Chapter – 1

INTRODUCTION

1.1 Inventory Management:

Importance of inventory management in overall production as well as maintenance management or even in management of services need not be elaborated. Inventory, defined as 'an idle resource of any kind which has economic values' by Fred Hansmann is one of the most important entities in business management. Minimization of inventory cost has always been an important objective of business management. The aim of any inventory policy is to derive optimal levels of resources to be kept idle in order to meet the demand. Since demand results in the withdrawal of items from stock on hand it is necessary to replenish the stock from time to time. Holding a large stock means avoidable cost, on the other hand running short of stocks also has great financial implications. Thus, the two most important questions that arise in maintenance and control of inventory are

i.) when to place an order(reorder time)
ii.) how much to order at a time(order quantity)

Inventory management may also decide upon frequency of order, reorder interval, order preponement time other than the two mentioned in i) and ii). The answers to these questions depend on variety of factors; important among them being (a) nature of the item to be stocked, (b) costs involved, (c) pattern of demand (d) effect of lead time (time lag between placing an order and receiving the supply).

Inventory models take into account special features of certain situations e.g. when the items held in stock deteriorate or are perishable in nature and hence get lost after a random time from acquisition in storage. The lifetime of such items may not be fixed but may vary according to some probability law. A model may take account substitution of items. Some model takes into account defectiveness and otherwise of the items and substitution of the earlier by latter. Models involving pricing with quantity discount depending on lot size are also considered. Some inventory models take into account of multiple item, other consider multiple storage and so on.

The costs involved in any inventory model have very big implication and main decisive role for controlling the inventory situation. Costs may vary with time also. Following are the different costs primarily associated with any inventory system.

I. Procurement cost
    - Ordering cost
    - Purchase or production cost

II. Carrying cost or holding cost
    - Cost of lost opportunity
    - Insurance cost
    - Storage cost
    - Handling cost
    - Obsolescence cost

III. Shortage cost

IV. Excess cost

The nature of the demand or demand pattern is essential part of maintaining inventory. Sometimes demand depends on time. It may even depend on the quantity held in stock, promotional activities to boost the demand, prices offered and similar other factors. Demand may sometimes be considered as purely random varying with some probability law and hence the model becomes stochastic in nature.

Problems of purchase inventory model are different from those of production inventory model where production function, time to start the production are the important questions to be answered. Production inventory model for decaying or non decaying materials for single and/or multiple finished product, finite and constant production rate vis-à-vis time varying production function are different choices of such production inventory models.

Effects of centralization on expected cost are also important in inventory models where demand arises in different locations. Such multi-location problem for single item, single period case as well as for others, benefits and disbenefits of centralization of such cases are the common variation of such models.

1.2 Review of Past Work :

The traditional and most commonly used procedure to arrive at an optimum order quantity (on any other related decision variable) is to minimize the total (expected) cost or to maximize the total (expected) profit. In some cases, cost per unit investment is also considered as objective function. Classical calculus along with the use of Lagrange multiplier, where needed generally enables us to tackle such optimization problems.

Randomness in the objective function due to random demand and / or lead time and / or supply is obviated and the objective function is reduced to a deterministic one by taking expectation of the random objective function over random variations in the random components of the model. Thus the expected total effective cost or profit is only a deterministic function of the policy variable(s) like order quantity, bin size, order level etc (sometimes reorder time or other variable also). Cost coefficients are usually assumed constant, through effects of non-random variations in them are examined through sensitivity analysis. Except when stated otherwise, we assume the decision variable(s) to be non-random. Thus the random objective function is usually transformed to a non-random one to derive the value(s) of the decision variable(s) optimally.

The earliest work on dimensional analysis is due to Naddor (1966) who first illustrated the use of dimensional analysis in queuing, inventory and linear programming. Some more useful applications of dimensional analysis have been reported in the literature on inventory analysis. Ehrhardt (1979) and Ehrhardt and Mosier (1984) suggested some efficient power approximation for comuting (s, S) policies in a periodic review, single item, back logged inventory system with a constant lead time and its revision thereafter. Later Vignaux (1986) and Vignaux and Jain (1988) suggested some modification and gave an approximation on the basis of dimensionless products of original variables and parameters. Reported work on dimensional analysis has been solely based on (s,S) policies. Other dynamic risk models with lead-time have so far not been considered.

A single item EOQ (Economical Order Quantity) model under permissible delay in payments was first considered by Goyal (1985) and later by Chand & Ward (1987). The model was considered without shortage, instantaneous replenishment of stock and investment of sales revenue to earn interest during the permissible settlement period. Later, Mandal and Phaujdar (1989) worked out EOQ with instantaneous replenishment of stock, where shortages are not allowed to occur. EOQ with a finite rate of replenishment under shortage was also worked out. The case with random lead time has not yet been considered. Different patterns of demand in such cases of permissible delay in payment like time dependent demand, random demand are some of the interesting and realistic extensions which deserve consideration.

Production scenario in inventory analysis was done by Kok (1988), who, among the few others worked with dynamic version of classical inventory models. Approximation and algorithm in production – inventory control model was widely discussed by him. Later similar single period, single product inventory models with several individual sources of demand and an opportunity of centralization was considered by M-S. Chen and C-T. Lin (1989). Both the centralized and deecentralized system on expected cost are considered and the condition of consolidation of demand was verified along with the effect of centralization on the total expected cost. Later, an extension on the disbenefits of centralized stocking was also considered by the same author (1990) where the demands were filled on a first come, first served basis. Effects of centralization on expected cost in multi-location

Newsboy problem was considered by Eppen (1979) Stulman (1987) and Chen and Lin (1989). Gabby (1979) also discussed the multi-stage production planning problem.

The earliest work on stocking policies for deteriorating items is due to Ghare and Schrader (1963). They considered EOQ model for items subject to fixed rate of decay. Their model was extended to more general types of deterioration by authors like Covert and Philip (1973) and Shah (1977) and Shine Yih-chearug (1990) with different additional characterizations like Weibull distribution deterioration, quantity discounts etc. Other models developed for items with variable life time include a heuristic lot size re-order point model by Nahimas and Wang (1978) and Dev and Chaudhuri (1987), an order level production inventory model by Roy Chowdhury and Chaudhuri (1982) etc. Cohen (1977) first considered the joint pricing and ordering policy for exponentially decaying inventory with known demand. Mukherjee (1987) extended this model with a linearly time dependent decay rate of inventory, which was subsequently extended by Bhunia and Maiti (1997) where demand rate is linearly dependent on selling price, time and frequency of advertisement. Some more notable inventory problems with time dependent demand pattern were developed by Dave and Patel (1981), Mitra et al. (1984), Sachan (1984), Bahari-Kashani (1989) and Haiping and Wang (1990). Mudreswar (1988) and Goswami and Chaudhuri (1991) developed some extensions of the earlier models allowing shortages to occur.

1.3Preview of Present Work:

The present work attempts to make a modest contribution to the existing literature on development and solution of inventory models. The work is primarily concerned with stochastic inventory models, though a few specific deterministic models have also been discussed. Similarly, purchase inventory models have been generally considered, though in chapter 5 a production inventory models has been developed and solved.

The work can be divided into two distinct parts (viz. Part II and Part III), the first dealing with some new solutions to stochastic inventory models illustrated in the context of some classical models and the second devoted to some extensions of a few existing models, some stochastic and some deterministic.

The first part offers two new optimality criteria for determining the Economic Order Quantity (EOQ) or Optimal Order Quantity for the classical Newsboy Problem (Chapter 2 and Chapter 3). It also deals with dimensional analysis and its application in (R,T) policy (Chapter 4). In the second part, the classical newsboy model has been extended to a production situation in chapter 5. Chapter 6 deals with situations where delay in payments is permitted in a profit model while chapter 7 discusses a few models for deteriorating items.

In chapter 2, randomness in the objective function viz. total effective cost in the Newsboy Problem has been recognized explicitly (and not just in terms of its expected value which is a deterministic function of the order quantity), the distribution of this cost has been worked out for each order quantity and the order quantity that corresponds to the stochastically smallest cost has been proposed as the optimal solution. The difficulty in deriving such a solution dealing with a general demand distribution has been pointed out, and necessary conditions for the existence for such a solution have been worked out. The solution has been illustrated in terms of beta distributed demand.

Chapter 3 proposes another solution to the previous model – a solution that can be spoken of in the context of any other stochastic inventory models. Here, it has been argued that an order quantity for which the effective cost distribution has the lowest mode (most probable value) can be taken as the optimal solution. Several demand distributions have been assumed to examine relative merits and demerits of this solution compared to the classical mean minimizing solution. Conceptually analogous to a 'Minimax' solution, this approach can be adopted to maximize the mode of the total profit to derive optimum solution, in the case of a profit model.

In chapter 4, the principle of dimensional balancing in physical science has been applied to identify appropriate functions of model parameters which are dimensionless and to use this principle to derive data based solutions to some models. The principle has been applied to derive an approximate solution to the reorder quantity (R) in the (R, T) policy with a known T.

Chapter 5 treats a simple production inventory model where demand arises and sales occur at one given point of time, production starts earlier, and the cumulative production up to the demand point is used to meet demand, assumed to be random. The problem is to determine when to start production and at which rate. Two forms of unit production cost have been assumed to illustrate the solution..

In the next chapter, EOQ under permissible delay in payments has been taken up. A time dependent non-random demand rate has been discussed in section 6.2 while section 6.3 considers the case of random one point demand. Section 6.4 accounts for a random lead time against a constant rate of demand.

The last chapter is concerned with models for deteriorating items. Section 7.2 considers a model with time dependent decay rate and random demand. An extension of EOQ involving an optimal number of reorder intervals to deal with items deteriorating at a constant rate to meet the demand which depends on selling price as well as time, has been worked out in section 7.3. The next section deals with a similar problem involving random demand.