*}
\]

Substituting from equations (2.26) to (2.29) and (3.75) to (3.81) into equation (3.74) gives

\[
\begin{array}{|c|c|c|c|c|}
\hline
p' & q' & r' & s' & t' \\
\hline
p' & \left( R - \frac{j}{\omega C} \right) & -\frac{h_{bc} h_{bb}}{h_{cc}} & -\frac{h_{bc}^*}{h_{cc}^*} & \frac{j}{\omega C} \\
\hline
q' & \frac{h_{bc}^*}{h_{cc}^*} & \frac{R}{n} + \frac{1}{h_{cc}} & -\frac{R}{n} & & \\
\hline
z' = r' & & -\frac{R}{n} & \frac{R}{n} - \frac{j}{\omega C} & \frac{j}{\omega C} \\
\hline
s' & & \frac{j}{\omega C} & R - j\omega C & \frac{j}{\omega C} \\
\hline
t' & \frac{j}{\omega C} & \frac{j}{\omega C} & R - j\omega C & & \ldots (3.75)
\end{array}
\]
The equation of performance of the network under oscillatory condition is obtained by equating determinant $D$ of the impedance tensor $Z'$ to zero. The frequency as well as the condition of oscillations can be obtained by equating real and imaginary parts to zero.

Hence under oscillatory condition,

$$D = \det (Z') = \det \begin{array}{c|c|c|c|c}
p' & q' & r' & s' & t' \\
\hline
p' & \left( R- \frac{j}{\omega C} \right) - \frac{h_{b}^{c}h_{c}^{c}}{h_{c}^{c}} & - \frac{h_{b}^{c}}{h_{c}^{c}} & \frac{j}{\omega C} & \\
q' & \frac{h_{b}^{c}}{h_{c}^{c}} & \frac{R}{n} + \frac{1}{h_{c}^{c}} & - \frac{R}{n} & \\
r' & \frac{-R}{n} & \frac{R}{n} - \frac{j}{\omega C} & \frac{j}{\omega C} & \\
s' & \frac{j}{\omega C} & R - \frac{j}{\omega C} & \frac{j}{\omega C} & \\
t' & \frac{-j}{\omega C} & \frac{-j}{\omega C} & R - \frac{j}{\omega C} & \\
\end{array}$$

... (3.76)

If, now, it is assumed that $h_{b}^{c} << 1$ and $\frac{1}{h_{c}^{c}}$ is very large, the equation (3.76) becomes

$$\frac{R}{n} - \frac{6R^2}{\omega^2 C^2} - \frac{4R^3}{2\omega^2 C^2} - \frac{j}{\omega C} - \frac{5R^3}{n} + R^3 - \frac{R}{\omega^2 C^2} - \frac{3R}{\omega^2 C^2}$$

$$- \frac{R}{\omega^2 C^2} \cdot h_{b}^{c} = 0 \quad \ldots (3.77)$$
Equating real part of equation (3.77) to zero gives the frequency of oscillation as

\[ \omega = \frac{1}{RC} \sqrt{\delta + \Delta} \]  \hspace{1cm} \text{... (3.78)}

Equating imaginary part of equation (3.77) to zero gives

\[ \frac{5R^3}{n} + R^3 - \frac{R}{n\omega^2C^2} - \frac{3R}{\omega^2C^2} - h_c b = 0 \]  \hspace{1cm} \text{... (3.79)}

Substituting the value of \( \omega \) from equation (3.78) into equation (3.79) gives the condition of oscillation as

\[ h_c b = 29 + 23n + 4n^2 \]  \hspace{1cm} \text{... (3.80)}

3.3c Two Stage RC Oscillator

Wien Bridge Oscillator

The two-stage RC oscillator circuit Fig.3.22 uses a feedback network which is the inverse of a Wien bridge and its Y-equivalent is shown in Fig.3.23. The active network of the Wien bridge oscillator as shown in Fig.3.23 consists of a common-collector stage followed by a common-base stage transistors. The by-pass condensor C' (Fig.3.22) is used to short circuit the terminals of \( R_7 \) and \( R_8 \) for a.c. So in a.c. analysis they are neglected.
Fig. 3.22. The Wien bridge Oscillator.

Fig. 3.23. Y-equivalent of Wien bridge Oscillator.
Analysis of Active Networks:

The old admittance tensor ($Y_1$) of the passive elements is given by

\[
\begin{array}{cccc}
3 & 4 & 5 & 6 \\
3 & Y^3 & & \\
4 & & Y^4 & \\
5 & & & Y^5 \\
6 & & & Y^6 \\
\end{array}
\]

where $Y^3 = \frac{1}{R_3}$, $Y^4 = \frac{1}{R_4}$, $Y^5 = \frac{1}{R_5}$ and $Y^6 = \frac{1}{R_6}$.

The admittance tensor ($Y_2$) of the common-collector and common-base stage transistors are given by

\[
\begin{array}{cccc}
b_1 & e_1 & e_2 & c_2 \\
b_1 & y_{bb_1} & y_{be_1} & \\
e_1 & y_{eb_1} & y_{ee_1} & \\
e_2 & & y_{ee_2} & y_{ec_2} \\
c_2 & & y_{ce_2} & y_{cc_2} \\
\end{array}
\]
Fig. 3.24. Junction equivalent of the active network of Wienbridge Oscillator.
The old admittance tensor \( Y \) of the active network is given by \( Y = (Y_1 + Y_2) \) as

\[
Y = \begin{array}{cccccc}
3 & 4 & 5 & 6 & b_1 & e_1 & e_2 & c_2 \\
3 & Y^3 & & & & & \\
4 & & Y^4 & & & & \\
5 & & & Y^5 & & & \\
6 & & & & Y^6 & & \\
b_1 & & & & & y_{bb_1} & y_{be_1} & \\
e_1 & & & & & y_{eb_1} & y_{ee_1} & \\
e_2 & & & & & y_{ee_2} & y_{ec_2} & \\
c_2 & & & & & y_{ce_2} & y_{cc_2} & \\
\end{array}
\]

... (3.51)

Impressed current tensor in the branches of the active network is given by

\[
\Delta I = \begin{array}{c}
1 & \Delta I_f \\
2 & \\
3 & \\
4 & b_1 \\
e_1 & \\
e_2 & \\
c_2 & \Delta I_g \\
\end{array}
\]

... (3.82)
where $\Delta I^f$ and $\Delta I^g$ are the currents impressed in the active network.

The assumed junction-pair voltage tensor ($\Delta E'$) is given by

$$
\begin{pmatrix}
\Delta E'_{f'} \\
\Delta E'_{g'} \\
\Delta E'_{h'} \\
\Delta E'_{i'}
\end{pmatrix} = \begin{pmatrix}
\Delta E_{f} \\
\Delta E_{g} \\
\Delta E_{h} \\
\Delta E_{i}
\end{pmatrix}
$$

... (3.63)

where $\Delta E_{f}$, $\Delta E_{g}$, $\Delta E_{h}$, $\Delta E_{i}$ are the assumed junction-pair voltages.

Fig. 3.24 shows the branch voltages and junction-pair response voltages. The corresponding equations expressing the relationships between branch and junction-pair voltages give the junction-pair voltage transformation tensor $A$ as

$$
\begin{array}{c}
\Delta E_3 = \Delta E_{f} \\
\Delta E_4 = \Delta E_{f} \\
\Delta E_5 = \Delta E_h, + \Delta E_{i} \\
\Delta E_6 = \Delta E_h \\
\Delta E_{b1} = \Delta E_{f}, - \Delta E_h \\
\Delta E_{e1} = \Delta E_{f} \\
\Delta E_{e2} = \Delta E_h, + \Delta E_{i}, \\
\Delta E_{c2} = \Delta E_g
\end{array}
$$

$$
\begin{array}{cccc}
f' & g' & h' & i' \\
3 & 1 & & \\
4 & 1 & & \\
5 & & 1 & 1 \\
6 & & 1 & \\
b_1 & 1 & & -1 \\
\varepsilon_1 & & & 1 \\
\varepsilon_2 & & & 1 \\
\varepsilon_2 & & 1 & \\
\end{array}
$$

... (3.64)
The transpose of $A$, obtained by interchanging the columns and rows of the right hand member of equation (3.84), is

$$
A_t = \begin{bmatrix}
3 & 4 & 5 & 6 & b_1 & e_1 & e_2 & c_2 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
$$

... (3.85)

The admittance tensor $Y'$ of the active network is as obtained from the relation $Y' = A_t \cdot (Y \cdot A)$ is given by

$$
Y' = \begin{bmatrix}
f' & g' & h' & i' \\
3 + y & y & -y & y \\
y & y & y & -y \\
-y & y & y & -y \\
y & y & y & -y \\
\end{bmatrix}
$$

... (3.86)
The impressed current tensor \( \Delta I' \) in the actual network is given by

\[
\Delta I' = \begin{bmatrix}
    f' & \Delta I_f' \\
    g' & \Delta I_g' \\
    h' & \\
    i' & 
\end{bmatrix}
\]

... (3.87)

The equation of performance of the complete active network is

\[
\begin{array}{cccc}
    f' & g' & h' & i' \\
    \Delta I_f' & y^{3}+y^{4}+y^{bb_1} & -y^{bb_1} & y^{be_1} \\
    \Delta I_g' & y^{cc_2} & y^{ce_2} & y^{ce_2} \\
    \Delta I_h' & -y^{bb_1} & y^{ec_2} & y^{5+y_6+y^{bb_1}+y^{ee_2}} \\
    \Delta I_i' & y^{eb_1} & y^{ec_2} & y^{5-y^{eb_1}+y^{ee_2}} & y^{5+y^{ee_1}+y^{ee_2}} \\
\end{array}
\]

... (3.88)
The equations of the partitioned equation (3.08) are

\[ \Delta I^j = Y^{jm} \Delta E_m + Y^{jn} \Delta E_n \quad \ldots (3.89) \]

\[ \Delta I^k = Y^{km} \Delta E_m + Y^{kn} \Delta E_n \quad \ldots (3.90) \]

From equation (3.90), \( \Delta E_n = (Y^{kn})^{-1} (\Delta I^k - Y^{km} \Delta E_m) \)

\ldots (3.91)

Substituting for \( \Delta E_n \) from equation (3.91) into equation (3.89) gives

\[ \Delta I^j = Y^{jm} \Delta E_m + Y^{jn} (Y^{kn})^{-1} (\Delta I^k - Y^{km} \Delta E_m) \]

\[ = Y^{jm} \Delta E_m + Y^{jn} (Y^{kn})^{-1} Y^{km} \Delta E_m + Y^{jn} (Y^{kn})^{-1} \Delta I^k \]

\ldots (3.92)

It is obvious, from equation (3.88), that the components of \( \Delta I^k \) are each zero, therefore, the term \( Y^{jn} (Y^{kn})^{-1} \Delta I^k \) of equation (3.92) is zero. Also, from equation (3.88)

\[ Y^{jm} = \begin{array}{c|c|c|c}
  & f' & g' \\
\hline j & m \\
\hline f' & 3 & 4 & bb_1 \\
\hline g' & & & c_{c2} \\
\end{array} \]

\[ Y^{jn} = \begin{array}{c|c|c|c}
  & f' & i' \\
\hline j & n \\
\hline f' & -bb_1 & y_{be_1} \\
\hline g' & y_{ce_2} & y_{ce_2} \\
\end{array} \]
\( Y^{km} = \begin{array}{c|c}
  f' & g' \\
  \hline 
  -y_{bb1} & y_{ec2} \\
  y_{eb1} & y_{ec2} \\
\end{array} \)

\( Y^{kn} = \begin{array}{c|c}
  h' & i' \\
  \hline 
  y_{5+y_{bb1+y_{ee2}}} & y_{5+y_{be1+y_{ee2}}} \\
  y_{5-y_{eb1+y_{ee2}}} & y_{5+y_{ee1+y_{ee2}}} \\
\end{array} \)

\( (Y^{kn})^{-1} = \frac{1}{\text{Det}} X \begin{array}{c|c}
  h' & i' \\
  \hline 
  y_{5+y_{ee1+y_{ee2}}} & y_{be1-y_{ee2}-y_{5}} \\
  y_{be1-y_{ee2}-y_{5}} & y_{5+y_{bb1+y_{ee2}}} \\
\end{array} \)

where \( \text{Det} = \det(Y^{kn}) \)

\( = 6 \cdot y_{(Y^{5+y_{ee1+y_{ee2}}}+y_{bb1+y_{ee1-y_{be1+y_{eb1}}}})} + (Y^{5+y_{ee2}})(y_{bb1+y_{be1+y_{eb1+y_{ee1}}}}) \)

\( (Y^{kn})^{-1} Y^{km} = \begin{array}{c|c|c}
  h' & i' & k' \\
  \hline 
  y_{5+y_{ee1+y_{ee2}}} & y_{be1-y_{ee2}-y_{5}} & h' \\
  y_{be1-y_{ee2}-y_{5}} & y_{5+y_{bb1+y_{ee2}}} & i' \\
\end{array} \)

\( \frac{1}{\text{Det}} X \begin{array}{c|c|c}
  h' & i' & k' \\
  \hline 
  y_{5+y_{ee1+y_{ee2}}} & y_{be1-y_{ee2}-y_{5}} & h' \\
  y_{be1-y_{ee2}-y_{5}} & y_{5+y_{bb1+y_{ee2}}} & i' \\
\end{array} \)
\[ \frac{1}{\text{Det} \, X} = \begin{array}{c|c}
\text{h}' & \text{i}' \\
\hline
-y \, \text{bb}_1(y^5 + y \, \text{ee}_1 + y \, \text{ee}_2) & y \, \text{ec}_2(y \, \text{ee}_1 + y \, \text{be}_1) \\
+y \, \text{eb}_1(y \, \text{be}_1 - y \, \text{ee}_2 - y^5) & \\
\end{array} \]

\[ y \, \text{jn} \, (Y^{kn})^{-1} \, Y^{km} = \begin{array}{c|c}
\text{f}' & \text{g}' \\
\hline
-y \, \text{bb}_1 & y \, \text{be}_1 \\
y \, \text{be}_1 & y \, \text{co}_2 \\
\end{array} \]

\[ \frac{1}{\text{Det} \, X} = \begin{array}{c|c}
\text{f}' & \text{g}' \\
\hline
-y \, \text{bb}_1(y^5 + y \, \text{ee}_1 + y \, \text{ee}_2) & y \, \text{ec}_2(y \, \text{ee}_1 + y \, \text{be}_1) \\
+y \, \text{eb}_1(y \, \text{be}_1 - y \, \text{ee}_2 - y^5) & \\
\end{array} \]

\[ \frac{1}{\text{Det} \, X} = \begin{array}{c|c}
\text{f}' & \text{g}' \\
\hline
y \, \text{bb}_1(y \, \text{bb}_1 \, y \, \text{ee}_1 - y \, \text{be}_1 \, y \, \text{eb}_1) & y \, \text{ec}_2(y \, \text{ee}_1 + y \, \text{be}_1 - (y \, \text{bb}_1 \, y \, \text{ee}_1 - y \, \text{be}_1 \, y \, \text{eb}_1)) \text{\_\_} \\
y \, \text{bb}_1(y^5 + y \, \text{ee}_2)(y \, \text{bb}_1 + y \, \text{eb}_1) & +y \, \text{be}_1(y^5 + y \, \text{ee}_2)(y \, \text{bb}_1 + y \, \text{eb}_1) + y \, \text{be}_1 \, y \, \text{eb}_1 \, y^6 \text{\_\_} \\
\end{array} \]

\[ \frac{1}{\text{Det} \, X} = \begin{array}{c|c}
\text{f}' & \text{g}' \\
\hline
y \, \text{ce}_2(y \, \text{ee}_1 + y \, \text{be}_1 - (y \, \text{bb}_1 \, y \, \text{ee}_1 - y \, \text{be}_1 \, y \, \text{eb}_1)) \text{\_\_} & y \, \text{ce}_2(y \, \text{ee}_1 + y \, \text{be}_1 + y \, \text{be}_1 + y \, \text{ee}_1 + y^6) \text{\_\_} \\
\end{array} \]
\[
\begin{align*}
\mathbf{y}^\mathbf{jm} - \mathbf{y}^{jn} & = \mathbf{k}^{n-1} \mathbf{y}^{\mathbf{kn}} = \\
\begin{array}{c|cc}
\mathbf{f}' & \mathbf{f}'' \\
\hline
\mathbf{f}' & \mathbf{f}'' \\
\mathbf{g}' & \mathbf{g}'' \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\mathbf{f}' &= \mathbf{f}'' = \\
&= \frac{y^{bb_1(y^{bb_1 y ee_1 y} \ y^{bb_1 y eb_1}) + y^{bb_1 (y^{5 y ee_2} (y^{bb_1 y eb_1}) + y^{be_1 (y^{5 y ee_2} (y^{bb_1 y eb_1}) + y^{be_1 y eb_1})} }{\text{Det}} \\
\mathbf{g}' &= \mathbf{g}'' = \\
&= -\frac{y^{ce_2 (y^{be_1 y eb_1} (y^{bb_1 y ee_1 y} \ y^{bb_1 y eb_1})}{\text{Det}} \\
&= \frac{y^{cc_2 (y^{be_1 y eb_1} (y^{5 y ee_2} (y^{bb_1 y eb_1}) + y^{be_1 (y^{5 y ee_2} (y^{bb_1 y eb_1}) + y^{be_1 y eb_1})}}{\text{Det}}
\end{align*}
\]

\[\text{... (3.93)}\]
The equation (3.93) characterizes the complete active network reduced to a simple two-port network and can be written as

$$f' \Delta f, g' \Delta g = f' Y_{11} Y_{12} g' Y_{21} Y_{22} g' \Delta E_y, \Delta E_g,$$

where

$$Y_{11} = Y + 4 y b b_1 - \frac{y \delta y b 1 (y b b_1 y e e_1 - y b 1 y e b_1)}{6 (y e e_1 + y e e_2) (y b b_1 y e e_1 - y b 1 y e b_1)}$$

$$Y_{12} = \frac{y \delta y e 2 (y b b_1 y e 1) - (y b b_1 y e e_1 - y b 1 y e b_1)}{y^{y 6} (y e e_1 + y e e_2) + (y b b_1 y e e_1 - y b 1 y e b_1) + (y b b_1 y e e_1 + y b b_1 y e b_1 + y e e_1)}$$

$$Y_{21} = \frac{y \delta y e 2 (y b b_1 y e 1) - (y b b_1 y e e_1 - y b 1 y e b_1)}{y^{y 6} (y e e_1 + y e e_2) + (y b b_1 y e e_1 - y b 1 y e b_1) + (y b b_1 y e e_1 + y b b_1 y e b_1 + y e e_1)}$$
\[ y^{22} = y^{cc2} - \frac{y^{ee}_2 y^c e_2 \left( -y^{bb1+ y^{eb1} + y^{ee1} + y^b} \right)}{Y^6 \left( y^{ee1} + y^{5+ y^{ee2}} + (y^{bb1} + y^{ee1} + y^{eb1}) \right)} \]

But \( y^{ee2} \) and \( y^{cc2} \) are very small compared to the other parameters of the transistors. Hence it can be written as:

\[ y^{bb1}(y^{bb1} + y^{ee1} + y^{eb1}) + y^{bb1}(y^{5+ y^{ee2}}) \]

\[ y^{11} = y^{3+ y^{4+ y^{bb1}}} \]

\[ y^{12} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (3.95) \]

\[ y^{21} = -\frac{y^{ee2} y^e_{bb1} y^b_1 \left( y^{b_{bb1}} e_{ee1} - y^{b_{eb1}} \right)}{Y^6 \left( y^{ee1} + y^{5+ y^{ee2}} + (y^{bb1} + y^{ee1} + y^{eb1}) \right)} \]

\[ y^{22} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (3.97) \]
Analysis of the complete Wien Bridge Oscillator Network:

The junction equivalent of the Wien bridge oscillator network is shown in Fig. 3.25.

The old admittance tensor $Y$ of the interconnected network is given by

$$
Y = \begin{bmatrix}
\alpha & \beta & 2 & 3 & 4 & 5 & 6 & b_1 & e_1 & e_2 & c_2 \\
\alpha & \beta & 2 & 3 & 4 & 5 & 6 & b_1 & e_1 & e_2 & c_2 \\
& & Y^2 & & & & & & & & \\
& & Y^3 & & & & & & & & \\
& & Y^4 & & & & & & & & \\
& & Y^5 & & & & & & & & \\
& & Y^6 & & & & & & & & \\
& & & & & & & y_{bb_1} & y_{be_1} & & \\
& & & & & & & y_{eb_1} & y_{ee_1} & & \\
& & & & & & & y_{ee_2} & y_{ec_2} & y_{cc_2} & \\
& & & & & & & y_{ce_2} & y_{cc_2} & & \\
\end{bmatrix}
$$

... (3.98)
Fig. 3.25. Junction equivalent of Wienbridge Oscillator.
where \( Y^\alpha = \frac{1}{R_1}, \quad Y^\beta = j\omega C_1, \quad Y^2 = \frac{j\omega C_2}{1+j\omega C_2 R_2}, \quad Y^3 = \frac{1}{R_3}, \quad Y^4 = \frac{1}{R_4}, \quad Y^5 = \frac{1}{R_5}, \) and \( Y^6 = \frac{1}{R_6} \)

\[ \text{... (3.99)} \]

Fig. 3.25 shows the branch voltages and the junction-pair response voltages. The corresponding equations expressing the relationships between branch and junction-pair voltages give the junction-pair voltage transformation tensor \( A \) as

\[
\begin{align*}
\Delta E_\alpha &= \Delta E_s, \\
\Delta E_\beta &= \Delta E_s, \\
\Delta E_2 &= \Delta E_p + \Delta E_q - \Delta E_s, \\
\Delta E_3 &= \Delta E_p + \Delta E_q, \\
\Delta E_4 &= \Delta E_p + \Delta E_q, \\
\Delta E_5 &= \Delta E_q + \Delta E_r, \\
\Delta E_6 &= \Delta E_q, \\
\Delta E_b &= \Delta E_p, \\
\Delta E_{e_1} &= \Delta E_r, \\
\Delta E_{e_2} &= \Delta E_q + \Delta E_r, \\
\Delta E_{c_2} &= \Delta E_s.
\end{align*}
\]
The transpose of $A$, obtained by interchanging the columns and rows of the right hand member of equation (3.100), is

$$A_t = \begin{array}{cccccccc}
\alpha & \beta & 2 & 3 & 4 & 5 & 6 & b_1 & e_1 & e_2 & e_2 \\
p' & 1 & 1 & 1 & 1 & 1 & 1 \\
q' & 1 & 1 & 1 & 1 & 1 & 1 \\
r' & 1 & 1 & 1 & 1 & 1 & 1 \\
s' & 1 & 1 & -1 & 1 & 1 & 1 \\
\end{array} \quad \ldots \quad (3.101)$$
The admittance tensor $Y'$ of the actual network is obtained from the relation $Y' = A^t(Y . A)$ as

$$p' = \begin{bmatrix} y^{2} + y^{3} + y^{4} + y^{bb} & y^{2} + y^{3} + y^{4} & y^{be} & -y^2 \\ y^{2} + y^{3} + y^{4} & y^{5} + y^{6} + y^{ee} & y^{5} + y^{ee} & -y^2 + y^{ec} \\ y^{eb} & y^{5} + y^{ee} & y^{ee} + y^{5} + y^{ee} & y^{ec} \\ -y^2 & -y^{2} + y^{ce} & y^{ce} & y^{ce} \\ y^{2} + y^{3} + y^{4} & y^{5} + y^{6} + y^{ee} & y^{5} + y^{ee} & -y^2 + y^{ec} \\ -y^2 & -y^{2} + y^{ce} & y^{ce} & y^{ce} \end{bmatrix}$$

The equation of performance of the network under oscillatory condition is obtained by equating the determinant $D'$ of $Y'$ to zero. Hence under oscillatory condition

$$D = \det (Y') =$$

$$p' = \begin{bmatrix} y^{2} + y^{3} + y^{4} + y^{bb} & y^{2} + y^{3} + y^{4} & y^{be} & -y^2 \\ y^{2} + y^{3} + y^{4} & y^{5} + y^{6} + y^{ee} & y^{5} + y^{ee} & -y^2 + y^{ec} \\ y^{eb} & y^{5} + y^{ee} & y^{ee} + y^{5} + y^{ee} & y^{ec} \\ -y^2 & -y^{2} + y^{ce} & y^{ce} & y^{ce} \end{bmatrix}$$

$$= 0$$

$$... (3.103)$$
If it is assumed that $y^e_2$ and $y^c_2$ are very small compared to other parameters, equation (3.103) reduces to

$$\begin{align*}
\left( Y + Y + Y \right) \\
\left( Y + Y + Y \right) + y^b_1 - \frac{y^b_1(y^b_1 y^c_1 y^e_1 y^b_1) + y^b_1(y^5 y^e_2)}{Y^6 (y^c_1 y^e_1 y^e_2) + (y^b_1 y^c_1 y^e_1 y^b_1)}
\end{align*}$$

... (3.104)

With the help of equations (3.94) and (3.96), the equation (3.104) can be written as

$$(Y + Y + Y)^2 \left[ \frac{Y^6 y^b_1 y^e_1 y^c_2 y^e_2}{Y^6 (y^c_1 y^e_1 y^e_2) + (y^b_1 y^c_1 y^e_1 y^b_1)} \right] + (Y + Y + Y)^2 = 0 \quad \ldots (3.105)$$

Substituting the values of $Y^a$, $Y^b$ and $Y^2$ from equation (3.99) into equation (3.105) gives

$$\left( \frac{1}{R_1} + j \omega C_1 + \frac{j \omega C_2}{1 + j \omega C_2 R_2} \right) Y^{11} + \frac{j \omega C_2}{1 + j \omega C_2 R_2} Y^{21} + \left( \frac{1}{R_1} + j \omega C_1 \right)
\left( \frac{j \omega C_2}{1 + j \omega C_2 R_2} \right) = 0 \quad \ldots$$
or

\[
\begin{align*}
\angle Y^{11}( \frac{1}{R_1} + \frac{(\omega C_2)^2 R_2}{1+\omega C_2 R_2^2} )^2 + \frac{(\omega C_2)^2 \frac{R_2}{R_1} - (\omega)^2 C_1 C_2}{1 + (\omega C_2 R_2^2)^2} + \frac{(\omega C_2)^2 R_2^2 Y^{21}}{1 + (\omega C_2 R_2^2)^2} & = 0 \\
+ j \omega \angle Y^{11}(C_1 + \frac{C_2}{1+\omega C_2 R_2^2} )^2 + \frac{(\omega C_2)^2 C_1 R_2 + \frac{C_2}{R_1}}{1 + (\omega C_2 R_2^2)^2} + \frac{C_2 Y^{21}}{1 + (\omega C_2 R_2^2)^2} & = 0 \\
\end{align*}
\]

(3.106)

Equating real and imaginary parts to zero gives

\[
\begin{align*}
Y^{11}( \frac{1}{R_1} + \frac{(\omega C_2)^2 R_2}{1+\omega C_2 R_2^2} )^2 + \frac{(\omega C_2)^2 \frac{R_2}{R_1} - (\omega)^2 C_1 C_2}{1 + (\omega C_2 R_2^2)^2} + \frac{(\omega C_2)^2 R_2^2 Y^{21}}{1 + (\omega C_2 R_2^2)^2} & = 0 \\
\end{align*}
\]

(3.107)

and

\[
\begin{align*}
Y^{11}(C_1 + \frac{C_2}{1+\omega C_2 R_2^2} )^2 + \frac{(\omega C_2)^2 C_1 R_2 + \frac{C_2}{R_1}}{1 + (\omega C_2 R_2^2)^2} + \frac{C_2 Y^{21}}{1 + (\omega C_2 R_2^2)^2} & = 0 \\
\end{align*}
\]

(3.108)

Multiplying equation (3.108) by \( \omega C_2 R_2 \) and subtracting from equation (3.107) gives

\[
\begin{align*}
Y^{11}( \angle - \frac{1}{R_1} - (\omega)^2 C_1 C_2 R_2 \angle - \frac{\omega^2 C_1 C_2}{1 + (\omega C_2 R_2^2)^2} \angle Y^{21} = 0 \\
\end{align*}
\]
or

\[ \omega^2 c_1 c_2 r_1 r_2 = \frac{1}{1 + \frac{1}{y_{11} r_2}} \]

or

\[ \omega = \sqrt{c_1 c_2 r_1 r_2 \left(1 + \frac{1}{y_{11} r_2}\right)} - \frac{1}{2} \] ... (3.109)

From equation (3.107),

\[ y_{21} = -y_{11} \left[ \frac{1}{(\omega c_2)^2 r_1 r_2} + 1 - \frac{1}{r_1} + \frac{c_1}{c_2 r_2} \right] \]

or

\[ y_{21} = -y_{11} \left[ \frac{1}{(\omega c_2)^2 r_1 r_2} + \frac{r_2}{r_1} + 1 - \frac{1}{r_1} + \frac{c_1}{c_2 r_2} \right] \] ... (3.110)

From equations (3.109) and (3.110),

\[ y_{21} = -y_{11} \left[ \frac{c_1}{c_2} + \frac{r_2}{r_1} + 1 - \frac{1}{r_1} + \frac{c_1}{c_2 r_2} \right] \]

... (3.111)

Rearranging equation (3.111) gives

\[ y_{21} = -\frac{1}{c_2 r_1} \left[ -y_{11} (c_1 r_1 + c_2 r_2 + c_2 r_1) + c_2 \right] \] ... (3.112)
The equation (3.109) gives the angular frequency of oscillation of Wien bridge oscillator and the equation (3.112) gives the condition for sustained oscillation in the circuit.

3.4 DISCUSSIONS

The expressions of frequency and condition of oscillations, as derived in this chapter by tensor method of approach, are found to be in perfect agreement with those given by earlier workers\[^{16,25,26}\]. In deriving final expressions similar approximations are made as used by other workers. But these approximations are quite justified from the practical point of view.