Chapter Four: 
Measuring between-group inequality in multidimensional space

4.1. Motivation and foundational issues in measurement

In the last two chapters we examined alternative measures of horizontal inequality in the unidimensional space. Using these measures on Indian data we have found significant disparity between the social groups, between religious groups and between sectors in the dimensions of consumption expenditure and education. Not unexpectedly, the degree of inequality differs across groupings as well as dimensions. For instance, in terms of simple group averages of per capita consumption expenditure and average years of schooling for the major states of India we observe that in rural Gujarat the average years of schooling of SCs is greater than OBCs, but the average per capita consumption expenditure of OBCs is greater than that of SCs. Likewise in urban Gujarat the average years of schooling of STs is greater than that of OBCs although the average per capita consumption expenditure of OBCs is greater than that of STs. This is similar to the observations made by Thorat (2007), as he shows that relative positions of the groups varied in different dimensions across the major states of India. Given the array of numbers representing inequality in different dimensions it is hard to get an overall understanding of group inequality in this context unless one uses an index that meaningfully combines inequality in several dimensions into a scalar number. Therefore, in this chapter we develop a discussion on multidimensional between-group inequality measurement and apply it to assess horizontal inequality in India.

Fisher (1956) first introduced the theoretical foundation of multidimensional interpersonal inequality by incorporating the idea of the multidimensional distribution matrix. However, pioneering work on the formal analysis of multidimensional interpersonal inequality and its foundational issues was done by Kolm (1977). He developed the dominance criteria by multidimensional generalization of the Pigou-Dalton transfer principle. The multidimensional inequality measures which satisfy these dominance criteria enable us to compare the degrees of multidimensional inequality between two or more distributions

31The dominance criteria are discussed in Appendix I.
Maasoumi (1986) developed a multidimensional measure of inequality using an aggregation function, which converts the multidimensional distribution into a distribution of utilities. The multidimensional inequality index is then obtained by applying a univariate index of inequality to this distribution. Later, Tsui (1995) generalized the univariate Atkinson-Kolm-Sen approach, which is based on a social welfare function, and developed some multidimensional interpersonal inequality measures. Tsui first axiomatically characterized a class of social evaluation functions (SEFs), which are multi-attribute generalizations of a class of SEFs for univariate distribution introduced by Blackorby, Donaldson and Auersperg (1981). He then developed the measures of multidimensional inequality on the basis of this multidimensional SEFs. The measures developed by Tsui (1995) satisfy only those dominance criteria which are developed by the multidimensional generalization of the Pigou-Dalton transfer principle.

Multidimensional inequality can be viewed as a combination of two kinds of effects. One is due to the degree of inequality in each dimension and the other is due to the possible intensification because of the high degree of correlation between inequalities in different dimensions (List, 1999). Therefore, the relative advantages and disadvantages of the persons across dimensions or local inequalities in different dimensions can cancel each other out, which can bring down inequality in the multidimensional distribution (Atkinson and Bourguignon, 1982; Walzer, 1983)\textsuperscript{32}. The dominance criterion, which was developed from the Pigou-Dalton principle of transfer, cannot develop a measure of multidimensional inequality which would incorporate the idea of cross-correlation\textsuperscript{33}. For this reason, Tsui (1999) and List (1999) have introduced another

\textsuperscript{32} The relative advantage of a group in one dimension can weaken the relative disadvantages of that group in other dimensions, which can bring down the degree of total inequality derived by simple additions of the degrees of inequality across dimensions of the multidimensional distribution. On the contrary, consistent relative disadvantages of a certain group across dimensions can intensify the overall degrees multidimensional inequality which is over and above simple addition of the degrees of inequality in different dimensions of the multidimensional distribution.

\textsuperscript{33} Let there be three multidimensional distributional matrices for two groups and two attributes, A, B and C, where groups are measured along rows and attributes are measured along columns, and the cell elements of the matrices are the group averages of the attributes.

\[
A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 4 \\ 5 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 1 \\ 1 & 8 \end{bmatrix}
\]

In the distribution A the first group is advantaged in the first dimension and the second group is disadvantaged in the second dimension. In distribution B, the first group is disadvantaged in both dimensions and the second group is advantaged in all dimensions. In C, like in A, the first group is advantaged in the
dominance criterion, i.e. *correlation increasing majorization*, which can take care of the systematic cross-correlation between inequalities in different dimensions. They have developed some measures of multidimensional interpersonal inequality, which satisfy the dominance criteria developed from the multidimensional generalization of transfer principle as well as the *correlation increasing majorization*. These measures of multidimensional inequality can take into account both degrees of inequality in different dimensions and systematic cross-correlation between inequalities in different dimensions.

The first objective of this study is to develop the group analogue of the multidimensional interpersonal inequality measures by reviewing the existing multidimensional interpersonal inequality measures. To examine how the assessed multidimensional between-group inequality differs when we take into account the systematic cross-correlation between between-group inequalities in different dimensions, we take the group analogue of two multidimensional inequality measures. The first measure satisfies the dominance criteria developed from transfer principle along with other common axiomatic properties relevant in the multidimensional between-group inequality analysis, and the second measure satisfies the dominance criteria based on the transfer principle and the *correlation increasing majorization*.34

Underlying any standard inequality measure there is some value judgment. On the basis of this value judgment distinction is made between the relative and absolute measures of inequality. While the relative inequality measures are the ones which satisfy the ‘scale invariance’ property, the absolute measures satisfy the ‘translation invariance’ property. If these two value positions indicate two extremes, an intermediate position can also be taken which would yield an ‘intermediate’ inequality measure (Kolm, 1976a and 1976b). The second objective of this study is to convert the initially developed relative multidimensional between-group inequality into intermediate multidimensional between-group inequality by incorporating some centrist value judgment in the analysis.

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34The axiomatic properties and the dominance criteria are discussed in Appendix I.
Finally, we apply the group analogue of multidimensional relative and intermediate inequality measures to assess inequality among the social groups in rural and urban areas of fourteen major states in India and at the all-India level\(^{35}\). We explore here if there is any systematic differences in the between-group inequality in multidimensional space across the major states of India.

In the next section we develop two multidimensional between-group inequality measures based on two multidimensional interpersonal inequality measures. Section three discusses the importance of the intermediate attitude to inequality, and in this section we develop two between-group multidimensional intermediate inequality measures. Section four describes the empirical applications of the multidimensional between-group relative and intermediate inequality measures to assess inequality among the social groups in fourteen major states of India, and section five concludes.

### 4.2. Group analogue of the multidimensional inequality measures

The first multidimensional between-group inequality measure is developed from the AKS index developed in Tsui (1995). This measure satisfies the *uniform Pigou-Dalton majorization* (UPDM) and *uniform majorization* (UM). The second measure is developed by converting the Generalized Gini index (List, 1999) to adapt it to suit between-group inequality. This measure satisfies the UM, UPDM and CIM criteria, along with other relevant properties of multidimensional inequality measures.

#### 4.2.1. Group analogue of multidimensional AKS measure

First we develop the group analogue of the AKS index of multidimensional interpersonal inequality following Tsui (1995). This measure is based on the social welfare approach to inequality (Atkinson, 1970; Kolm, 1966; and Sen, 1973).

Let there be \(K\) well-defined groups and \(M\) attributes, and the social welfare/evaluation function (SEF) be continuous, increasing, concave and additively separable\(^{36}\). Assume that all individuals

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35 We assess between-group inequality in the distribution of consumer expenditure and years of education in Andhra Pradesh, Assam, Bihar, Gujrat, Karnataka, Kerala, Madhya Pradesh, Maharastra, Orissa, Punjab, Rajasthan, Tamil Nadu, Uttar Pradesh and West Bengal.

included in a group are alike. This assumption enables us to view the groups as individuals, and we can derive the group analogue of AKS index, which has the following form\(^{37}\):

\[
I_{\text{AKS}} = 1 - \left( \sum_{i=1}^{K} \lambda_i \left( \prod_{j=1}^{M} \left( \frac{\mu_{ij}}{\bar{\mu}_j} \right)^{r_j} \right) \right)^{\frac{1}{r_j}} \tag{1}
\]

Where \(\lambda_i\) is the population share of the \(i\)-th group, \(\mu_{ij}\) is the mean of the \(j\)-th attribute for the \(i\)-th group and \(\bar{\mu}_j\) is the mean value of the \(j\)-th attribute.

Index (1) satisfies the normalization, within-group anonymity, between-group anonymity, scale invariance, total population size and group replication invariance principles. However, this index of multidimensional between-group inequality measure does not satisfy the population composition invariance principle as it is weighted by the population shares of the groups. In addition this measure satisfies the uniform majorization and uniform Pigou-Dalton majorization criteria. However, this measure does not satisfy the correlation increasing majorization criterion, as it fails to take into account the systematic cross-correlation between between-group inequalities in different dimensions.

4.2.2. Group analogue of multidimensional Generalized Gini Index (GGI)

Following the method of formation of multidimensional Generalized Gini (List, 1999), we develop its group analogue form, by assuming that the individuals in a groups are alike, i.e. we can view groups as individuals. If there are \(K\) groups and \(M\) attributes in the multidimensional distribution, then the group analogue of Generalized Gini Index (GGI) has the following form\(^{38}\):

\[
I_{\text{GGI}} = 1 - \eta + \theta \tag{2}
\]

Where \(\eta = \left( \frac{1}{M} \right) \sum_{i=1}^{K} \lambda_i \left( \sum_{j=1}^{M} \left( \frac{\mu_{ij}}{\bar{\mu}_j} \right)^{r_j} \right) \) and

\[
\theta = \left( \frac{1}{MK(K-1)} \right) \sum_{i>j}^{K} \lambda_i \sum_{j=1}^{K} \left[ \left( \sum_{j=1}^{M} \left( \frac{\mu_{ij}}{\bar{\mu}_j} \right)^{r_j} \right) - \left( \sum_{j=1}^{M} \left( \frac{\mu_{ij}}{\bar{\mu}_j} \right)^{r_j} \right) \lambda_i \lambda_j \right] \]

\(^{37}\) The derivation of the group analogue of multidimensional AKS index is described in Appendix II.

\(^{38}\) The derivation of the group analogue of Generalized Gini coefficient is described in Appendix II.
Where $\lambda_i$ and $\lambda_1$ are the population shares of the $i$-th and $l$-th groups; $\mu_{ij}$ and $\mu_j$ are the average value of the $j$-th attribute for $i$-th group and the average value of the $j$-th attribute; and $r_j$ is a parameter representing the concavity of the function from which we this index has been developed. The minimum value of the index $I^{GGI}$ is 0, and its value rises with the rise in between-group inequality.

This index satisfies within-group anonymity, scale invariance, total population size invariance and group replication invariance principles. However, it does not satisfy population composition invariance and between-group anonymity principles. Moreover, this index also satisfies the uniform majorization, uniform Pigou-Dalton majorization and correlation increasing majorization criteria. Therefore, this index can order larger set of multidimensional distributions according to the degrees of between-group inequality as well as on the basis of the systematic cross-correlation between between-group inequalities in different dimensions of the multidimensional distributions.

The computed value of the index $I^{GGI}$ varies across settings depending on the relative strengths of the degrees of between-group inequalities across dimensions and the systematic cross-correlation between between-group inequalities in different dimensions. If one distribution is obtained by progressive transfers from another, then the between-group multidimensional inequality in the former distribution might be greater if there is identical cross-correlation between inequalities across different dimensions. Again, if in one distribution systematic cross-correlation between between-group inequalities in different dimensions is greater than the other, then with equal degrees of inequality across dimensions the degrees of multidimensional inequality in the former is greater than the latter.

4.3. Intermediate measure of multidimensional between-group inequality
4.3.1. Relative and absolute approaches to between-group inequality
In the earlier part of the thesis we have mentioned that there are two broad categories of inequality measures, which are defined by the value judgements associated with inequality measurement. One category comprises relative inequality measures which satisfy the scale invariance property of the inequality measure, i.e., equal proportional rise in the value of the
attribute cannot influence the value of inequality. Second class includes the absolute measures of
inequality which satisfy the translation invariance property, i.e., equal absolute rise in the value
of the attribute doesn’t influence the value of inequality.

Kolm (1997a & 1976b) has designated the relative inequality indices as rightist measures of
inequality and absolute index of inequality index as leftist measures of inequality as equal
proportional rise in income raises the absolute difference between the incomes and the actual
inequality condition will be aggravated and equal absolute rise may reduce the percentage
difference between the income levels of a distribution. According to his view, if all observations
increase by equal proportion (say, \(\lambda\)), the dispersion of the distribution will increase. On the
contrary, when all observations increase by equal absolute amount, the lowest income will rise
by the largest percentage or rate and the highest income will rise by the lowest percentage, i.e.,
dispersion of the distribution declines.

Based on this view, Bossert and Pfingsten used the compromise property: (i) \(I(x) < I(\lambda x)\), and (ii)
\(I(x) > I(x + \delta)\). This compromise condition is less conservative compared to the conditions
applied in the case of relative measures of inequality. In case of relative inequality measures the
condition is: \(I(\lambda x) = I(x)\); where \(n \geq 2\) and \(x \in \mathbb{D}^n\) and \(\lambda \in \mathbb{R}\) (\(\mathbb{R}\) is the set of all real numbers).
Likewise, for the absolute inequality measures: \(I(x + \delta) = I(x)\); where \(n \geq 2\) and \(x \in \mathbb{D}^n\) and \(\delta \in \mathbb{R}\)
(\(\mathbb{R}\) is the set of all real numbers).

On the basis of this view regarding the relative and absolute measures of inequality, Kolm
(1997a & 1976b) proposed a less conservative approach to inequality and termed this new
approach as ‘intermediate approach’ to inequality. The inequality measures which are based on
the intermediate approach to inequality satisfy the aforementioned compromise condition.
Different scholars have proposed different forms of the intermediate inequality measures and the
fundamental similarity among all these measures is that they all satisfy the compromise
condition\(^{39}\).

\(^{39}\) Different intermediate measures are explained in the Appendix III.
4.3.2. Group analogue of multidimensional intermediate inequality measures

Following the intermediate approach we develop two intermediate measures of multidimensional between-group inequality by transforming two multidimensional relative measures of between-group inequality developed in the earlier sections, by adding a constant ‘α’ with each observation of all attributes (Aczel, 1966), i.e., in the numerator and denominator of the indices. These measures satisfy the compromise condition stated above.

If there are K well-define groups and M attributes, then the forms of the indices are:

\[ I_C^{AKS} = 1 - \left( \sum_{i=1}^{M} \lambda_i \left( \prod_{j=1}^{K} \left( \frac{\mu_{ij} + \alpha_k}{\mu_{ij} + \alpha_k} \right)^{r_{ij}} \right) \right) \sum_{j=1}^{r_{ij}} \frac{1}{r_{ij}} \]  \hspace{1cm} (3)

and

\[ I_C^{GGI} = 1 - \gamma + \delta \]  \hspace{1cm} (4)

Where \( \gamma = \left( \frac{1}{M} \right) \sum_{i=1}^{K} \lambda_i \left( \sum_{j=1}^{M} \left( \frac{\mu_{ij} + \alpha_k}{\mu_{ij} + \alpha_k} \right)^{r_{ij}} \right) \) and \( \delta = \left( \frac{1}{MK(K-1)} \right) \sum_{i=1}^{K} \sum_{l=1}^{K} \left[ \sum_{j=1}^{M} \left( \frac{\mu_{ij} + \alpha_k}{\mu_{ij} + \alpha_k} \right)^{r_{ij}} - \sum_{j=1}^{M} \left( \frac{\mu_{il} + \alpha_k}{\mu_{il} + \alpha_k} \right)^{r_{ij}} \right] \lambda_i, \lambda_l \)

Where \( \lambda_i \) and \( \lambda_l \) are the population shares of the i-th and l-th groups. The value of \( \alpha_k \) shouldn’t be very large, as the larger value of \( \alpha_k \) reduces the importance of the observations. Therefore, \( \alpha_k \) should be chosen carefully according to the observations of the attribute/s. These measures are less conservative and satisfy the normalization, within-group anonymity (index (3) satisfies the between-group anonymity, but index (4) does not satisfy total population size invariance and group replication invariance principles. These measures do not satisfy the population composition invariance principle as these are population weighted form of the group specific aggregate achievement across dimensions. Like the multidimensional relative inequality measure, (3) satisfies only uniform majorization criterion, though (4) satisfies uniform majorization and correlation increasing majorization criteria.

4.4. Applications of the multidimensional measures

4.4.1. Data source and methods

We use the data set from Indian Human Development Survey (IHDS; 2004-05), conducted by National Council of Applied Economic Research (NCAER), New Delhi, India, in collaboration
**Table 4.1: Average ‘mpce’ of the social groups in India and in its fourteen major states (in Rupees)**

<table>
<thead>
<tr>
<th>States</th>
<th>Rural (A)</th>
<th>Urban (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Others</td>
<td>OBC</td>
</tr>
<tr>
<td>Andhra Pradesh</td>
<td>976.83</td>
<td>778.24</td>
</tr>
<tr>
<td>Assam</td>
<td>1127.55</td>
<td>646.03</td>
</tr>
<tr>
<td>Bihar</td>
<td>732.56</td>
<td>613.06</td>
</tr>
<tr>
<td>Gujrat</td>
<td>863.2</td>
<td>736.72</td>
</tr>
<tr>
<td>Karnataka</td>
<td>865.26</td>
<td>776.92</td>
</tr>
<tr>
<td>Kerala</td>
<td>1069.07</td>
<td>926.08</td>
</tr>
<tr>
<td>Madhya Pradesh</td>
<td>637.62</td>
<td>518.95</td>
</tr>
<tr>
<td>Maharasra</td>
<td>691.53</td>
<td>702.07</td>
</tr>
<tr>
<td>Orissa</td>
<td>638.62</td>
<td>485.72</td>
</tr>
<tr>
<td>Punjab</td>
<td>1183.68</td>
<td>988.16</td>
</tr>
<tr>
<td>Rajasthan</td>
<td>745.56</td>
<td>689.53</td>
</tr>
<tr>
<td>Tamil Nadu</td>
<td>958.79</td>
<td>819.69</td>
</tr>
<tr>
<td>Uttar Pradesh</td>
<td>878.04</td>
<td>701.71</td>
</tr>
<tr>
<td>West Bengal</td>
<td>878.57</td>
<td>790.75</td>
</tr>
<tr>
<td>All India</td>
<td>852.99</td>
<td>760.88</td>
</tr>
</tbody>
</table>

*Source: Indian Human Development Survey Data, 2004-05.*
Table 4.2: Average years of schooling of the social groups in India and its fourteen major states

<table>
<thead>
<tr>
<th>States</th>
<th>Rural (A)</th>
<th>Urban (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Others</td>
<td>OBC</td>
</tr>
<tr>
<td>Andhra Pradesh</td>
<td>4.68</td>
<td>3.03</td>
</tr>
<tr>
<td>Assam</td>
<td>6.9</td>
<td>5.37</td>
</tr>
<tr>
<td>Bihar</td>
<td>4.87</td>
<td>2.43</td>
</tr>
<tr>
<td>Gujrat</td>
<td>4.72</td>
<td>3.69</td>
</tr>
<tr>
<td>Karnataka</td>
<td>5.05</td>
<td>4.37</td>
</tr>
<tr>
<td>Kerala</td>
<td>7.71</td>
<td>7.07</td>
</tr>
<tr>
<td>Madhya Pradesh</td>
<td>4.51</td>
<td>3.08</td>
</tr>
<tr>
<td>Maharastra</td>
<td>4.92</td>
<td>4.81</td>
</tr>
<tr>
<td>Orissa</td>
<td>4.88</td>
<td>4.11</td>
</tr>
<tr>
<td>Punjab</td>
<td>4.78</td>
<td>4.58</td>
</tr>
<tr>
<td>Rajasthan</td>
<td>3.83</td>
<td>3.13</td>
</tr>
<tr>
<td>Tamil Nadu</td>
<td>5.43</td>
<td>5.12</td>
</tr>
<tr>
<td>Uttar Pradesh</td>
<td>4.96</td>
<td>3.17</td>
</tr>
<tr>
<td>West Bengal</td>
<td>5.38</td>
<td>4.69</td>
</tr>
<tr>
<td>All India</td>
<td>5.06</td>
<td>3.71</td>
</tr>
</tbody>
</table>

*Source: Indian Human Development Survey Data, 2004-05.*
with the University of Maryland to compute the value of multidimensional between-group inequality in India. The data were collected on the basis of stratified random sampling procedure, which covered 41554 households and 215784 individuals in rural and urban areas of India. This survey dataset provides information on incomes (income from different sources, such as income from farms, income from salary, income from agricultural and non-agricultural wages and income from business), consumer expenditures, and educational achievements in terms of years of schooling, and so forth. It also provides some demographic details like sex, caste, religion, family background, etc. The sample includes individuals belonging to different religious groups, and we take only those individuals who report themselves as Hindu as our objective is to evaluate the inequalities among the social groups in multidimensional space. Moreover, we include the persons aged above 24 years in the sample. Therefore, finally the restricted sample came down to 91214 individuals (59429 in rural and 31785 in urban).

We take two attributes - monthly per capita consumer expenditure (‘mpce’) and educational achievements (measured in terms of ‘years of schooling’) of the individuals, and compute the multidimensional inequality among four social groups, namely ST, SC, OBC and ‘others’ in rural and urban areas across fourteen major states of India. The relative and intermediate forms of two aforementioned multidimensional between-group inequality measures are also used in the analysis. To compute the group analogue of the multidimensional AKS index ($I^{AKS}$) and group analogue of the multidimensional GGI ($I^{GGI}$), we take the value of the inequality aversion parameter $r_j = 0.5$, $\forall j = 1, 2, \ldots, M$.

4.4.2. Between-group inequality in the unidimensional spaces in rural and urban areas of India

Tables 4.1 and 4.2 report the average values of ‘mpce’ and ‘years of schooling’ of four social groups separately for rural and urban sectors across fourteen major states of India.

Mutatkar (2005) and Thorat (2007) have conducted studies on between-group inequality across states in the unidimensional and multidimensional spaces by using ‘disparity ratio’. Thorat (2007) found that the disparity ratios of average ‘mpce’, infant mortality rate and the rate of literacy varied across the states. Based on the average ‘mpce’ in 1999-2000, the relative positions of STs and SCs compared to ‘others’ (including OBCs) were the worst in Maharashtra and Tamil
Nadu. It is worth noting that the deprived groups in some states regained their positions in other states. However, in this study there was no systematic disparity among the concerned states or regions on the basis of the inequalities among the social groups in the distribution of consumer expenditure, or in terms of any other attributes of interest. The observations made by Mutatkar (2005) are also not significantly different from Thorat (2007) on this.

Table 4.3: Representational inequality in consumer expenditure and in years of schooling

<table>
<thead>
<tr>
<th>States</th>
<th>$D^R_{mpce}$</th>
<th>$D^R_{edu}$</th>
<th>$D^U_{mpce}$</th>
<th>$D^U_{edu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andhra Pradesh</td>
<td>0.0939</td>
<td>0.1705</td>
<td>0.1011</td>
<td>0.1190</td>
</tr>
<tr>
<td>Assam</td>
<td>0.0924</td>
<td>0.1089</td>
<td>0.1392</td>
<td>0.0472</td>
</tr>
<tr>
<td>Bihar</td>
<td>0.1860</td>
<td>0.2401</td>
<td>0.2292</td>
<td>0.1619</td>
</tr>
<tr>
<td>Gujarat</td>
<td>0.1089</td>
<td>0.1421</td>
<td>0.2476</td>
<td>0.2234</td>
</tr>
<tr>
<td>Karnataka</td>
<td>0.1544</td>
<td>0.1601</td>
<td>0.1330</td>
<td>0.1220</td>
</tr>
<tr>
<td>Kerala</td>
<td>0.1153</td>
<td>0.0791</td>
<td>0.1877</td>
<td>0.0674</td>
</tr>
<tr>
<td>Madhya Pradesh</td>
<td>0.1845</td>
<td>0.2192</td>
<td>0.2420</td>
<td>0.2740</td>
</tr>
<tr>
<td>Maharashtra</td>
<td>0.1057</td>
<td>0.0753</td>
<td>0.1088</td>
<td>0.1094</td>
</tr>
<tr>
<td>Orissa</td>
<td>0.1782</td>
<td>0.2197</td>
<td>0.2702</td>
<td>0.2402</td>
</tr>
<tr>
<td>Punjab</td>
<td>0.1455</td>
<td>0.1450</td>
<td>0.1344</td>
<td>0.2300</td>
</tr>
<tr>
<td>Rajasthan</td>
<td>0.1210</td>
<td>0.1809</td>
<td>0.1885</td>
<td>0.1820</td>
</tr>
<tr>
<td>Tamil Nadu</td>
<td>0.1278</td>
<td>0.1069</td>
<td>0.1085</td>
<td>0.0996</td>
</tr>
<tr>
<td>Uttar Pradesh</td>
<td>0.1736</td>
<td>0.2032</td>
<td>0.2532</td>
<td>0.2403</td>
</tr>
<tr>
<td>West Bengal</td>
<td>0.1429</td>
<td>0.2257</td>
<td>0.1964</td>
<td>0.1836</td>
</tr>
<tr>
<td>All India</td>
<td>0.1455</td>
<td>0.1834</td>
<td>0.1809</td>
<td>0.1614</td>
</tr>
</tbody>
</table>

Note: $D^R_{mpce}$ and $D^U_{mpce}$ are the representational inequality indices in the distribution of ‘mpce’ in rural and urban areas; and $D^R_{edu}$ and $D^U_{edu}$ are the representational inequality indices in rural and urban areas.

Source: Indian Human Development Survey Data, 2004-05.

However, by using a unidimensional measure of between-group inequality based on the concept of ‘representational inequality’ we have got some systematic differences among the states of India on the basis of the between-group inequality in the distribution of ‘mpce’ and years of schooling (reported in Table 4.3). It is observed that both in rural and urban areas of Bihar, Madhya Pradesh, Orissa and Uttar Pradesh (which are relatively poorer states in terms of average
inequalities among the social groups were greater than the relatively richer states/regions\textsuperscript{40}.

4.4.2. \textit{Multidimensional between-group inequality in major states of India}

Thorat (2007) assessed multidimensional inequality among STs, SCs and ‘others’ by the disparity ratio of the group specific Human Development Index (HDI)\textsuperscript{41}. According to his finding, there was no systematic disparity among the states in this regards. However, in our study using two indicators ‘mpce’ and ‘years of schooling’, and invoking the multidimensional group analogue of AKS index for the purpose of assessing between-group inequality in the multidimensional space we can identify some systematic difference in between-group

\begin{table}[h]
\centering
\begin{tabular}{|l|cccc|}
\hline
\textbf{States} & \textbf{$I^{\text{AKS}}$} & \textbf{$I^{\text{cAKS}}$} & \textbf{$I^{\text{GHI}}$} & \textbf{$I^{\text{cGHI}}$} \\
& $r_j(\forall j) = 0.5$ & $r_j(\forall j) = 0.5$ & $r_j(\forall j) = 0.5$ & $r_j(\forall j) = 0.5$ \\
& (2) & (3) & (4) & (5) \\
\hline
Andhra Pradesh & 0.075 (6) & 0.062 (6) & 0.069 (8) & 0.072 (8) \\
Assam & 0.078 (5) & 0.075 (5) & 0.065 (9) & 0.064 (9) \\
Bihar & 0.157 (1) & 0.133 (1) & 0.174 (1) & 0.179 (1) \\
Gujarat & 0.041(12) & 0.037(12) & 0.058 (11) & 0.056 (10) \\
Karnataka & 0.049 (9) & 0.053 (9) & 0.083 (6) & 0.08 (6) \\
Kerala & 0.045(10) & 0.041(11) & 0.052 (12) & 0.041 (12) \\
Madhya Pradesh & 0.102 (4) & 0.091 (4) & 0.133 (3) & 0.137 (3) \\
Maharashtra & 0.043(11) & 0.043(10) & 0.044 (13) & 0.04 (13) \\
Orissa & 0.129 (2) & 0.119 (2) & 0.148 (2) & 0.145 (2) \\
Punjab & 0.031(13) & 0.032(13) & 0.038 (14) & 0.037 (14) \\
Rajasthan & 0.069 (7) & 0.059 (7) & 0.098 (5) & 0.105 (4) \\
Tamil Nadu & 0.029 (14) & 0.022(14) & 0.062 (10) & 0.055 (11) \\
Uttar Pradesh & 0.101 (3) & 0.097(3) & 0.119 (4) & 0.102 (5) \\
West Bengal & 0.051 (8) & 0.056(8) & 0.078 (7) & 0.079 (7) \\
All India & 0.106 & 0.092 & 0.117 & 0.091 \\
\hline
\end{tabular}
\caption{Multidimensional inequality among the social groups in rural areas}
\end{table}

Source: \textit{Indian Human Development Survey Data, 2004-05.}

\textsuperscript{40} Although this study is based on the IHDS dataset (2004-05), since the dataset is nationally representative, the findings are comparable with the findings of the study of Thorat (2007), which is based on NSSO, 55\textsuperscript{th} Round (1999-2000).

\textsuperscript{41} Stanton (2007) has also suggested this method to assess between-group inequality.
inequality between the states/regions in two dimensions – educational attainment and the standard of living.

Table 4.5: Multidimensional inequality among the social groups in urban areas

<table>
<thead>
<tr>
<th>States</th>
<th>1_{AKS}</th>
<th>1_{GSI}</th>
<th>1_{GSI}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r(j) = 0.5 )</td>
<td>( r(j) = 0.5 )</td>
<td>( r(j) = 0.5 )</td>
</tr>
<tr>
<td>Andhra Pradesh</td>
<td>0.073 (10)</td>
<td>0.063 (10)</td>
<td>0.084 (11)</td>
</tr>
<tr>
<td>Assam</td>
<td>0.105 (5)</td>
<td>0.101 (5)</td>
<td>0.067 (14)</td>
</tr>
<tr>
<td>Bihar</td>
<td>0.169 (2)</td>
<td>0.151 (2)</td>
<td>0.175 (2)</td>
</tr>
<tr>
<td>Gujarat</td>
<td>0.058 (11)</td>
<td>0.053 (11)</td>
<td>0.069 (12)</td>
</tr>
<tr>
<td>Karnataka</td>
<td>0.074 (9)</td>
<td>0.067 (9)</td>
<td>0.102 (7)</td>
</tr>
<tr>
<td>Kerala</td>
<td>0.042 (14)</td>
<td>0.041 (14)</td>
<td>0.068 (13)</td>
</tr>
<tr>
<td>Madhya Pradesh</td>
<td>0.184 (1)</td>
<td>0.162 (1)</td>
<td>0.201 (1)</td>
</tr>
<tr>
<td>Maharashtra</td>
<td>0.076 (8)</td>
<td>0.069 (8)</td>
<td>0.097 (9)</td>
</tr>
<tr>
<td>Orissa</td>
<td>0.132 (4)</td>
<td>0.136 (4)</td>
<td>0.157 (4)</td>
</tr>
<tr>
<td>Punjab</td>
<td>0.091 (7)</td>
<td>0.072 (7)</td>
<td>0.01 (8)</td>
</tr>
<tr>
<td>Rajasthan</td>
<td>0.098 (6)</td>
<td>0.09 (6)</td>
<td>0.109 (6)</td>
</tr>
<tr>
<td>Tamil Nadu</td>
<td>0.044 (13)</td>
<td>0.051 (12)</td>
<td>0.135 (5)</td>
</tr>
<tr>
<td>Uttar Pradesh</td>
<td>0.138 (3)</td>
<td>0.149 (3)</td>
<td>0.159 (3)</td>
</tr>
<tr>
<td>West Bengal</td>
<td>0.055 (12)</td>
<td>0.044 (13)</td>
<td>0.09 (10)</td>
</tr>
<tr>
<td>All India</td>
<td>0.116</td>
<td>0.112</td>
<td>0.185</td>
</tr>
</tbody>
</table>

Source: Indian Human Development Survey Data, 2004-05.

Tables 4.4 and 4.5 present the computed values of the group analogue of the relative AKS index. We find a systematic difference among the states of India. Rural areas of Bihar (1_{AKS} = 0.157), Orissa (1_{AKS} = 0.129), Uttar Pradesh (1_{AKS} = 0.101) and Madhya Pradesh (1_{AKS} = 0.102) respectively, have the first, second, third and fourth highest incidences of multidimensional inequality among the social groups in India. Relatively richer states occupy lower positions. Multidimensional inequality among the social groups is the least in Tamil Nadu (1_{AKS} = 0.029) and the second lowest position is occupied by Punjab (1_{AKS} = 0.031), which are relatively richer states. Likewise, in the urban areas relative poorer states, Madhya Pradesh (1_{AKS} = 0.184), Bihar (1_{AKS} = 0.129), Uttar Pradesh (1_{AKS} = 0.108) and Orissa (1_{AKS} = 0.102) respectively, have the first, second, third and fourth highest incidences of multidimensional inequality among the social
groups in urban India. Kerala and Tamil Nadu occupy the bottom and next to the bottom positions in terms of the relative group analogue of the AKS index.

Figure 4.1: Association between ‘mpce’ and multidimensional AKS index (Rural)

Figure 4.2: Association between ‘mpce’ and multidimensional AKS index (Urban)
Therefore, according to the relative group analogue of the AKS measure, systematic differences are found in the multidimensional between-group inequality between the rich and poor states of India. We present in Figures 4.1 and 4.2, the scatter plots of the state average ‘mpce’ and multidimensional inequality separately for rural and urban areas. They all show negative relationships between the state average ‘mpce’ and multidimensional inequality.

Interestingly, invoking the centrist attitude to inequality and taking the intermediate group analogue of AKS index into account, this relation is not altered significantly. Based on the values of $I^\text{AKS}_c$ (in rural areas) the rankings of the states do not differ significantly from $I^\text{AKS}$, and a significant number of states (12 states) are able to retain their ranks. There is a positive and stronger correlation between the ranking of the states by $I^\text{AKS}_c$ and $I^\text{AKS}$ (Spearmen’s rank correlation coefficient is 0.995, at 1% level of significance). Likewise, on the basis of the values of intermediate group analogue of AKS index assessed in the urban areas the rankings of the states change in a similar way (Spearmen’s rank correlation coefficient is 0.987, at 1% level of significance) and twelve states can also retain their ranks accordingly.

It is observed that in rural Andhra Pradesh, Assam, Kerala, Maharashtra, and Punjab, some of the groups are advantaged in one dimension and relatively disadvantaged in other. So, the systematic cross-correlation between between-group inequalities in different dimensions is not so strong in these states. So, these states slip down in the league table if we use $I^\text{GGI}$ instead of $I^\text{AKS}$, except Gujrat (move up by one rank) if we use $I^\text{GGI}$ instead of $I^\text{AKS}$. It gets reversed in some other states due to the existence of stronger systematic cross-correlation between between-group inequalities in different dimensions. Existence of systematic cross-correlation between between-group inequalities in different dimensions existing in the multidimensional distribution can be identified by the deviation of the ranks of the states between $I^\text{GGI}$ and $I^\text{AKS}$. The ranks of almost all states taken into account according to $I^\text{GGI}$ instead of $I^\text{AKS}$ differ significantly (Spearmann’s rank correlation coefficient being 0.862, significant at 1% level). In this case only two states are able to retain their ranks in the league table when we use $I^\text{GGI}$ instead of $I^\text{AKS}$. This implies that local inequalities among the social groups vary significantly across states. Even after the significant change in the ranks of the states on using the index $I^\text{GGI}$, the positions of the poorest states do not change from the top according to the multidimensional between-group inequality.
Likewise in the urban areas, due to the existence of stronger systematic cross-correlation between inequalities in different dimensions, some states move up along the league table in case of ranking by $I^{GGI}$ instead of $I^{AKS}$. For instance, Karnataka, Tamil Nadu, and West Bengal move up in the league table if we use $I^{GGI}$ instead of the relative AKS index. Some of other states slip down in the league table due to weak systematic cross-correlation between between-group inequalities across dimensions. The ranking of the states differ significantly, and only five states can retain their ranks between these two measures of multidimensional inequalities. The correlation between the inter-state rankings determined by $I^{AKS}$ and $I^{GGI}$ is positive but weak (Spearman’s rank correlation coefficient being 0.659, significant at 1% level). In spite of the change in the computed values of between-group inequality and ranks of the states, the positions of the poorest states Bihar, Orissa, Madhya Pradesh and Uttar Pradesh have not changed significantly according to $I^{GGI}$, i.e., these states are still occupying the top positions of the league table according to the computed values of between-group inequality in the urban areas.

By invoking the centrist attitude to inequality instead of the rightist attitude, in case of the group analogue of AKS index in the rural areas the ranking of the states do not differ significantly. A significant number of states (12 states) are able to retain their ranks with $I^{AKS}_C$ instead of $I^{AKS}$, and the correlation between the ranking of the states between these two measures is positive and strong (Spearman’s rank correlation coefficient is 0.995, significant at 1% level). By using $I^{AKS}_C$ instead of $I^{AKS}$, twelve states can also retain their ranks in the urban areas, and the Spearman’s rank correlation coefficient between the rankings of the states is 0.987 (significant at 1% level).

Likewise the group analogue of AKS measure, with the change in the attitude to inequality in case of group analogue of generalized Gini in the rural areas, we cannot find any significant difference between the ranking of the states. For instance, for using $I^{GGI}_C$ instead of $I^{GGI}$, there is no spectacular change in the ranks of the states (Spearman’s rank correlation coefficient being 0.991, significant at 1% level), where ten states can retain their ranks. Likewise in the urban areas the correlation between $I^{GGI}_C$ instead of $I^{GGI}$ is strong enough and positive (Spearman’s rank correlation coefficient being 0.973, significant at 1% level) and eight states can retain their ranks. Despite the changes in the rankings of the states between $I^{GGI}_C$ and $I^{GGI}$, the economically poorer
states still occupy the top positions of the hierarchy according to the value of between-group inequality in India.

4.5. Conclusion
From the theoretical analysis of this chapter, it is apparent that between-group inequality in the multidimensional space should not be assessed only by adding the between-group inequalities in different dimensions of interest. The multidimensional between-group inequality depends on two factors – one is the degrees of between-group inequality in different dimensions and other is the systematic cross-correlation between between-group inequalities in different dimensions. The measures which assess multidimensional between-group inequality by taking account both these factors explore the degrees of between-group inequality in the multidimensional distribution according to the relative strengths of these factors. If some groups are advantaged in some dimensions and disadvantaged in some other dimensions, then this weaken the degree of multidimensional between-group inequality.

It has been noted that the measure of multidimensional between-group inequality, which satisfies the UM and UPDM criteria, can only assess the degrees of between-group inequalities in different dimensions, and the measures which satisfy UM, UPDM and CIM, can capture the degrees of between-group inequality in different dimensions and systematic cross-correlation between between-group inequalities in different dimensions. Therefore, the ranking of the states varied significantly between these two measures of multidimensional between-group inequality.

Moreover, it has also been discussed that the attitude to inequality represented in terms of some value judgment is important in the study of inequality. There are three approaches to measuring inequality as far as this value judgment is concerned – rightist, intermediate and leftist approaches. If we choose a less conservative approach to inequality, i.e., intermediate approach to inequality instead of the rightist and assess between-group inequality in the multidimensional space, then the ranking differential of the states is not significant.

An important finding in the empirical section of this study is that in India there is a striking contrast in the between-group inequalities in the multidimensional distribution of ‘mpce’ and
'years of schooling' between rich and poor state-regions classified by the average ‘mpce’. In the relatively richer states the computed values of multidimensional between-group inequality assessed by some standard measures of inequality are less than the multidimensional between-group inequalities in the poorer states. This raises the important question, at least in the context of India, if inequality between groups comes down with economic prosperity.
Appendix I:
(A) Basic axioms in the analysis of multidimensional between-group inequality:
(i) Continuity (C)
A multidimensional inequality index \( I^M (X) \) should be continuous.

(ii) Normalization (N)
If all rows of the matrix representing the multidimensional distribution are identical, then there is no inequality in the multidimensional distribution and the value of the inequality index is zero, i.e., \( I^M (X) = 0 \).

(iii) Anonymity (A)
As in the case of unidimensional between-group inequality analysis, the anonymity principle has two sub-principles in the multidimensional between-group inequality analysis.

(a) Within-group anonymity (WIA)
A multidimensional between-group inequality measure satisfies this property if its value is invariant with the permutations of the values of all attributes assigned to different individuals within a group. For \( M \) attributes and \( K \) groups, a measure satisfies this axiom if the interchange in the possessions of each attribute by the individuals within each group do not influence the value of the index.

(b) Group-identity anonymity principle (GIA)
A multidimensional between-group inequality measure satisfies this property if it is invariant after any permutation of the group identities across the distributions of all attributes.

(iv) Population replication invariance (PI):
The population replication invariance principle has three sub-principles in multidimensional between-group inequality analysis: Population composition invariance principle (PCI), Total Population size invariance principle (TPI) and Group replication invariance principle (GRI). These sub-principles are similar with the unidimensional between-group inequality approach (see Chapter Two).
(v) Scale invariance (SI):
A multidimensional between-group inequality measure $I(X)$ satisfies this property, if and only if $I(X) = I(CX)$, where $C = \text{diag} (c_1, c_2, \ldots, c_k) \forall c_i > 0$, i.e., $C$ is a diagonal matrix. This implies that rescaling of the attributes does not affect the value of the index.

(B) Relevant dominance/majorization criteria:
(i) Uniform Pigou-Dalton majorization (UPD):
For two multidimensional distributions $X$ and $Y$ (with $K$ number of groups and $M$ number of attributes), $(X, Y) \in \text{UPD}$ and $X \text{ (upd)} Y$ (i.e., distribution $X$ dominates $X$ according to UPD if and only if $X = TY$, where $T$ is a finite product of the Pigou-Dalton matrices $(T = \lambda E + (1 - \lambda)Q)$, where $E$ is an identity matrix and $Q$ is a permutation matrix. It is also important to state that $X$ cannot be derived by permuting $Y$.

(ii) Uniform majorization (UM):
For two multidimensional distributions $X$ and $Y$, $(X, Y) \in \text{UM}$ and $X \text{ (um)} Y$ if and only if $X = BY$, where $B$ is a bistochastic matrix$^{42}$. It is also important that distribution $X$ cannot be derived by permuting distribution $Y$. The relation becomes strict if $X$ cannot be derived by permuting the rows of $Y$. According to the examples of the four matrices given in Appendix III(A), it is easily observed that $X_1$ is UM of $Y_1$ and $X_2$ is UM of $Y_2$.

(iii) Correlation Increasing Majorization (CIM):
For two multidimensional distributions $X$ and $Y$, $(X, Y) \in \text{CIM}$ and $X \text{ (cim)} Y$ if and only if $Y$ can be derived from $X$ by a permutation of rows and a finite sequence of correlation increasing transfers. If one of the correlation increasing transfers is strict, then $Y$ strictly dominates $X$.

(C) Correlation increasing transfer (Boland and Proschan, 1988; List, 1999):
Let there be two row vectors $a = (a_1, a_2, \ldots, a_K)$ and $b = (b_1, b_2, \ldots, b_K)$ of the matrices representing two multidimensional distributions $A$ and $B$. The distribution $B$ could be derived by a correlation increasing transfer, when for some row indices $i$ and $j$ (where $i$ and $j$ are not equal), $b_i = a_i \quad a_j$

$^{42}$ For any bistochastic $n \times n$ matrix, $B = (b_{ij})$, $\sum_{i=1}^{n} b_{ij} = 1$ and $\sum_{j=1}^{n} b_{ij} = 1$.
and \( b_j = a_i \lor a_j \), and for some \( s \notin \{i, j\} \) \( b_s = a_s \). This correlation increasing transfer is strict if \( a_i \neq b_i \) (Where \( a \land b = (\min\{a_1, b_1\}, \ldots, \min\{a_K, b_K\}) \) and \( a \lor b = (\max\{a_1, b_1\}, \ldots, \max\{a_K, b_K\}) \).

**(D) Local inequalities and correlation increasing/decreasing transfer:**

For instance, if there are three groups in the society and three dimensions, then \( X_1, X_2, Y_1 \) and \( Y_2 \) are four \( 3 \times 3 \) ordered multidimensional distributional matrices, where along the rows we measure the groups and along the columns we measure three attributes.

\[
[X_1] = \begin{bmatrix}
10 & 11 & 15 \\
6 & 8 & 7 \\
4 & 5 & 3
\end{bmatrix}
\]

\[
[X_2] = \begin{bmatrix}
6 & 11 & 3 \\
4 & 8 & 15 \\
10 & 5 & 7
\end{bmatrix}
\]

\[
[Y_1] = \begin{bmatrix}
8 & 9 & 10 \\
7 & 8 & 8 \\
5 & 7 & 6
\end{bmatrix}
\]

\[
[Y_2] = \begin{bmatrix}
7 & 9 & 6 \\
5 & 8 & 10 \\
8 & 7 & 8
\end{bmatrix}
\]

In the distributions \( X_1 \) and \( Y_1 \) first group is the most advantaged group in either dimension and third group is the most disadvantaged group in either dimension, and the second group occupies the intermediated position in terms of either disadvantage or advantage. In the distributions \( X_2 \) and \( Y_2 \) first group is advantaged in second dimension and occupies second and third places in other dimensions. Second group occupies the first place in the distribution of third attribute, and the second and third places in the distributions of second and first attributes. Third group respectively occupies the first, second and third positions in the distributions of the first, third and second attributes.

The matrices \( Y_1 \) are derived from \( X_1 \) by progressive inter-group transfers: from first to second and third groups in first dimension; first to third group in second dimension and first to second and third groups in third dimensions. Likewise, by the transfers of first attribute from third to first and second groups; second attribute from first group to third group and thidss attribute from second to first and third groups we can get the distribution \( Y_2 \) from \( X_2 \). Hence, we can compare the degrees of inequalities between the distributions \( Y_1 \) and \( X_1 \) (where inequality in the former is less than the latter), as well as between the distributions \( Y_2 \) and \( X_2 \) (where inequality in the former is less than the latter).
However, the degrees of inequalities between $X_1$ and $X_2$, or between $Y_1$ and $Y_2$ cannot be compared in a straight forward way, as the distributions $X_2$ and $Y_2$ are derived by ‘correlation decreasing transfers’ from $X_1$ and $Y_1$ (This transfers reduces the systematic cross-correlations between inequalities within different dimensions). In the former two distributions the correlation between advantaged and disadvantaged positions within different dimensions is positive and strong compared to the latter distributions. In other words, in $X_1$ and $Y_1$ someone who is well-off in one dimension is more likely to be well-off in other dimensions and the individuals who are badly-off in one dimension are more likely to be badly-off in other dimensions compared to $X_2$ and $Y_2$.

Therefore, even if degrees of inequalities in $X_1$ and $Y_1$ are less stronger than the degrees of inequalities within $X_2$ and $Y_2$, the estimated values of inequalities in $X_1$ and $Y_1$ may be greater than the latter distributions, due to higher systematic correlation within these distributions.

**Appendix II:**

**A) Derivation of group analogue of multidimensional AKS index**

Let there be $K$ well-defined groups and $M$ attributes. We assume that the individuals included in a group are alike and viewing the groups as individuals. Further, we assume that the welfare functions of the groups are alike, and we consider all assumptions and properties of social welfare function as in Tsui (1995). The group welfare function for a multi-attribute distribution may have the following form if it is continuous, increasing, concave and additively separable (as in Tsui, 1995):

$$u(. ) = a + b \prod_{j=1}^{M} \mu_{ij}^{r_j}$$  \hspace{1cm} (A4.1)

Where $\mu_{ij}$ is the mean value of the $j$-th attribute for $i$-th group. The inequality aversion parameter $r_j$ for $j$-th attribute represents the concavity the group welfare functions.

The social evaluation/welfare function (SEF) can be defined from this (A4.1), where weight are population shares of the group:

$$W(X) = \sum_{i=1}^{K} \lambda_i u(. )$$  \hspace{1cm} (A4.2)
where $\lambda_i$ is the population share of the $i$-th group and $\sum_{i=1}^{m} \lambda_i = 1$.

Therefore, from the group welfare function the SEF can be derived in the following way:

\[ W(X) = a + b \sum_{i=1}^{K} \lambda_i \left( \prod_{j=1}^{M} \frac{r_j}{\mu_{ij}} \right); \text{ for } r_j < 1 \text{ and } r_j \neq 0, \forall j = 1, 2, \ldots, M \quad (A4.3) \]

The form of the *equally distributed equivalent attribute* ($\eta_{je}$) for the SEF represented by (A4.3) is:

\[ \eta_{je} = \left( \frac{1}{r} \sum_{i=1}^{K} \lambda_i \prod_{j=1}^{M} \frac{r_j}{\mu_{ij}} \right)^{1/r_j} \quad (A4.4) \]

The formula of the relative inequality index based on the SEF suggested by Kolm (1966) and later on by Atkinson (1970) for a single attribute is:

\[ I^{AKS} = 1 - \frac{\text{Equally distributed equivalent attribute}}{\text{Mean value of the attribute}} \quad (A4.4) \]

Following the method of Tsui (1995), we can derive the multidimensional between-group inequality AKS index:

\[ I^{AKS} = 1 - \left( \sum_{i=1}^{K} \lambda_i \left( \prod_{j=1}^{M} \frac{\mu_{ij}}{\mu_j} \right)^{r_j} \right)^{\frac{1}{\Sigma r_j}} \quad (A4.5) \]

Where $\mu_j$ is the mean value of the $j$-th attribute.

**B) Derivation of group analogue of Generalized Gini coefficient**

Let there be $K$ well-defined groups and $M$ dimensions, then the multidimensional distribution matrices $X$ and $Y$ can be written as:

\[
X = \begin{bmatrix}
\mu_{11}^X & \cdots & \mu_{1M}^X \\
\vdots & \ddots & \vdots \\
\mu_{K1}^X & \cdots & \mu_{KM}^X
\end{bmatrix} \quad Y = \begin{bmatrix}
\mu_{11}^Y & \cdots & \mu_{1M}^Y \\
\vdots & \ddots & \vdots \\
\mu_{K1}^Y & \cdots & \mu_{KM}^Y
\end{bmatrix}
\]
Along the rows of these matrices, we measure K groups and along the columns, we measure M dimensions.

We develop the multidimensional Generalized Gini by following the approach used by List (1999). We first transform the multidimensional distributions into a unidimensional distribution by using a suitable one-dimensional aggregation function: \(u(\mu_j) : \mathbb{R}_+^K \rightarrow \mathbb{R}_+\). For this purpose a utility like aggregation function is introduced, which derives the vectors of the multidimensional distributions after aggregating group’s achievements across dimensions. By this aggregation function we add each group’s row vector of goods/attributes into an overall evaluation figure and we get a column vector from each multidimensional distribution. The vectors representing the multidimensional distributions are now: \(u(\mu_X) = (u(\mu_1^X), u(\mu_2^X), \ldots, u(\mu_K^X))\) and \(u(\mu_Y) = (u(\mu_1^Y), u(\mu_2^Y), \ldots, u(\mu_K^Y))\) for the distributions \(X\) and \(Y\).

In case of only uniform majorization, the aggregation function should be continuous, increasing and concave (see Tsui, 1999). For two vectors \(X\) and \(Y\), in this case \(X(um)Y\) implies \(X\) u-dominates \(Y\), i.e., according to the first intuition (Kolm, 1977) distribution \(X\) is more equal compared to the multidimensional distribution \(Y\).

However, for correlation increasing majorization criterion the aggregation function should be restricted more and continuous, increasing, concave and L-superadditive (see Tsui, 1999). The aggregation function \(u(\mu_i) : \mathbb{R}_+^K \rightarrow \mathbb{R}_+\), which satisfies continuity, increasing, concavity and L-superadditivity properties may have the following form:

\[
\begin{align*}
  u(\mu_i) &= \sum_{j=1}^{M} \mu_{ij}^{r_j}, \text{ for all } 0 < r_j < 1, j = 1, 2, \ldots, M 
\end{align*}
\]  

(A4.1)

Where \(\mu_{ij}\) is the mean value of the \(j\)-th attribute for \(i\)-th group and the parameter \(r_j\) represents the concavity of the aggregation function and might differ across the attributes.

An index of multidimensional inequality can be developed from the vector of the aggregation function \(u(\mu_X)\) developed from the multidimensional distribution \(X\). If the measure is relative and
satisfies the scale invariance property, then the groups’ aggregation functions should be derived from the compensation matrix of multidimensional distribution. The cell elements of the adjusted or compensated matrix are the ratios of the cell values of the original matrix and respective column averages. To capture the idea of relative inequality measurement represented by scale invariance property of the inequality measure, a multidimensional inequality index should be only sensitive to the relative distribution within each dimension and not to the total distribution.

So, we have to consider an adjusted matrix for each multidimensional distribution dividing the elements of each column (i.e., the group mean values in a dimension) by the population mean value of the particular attribute, i.e., by the respective column average. The matrix after this adjustment is called the compensation matrix. It is important to state that there is no change in the dominance criteria between original and adjusted matrices of multidimensional distribution. If X dominates (u-dominates) Y according to UM, UPD and CIM, then the multidimensional distribution represented by the matrix $X^C$ (adjusted matrix of X) also dominates (i.e., u-dominates) the multidimensional distribution represented by $Y^C$ (adjusted matrix of Y). The adjusted matrix of X and Y are given below:

$$X^C = \begin{bmatrix} \frac{\mu_{11}^X}{\mu_1^X} & \cdots & \frac{\mu_{1M}^X}{\mu_1^M} \\
\frac{\mu_{21}^X}{\mu_2^X} & \cdots & \frac{\mu_{2M}^X}{\mu_2^M} \\
\vdots & \ddots & \vdots \\
\frac{\mu_{M1}^X}{\mu_M^X} & \cdots & \frac{\mu_{MM}^X}{\mu_M^M} \end{bmatrix} \quad \quad Y^C = \begin{bmatrix} \frac{\mu_{11}^Y}{\mu_1^Y} & \cdots & \frac{\mu_{1M}^Y}{\mu_1^M} \\
\frac{\mu_{21}^Y}{\mu_2^Y} & \cdots & \frac{\mu_{2M}^Y}{\mu_2^M} \\
\vdots & \ddots & \vdots \\
\frac{\mu_{M1}^Y}{\mu_M^Y} & \cdots & \frac{\mu_{MM}^Y}{\mu_M^M} \end{bmatrix}$$

The aggregation function derived from the compensation matrix, which satisfies continuity, increasing, concavity and L-superadditivity properties, may have the following form:

$$u(\mu^C) = \sum_{j=1}^{M} \left( \frac{\mu_j}{\mu_j} \right)^{r_j}, \text{ for all } 0 < r_j < 1, \ j = 1, 2, \ldots, M \quad (A4.2)$$

The form of the multidimensional Generalized Gini coefficient with K number groups, M dimensions and $u(\mu^C)$ vector of the distribution, has the following form:

$$I^{GGI} = 1 - \frac{\sum_{i=1}^{K} \lambda_i u(\mu^C)}{M} \left( 1 - G(u(\mu^C), u(\mu^C), \ldots, u(\mu^C)) \right) \quad (A4.3)$$

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Where $\lambda_i$ is the population share of the $i$-th group of the society. We can use the form of the group analogue Gini coefficient given in Chapter two to derive the final form of the Generalized Gini coefficient. The form of the group analogue Gini coefficient with the aggregation function is:

$$ G(u(\mu_1^G), \ldots, u(\mu_K^G)) = \frac{1}{K(K-1)} \sum_{i=1}^{K} \lambda_i u(\mu_i^G) \sum_{j=1}^{K} \sum_{i=1}^{K} |u(\mu_j^G) - u(\mu_i^G)| \cdot \lambda_i \cdot \lambda_j $$

(A4.4)

Therefore, the group analogue of multidimensional generalized Gini coefficient has the following form:

$$ I_{GGI} = 1 - \left( \frac{1}{M} \right) \sum_{i=1}^{K} \lambda_i \cdot u(\mu_i^G) \cdot \sum_{j=1}^{M} \left( \frac{\mu_j}{\mu_i} \right)^{T_j} \cdot \sum_{i=1}^{K} \sum_{j=1}^{K} |u(\mu_j^G) - u(\mu_i^G)| \cdot \lambda_i \cdot \lambda_j $$

(A4.5)

On the basis of the specific form of the aggregation function given by (A2) the form of the index becomes:

$$ I_{GGI} = 1 - \eta + \theta $$

(A4.6)

Where

$$ \eta = \left( \frac{1}{M} \right) \sum_{i=1}^{K} \lambda_i \cdot \sum_{j=1}^{M} \left( \frac{\mu_j}{\mu_i} \right)^{T_j} $$

and

$$ \theta = \left( \frac{1}{M(K-1)} \right) \sum_{i=1}^{K} \sum_{j=1}^{K} \left| \sum_{j=1}^{M} \left( \frac{\mu_j}{\mu_i} \right)^{T_j} - \sum_{j=1}^{M} \left( \frac{\mu_j}{\mu_i} \right)^{T_j} \cdot \lambda_i \cdot \lambda_j \right| $$

Appendix III:

Different intermediate measures of inequality

Bossert and Pfingsten (1996; henceforth BP) have suggested a single parameter class of inequality measure, called $\mu$-inequality concept. An index of intermediate inequality based on this $\mu$-inequality concept satisfies the following condition:

$$ I^\mu(x + \tau(\mu x + (1 - \mu) 1^n)) = I^\mu(x); $$

where $n$ is the number of persons in the economy for vertical inequality measure and $n \geq 2$ and

$$ (x + \tau(\mu x + (1 - \mu) 1^n)) \in D^n; $$

where $D^n$ is the set of all $n$-dimensional vectors with only nonnegative components. BP have developed an index satisfying the intermediate attitude to inequality, which is based on the $\mu$-inequality concept. The $\mu$-inequality case is the intermediate
case of the extreme two cases relative and absolute measures of inequality. For \( \mu = 1 \) the intermediate view becomes the relative view and for \( \mu = 0 \) the intermediate view becomes absolute view. If \( \mu \) lies with 0 and 1 then the attitude to inequality becomes intermediate. However, with the rise in total income or attribute the value of the index approaches to the rightist or relative view of the inequality.

Pfingsten and Seidle (1997) have pointed out these limitations of the BP index and proposed a wide class of intermediate inequality invariant measure, where the inequality invariant ray does not merge with the relative inequality invariant ray. The P-S inequality measure is called \( \alpha \)-ray invariant inequality measure. In general \( \alpha \)-ray invariance requires an inequality measure not to change provided any income change is distributed according to the value judgment presented by the relative pattern \( \alpha \). For an example, if for only two person economy the incomes of these two persons are \( X_1 \) and \( X_2 \). The values of these incomes are 200 and 600. Then \( \alpha \)-ray invariant inequality measure states that the values of incomes will be 230 and 670, if income will rise by 100 units and \( \alpha = (0.3, 0.07) \).

Del Rio and Ruiz Castillo (1999; henceforth DRRC) have introduced another measure of intermediate inequality on the basis of invariance condition. This DRRC class of intermediate inequality measure is a subset of the P-S intermediate measure. DRRC inequality concept is called \((v, \pi)\) inequality; where \( v \) is the reference of the initial distribution and \( \pi \) is related with the distribution of additional income, where \( \pi \in (0, 1) \). Therefore, according to DRRC, if \( \pi \)100% of additional income will be allocated to individuals according to initial income shares and \((1 – \pi)\)100% in equal absolute amounts, then the intermediate attitude of inequality will be satisfied.