CHAPTER 1

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General topology - one of the most coherent doctrines in mathematics, was formulated in the early part of last century and owes its rise to two main sources: geometry and elements of analysis. The notion of geometry first started its development at the worthy hands of Euclid and other ancients and completed a full circle with the inaugural address of Riemann in 1851 on the hypothesis which serves as the foundation of geometry. Again, the concepts of limit and continuity dawned on the ancient Greek as they were trying to make precise the notion of numbers. It was then the turn of Cauchy (1821) and Abel (1823) to add the final touches of professionalism to these age old concepts.

Accordingly, topology, since its commencement, has derived inspiration and nourishment from these sources and proceeded in two main directions. One of these branches aims to place the study of formal Euclidean geometry on a generalized footing in terms of manifolds, homology theory and theory of dimensions. The other line of progress has been to pinpoint and analyse the abstract notions behind our so well-recognised compactness, convergence, connectedness, continuous functions etc. The language of topology is of paramount importance today in subjects as Integration Theory, Harmonic Analysis, Functional Analysis etc.
Since the advent of systematic study of Analysis, the enormous potentiality and applicability of concepts like compactness (characterized on $\mathbb{R}^n$ by the Heine-Borel Theorem) have been universally agreed upon. So work has poured in from all quarters to generalize 'compactness' and enunciate other compact-like covering properties as revealed in the study of paracompact spaces, almost compact spaces, nearly compact spaces and the like. A landmark in this development came with the introduction of the so-called $H$-closed spaces in 1924 by Alexandroff and Urysohn [4], which has generated immense interest over the years. According to Alexandroff and Urysohn, a Hausdorff space $X$ is called $H$-closed if it is closed in every Hausdorff space in which it is embedded. In this paper, they left an open question: Can every Hausdorff space be embedded in an $H$-closed space? The question was answered in the affirmative by Tychonoff [198] in 1930, which opened up a scope for a lot of further investigations along this line. In the ensuing periods of time, $H$-closedness was studied from various standpoints and in terms of different tools and techniques like (i) filter and filterbases, (ii) nets, (iii) cotopology, (iv) covering and proximate subcovering, (v) $\theta$-closure operator and allied concepts, (vi) closed graph techniques, (vii) through projectiveness and (viii) via grills. In view of the extensive investigations undertaken by a large number of researchers during the last eighty years or so, a huge amount of results has piled up many of which are of fundamental importance to topology and enormous potentiality for finding important applications in associated branches as well.

Several other weaker versions of compactness than that of $H$-closedness naturally came up for investigation. One such widely studied concept is that of near compactness introduced by Singal and Mathur [182]. They used the concept of regular open sets of M. H. Stone [194] to define near compactness in terms
of regular open sets. It was found that the class of nearly compact spaces is strictly lying between the classes of compact spaces and quasi \( H \)-closed spaces. Also known is the fact that the class of regular open sets in a topological space \( (X, \tau) \) forms a base for a weaker topology \( \tau_s \) on \( X \), called the semiregularization topology on \( X \), and \( (X, \tau) \) is nearly compact iff \( (X, \tau_s) \) is compact [121].

In [91] it has been shown by Joseph that a topological space \( X \) is nearly compact iff for every space \( Y \), the projection map from \( X \times Y \) to \( Y \) takes \( \delta \)-closed subsets onto \( \delta \)-closed subsets. In the same paper, Joseph has also improved the result by Singal and Mathur [182] that the topological products of nearly compact spaces is nearly compact to the following form: an arbitrary product of topological spaces is nearly compact iff each of its factor spaces is so. Further extensive studies on near compactness can be located in the work of Joseph [91, 92], Carnahan [19], Herrington[70], Singal and Mathur [182, 183] and Noiri [143].

The introduction of semiopen sets by Levine [108] in 1963 ushered in a new era for topologists who found a generalized setting for a newer study of already known concepts. Thompson [195] applied this concept successfully when he replaced open cover by semiopen cover in the definition of quasi \( H \)-closed spaces and formulated the so-called \( S' \)-closed space. Later investigations revealed that \( S \)-closedness implies quasi \( H \)-closedness, the converse requiring the additional condition of extremal disconnectedness [18]. As to the role of compactness in the study of \( S \)-closedness, Thompson [195] further showed that for compact Hausdorff spaces, the concept of \( S \)-closed space is equivalent to the concepts of extremally disconnectedness and projectiveness. Extensive information concerning \( S \)-closed spaces may be found in the works of Cameron [18], Ganster and Reilly [61], Mukherjee and Basu [124], Noiri [140, 141] etc.
Mayo and Noiri [113] interchanged the closure operator in the definition of $S$-closed spaces with semiclosure operator to innovate a new class of topological spaces called $s$-closed spaces. Further addition to this topic came from Ganster and Reilly [61], Ganguly and Basu [54, 55, 56] and Dorsett [47], who adopted a number of approaches for their study of $s$-closedness. Ganster and Reilly [61] have proved the interesting result that every infinite topological space can be embedded as a closed subspace of a connected $S$-closed space which is not $s$-closed. Dorsett [47] has shown among other things that $s$-closedness is a topological property. Ganguly and Basu [54, 55, 56] have also investigated extensively and contributed substantial results in this field. The concept of $S$-closedness and $s$-closedness have further been extended to arbitrary subsets of a space yielding even more generalized results. Noiri [140] and Mayo and Noiri [113] initiated respectively the investigations of $S$-closed and $s$-closed sets relative to a space. Several researchers have amply been motivated for further investigation of the said topics by these studies.

Apart from compactness and compact-like covering properties, the most coveted and celebrated weaker form of compactness is by far the notion of paracompactness. In 1944, Dieudonné [41] first introduced this most remarkable covering property viz. paracompactness which is known to be an extremely important generalization of compactness. Then followed in its wake a barrage of celebrated results as tributary concepts like countable paracompactness, near and almost paracompactness, m and mn-paracompactness etc. all of which have attracted diligent investigations over the years. During the last seven decades or so, the class of paracompact spaces and its allied notions have been studied extensively and the development of these kinds of covering properties, attained so far by many eminent mathematicians, is simply superb. A new class of paracompact
spaces viz. countable paracompact spaces was initiated independently by Dowker [48] and Katetov [103] in 1951. Hayashi [69] and Horne [77], contributed significantly to the study of countable paracompact spaces and derived different sorts of characterizations. The contribution of Iseki [82, 83, 84] is undoubtedly remarkable in this field. In the middle of the last century Giovani [63] and Morita [120] defined a new type of paracompactness termed as \(m\)-paracompactness which, by definition, requires every open cover of cardinality \(\leq m\) to admit of a locally finite open refinement. Many reputed mathematicians like Ishii [85, 86], Singal and Arya [179] and Mack [112] produced a good number of results in this area.

1.1. OPERATORS

The Kuratowski closure operator and its dual, the interior operator are the two pivotal concepts in set topology, which are instrumental for obtaining several crucial types of sets, various kinds of operators and different sorts of generalized types topological properties.

Among the various types of operators, the most popular one is \(\theta\)-closure operator initiated by Veličko [199]. Veličko introduced this concept, keeping an eye not only to investigate the \(H\)-closed spaces but at the same time to generalize Taimanov’s extension theorem in [200]. However, the \(\theta\)-closure operator proved efficient beyond expectation and in the periods to follow, it was made to bear successfully on widely researched objects like separation axioms, maximal and minimal spaces with respect to some topological properties and theorems of extensions of spaces and introduction and applications of different types of functions. Čech [20] particularly took an interest to initiate studying the closure spaces obtained from closure operator; and as this closure operator differs from
the widely known Kuratowski closure operator in not being idempotent, this was bound to produce interesting results. They were followed up by Dickman and Porter [39] who remarked that a Hausdorff space $X$ is Urysohn if the $\theta$-closure operator on $X$ is an idempotent one. However, they left aside the question of characterizing all spaces in which the $\theta$-closure operator is idempotent. This was later attacked by Herrman [74] and still later by Long and Herrington [111] who established that the collection of $\theta$-open sets (complements of $\theta$-closed sets) forms a topology on $X$, in spite of the fact that $\theta$-closure of a set need not be $\theta$-closed. Fomin [52] introduced a class of functions, called $\theta$-continuous, which was later on found to be very useful in some topological investigations of non-regular Hausdorff spaces. After more than three decades, Dickman and Porter [40] took up the study of $\theta$-continuous functions in terms of the $\theta$-closure operator and ultimately solved the problem relating to the extension of such functions between Hausdorff spaces.

Another useful operator, namely the $\delta$-closure operator, also owes its origin to Velicko [199]. He proved the following: (i) the $\delta$-closure operator is idempotent, (ii) it gives a topology on a topological space that coincides with the semi-regularization topology of the space, (iii) $\delta$-closure of any subset is contained in its $\theta$-closure, (iv) these two operators coincide in a regular space and with the closure operator of the space. Result (iv) found further upgradation at the hands of Noiri [144] where the regularity of the space was relaxed to almost regularity. The $\delta$-closure operator has also proved itself to be of much use in the investigation of $\delta$-continuous functions and nearly compact spaces.

Among other operators in vogue are semiclosure and its dual semiinterior operators. Levine [108] and Biswas [13] respectively initiated semiopen and semiclosed sets. Crossley and Hildebrand [34] took their lead and introduced semiclosure op-

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operators in a space. It is understood that both semiclosure operator and its variant \(\theta\)-semiclosure operator introduced by Noiri [147] are idempotent. Mayo and Noiri in [113] proposed another operator called the semi-\(\theta\)-closure operator. Semiopen sets and \(\theta\)-semiclosure operators are since then being used effectively to study \(S\)-closed spaces, while in the case of \(s\)-closed spaces, one prefers the semiclosure and semi-\(\theta\)-closure operators.

In comparatively recent time, another operator named preclosure operator, introduced by Mashhour et al. [114], has found wide acceptance among the researchers. This operator has been used effectively in the analysis of \(p\)-closedness. Chapter 3 of this thesis makes use of this operator and its dual the preinterior operator together with \(p(\theta)\)-closure and \(p(\theta)\)-interior operators.

1.2. GRILLS

The introduction of grills by Choquet [28] in 1947 was a landmark in the progress of topology. Subsequently researchers have revealed that the efficacy of grills has superseded that of its older counterparts, the nets and filters in certain areas. The theory of grills gained more ground in the works of Thron [197] who established the following pioneering results:

(i) Every grill is a union of ultrafilters contained in it.
(ii) Arbitrary union of ultrafilters is a grill.
(iii) An ultrafilter on a space \(X\) is a maximal proper grill on \(X\).
(iv) If \(\mathcal{F}\) and \(\mathcal{G}\) are respectively a filter and a grill on a set \(X\) with \(\mathcal{F} \subseteq \mathcal{G}\), there is an ultrafilter \(\mathcal{U}\) on \(X\) such that \(\mathcal{F} \subseteq \mathcal{U} \subseteq \mathcal{G}\).

With these proved, the stage was set for the concept of grills to find effective use in the investigation of proximity spaces, closure spaces, quasi \(H\)-closed spaces.
etc. This is exemplified in the papers [22, 24, 25, 197] by Chattopadhyay et al. Particularly in [25], they have considered adherence grills of points and Herlich [73] suggested the idea of convergence of the grills for their investigations of closure spaces and certain extension problems by sheer use of grills. They proved that the class of adherence grills of all points of $X$ completely determines the Kuratowski closure operator $K$ of a closure space $(X, K)$. Chattopadhyay and Thron [24] also introduced the $c$-grill with respect to the closure space. Thereafter Chattopadhyay, Guin and Thron [23] nicely adopted different types of grills e.g. linked grill and $c$-grill, and in [22] the authors also applied $c$-grills and adherent grills in the investigation of Wallman nearness compatible with a given $T_1$-space.

The ingenious use of grills in the study of quasi $H$-closed spaces can be found in Mukherjee and Debray [125] who also improved the theory of grills to some extent. In some recent papers also (see [127, 134, 135]) grills are being used as an effective appliance for certain important investigations. For example, in [172] Roy and Mukherjee presented a novel method of construction of the one-point compactification of a locally compact non-compact space.

In the major part of this thesis too the theory of grills has assumed to be an excellent machinery and a highly appropriate tool for manoeuvring certain investigations. In other words, this thesis adopts and uses the concept of grills with elan for manipulating and facilitating investigations in different chapters. Chapter 2 of this thesis comprises the findings on the unified theory concerning compactness and quasi $H$-closedness and certain other covering properties via the concepts of grills, as laid down in [129, 186]. Theory of grills has been extended to successfully suit our study of $s$-closed spaces in [187, 137]. Chapter 4 deals on this matter at length.
1.3. CLUSTER SETS

The concept of cluster sets was originated long back for paying serious attention in the arena of real and analytic function theory. A substantial degree of abstraction in formulating theorems on cluster sets was brought by the works of Hunter [78]. Real valued functions defined on the set of real numbers have notable symmetric properties that can be put nicely in terms of cluster sets, as obtained by Young [205]. A celebrated name in the field of cluster sets is Collingwood [29], who achieved a fundamental generalization of radial cluster sets and a very elegant theorem on the boundary behaviour of functions in the unit circle, where these fundamental results are independent of the properties peculiar to analytic functions. His classical book "The Theory of Cluster Sets" along with Lohwater [30] is a storehouse of information on the study of cluster sets of arbitrary functions. Also one must not forget the natural generalization by Gomonov [65] of cluster sets to gauge the behaviour of multivalued functions close to an arbitrary collection of elements in a domain.

In topological frameworks the study of cluster sets for arbitrary functions between topological spaces was initiated by Weston [203]. Subsequently many researchers have investigated the properties of variegated types of cluster sets of functions and multifunctions, and a good many substantial results have been obtained. In this connection one must mention the brilliant works of Hamlet [66, 67, 68] concerning cluster sets of functions between two topological spaces. The extension of cluster sets to multifunctions occurred at the hands of Joseph [98]. Extensions and generalizations were also effected by him of some characterizations of cluster sets of functions. Moreover, some valuable characterizations of properties of graph functions were done by him. In [132] by Mukherjee and Raychoudhuri, $\delta$-cluster sets of suitable functions and multifunctions have been pro-
posed and used expertly to manipulate investigations on nearly compact, nearly Lindelöf and almost regular spaces, δ-continuous functions and certain types of multifunctions.

Following their lead, we have tried to investigate s-closed spaces in terms of a kind of cluster sets of functions and multifunctions in [187]. In Chapter 4 of this thesis, we have listed our findings regarding the characterization of s-closed spaces, a number of results providing conditions for degeneracy of such cluster sets and equivalent condition for Hausdorffness of a space.

1.4. PREOPEN SETS AND

$p$-CLOSED SPACES

Mathematicians have over the years used concepts like semiopen sets [108], and regular open sets [194] to investigate semicompact spaces, $S$-closed and s-closed spaces, near compactness etc. Of late (last two decades) a new concept has been increasingly making itself prominent in this regard, namely the so-called preopen sets. The said concept, originally envisaged by Corson and Michael [31] as ‘locally dense’ or ‘nearly open’ underwent a change of nomenclature at the hands of Mashhour et al. [114] to ‘preopen’. Preclosed set, preclosure, preinterior etc also appeared in due time. El-Deeb, Hasanein, Mashhour and Noiri [49] proved the idempotence and increasing property of the preclosure operator, so that the preclosure of a set $A$ in a topological space is preclosed. It is known that the class of preopen sets contains the class of open sets. An important observation was made by Dontchev et al. [44] that when a certain topological property is inherited by both open and dense sets, it is often inherited by preopen sets.
The notion of preopen sets is extremely efficient in tackling different topological concepts, like Bourbaki's submaximal spaces [8]. Incidentally we may point out an interesting comment made by Dontchev et al. [44]: “a topological space is called submaximal if every (locally) dense subset is open or equivalently, if every subset is locally closed, i.e, the intersection of an open set and a closed set”. Another class of spaces commonly characterized in terms of preopen sets is the class of strongly irresolvable spaces introduced by Foran and Leibnitz [53]. Ganster [58] has pointed out that a space is strongly irresolvable iff every preopen set is open.

It is by now well known that although arbitrary union of preopen sets is preopen, the intersection of even two preopen sets need not be preopen, so that the collection of preopen sets in a topological space \( X \) does not form a topology in \( X \), in general. However, Noiri [146] has established a nice result that if \( A \) and \( B \) are respectively a preopen and a semiopen sets in a space \( X \), then \( A \cap B \) is semiopen in \( A \) and preopen in \( B \).

Among the attempts to replace the openness property in classical topological notions by preopenness one counts the works on strongly compact spaces initiated by Mashhour et al. in [116] and further followed up by Jankovic et al. [90] and Ganster [57]. Popa [155] studied preconnected spaces via preopen sets, and Noiri [146] and Ahmed and Noiri [2] have investigated hyperconnected spaces in terms of preopen sets.

In 1982, the notion of precontinuity was initiated by Mashhour et al. [116] using the preopen sets in place of open sets in the definition of continuity; although the same concept had been applied much earlier in different terminologies viz. near continuity by Ptak [164] and almost continuity by Husain [79]. Also ensued subsidiary concepts of preirresolute, \( p \)-almost continuous, faintly precontinuous
functions. Papers of Mashhour et al. [114, 115], Andrijevic [5, 6] and Noiri [146, 148] etc. contain valuable information in this regard. In recent time Dontchev [43] exhibited a brilliant survey taking into account a long list of papers relating to the preopen sets and allied notions, and covered a lot of results right from the beginning of such investigations.

In the definition of quasi $H$-closed spaces [4], if the roles of open sets and the closure operator are replaced by respectively those of preopen sets and pre-closure operator, one is led to another covering property, namely the so-called $p$-closedness. This concept was pioneered by Abo-Khadra [1]. From [44] we know that $p$-closedness implies $H$-closedness and is implied by strong compactness, but it is independent of compactness, near compactness, $S'$-closedness and $s$-closedness. Recently, $p$-closed spaces and subspaces, and $p$-closedness relative to a space have been investigated thoroughly by Dontchev [43].

Motivated by the above trend towards the rapidly flourishing area concerning $p$-closedness, we have ventured to add a few new things to the existing results on $p$-closed spaces. In [130] we have used different techniques involving $p(\theta)$-subclosed graphs and strong $p(\theta)$-closedness of graphs of functions to delve into the realm of $p$-closed spaces. Yet again in [136] we bring the tools of $p$-adherent points of sets, nets, filters, $p(\theta)$-complete adherent points of sets, $p(\theta)$-closed sets and order relation to bear upon $p$-closedness and its various characterizations. All these are laid down in detail in Chapter 3.

1.5. EXTENSIONS

Extension of a topological space is an age-old concept and such theories have been extensively investigated for a long time; special mention may be made of the
Alexander compactification of Tychonoff spaces, the one-point compactifications of $T_2$-locally compact, non-compact spaces and $H$-closed extensions of $T_2$-spaces. A comprehensive description specially regarding $H$-closed extensions can be had in the book of Porter and Woods [163]. Extension theory for $T_0$-topological spaces has also been developed (for instance see [9]). Closure spaces were introduced by Čech [20]. A general theory of extensions of closure spaces satisfying $G_0$-separation axiom was initiated by Chattopadhyay and Thron [24] by special use of grills, in which paper the authors found necessary and sufficient conditions for an extension of a $G_0$-closure space to become a compactification of it. In [25], Chattopadhyay et al. considered principal extensions of $T_1$-closure spaces by magnificent application of grills and found a characterizing condition for such an extension to become compact. They also introduced and treated conjointly compactness and linkage compactness in the same paper, and ultimately used all these ideas in connection with the study of certain nearness structures. In fact, they investigated the one-one correspondence between the class of concrete Riesz merotopic spaces on a given $T_1$-closure space and the class of principal $T_1$-extensions of the space. Again in [21], Chattopadhyay introduced regular extensions of closure spaces and established a one-one correspondence between the class of all concrete $R_1$-nearness on a given $R_1$-closure space and the class of all regular extensions of the space. Wallman-type extensions were introduced in [170], while Chattopadhyay, Guin and Thron [23] could successfully characterize those Wallman-type extensions which are compact or linkage compact.

Rather recently, a type of extensions, called $\theta$-extension of a topological space, was introduced by Mukherjee and Debray [128], the notion of $\theta$-principal extension was also initiated therein. After constructing a typical $\theta$-principal extension they achieved some characterizations of $H$-closedness of a Hausdorff space.
Motivated by all these investigations, we tried in [138] to initiate a kind of extensions, termed \( \delta \)-extension and a special type of it viz. \( \delta \)-principal extensions, with a similar line of approach of our predecessors. By considering the notions of \( \delta \)-equivalence, \( \delta \)-trace systems and certain allied ones, we have successfully constructed a \( \delta \)-extension of a topological space which serves for any \( \delta \)-principal extension of the space up to \( \delta \)-homeomorphism. The question of linkage near compactness of such an extension is also addressed; however, we leave the part concerning the investigation of near compactness vis-a-vis such an extension for future study. In Chapter 6 we discuss our findings regarding \( \delta \)-extensions.

1.6. MULTIVALUED FUNCTIONS

One comes across multivalued functions, playing pivotal roles in as diverse fields as graph theory, combinatorics, cluster set theory, theory of selections etc. and it has been worked upon from as many diverse angles by reputed mathematicians. Specials mention must be made here of the three remarkable papers by Joseph [91, 96, 98]. Detailed research has also been carried out by Smithson [188, 189, 190, 191]. In [188] Smithson studies some general properties of multifunctions and particularly concentrates upon the appropriate classes of multivalued functions to which the already known results of single valued function theory may be extended. In [190] he establishes the notion of subcontinuity for multifunctions and characterizes compact preserving multifunctions in terms of subcontinuity, and also tries to set forth a plausible condition to imply upper semicontinuity. Smithson [189] considers the uniform convergence of a class of multifunctions from a topological space to a uniform space; it also establishes that compact open topology and topology of uniform convergence are equivalent.
under certain constraints, which ultimately result in the derivation of the Ascoli theorem for multifunctions. Engelking [50] has characterized the regularity and normality of a topological space in terms of multifunctions, which Joseph [96] also achieved in terms of weakly upper semicontinuous multifunctions. A characterization of compactness via upper semicontinuous multifunctions may be found in [51] by Espelie and Joseph.

The notions of closed, strongly closed and $\theta$-closed graphs of multifunctions have been defined by Joseph [95] for the generalizations and extensions of different known results in multivalued setting, which produced a number of sufficient conditions for multifunctions to become upper semicontinuous. Some extensions of the uniform boundedness principle from analysis have also been found therein. Afterwards Popa [152, 153, 154, 157] and some others studied extensively multivalued version of weak continuity and characterized several concepts. Noiri and Popa [151] defined and characterized the notion of strongly irresolute multifunction.

The process of development of the theory of multifunctions, as detailed above, has been enough source of inspiration for us to get motivated towards some investigations in this area. We have thus in [133] considered at length the upper and lower $\theta^*$-continuous multifunctions and could successfully establish a good number of characterizations of such multifunctions. We also obtained some applications of certain real valued functions as special cases of the said concepts. Chapter 5 of this thesis provides a comprehensive description of our findings regarding the new classes of multifunctions. And also in the last section of Chapter 3, a few characterizations of $p$-closedness have been obtained under compatibility condition of certain preorder relation on a topological space, where another kind of multifunctions termed $p(\theta)$-continuous multifunctions comes into play. Ulti-
mately we have successfully achieved a fixed set theorem for multifunction on a $p$-closed space.

### 1.7. SOME NOTATIONS, TERMINOLOGIES AND DEFINITIONS

This section is meant for clarifying certain notations and terminologies to be followed all through and for elucidating some definitions.

Henceforth unless otherwise is stated explicitly, by a space $X$ (or $Y$), we shall mean a topological space $(X, \tau)$ (or $(Y, \sigma)$). The term neighbourhood will be abbreviated as 'nbd' and the neighbourhood system at a point $x$ in $X$ will be denoted by $\eta_x$. When $\tau$ stands for the topology on $X$, $\tau(x)$ stands for the collection of all open sets containing $x$.

The notations $clA$, $intA$ for a subset $A$ of a topological space $X$ will stand for the closure and interior of $A$ respectively. The symbols $\Lambda$, $\mathbb{R}$, $\mathbb{N}$ and $\Phi$ shall always denote respectively some indexing set, the set of all real numbers, the set of all natural numbers and the null set, if otherwise is not mentioned explicitly.

For a space $X$, by $P(X)$ will always be meant the power set of $X$.

**Definition 1.7.1.** Let $A$ be any subset of a topological space $(X, \tau)$. Then $A$ is called

(a) a regular open set [194] if $A = intclA$,
(b) a regular closed set if its complement is regular open,
(c) a semiopen set [108] if $A \subseteq clintA$,
(d) a semiclosed set [13] if its complement is semiopen,
(e) a semiregular set [113] if it is semiopen as well as semiclosed.

The set of all semiopen sets in a space $X$ will be denoted by $SO(X)$. 

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