Chapter – 3

SOLUTIONS THROUGH MODE MINIMIZATION (MAXIMIZATION)

3.1 Introduction

As discussed in chapter-2, it is quite convincing that the order quantity which corresponds to the stochastically smallest cost (or largest profit) is the best possible solution for any inventory model. Unfortunately for any arbitrary demand distribution it is not trivial to prove the existence or otherwise of such a solution.

From practical point of view, some Q_0, for which the most probable value of the total cost is the least, has some important appeal particularly to a risk averting decision maker. Thus the solution that minimizes the mode of the cost distribution merits some consideration. In fact, the 'Mode minimizing solution' Q_0 represents that Y for $Q = Q_0$ has the smallest mode among all $Q \neq Q_0$, and thus it has some conceptual similarity with the 'Minimax' solution. Similarly, we can speak of a 'Mode-maximizing solution,' in the context of a profit model.

It may not be easy to examine the superiority or otherwise of the mode minimizing (of maximizing) solution proposed here compared to the classical mean minimizing (maximizing) solution in terms of cost (profit). But sensitivity analysis may throw some light in favour of one or the other. In fact, mode minimizing solution is usually likely to be independent of cost components and that way to be more robust than the mean minimizing solution.

3.2 Mode-Minimizing Solution for Classical Newsboy Problem.

This model is a static risk model where demand is a random variable and only one order is placed during the whole planning horizon. A newspaper vendor starts his day with Q newspapers on hand, the number of newspapers demanded that day being a random variable. At the end of the day he either has some newspapers in excess or faces a shortage.

Thus the total effective cost Y_Q is given by

$$Y_Q = c_1 (X-Q) \text{ for } X \geq Q$$
$$= c_2 (Q-X) \text{ for } X \leq Q$$

where $c_1 =$ cost of running short of one unit

$c_2 =$ excess cost per unit left unsold

$X =$ demand with cumulative distribution function F(x) and probability

density function f(x).

As derived in chapter 2, the probability density function of Y_Q is

$$g_Q(y) = \frac{1}{c_1} f\left(Q+\frac{y}{c_1}\right)+\frac{1}{c_2} f\left(Q-\frac{y}{c_2}\right), \qquad y \leq c_2Q$$

$$= \frac{1}{c_1} f\left(Q+\frac{y}{c_1}\right) \qquad y \geq c_2Q$$

The mode of Y_Q, $M_o(Y_Q)$, is such that

$g_Q'(M_0(Y_Q)) = 0$ where

$$g_Q'(y) = \frac{1}{c_2^2} f'\left(Q + \frac{y}{c_1}\right) - \frac{1}{c_2^2} f'\left(Q - \frac{y}{c_2}\right), \quad y \le c_2Q$$

$$= \frac{1}{c_2^2} f'\left(Q + \frac{y}{c_1}\right) \qquad\qquad y \ge c_2Q$$

In chapter-2 distribution of total effective cost Y_Q has been derived explicitly for some chosen demand distributions. Here some analytical results have been obtained in the case of gamma distributed demand and beta-distributed demand as these two distributions cover a large family of distributions as their special cases.

3.2.1 Case of Gamma Demand

For the sake of simplicity demand distribution is assumed to have the density function $f(x) = \frac{1}{\Gamma p} e^{-x} x^{p-1} \quad 0<x<\infty$

and cumulative distribution function

$$F(x) = \int_0^x f(t)dt = I(u, p), \qquad u = \frac{x}{\sqrt{(p+1)}}$$

where I(u,p) is the Incomplete gamma function

Mean demand = p

Let us now consider the mode minimizing solution. Probability density function of total effective cost for such a demand distribution may be written as

$$g_Q(y) = h_1(y) + h_2(y); \; y \le c_2Q$$

$$= h_1(y); \qquad y \ge c_2Q$$

where $h_1(y) = \frac{1}{c_1 \Gamma p} e^{-\left(Q + y/c_1\right)} \left(Q + \frac{y}{c_1}\right)^{p-1}$

and $h_2(y) = \frac{1}{c_2 \Gamma p} e^{-\left(Q - y/c_2\right)} \left(Q - \frac{y}{c_2}\right)^{p-1}$.

Behaviour of the functions h_1 and h_2 may be studied first, as explained by the following facts :

(i) for $Q<p-1$ (assumed positive),

$h_1'(y) \gtrless 0$ according as $y \gtrless c_1(p-1-Q)$

$h_2'(y) < 0$ for all $y \geq 0$

(ii) for $Q = p-1$

$h_1'(y) < 0$ for all $y \geq 0$

$h_2'(y) < 0$ for all $y \geq 0$

(ii) for $Q > p-1$

$h_1'(y) < 0$ for all $y \geq 0$

$h_2'(y) < 0$ if $c_2Q \geq y > c_2(Q-p+1)$;

$= 0$ if $y = c_2(Q-p+1)$

> 0 if $c_2(Q-p+1) > y \geq 0$.

Over the range $y \geq c_2Q$, $g_Q(y) = h_1(y)$ and we notice that

$c_1(p-1-Q) \geq c_2Q$

$\Rightarrow Q \leq \frac{c_1}{c_1 + c_2}(p-1)$

Thus, we need to consider four possible cases viz.

1. (a) $\frac{c_1}{c_1+c_2}(p-1) \le Q \le p-1$

   (b) $Q < \frac{c_1}{c_1+c_2}(p-1)$

2. $Q > p-1$

3. $Q = p-1$

Figures for different ranges :

From the graphical representation (next page), it is seen that

(i) for 1(a) (fig.1a), the mode of Y_Q is either at $c_1(p-1-Q)$ or some where earlier.

Since both $h_1(y)$ and $h_2(y)$ are decreasing beyond $c_1(p-1-Q)$, mode cannot exceed $c_1(p-1-Q)$.

(ii) for 1(b)(Fig. 1b.), the mode is either at $c_1(p-1-Q)$ or some where between 0 to c_2Q.

(iii) When $Q>p-1$, the mode is at $c_2(Q-p-1)$ or some where earlier (fig. 2)

(iv) For case 3, (Fig-3), when $Q = p-1$, the mode is at 0 as both $h_1(y)$ and $h_2(y)$ are decreasing beyond 0

Thus considering all these possibilities, one concludes that $Q = p-1$ is the mode minimizing solution, irrespective of c_1 and c_2. Incidentally (p-1) is the mode of the demand distribution. Thus, the mode minimizing solution will not generally coincide with the classical mean minimizing solution

Figure 1(a)

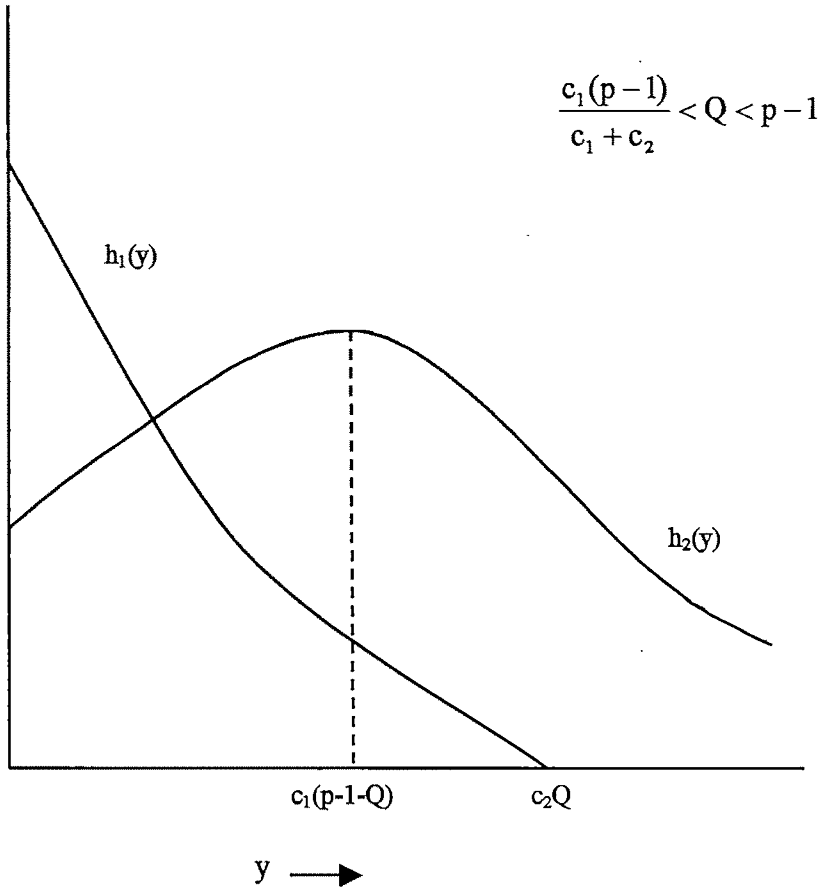
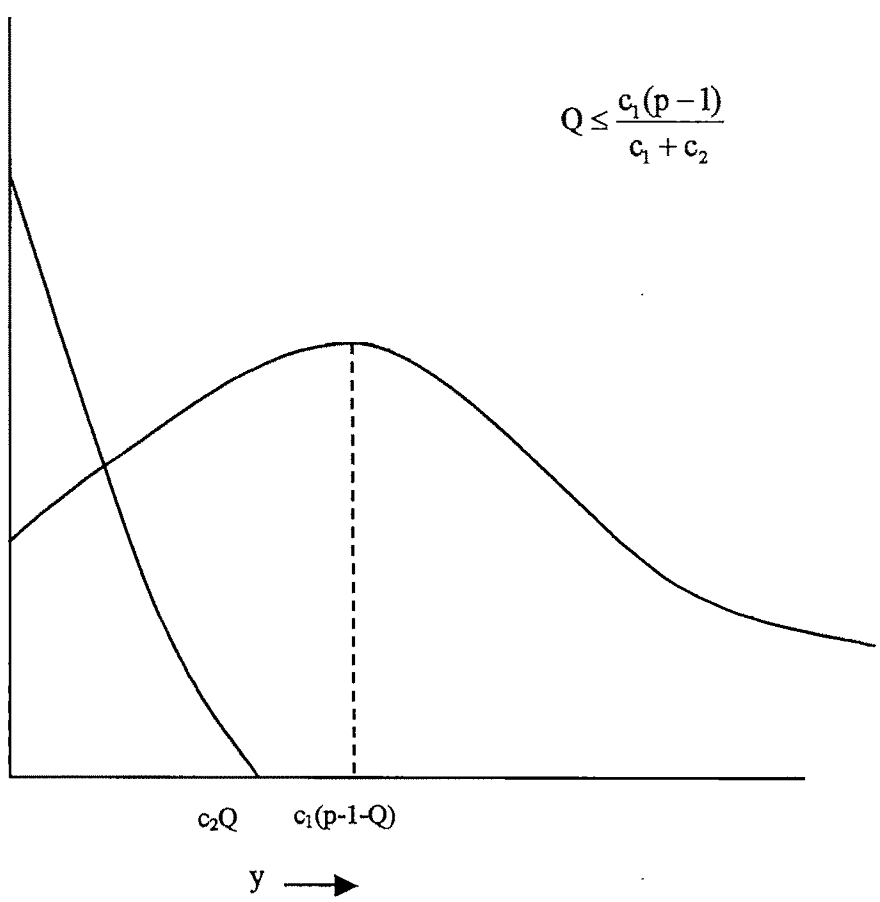


Figure 1(b)



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$Q > p-1$ $\quad$ $p \geq 1$, m, n $\geq$

Figure 2

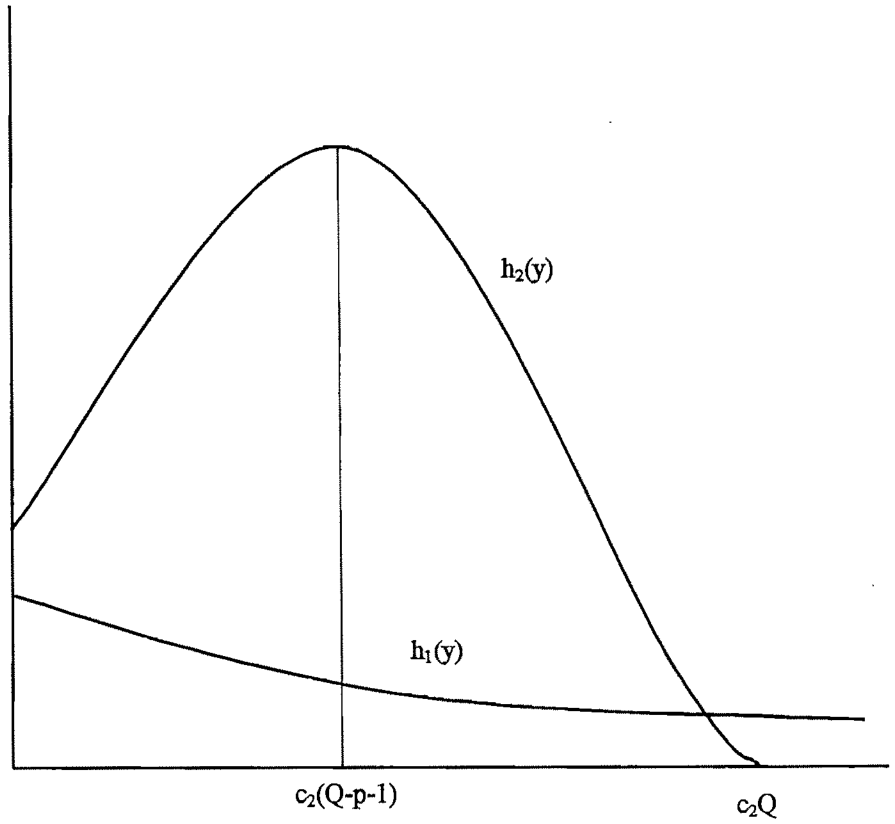
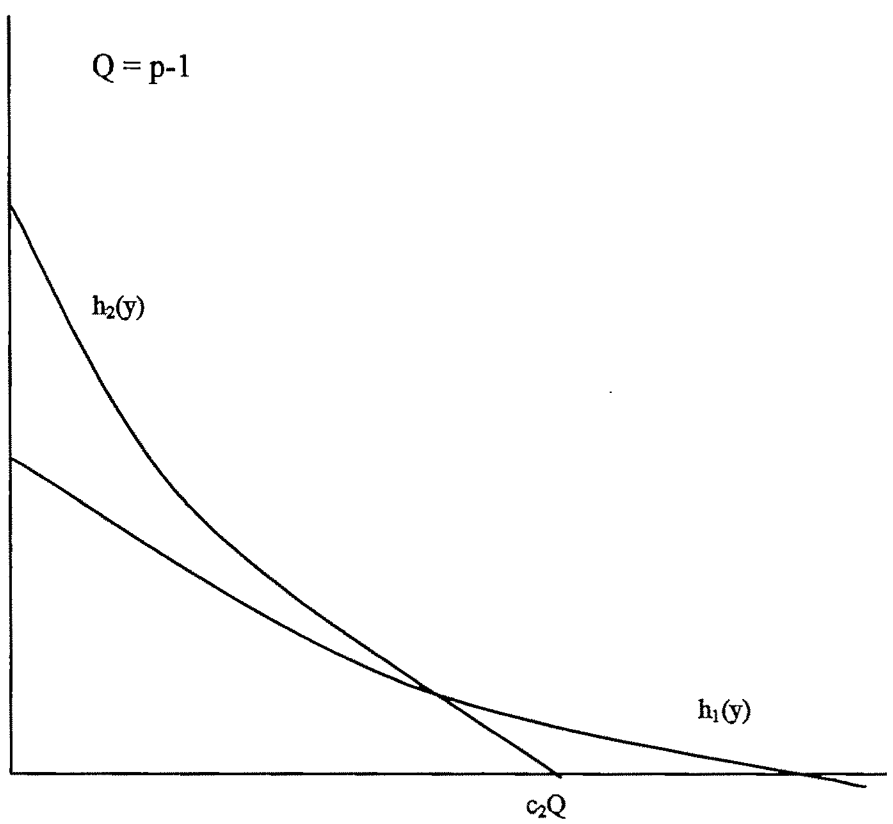


Figure 3



Numeric examples with different choices of p, c_1, c_2 are shown in Table 3.1 where Q_0 and Q_* represents the mode-minimizing solution and the classical solution through minimization of expected cost and EC is the average of Y_{Q_0} or Y_{Q^*}.

Table 3.1

Mean demand (p)	cost components c_1, c_2 $v = c_1 / c_{1+} c_2$	Mean Minimizing Solution(Q*)	Expected cost EC(Q*)	Mode Minimizing Solution(Q_0)	Expected cost EC(Q_0)
2	1,2.5, v= 0.80	4.2807	0.4522	1.0	1.05353
	1,1, v = 0.5	2.6750	.87807	1.0	1.08565
	1,1.2 v= 0.4546	2.4922	0.9397	1.0	1.09421
	1, 1.5 v =0.40	2.2854	1.01002	1.0	1.10706
	1,2, v = 0.3333	2.0370	1.1554	1.0	1.05263
	1,3 v= 0.25	1.7272	1.5174	1.0	1.45570
	1,4, v = 0.20	1.5344	1.2986	1.0	1.21412
4	1,2.5, v= 0.80	6.7215	0.6899	3	1.2726
	1,1, v = 0.5	4.6712	1.3873	3	1.4361

	1,2, v = 0.3333	3.8060	1.7718	3	1.6542
	1,4, v = 0.20	3.0884	2.1288	3	2.09025
6	1,2.5, v= 0.80	9.0779	0.86208	5	1.07171
	1,1, v = 0.5	6.6703	1.7717	5	1.11473
	1,2, v = 0.3333	5.6223	2.2921	5	1.1721
	1,4, v = 0.20	4.7327	2.7800	5	1.28683
10	1,2.5, v= 0.80	13.6527	1.1334	9	1.2100
	1,1, v = 0.5	10.6690	2.3736	9	1.3360
	1,1.2 v= 0.4546	10.3017	2.5618	9	1.36963
	1, 1.5 v =0.40	9.8642	2.7950	9	1.4199
	1,2, v = 0.3333	9.3260	3.1061	9	1.5039
	1,3 v= 0.25	8.6197	3.5248	9	1.672
	1,4, v = 0.20	8.1540	3.8122	9	1.8398

For different choices of cost components solutions are obtained through mean minimization as well as mode minimization. Shortage cost c_1 is normalized as 1 and excess cost is expressed as a multiple of shortage cost = $k\, c_1$, (say) where k may take any value; v is defined as $1/\ 1+k = c_1\ /\ (c_2 + c_1)$

The Table 3.1 gives solutions through Mean Minimization (Q^*) and the expected costs associated with those as also solutions through Mode Minimization (Q_0) and expected costs associated with such solutions.

It is evident that as p increases, EC is smaller for mode minimizing solution. Again for smaller p also, for $c_2 > c_1$, EC is less for mode minimizing solution. Thus, the mode minimizing solution is not only robust (independent of c_1 and c_2) and easily obtainable, but also gives smaller cost for the particular items with higher demand and when excess cost is higher. Usually for inventory models like Newsboy problem, Christmas tree problem, high demand occurs for only a very short period, and quantity left over after that time period is really of no use, mode minimization may prove itself very useful.

3.2.2 Case of Beta demand

A second situation with a suitable normalizing number is considered where $x' = x/p$ follows beta (m, n); $0<x'<1$, where p is any positive number.

Thus, x has a p.d.f $f(x) = \frac{1}{B(m,n)p^{m+n-1}} x^{m-1}(p-x)^{n-1}$ $\quad o<x<p$

$p \geq 1,\ \ m, n \geq 1$

and the cumulative distribution function F(x) is

$$F(x) = \int_{o}^{x} \frac{1}{B(m,n).p^{m+n-1}} x^{m-1}(p-x)^{x-1}dx$$

$$= \int_{o}^{x/p} \frac{1}{B(m,n)} x^{m-1}(1-x)^{x-1}dx$$
$$= I_{x/p}(m,n)$$

where $I_{x/p}$ (m,n) = incomplete beta function

The mean demand for such a case becomes $\frac{mp}{(m+n)}$

Stochastically smallest cost:

When $c_1 = c_2$ (≥ 2) and m=n, then

Y_{Qo} where Q_o = p/2 corresponds to the stochastically smallest cost and p/2 could be the best possible order quantity if such a demand distribution is encountered.

Solutions through Mode Minimization:

However, if m $\neq$ n and $c_1 \neq c_2$, it is not easy to derive the stochastically smallest solution and hence a solution through mode-minimization is arrived at in, as in the 'Min Max' solution, in the spirit that this solution minimizes the most frequently encountered cost when such a demand distribution is experienced.

The probability density function of the cost Y_Q becomes

$g_Q(y) = h_1(y) + h_2(y)$ $\qquad y \leq c_2Q$

$\quad = h_1(y)$ $\qquad y \geq c_2(Q)$

where $h_1(y) = \dfrac{1}{B(m,n)p^{m+n-1}.c_1}\left(Q + \dfrac{y}{c_1}\right)^{m-1}\left(p - Q - \dfrac{y}{c_1}\right)^{n-1}$; $\qquad o<y<\alpha$

and $h_2(y) = \dfrac{1}{B(m,n)p^{m+n-1}.c_2}\left(Q - \dfrac{y}{c_2}\right)^{m-1}\left(p - Q + \dfrac{y}{c_2}\right)^{n-1}$; $\qquad y \leq c_2Q$

The nature of these functions $h_1(y)$ and $h_2(y)$ may be studied as done in the case of gamma demand.

Let us take $c = \dfrac{1}{B(m,n)p^{m+n-1}}$

Then,

$$h_1'(y) = \frac{c}{c_1^2}\left(Q + \frac{y}{c_1}\right)^{m-2}\left(p - Q - \frac{y}{c_1}\right)^{n-2}\left\{(m-1)\left(p - Q - \frac{y}{c_1}\right) - (n-1)\left(Q + \frac{y}{c_1}\right)\right\}$$

$$= \frac{c}{c_1^2}\left(Q + \frac{y}{c_1}\right)^{m-2}\left(p - Q - \frac{y}{c_1}\right)^{n-2}\left\{(m-1)p - Q(m+n-2) - \frac{y}{c_1}(m+n-2\right\}$$

Hence $\qquad h_1'(0) > o$ if

$(m-1)p - Q(m+n-2) - \dfrac{y}{c_1}(m+n-2) > 0$

i.e. $\quad y < \dfrac{c_1}{(m+n-2)}\{(m-1)p - Q(m+n-2)\}$

i.e. $\quad y < c_1\left\{\dfrac{(m-1)p}{m+n-2} - Q\right\}$

i.e. $y < Q_1^*$ (say)

Thus $h_1(y)\uparrow y \qquad o<y< Q_1^*$

And (i) $Q_1^*>0$ for $Q < \frac{m-1}{m+n-2}p$

(ii) $Q_1^*<c_2Q$ for $Q > \frac{c_1}{c_1+c_2}\frac{(m-1)}{m+n-2}p$

Again

$$h_2'(y) = \frac{c}{c_2^{\,2}}\left(Q-\frac{y}{c_2}\right)^{m-2}\left(p-Q+\frac{y}{c_2}\right)^{n-2}\left\{-(m-1)\left(p-Q+\frac{y}{c_2}\right)(n-1)\left(Q-\frac{y}{c_2}\right)\right\}$$

$$=\frac{c}{c_2^{\,2}}\left(Q-\frac{y}{c_2}\right)^{m-2}\left(p-Q+\frac{y}{c_2}\right)^{n-2}\left\{-(m-1)p+Q(m+n-2)-\frac{y}{c_2}(m+n-2)\right\}$$

Hence

$$h_2'(y)>0 \quad \text{if} \quad y<c_2\left\{Q-\frac{(m-1)p}{(m+n-2)}\right\}$$
$$= Q_2^* \qquad \text{(say)}$$

obviously $Q_2^* < c_2Q$

and $Q_2^* > 0$ for $Q > \frac{(m-1)p}{(m+n-2)}$

Now the following four cases may be studied to derive the mode minimizing solution for this case.

1 (a) $0<Q<\frac{c_1}{(c_1+c_2)}\cdot\frac{(m-1)p}{(m+n-2)}$

1 (b) $\frac{c_1}{(c_1+c_2)}\cdot\frac{(m-1)p}{(m+n-2)}<Q<\frac{(m-1)p}{(m+n-2)}$

2. $Q = \frac{(m-1)p}{m+n-2}$

3. $Q > \frac{(m-1)p}{m+n-2}$

Taking these cases into consideration, we find that the mode minimizing solution is $\frac{(m-1)p}{m+n-2}$ irrespective of c_1 and c_2.

Following Table gives a comparative study of Q_0 and Q_* where Q_0 and Q_* represent the mode-minimizing solution and the classical solution (through minimization of expected cost) and EC is the average cost for the distributions of Y_{Q0} and Y_{Q*}.

Table 3.2

m	n	cost components $k = c_1/c_2$ $v = 1/(1+k)$	Mean Minimizing Solution(Q*)	Expected cost EC(Q*)	Mode Minimizing Solution(Q_0)	Expected cost EC(Q_0)
2	1	1,0.5	7.07092	5.3389	10	7.8932
		2,0.3333	5.7733	6.15063	10	6.6667
		0.25,0.50	8.94413	0.70363	10	0.83333
3	1	1,0.5	7.93671	4.0681	10	5.0
		0.25,0.80	9.28303	3.03738	10	3.125

4	2	1,0.5	6.86176	1.45504	6	1.53972
		2,0.3333	5.9784	2.06685	6	2.0786
		4,0.20	5.09804	2.70519	6	3.456
		0.25,80	8.31406	3.89913	6	3.684
6	2	1,0.5	7.71497	3.66583	7.14286	3.88034
		2,0.3333	7.01815	4.17295	7.14286	432434
		4,0.20	6.29117	4.73858	7.14286	4.20085
		0.25,80	8.83224	2.86610	7.14286	2.55022
10	4	1,0.5	7.24705	3.79405	7.5	3.31608
		2,0.3333	6.70606	4.18243	7.5	4.97412
		4,0.20	6.21095	4.2745	7.5	4.29019
		0.25,80	8.17863	3.2333	7.5	3.82255
10	1	1,0.5	9.32932	1.51673	10	1.81818
		2,0.3333	8.95837	1.85003	10	2.72727

No trend is visible for this case. It may be noted that for some of the combinations one solution outsmarts the other.

3.3 Case for Profit Maximization.

We now consider a case of profit model in which the unit selling price and the unit purchase cost are the additional given variable in the classical newsboy problem.

$$
\begin{aligned}
Y_Q &= X(p-c_o)-c_1(X-Q); \qquad X \geq Q \\
&= X(p-c_o)-c_2(Q-X); \qquad X \leq Q
\end{aligned}
$$

where c_0 = purchase price per unit

c_1 = cost of running short of one unit

c_2 = excess cost per unit left unsold

X = random demand with cumulative distribution function $F(x)$ and probability density function $f(x)$.

p = selling price per unit.

The cumulative distribution function $G_Q(y)$ of Y_Q may be derived as

$$G_Q(y) = P[Y_Q \leq y;\ 0<x<Q] + P[Y_Q \leq y,\ Q<x<\infty]$$

$$= P\left[X < \frac{y+c_2Q}{p-c_0+c_2}, X<Q\right] + P\left[X < \frac{y-c_1Q}{p-c_0-c_1}, X<Q\right],$$

$$= P\left[X < \mathrm{Min}\left\{\frac{y+c_2Q}{p-c_0+c_2}, Q\right\}\right] + P\left[Q < X < \frac{y-c_1Q}{p-c_0-c_1}\right]$$

$$= P_1 + P_2$$

For P_1, the range of y is to be specified to determine the minimum among Q and $\frac{y+c_2Q}{p-c_0+c_2}$ i,e Min $\left\{\frac{y+c_2Q}{p-c_0+c_2}, Q\right\}$

Now

$$Q > \frac{y+c_2Q}{p-c_0+C_2} \qquad \Rightarrow \ (p\text{-}c_0)Q > Y$$

$$\text{Min}\left\{\frac{y+c_2Q}{p-c_0+c_2}, Q\right\} = \frac{y+c_2Q}{p-c_0+c_2} \qquad \text{if } y < (p\text{-}c_0)\,Q$$

$$= Q \qquad \text{if } y > (p\text{-}c_0)\,Q$$

Similarly $P_2 \geq 0$ if $Q \leq \frac{y-c_1Q}{(p-c_0-c_1)}$ i.e. $y \geq (p\text{-}c_0)\,Q$

Thus

$$G_Q(y) = F\left(\frac{y+c_2Q}{p-c_0+c_2}\right), \qquad -c_2Q < y < (p\text{-}c_0)Q$$

$$= F(Q) + F\left(\frac{y-c_1Q}{p-c_0-c_1}\right) - F(Q), \qquad y>(p\text{-}c_0)\text{-}Q$$

Finally,

$$G_Q(y) = F\left(\frac{y+c_2Q}{p-c_0+c_2}\right), \qquad -c_2Q < y < (p\text{-}c_0)Q$$

$$= F\left(\frac{y-c_1Q}{p-c_0-c_1}\right), \qquad y>(p\text{-}c_0)\text{-}Q$$

and hence the probability density function becomes

$$g_Q(y) = \frac{1}{p-c_0+c_2} f\left(\frac{y+c_2Q}{p-c_0+c_2}\right) \qquad -c_2Q<y<(p\text{-}c_0)\,Q$$

$$= \frac{1}{p-c_0+c_2} f\left(\frac{y+c_1Q}{p-c_0+c_1}\right) \qquad y>(p\text{-}c_0)$$

Properties of $G_Q(y)$

It is clear that $g_Q(y)$ has a discontinuity point at $(p-c_0)Q$ if $c_1 \neq c_2$. The mean if Y_Q may be derived as

$$M_Q(y) = \int_{-c_2Q}^{(p-c_0)Q} y.\frac{1}{p-c_0+c_2} f\left(\frac{y+c_2Q}{p-c_0+c_2}\right)dy + \int_{(p-c_0)Q}^{\infty} y.\frac{1}{p-c_0+c_1} f\left(\frac{y+c_1Q}{p-c_0+c_1}\right)dy$$

$$= \int_0^Q (c_3x - c_2Q)f(x)dx + \int_Q^\infty (c_4x + c_1Q)f(x)dx$$

where $c_3 = (p-c_0+c_2)$

$c_4 = (p-c_0-c_1)$

$$= c_3\int_0^Q xf(x)dx - c_2Q\int_0^Q f(x)dx + c_4\int_Q^\infty xf(x)dx + c_1Q\int_Q^\infty f(x)dx$$

$$= (c_3 - c_4)\int_0^Q xf(x)dx + c_4E(x) + c_1Q - (c_1 + c_2)QF(Q)$$

$$= (c_1 + c_2)\left[\int_0^Q xf(x)dx - QF(Q)\right] + (p - c_0 - c_1)E(x) + c_1Q$$

It may be noted that the solution for $\frac{d}{dQ}$ $M_Q(y) = 0$ will represent the usual economic order quantity corresponding to maximum expected profit.

Similarly mode of the profit function also may be derived as $M_0(y_Q)$ which satisfies

$g_Q'(M_0(Y_Q)) = 0$

Now $g_Q'(y) = \frac{1}{(p-c_0+c_2)^2} f'\left(\frac{y+c_2Q}{p-c_0+c_2}\right) -c_2Q < y < (p-c_0)Q$

$$= \frac{1}{(p-c_0-c_1)^2} f'\left(\frac{y-c_1Q}{p-c_0-c_1}\right), \qquad y > (p-c_0)\,Q$$

3.3.1 Case of Gamma Demand

Let demand X follow a gamma distribution with p.d.f

$$f(x) = \frac{1}{\Gamma k} e^{-x} x^{k-1} \qquad o<x<\alpha$$

and the cumulative distribution function as

$$F(x) = \int_o^x f(x)dx = I(u,k); \qquad u = \frac{x}{\sqrt{(k+1)}}$$

where I(u, k) is the incomplete gamma function.

Mean demand is k

To derive the mode-maximizing solution, the probability density function of profit Y_Q is noted to be

$$g_Q(y) = \begin{cases} h_1(y) & -c_2Q<y<(p-c_0)Q \\ h_2(y) & y>(p-c_0)Q \end{cases}$$

where $h_1(y) = \dfrac{1}{p-c_0+c_2} e^{-\left(\frac{y+c_2Q}{p-c_0+c_2}\right)} \left(\dfrac{y+c_2Q}{p-c_0+c_2}\right)^{k-1}$

and

$$h_2(y) = \frac{1}{p-c_0-c_1} e^{-\left(\frac{y+c_1Q}{p-c_0-c_1}\right)} \left(\frac{y+c_1Q}{p-c_0+c_1}\right)^{k-1}$$

Now $h_1(y) > 0$ as $y<(k-1)(p-c_0+c_2) - c_2Q = Y_1$ (say)

and $h_2(y) > 0$ as $y<(k-1)(p-c_0+c_1) - c_1Q = Y_2$ (say)

Now $Y_1 \leq (p-C_0)\,Q$ means $Q \geq (k-1)$

and $Y_2 \geq (p-c_0)\,Q$ means $Q \leq (k-1)$

Thus considering $g_Q(y)$ in the whole range of $-c_2Q<y<\infty$ it is conceived that if $Q = k-1$. $g_Q(y)$ has the maximum mode at $Y_Q = k-1$. Hence $Q = k-1$ is the optimum solution for the order quantity to be stocked which ensures to maximize the mostly encountered profit.

3.3.2 Case of Beta demand

Now we assume $X' = X/K$ to follow a beta (m,n) distribution, where K is a suitable normalizing number.

Thus X has the probability density function

$$f(x) = \frac{1}{B(m,n)K^{m+n-1}} x^{m-1}(K-x)^{n-1} \qquad 0<x<K \text{ and } K>0,\ m,n \geq 1$$

and the cumulative distribution function F(x) is

$$\begin{aligned} F(x) &= \int_0^x \frac{1}{B(m,n)K^{m+n-1}} \cdot x^{m-1}(K-x)^{m-1}dx \\ &= \int_0^{x/K} \frac{1}{B(m,n)} \cdot x^{m-1}(1-x)^{m-1}dx \\ &= I_{x/K}(m,n) \end{aligned}$$

where I_x (m,n) = incomplete beta function. The mean demand for such a case is mK/(m+n).

To derive mode maximizing solution in this case, we note that $g_Q(y)$, the probability density function of Y_Q is given by

$$\begin{aligned} g_Q(y) &= h_1(y) \qquad && -c_2Q<y<(p-c_0)Q \\ &= h_2(y) && y>(p-c_0)Q \end{aligned}$$

where $h_1(y) = \frac{1}{(p-c_0+c_2)} f\left(\frac{y+c_2Q}{p-c_0+c_2}\right)$

$$= \frac{1}{B(m,n)K^{m+n-1}(p-c_0+c_2)}\left(\frac{y+c_2Q}{p-c_0+c_2}\right)^{m-1}\left(K-\frac{y+c_2Q}{p-c_0+c_2}\right)^{n-1}$$

and

$$h_2(y) = \frac{1}{(p-c_0+c_1)} f\left(\frac{y-c_1Q}{p-c_0-c_1}\right)$$

$$= \frac{1}{B(m,n)K^{m+n-1}(p-c_0-c_1)}\left(\frac{y-c_1Q}{p-c_0-c_1}\right)^{m-1}\left(K-\frac{y-c_1Q}{p-c_0-c_1}\right)^{n-1}$$

Now

$$h_1'(y) = \left[\frac{c}{(p-c_0+c_2)}\cdot\left(\frac{y+c_2Q}{p-c_0+c_2}\right)^{m-2}\left(K-\frac{y+c_2Q}{p-c_0+c_2}\right)^{n-2}\right]$$

$$\left[(m-1)K-(m+n-2)\frac{(y+c_2Q)}{(p-c_0+c_2)}\right]$$

and

$$h_2'(y) = \frac{c}{(p-c_0-c_1)}\cdot\left(\frac{y-c_1Q}{p-c_0-c_1}\right)^{m-2}\left(K-\frac{y-c_1Q}{p-c_0-c_1}\right)^{n-2}$$

$$\left[(m-1)K-(m+n-2)\frac{(y-c_1Q)}{(p-c_0-c_1)}\right]$$

where $c = \frac{1}{B(m,n)K^{m+n-1}}$

Thus $h_1(y) > 0$ as $y < -c_2Q+(p-c_0+c_2)\left(\frac{m-1}{m+n-2}\right)K = Y_1$ (say)

and

$h_2(y)>0$ as $y< c_1Q+(p-c_0-c_1)\left(\frac{m-1}{m+n-2}\right)K = Y_2$ (say)

Again

$Y_1 < (p-c_0)\,Q \quad \Rightarrow \; Q > \frac{(m-1)K}{(m+n-2)}$

and $Y_2 > (p-c_0)\,Q \quad \Rightarrow \; Q < \frac{(m-1)K}{(m+n-2)}$

Thus when $g_Q'(y)$ is considered in the whole range of $-c_2Q<y<\infty$, it is observed that $Q = \frac{(m-1)K}{m+n-2}$ is the optimum solution as Y(Q) has the maximum mode if $Q = \frac{(m-1)K}{m+n-2}$. Thus for such a beta type demand, which in turn covers a wide family of distributions, the optimum quantity to be stocked is $\frac{(m-1)K}{m+n-2}$, and this stock results in the maximum value of the most frequently realized profit.