CHAPTER - I

STUDIES ON POLARIZATION SPECTRAL FILTERS
INTRODUCTION

Sole and Lyot filter are wellknown examples of linear birefringent networks. The state of polarization and the spectral distribution of intensity are the two properties of the wave that are simultaneously operated upon by such polarizing optical systems. These types of filters, which depend for their action on the interference of polarized light, can be designed for narrow-pass band (about fraction of an angstrom) spectral intensity transmittance. An useful discussion on Lyot and Sole filters and a review of the elementary theory can be found in the works of Evans [1,2]. The comparison between Lyot and Sole filters can be found in Refs. 2 and 3. In both fan and folded type Sole filters, the azimuths of different retarders are obtained by simple mathematical relations. The intensity transmission at the principal pass gets sharper as the number of stages increase, but this will enhance the neighbouring maximas. An interesting development in the birefringent filter system was described by Harris, Ammann and others [3]. They were able to design for any arbitrarily prescribed amplitude transmittance, by impulse response technique, subject to the conditions that the amplitude transmittance must be periodic with frequency and the corresponding impulse response must be real and causal. Sole and Lyot filters may be considered as the special cases of this generalized filter system. Recently the frequency dependent
rotation introduced by an optically active medium i.e. rotator has been shown to be useful for modifying the spectral intensity transmittance. Chakraborty, Ghosh [30,31] have shown a way of using an optically active medium i.e. rotators in the linear birefringent filter system. Section-I of this chapter deals with the effect of using an optically active medium as an interstage element of linear birefringent networks. The spectral intensity transmittance characteristics of different types of transmission type polarization filters are studied by Jones matrix technique [32] where the length of the elements of the successive stages are equal. The well known impulse response technique is also used to design a linear birefringent network. A new type of spectral intensity transmittance and its application are proposed. Section-II deals with the intensity transmittance characteristics of different types of polarization filters where the length of the elements of the successive stages are different.

Tunability of the birefringent filter is an important criterion from an application point of view. Billings [33] showed a series of methods by which a Lyot-Ohman polarization filter could be tuned over a wide range of wavelengths. In the case of a generalized birefringent networks, the use of an electro-optic cell in place of a quartz or Calcite retarder is of great advantage where variable tunability is required [3,34]. Alternatively the proportional changes in the
retardation of each component can be brought about by adding an adjustable phase shifter to each retardation plate. This could consist of a sheet of photo-elastic material under adjustable tension or an electro-optic material in which required retardation is obtained by the application of electric field [33,35,36]. Both the sections of Chapter-I include the discussions on the tunning procedures of the fixed type filter systems.

SECTION-I TRANSMISSION CHARACTERISTICS OF A BIREFRINGENT FILTER SYSTEM WHERE RETARDER-ROTATOR COMBINATION IS USED AT EACH STAGE AND THE LENGTH OF ELEMENTS OF THE SUCCESSIVE STAGES ARE EQUAL

1.1.1 Evaluation of the analytical expressions for the intensity transmittance of a narrow band pass birefringent filter and the birefringent band suppression filter (BBSF) where the thickness of each retarder is same and the thickness of each rotator is same

In order to study the intensity transmittance characteristics of the birefringent filter system, where a retarder-rotator combination is used at each stage, we first start with a fan-type Sole filter, where a rotator is placed after each retarder as shown in the fig.(1.1.1).

Let the Jones vector of the input beam be represented by

$E_i = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$ \hspace{1cm} \ldots (1.1.1)
With the help of Fig.(1.1.1) the Jones vector of the output beam can be written as

\[ E_0 = P_{\text{out}(0)} \ R(\theta/2) \ R(n\theta) \left[ R(\alpha-\theta) \ C(\delta) \right]^n \ R(-\theta/2) \ P_{\text{in}(0)} \ E_1 \]

\[ \ldots (1.1.2) \]

where \( n \) is the number of retardation plates which are equal to the number of rotators. In the fan-type Sole filter the azimuth of the \( j \)th retarder is given by

\[ \theta_j = (\theta/2) + (j-1)\theta \]

\[ \ldots (1.1.3) \]

where \( \theta \) can be obtained by the condition \( n\theta \approx \pi/2 \). Here \( C(\delta) \) and \( R(\alpha) \) represent the Jones matrices of the retarder and rotator respectively [Appendix-II]. \( P_{\text{in}(0)} \) and \( P_{\text{out}(0)} \) represent the Jones matrices of the input and output ideal linear polarizers [Appendix-I] respectively, both have the transmission axes parallel to the x-axis of the reference co-ordinate system.

Now we will use the well known property of an unimodular matrix [30,31,13,14] in order to obtain the expression for intensity transmittance of the filter system. The unimodular matrix \( m_0 \) is given by
\[ m_0 = R(\alpha - \theta)C(\delta) = \begin{pmatrix}
  e^{i\delta/2}\cos(\alpha - \theta) & -\sin(\alpha - \theta) e^{-i\delta/2} \\
  e^{i\delta/2}\sin(\alpha - \theta) & \cos(\alpha - \theta) e^{-i\delta/2}
\end{pmatrix} \]

\[ = \begin{pmatrix}
  m_{11} & m_{12} \\
  m_{21} & m_{22}
\end{pmatrix} \quad \text{... (1.1.4)} \]

The \textit{n}th power of an unimodular matrix is

\[
\begin{pmatrix}
  m_{11} P_{n-1}(x) - P_{n-2}(x) & m_{12} P_{n-1}(x) \\
  m_{21} P_{n-1}(x) & m_{22} P_{n-1}(x) - P_{n-2}(x)
\end{pmatrix} \quad \text{... (1.1.5)}
\]

where \( x = \frac{1}{2}(m_{11} + m_{22}) \) \quad \text{... (1.1.6)}

and \( P_n(x) \) is the Chebyshev polynomial of the second kind, given by

\[
P_n(x) = \sin \left[ (n + 1) \cos^{-1} x \right] / \sqrt{1 - x^2} \quad \text{... (1.1.7)}
\]

Using the above result for \( m_0^n \) in expression (1.1.2), we have

\[
E_o = \begin{pmatrix} P \\ E_x \\ 0 \end{pmatrix} \quad \text{... (1.1.8)}
\]
where
\[ p = \frac{\sin(nX)}{\sin X} \cos \left(\frac{\theta}{2}\right) \cos (\alpha + n\theta - \frac{\theta}{2})e^{i\delta/2} \]
\[ - \frac{\sin[(n-1)X]}{\sin X} \cos (n\theta) \]
\[ + \frac{\sin(nX)}{\sin X} \sin \left(\frac{\theta}{2}\right) \sin(\alpha + n\theta - \frac{\theta}{2})e^{-i\delta/2} \]

Now making use of the condition \( n\theta \approx \pi/2 \) in the above expression, the intensity transmittance of the filter, apart from a constant photometric factor is given by

\[ T = \left(\frac{\sin(nX)}{\sin X}\right)^2 \left[\sin^2(\alpha - \theta) \cos^2 \left(\frac{\delta}{2}\right) + \sin^2 \alpha \sin^2 \left(\frac{\delta}{2}\right)\right] \]

... (1.1.10)

where \( X = \cos^{-1}\left[\cos \left(\delta/2\right) \cos(\alpha - \theta)\right] \)

... (1.1.11)

Putting \( \alpha = 0 \) in the expression (1.1.10), we get the intensity transmittance of an original Sole filter \([2]\), given by

\[ T = \left[\frac{\sin(nX)}{\sin X} \cos X \tan \theta\right]^2 \]

... (1.1.12)

where
\[ \chi = \cos^{-1} \left[ \cos \left( \frac{\delta}{2} \right) \cos \theta \right] \] ... (1.1.13)

Now we derive an expression for the intensity transmittance of the filter, where the slow axis of each retarder is parallel to the transmission axes of the polarizers. In this case the number of retarder is equal to one less than the number of rotator.

Substituting \( \theta = 0 \) in the expression (1.1.9), the intensity transmittance of the filter is found as

\[ T = 1 - \left[ \frac{\sin(n\chi)}{\sin \chi} \sin \alpha \right]^2 \] ... (1.1.14)

where \( \chi = \cos^{-1} \left[ \cos \alpha \cos \left( \frac{\delta}{2} \right) \right] \) ... (1.1.15)

Here \( n \) is the number of rotator in the filter system. The retarder placed just after \( P_{in}(\alpha) \) [Fig. (1.1.1)] can be dropped. The expression (1.1.14) gives the transmission characteristics of a Birefringent Band Suppression Filter (BBSF).

1.1.2 Computation and the key idea of the tuning procedure of the narrow band pass birefringent filter and BBSF where the thickness of each retarder is same and the thickness of each rotator is same

For the purpose of calculations, the rotation \( \alpha \) produced by a quartz plate is given by the well known Biot's law [Appendix-II]
\[ \alpha = A + Bv^2 \]

where \( A = -1.845 \text{ deg./mm.} \) and \( B = 9.00 \times 10^{-29} \text{ deg.} S^2/\text{mm} \).

Neglecting the dispersion of birefringence \((\Delta n)\), the phase difference \( \delta \) introduced between two orthogonal components of the light vibration of wavelength \( \lambda \) by a retardation plate of thickness \( d \), is given by [Appendix-II]

\[
\delta = \frac{2\pi}{\lambda} \Delta n \cdot d
\]

\[ = \frac{c_1}{\lambda} \]

where \( c_1 \) is a constant.

In the case of a narrow pass band filter, the retarder and the rotator thicknesses are chosen in such a way that at principal pass wavelength \( \lambda_{OP} \) the phase difference \( \delta \) introduced by each retarder is 360 deg. and the linearly polarized component \( \lambda_{OP} \) of the light vibration comes out of the filter system, being unobstructed by the output polarizer \( P_{out(o)} \). On the other hand in the case of BBSF the linearly polarized component \( \lambda_{OR} \) of the light vibration is completely cut off by the output polarizer \( P_{out(o)} \). At \( \lambda_{OR} \) the phase difference \( \delta \) introduced by each retarder is also 360 deg. and we may call \( \lambda_{OR} \) the principal rejection wavelength.
1.1.3 Discussions on the transmission curves of a narrow band pass birefringent filter where the thickness of each retarder is same and the thickness of each rotator is same.

Figures (1.1.2), (1.1.3) and (1.1.4) show the variation of intensity transmittance $T$ with wavelength $\lambda$ of a 10-stage ($n = 10$) narrow band pass filter. Each stage consists of a retarder and a rotator as shown in the fig. (1.1.1) and the curves are drawn with the help of the equations (1.1.10) and (1.1.11). For figures (1.1.2), (1.1.3) and (1.1.4) the principal pass wavelengths are 509 nm, 453 nm, and 640 nm respectively. The azimuth of each retarder is given by the expression (1.1.3). The rotators thicknesses are obtained with the help of the relation $\alpha_{OP} = m\pi/2n$, where $\alpha_{OP}$ is the amount of rotation in degrees introduced by each rotator for the principal pass wavelength $\lambda_{OP}$, $n$ is the number of stage and $m = 2, 4, 6, 8, 10, \ldots$ etc. For the plots of the figures (1.1.2), (1.1.3) and (1.1.4) we take $m = 2$, which gives the rotators thicknesses as 0.60, 0.48 and 1 nm respectively. Figures (1.1.2), (1.1.3) and (1.1.4) show that the effect of secondary maxima are more pronounced on the left hand side of the principal maxima than on the right hand side. If now the thickness of each rotator of this ten stage ($n = 10$) network is changed by the same amount we get the plots of the figures (1.1.5) and (1.1.6) which are obtained with $m = 6$ and $m = 10$ respectively and are tuned at the principal
pass wavelength $\lambda_{0p} = 509$ nm. For $m = 6$ and $m = 10$ the rotators thicknesses are obtained as 1.83 mm and 3 mm, respectively. Thus we see that the amplitudes of the secondary maxima on both sides of the principal maxima increase as we take higher values of $m$. If we take odd value of $m$ such as $m = 1$ (as in a BBSF system, discussed later) we get the plot of the fig. (1.1.7), where a small amount of transmission occurs at some discrete wavelengths and virtually a cut-off is obtained over a wide region in the visible spectrum. Fig. (1.1.8) shows the intensity transmittance of a 10-stage Solc filter tuned at $\lambda_{0p} = 509$ nm. A comparison of plot (1.1.2) with the plot (1.1.8) shows that in plot (1.1.2) the amplitude of the secondary maxima on the right hand side of the principal maxima is lower, whereas the secondary maxima on the left hand side of the principal maxima is much higher than that of Solc filter. The widths of the principal maxima are more or less same in both the cases. If now, the number of stages of the narrow pass band filter as discussed above is increased to $n = 14$, we get the plot of the figure (1.1.9) which is obtained with $m = 2$ and $\lambda_{0p} = 509$ nm. The rotator's thickness is 0.44 mm. A comparison of fig. (1.1.9) with the plot of fig. (1.1.2) shows a small decrease in the amplitudes of the secondary maxima on both sides of the principal maxima as well as in the width of the principal maxima as the number of stage is increased to $n = 14$. The thickness of each rotator plate is also found to decrease
as \( n \) increases. This situation is particularly interesting because of the fact that in a well known linear birefringent narrow pass band filter, such as Solc filter, as the number of stages increase, the amplitudes of the secondary maxima on both sides of the principal maxima increase while the width of the principal maxima decreases. Another comparison of fig. (1.1.9) with the \( T \) vs. \( \lambda \) plot of an original 14-stage \((n = 14)\) Solc filter (fig. (1.1.10)) shows that in fig. (1.1.9) the amplitude of the secondary maxima on the higher wavelength side of the principal maxima is lower, whereas the secondary maxima on the lower wavelength side of the principal maxima is much higher than the corresponding Solc filter. Again the widths of the principal maxima are more or less same in both the cases.

1.1.4 Discussions on the transmission curves of a BESF where the thickness of each retarder is same and the thickness of each rotator is same

Plots of the figures (1.1.11), (1.1.12) and (1.1.13) show the variation of intensity transmittance \( T \) with the variation of wavelength \( \lambda \) of a 10-stage BBSF \((n = 10)\) system. Here the slow axis of each retarder is parallel to the transmission axes of the polarizers. The number of retarder is 9 and the number of rotator is 10. The transmittance curves are drawn with the help of the equations (1.1.14) and (1.1.15). For figures (1.1.11), (1.1.12) and (1.1.13), the principal
rejection wavelengths are 509, 640 and 453 nm and the rotators thicknesses are 0.31, 0.50 and 0.24 mm respectively. The rotators thicknesses are obtained with the help of the relation
\[ \alpha_{OR} = \frac{m\pi}{2n} \]
where \( \alpha_{OR} \) is the amount of rotation in degrees introduced by each rotator for the principal rejection wavelength \( \lambda_{OR} \), \( n \) is the number of rotator and \( m = 1, 3, 5, 7, 9, \ldots \) etc. The curves of the figures (1.1.11), (1.1.12) and (1.1.13) are plotted for \( m = 1 \). The above mentioned plots show that the fluctuations on the lefthand side pass band are much higher than the right hand side pass band of the principal rejection wavelength \( \lambda_{OR} \). When \( m \) increases i.e. the thickness of each rotator increases the fluctuations on the pass band of both sides of \( \lambda_{OR} \) increase. Plots of the figures (1.1.14) and (1.1.15) are drawn with \( n = 10 \), \( \lambda_{OR} = 509 \) nm and for \( m = 5 \) and 7 respectively. The rotators thicknesses are 1.53 mm and 2.14 mm respectively. However when \( m = 2 \) (as in narrow band pass birefringent filter) and \( n = 10 \) are used to draw the \( T \) vs. \( \lambda \) curve with the help of the equations (1.1.14) and (1.1.15), we get the plot of fig.(1.1.16) where the intensity transmittance reaches its peak value at 509 nm. Plot of the fig.(1.1.17) is obtained with \( m = 1 \), \( n = 14 \) and for \( \lambda_{OR} = 509 \) nm. In this case the thickness of the rotator is 0.22 mm. Comparing the \( T \) vs. \( \lambda \) curve of the fig.(1.1.17) with that of the fig.(1.1.11) we see that the amplitudes of fluctuations over the pass band of both side of \( \lambda_{OR} = 509 \) nm decreases slightly and the width of
the rejection band decreases too as $n$ increases. Here again the ultimate constraint on the value of $n$ is imposed by the thickness of the rotator which decreases when $n$ increases. The BBSF system may be used with other filter systems for modifying the spectral intensity transmittance characteristics. Fig. (1.1.18) shows the transmittance characteristics of two filter system in combination whose transmission characteristics are represented by the figures (1.1.2) and (1.1.13). Comparing the plot of the fig. (1.1.18) with that of fig. (1.1.2) we note that in the fig. (1.1.18) the secondary maxima on the lefthand side of the principal maxima is much reduced and the transmission at the principal pass wavelength occurs at 98 per cent of its ideal maximum value.

1.1.5 The impulse response technique to synthesize a band suppression filter by employing equal length linear retarder plates as core elements and two linear polarizers

Like Solo filter where equal length linear retarder plates are used as core elements for narrow band pass purpose, a birefringent network can also be synthesized by impulse response technique [3], which uses equal length linear retarder plates as core elements, for band suppression purpose. A five stage linear birefringent network having an approximated periodic rectangular amplitude transmittance (obtained by six term exponential Fourier series and is shown in fig. (1.1.19)) has been synthesized by the simple synthesis process described
by Harris, Ammann and Chang [37]. The absolute angles (measured with respect to the x-axis) of the 1st, 2nd, 3rd, 4th, 5th retarder and the output linear polarizer are \( \theta_1 = -21°36', \theta_2 = -2°32', \theta_3 = -56°34', \theta_4 = -11°037', \theta_5 = -91°19' \) and \( \theta_p = -23°10' \) respectively [Appendix-III]. The transmission axis of the input linear polarizer is assumed to be parallel to the x-axis (horizontal direction) of a cartesian co-ordinate system. The normalized intensity transmittance of the five stage linear birefringent network having an approximated periodic rectangular amplitude transmittance is shown in Fig.(1.1.20). The principal rejection wavelength corresponds to \( \delta = \pi/2 \) or \( 3\pi/2 \) where \( \delta \) is the frequency dependent retardance introduced by each linear retarder plate. Although, ideally, full transmission is desirable at each wavelength of the pass band, here we note some fluctuations over the pass band of the band suppression filter. By increasing the number of stages the rejection band-width can be made narrower and the amplitudes of the ripple over the pass band can also be reduced. But as the number of stages increase the number of terms in the approximated amplitude transmittance increases too (the mean square error comes down) while the approximated function shows a large amount of ripple at the point of discontinuity. This causes lowering of the entire pass band in the normalized intensity transmittance curve. Fig.(1.1.21) shows the normalized intensity transmittance curve of a thirteen stage band-suppression linear birefringent network, obtained
from fourteen term exponential Fourier series \[\text{Expression 2b of Appendix-III}\]. The plot shows a lowering of the pass band at the higher wavelength side of the principal rejection wavelength at 448 nm. Fig. (1.1.8) shows the typical intensity transmittance curve of a ten stage Solc filter with principal pass wavelength at 509 nm and the secondary maxima at 448 nm and 589 nm. If we use this ten stage Solc filter with the thirteen stage band suppression filter (discussed here) in cascade, the intensity transmittance of the combination will take shape as shown in fig. (1.1.22). The secondary maxima at 448 nm and 589 nm show 100% and 40% reduction respectively and the transmission at 509 nm occurs at 96% of its ideal value.

SECTION-II TRANSMISSION CHARACTERISTICS OF A BIREFRINGENT FILTER SYSTEM WHERE THE LENGTH OF THE ELEMENTS OF THE SUCCESSIVE STAGES ARE DIFFERENT

1.2.1 Birefringent filter system where the elements of the successive stages get double in thickness

We now study the transmission characteristics of both narrow pass band filter and birefringent band suppression filter where the elements of the successive stages get double in thickness (fig.1.2.1). The generalized expression for the output Jones vector $E_0$ of a $n$-stage network where the retarder axes are oriented like a $n$-stage fan type Solc filter is given by
\[
E_o = P_{\text{out}(\theta)} R(\theta/2) R(2^{n-1} \alpha - \theta) C(2^{n-1} \delta) R(2^{n-2} \alpha - \theta) \\
x C(2^{n-2} \delta) \ldots R(\alpha - \theta) C(\delta) R(-\theta/2)P_{\text{in}(1)} E_i \\
\ldots (1.2.1)
\]

where \( E_i = \begin{bmatrix} E_x \\ E_y \end{bmatrix} \) represents the Jones vector of the input beam and \( C(\delta), R(\alpha), P(\theta) \) represent the Jones matrices of the retarder, rotator and linear polarizer respectively. Here unlike the cases of the subsection (1.1.1) where closed form expressions were used to obtain the intensity transmittance plots, a rigorous computer software (multiplication of a series of \( 2 \times 2 \) complex matrices) is used to obtain the intensity transmittance values at different wavelengths with the help of the expression (1.2.1) and with the aid of an IBM 1130 computer. For computational purpose we use Biot's law in order to realize the optical rotation produced by a quartz rotator plate and neglect the dispersion of birefringence of a linear retarder plate due to the reason indicated in Appendix-II.

1.2.2 Discussions on the transmission curves where the retarder and the rotator of the successive stages get double in thickness

Fig. (1.2.2) shows the T vs. \( \lambda \) curve of a ten stage network tuned for narrow band pass purpose. Here the 1st rotator (from left) rotates the plane of polarization of the principal
pass wavelength component through an angle determined by the relation

$$\alpha_{OP} = \frac{m\pi}{\sum_{n=1}^{m} 2^{n-1}} \quad \ldots (1.2.2)$$

where, $m = 1, 2, 3, 4, 5 \ldots$ etc.

For $n = 10$ and $\lambda_{OP} = 509$ nm. if we choose any value of $m$ between 1 to 19, the thickness of first few rotators will be impossibly small. So, we prefer to choose $m = 20$ for computational purpose which determines the thickness of 1st rotator as 0.12 mm and the thickness of last rotator as 6.13 cm. The 1st retarder (just after $P_{in(o)}$) introduces a phase difference of $360^\circ$ at $\lambda_{OP} = 509$ nm. Fig.(1.2.2) shows sharp peaks of different amplitudes and shapes where the azimuths of the retarders are arranged like a ten stage Sole filter (that yields $\theta = 9^\circ$ from $n\theta \approx \pi/2$). At $\lambda_{OP} = 509$ nm, the intensity transmittance is at its ideal maximum. Fig.(1.2.3) shows the $T$ vs. $\lambda$ curve of the same set up with the only difference that the slow axis of each retarder is parallel to the transmission axes of the polarizers i.e. $\theta = 0^\circ$. Here obviously the number of retarder is one less than the number of rotator and the first retarder (after first $R(a)$ from left; fig.(1.2.1)) introduces a phase difference of $720^\circ$ at $\lambda_{OP}$. Fig.(1.2.3) also shows a number of sharp peaks of different
shapes. If now for ten stage network, the thickness of each retarder is same and is made to introduce a phase difference of $360^\circ$ at $\lambda_{OP}$ and the rotators are chosen according to the relation (1.2.2) (with $m = 20$), then for $\lambda_{OP} = 509$ nm, we get the $T$ vs. $\lambda$ variation of fig.(1.2.4) and fig.(1.2.5) for both $\theta = 9^\circ$ and $0^\circ$ respectively. These curves also show an irregular distribution of sharp peaks. A large number of transmission peaks of fig.(1.2.5) found to be sharper than those obtained in figures (1.2.2), (1.2.3) and (1.2.4).

1.2.3 Discussions on the transmission curves of the Sole filter where the retarder of the successive stages get double in thickness

We now study the $T$ vs. $M$ curves of a fan type Sole filter where the successive retarders get double in thickness (substituting $\alpha = 0$ in expression (1.2.1)). The transmission curves of the six stage ($n = 6$) and the ten stage ($n = 10$) filter of this type, tuned at $\lambda_{OP} = 509$ nm, are shown in the figures (1.2.6) and (1.2.7) respectively. Comparing the plot of fig.(1.2.7) with that of a ten stage fan type Sole filter (fig.(1.1.8)) we see that much narrower transmission peak can be obtained at the principal pass wavelength $\lambda_{OP} = 509$ nm, but the amplitudes of the secondary peaks get higher and the side bands are of irregular shape as the successive stages get doubled in thickness.
1.2.4 Discussions on the transmission curves where the thickness of each rotator is same and the retarder of the successive stages get double in thickness.

With the help of expression (1.2.1) we now study the $T$ vs. $\lambda$ curve of a birefringent network where equal thickness rotator plates are used as interstage elements and the retarders of successive stages get double in thickness. Here again we use the expressions $\alpha_{op} = \frac{m\pi}{2n}$ (where $m = 2, 4, 6, \ldots$) and $\alpha_{OR} = \frac{m\pi}{2n}$ (where $m = 1, 3, 5, \ldots$) in order to find the thickness of each rotator plate. The plots of the figures (1.2.8'), (1.2.9) and (1.2.10) show the $T$ vs. $\lambda$ curves of a ten stage ($n = 10$) filter of this type tuned at $\lambda_{gp} = 509$ nm, 453 nm and 640 nm respectively. The curves of the figures (1.2.8), (1.2.9) and (1.2.10) are drawn for $m = 2$ and the thickness of the rotators are obtained as 0.60, 0.48 and 1 mm respectively. The 1st retarder (just after $P_{in}(o)$, fig.(1.2.1)) behaves as a full wave plate for the principal pass wavelength and the azimuths of the retarders are arranged like a fan type Sokol filter. Comparing $T$ vs. $\lambda$ curve of fig. (1.2.8) with that of fig. (1.2.7) we see that in fig. (1.2.8) the secondary peaks at the higher wavelength side of principal pass are greatly reduced where as the secondary peaks at the lower wavelength side increase. If now the number of stages are decreased to $n = 6$, we get the transmittance curve of fig. (1.2.11) (tuned at $\lambda = 509$ nm) which when compared to that of fig. (1.2.8)
reveals that the increase in the number of stages decrease the amplitudes of the peaks on the higher wavelength side than on the lower wavelength side of $\lambda_{QP}$. Again if the thickness of each rotator is changed the $T$ vs. $\lambda$ curve thus takes the shape (for $n=10$) shown by the plot of fig. (1.2.12). Here we use $m = 4$ and the thickness of each rotator is obtained as 1.22 mm. for $\lambda_{QP} = 509$ nm. Comparing the plot of fig. (1.2.12) with the plot of the fig. (1.2.8) we see that the width of primary maxima as well as the number of secondary maxima increase as the thickness of each rotator increases. Fig. (1.2.13) shows the $T$ vs. $\lambda$ curve of a ten stage (no. of retarder is 9) network where the slow axis of each retarder is parallel to the transmission axes of the polarizers and $m = 2$, $\lambda_{QP} = 509$ nm. The 1st retarder introduces a phase difference of $720^\circ$ at $\lambda_{QP}$. Here again we see a number of transmission peaks of different shapes.

We now present two cases of a ten stage network tuned at principal rejection wavelength $\lambda_{QR} = 509$ nm. at which the value of intensity transmittance is ideally zero. The thickness of each rotator is same and the first retarder introduces a phase difference of $360^\circ$ at $\lambda_{QR}$. Fig. (1.2.14) shows the $T$ vs. $\lambda$ curve of the band suppression network where the axes of the retarders are oriented like a ten stage Solc filter. Fig. (1.2.15) shows the $T$ vs. $\lambda$ curve of the same network where the slow axis of each retarder is parallel to the
transmission axes of the polarizers. In this set up the number of retarder is one less than the number of rotator and first retarder introduces a phase difference of 720° at $\lambda_{OR}$. In both the cases the thickness of each rotator is obtained as 0.31 mm for $m = 1$. It is interesting to note that in the plot of fig. (1.2.14) we get a complete suppression over a wide spectral range around $\lambda_{OR}$ with little transmission at both ends. Fig. (1.2.15) shows large amplitude of fluctuation over the lower wavelength side pass band than on the higher wavelength side pass band of $\lambda_{OR}$. In both the cases discussed above, the fluctuations over the pass band of either side of $\lambda_{OR}$ increase as we take higher value of $m$ or higher number of stages.

1.2.5 Birefringent filter system where the thickness of each retarder is same and the thickness of interstage rotators are different

In the preceding subsections the spectral intensity transmittance of different types of birefringent filters are studied where the slow axes of different retarders are either arranged by a simple mathematical relation or parallel to the transmission axes of the polarizers. The thickness of different core elements are shown to be same or increase in a regular way. In the following studies we investigate the effect of substituting the mechanical rotations of the linear retarder plates of a linear birefringent network by the frequency dependent rotations introduced by the optically active medium.
Fig. (1.2.16) shows a five stage linear birefringent network. Following Jones formalism the Jones vector $E_0$ of the output beam can be written as

$$E_0 = P(\theta_0) R(\theta_5) C(\delta) R(\theta_4 - \theta_5) C(\delta) R(\theta_3 - \theta_4) C(\delta)$$

$$\times R(\theta_2 - \theta_3) C(\delta) R(\theta_1 - \theta_2) C(\delta) R(-\theta_1) P(\gamma) E_1$$

... (1.2.3)

where $E_1$ is the Jones vector of the input beam represented by the expression (1.1.1).

We now study the effect of inserting rotator plates of different thickness in a five stage linear birefringent network having an approximated periodic triangular amplitude transmittance.

The azimuths of linear retarders and output polarizer of a linear birefringent network having an approximated periodic triangular amplitude transmittance are given by [37]

$$\theta_1 = 88^0 45', \ \theta_2 = 84^0 52', \ \theta_3 = 55^0 31'$$

$$\theta_4 = 26^0 10', \ \theta_5 = 22^0 18', \ \theta_p = 21^0 3'$$

... (1.2.4)

substituting the values of the equations (1.2.4) in expression (1.2.3) we get the $T$ vs. $\lambda$ curve of fig. (1.2.17) by the direct matrix multiplication. At $\lambda = 509$ nm each retarder behaves as a full wave plate and we get the intensity maxima at this wavelength.
If now the slow axis of each retarder is made parallel to the transmission axis of the input polarizer $P(\theta)$ and the rotator plates are placed in such a way that the 1st rotator $R(\alpha_1)$ rotates the plane of polarization of the spectral component $\lambda = 509$ nm through an angle $-\theta_1$ deg., the second rotator $R(\alpha_2)$ rotates the plane of polarization of $\lambda = 509$ nm through an angle $(\theta_1 - \theta_2)$ deg. and so on, the Jones vector $E_0$ of the output beam can be written with the help of expression (1.2.3) as

$$E_0 = P(\theta_p) R(\alpha_6) C(\delta) R(\alpha_5) C(\delta) R(\alpha_4) C(\delta) R(\alpha_3) C(\delta)$$

$$\times R(\alpha_2) C(\delta) R(\alpha_1) P(\theta) E_1 \quad \cdots (1.2.5)$$

The arrangement of the elements of the network that corresponds to expression (1.2.5) is shown in fig.(1.2.18). With the help of Biot's law the thickness of rotators $R(\alpha_1)$, $R(\alpha_2)$, $R(\alpha_3)$, $R(\alpha_4)$, $R(\alpha_5)$ and $R(\alpha_6)$ are obtained as $t_1 = 3$ mm, $t_2 = 0.13$ mm, $t_3 = 1$ mm, $t_4 = 1$ mm, $t_5 = 0.13$ mm, and $t_6 = 0.76$ mm respectively for $\lambda = 509$ nm as tuning wavelength. Here the rotator $R(\alpha_1)$ rotates the plane of polarization of the wavelength components through clockwise direction and all other rotators introduce anticlockwise rotation. Fig.(1.2.19) shows the $T$ vs. $\lambda$ curve of the network (fig.1.2.18) for different rotator thickness shown above and for the same thickness of each retarder plate. Comparing $T$ vs. $\lambda$ curve
of fig.(1.2.17) with that of fig.(1.2.19) we notice a marked difference between the two distributions. Although the intensity transmittances at $\lambda = 509$ nm are same in both the cases, the plot of fig.(1.2.19) shows an increase at lower wavelength side and rather sharp decrease at higher wavelength side of $\lambda = 509$ nm, showing a more or less flat top intensity transmittance over a wide range of visible region. If now we tune the network at some higher wavelength $\lambda = 546$ nm, the $T$ vs. $\lambda$ curve thus obtained is shown in the plot of fig.(1.2.20). At $\lambda = 546$ nm each retarder behave as a full wave plate and the thickness of the rotators are obtained as $t_1 = 3.5$ mm, $t_2 = 0.15$ mm, $t_3 = 1.16$ mm, $t_4 = 1.16$ mm, $t_5 = 0.15$ mm and $t_6 = 0.88$ mm. Here again the 1st rotator introduces clockwise rotation and all other rotators introduce anticlockwise rotation. Similarly tuning the network at some lower wavelength $\lambda = 432$ nm, the rotators thickness are obtained as $t_1 = 2$ mm. (right handed rotator), $t_2 = 0.1$ mm, $t_3 = 0.71$ mm, $t_4 = 0.71$ mm, $t_5 = 0.10$ mm and $t_6 = 0.54$ mm. The $T$ vs. $\lambda$ curve thus obtained is shown in fig.(1.2.21). The curve resembles that of a band suppression one with large band width tuned at the principal rejection wavelength at $\lambda = 530$ nm.

In the network configurations discussed above we see that the 1st rotator introduces a clockwise rotation. If now we change the 1st right handed rotator by a left handed rotator which rotates the plane of polarization of the wavelength
component \( \lambda = 509 \text{ nm} \) through an angle \((360 - \theta_1) \text{ deg.}\) in the anticlockwise direction, the \( T \) vs. \( \lambda \) curve thus obtained (using other rotators as used for tuning at \( \lambda = 509 \text{ nm} \)) is shown in fig. (1.2.22). In this case the thickness of the 1st left handed rotator is \( t_1 = 2.7 \text{ cm} \).

The plot of fig. (1.2.22) shows the \( T \) vs. \( \lambda \) curve of a multi-band-pass filter with maxima at \( \lambda = 418 \text{ nm, } 453 \text{ nm, } 516 \text{ nm, } 563 \text{ nm, } \) and \( 629 \text{ nm} \). The peaks of all these maxima do not attain the ideal value of full transmission. The peaks at \( \lambda = 453 \text{ nm, } 516 \text{ nm, } \) and \( 629 \text{ nm} \) attain 98\% whereas the peaks at \( \lambda = 418 \text{ nm, } \) and \( 563 \text{ nm} \) attain 89\% and 55\% respectively of their ideal value of full transmission. Similarly we also note that the minima do not attain their ideal value of zero transmission. However any properly tuned band suppression filter can be combined to this set up for complete suppression of any desired wavelength.
Fig. (1.1.1) The diagram shows a fan type Solc filter where a Rotator $R(\alpha)$ is placed after each retarder $C(\delta, \theta)$. $P_{in}(0)$ and $P_{out}(0)$ represent linear polarizers with transmission axes parallel to the $X$-direction of the reference coordinate system. Here the thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.2) The plot shows the variation of the intensity transmittance $T$ with the variation of the wavelength $\lambda$ of a ten stage ($n=10$) birefringent narrow band pass filter with $m = 2$ and $\lambda_{0p} = 509$ nm. The azimuths of the retarders are arranged like a fan type Solc filter. The thickness of each retarder is same and the thickness of each rotator is same.
The azimuths of the retarders are arranged like a fan type Sole filter. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.4) $T$ Vs. $\lambda$ curve of a birefringent narrow band pass filter with $n = 10$, $m = 2$ and $\lambda_{op} = 640$ nm. The azimuths of the retarders are arranged like a fan type Sölic filter. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.5) $T$ Vs. $\lambda$ curve of a birefringent narrow band pass filter with $n = 10$, $m = 6$ and $\lambda_{0p} = 509$ nm. The azimuths of the retarders are arranged like a fan type Solc filter. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.6) T vs. $\lambda$ curve of a birefringent narrow band pass filter with $n = 10$, $m = 10$ and $\lambda_{0p} = 509$ nm. The azimuths of the retarders are arranged like a fan type Solc filter. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.7) T Vs. λ curve of a birefringent filter with n = 10, m = 1 and tuned at 509 nm. The azimuths of the retarders are arranged like a fan type Sole filter. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.8) $T$ Vs. $\lambda$ curve of a ten stage fan type Solc filter.
Fig. (1.1.9) \( T \) Vs. \( \lambda \) curve of a birefringent narrow band pass filter with \( n = 14 \), \( m = 2 \) and \( \lambda_{op} = 509 \) nm. The azimuths of the retarders are arranged like a fan type Sole filter. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.10) $T$ Vs. $\lambda$ curve of a 14 stage fan type Solc filter.
Fig. (1.1.11) T vs. λ curve of a Birefringent Band Suppression Filter (BBSF) where the slow axis of each retarder is parallel to the transmission axes of the polarizers and the number of rotator is ten (i.e. n = 10). Here m = 1 and the filter is tuned at λ₀R = 609 nm. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.12) T Vs. λ curve of a BBSF with $n = 10$, $m = 1$, $\lambda_{QR} = 640$ nm. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.13) T vs. λ curve of a BBSF with $n = 10$, $m = 1$, $\lambda_0 = 453$ nm. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.14) T vs. $\lambda$ for a WGGP with $n = 10$, $m = 5$, $\lambda_{QR} = 500 \text{ nm}$. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.15) $T$ vs. $\lambda$ curve of a BBSF with $n = 10$, $m = 7$, $\lambda_{OR} = 509$ nm. The thickness of each retarder is same and the thickness of each rotator is same.
tuned at 509 nm. The slow axis of each retarder is parallel to the transmission axes of the polarizers. The thickness of each retarder is the same and the thickness of each rotator is the same.

Fig.(1.1.16) T vs. λ curve of a birefringent filter with n = 10, m = 2 and tuned at 509 nm. The slow axis of each retarder is parallel to the transmission axes of the polarizers. The thickness of each retarder is the same and the thickness of each rotator is the same.
Fig. (1.1.17) $T$ vs. $\lambda$ curve of a BBSF with $n = 14$, $m = 1$, $\lambda_{CR} = 509$ nm. The thickness of each retarder is same and the thickness of each rotator is same.
Fig. (1.1.18) T vs. λ curve of a combined system comprises of the systems having T vs. λ curves represented by the figures (1.1.2) and (1.1.13).
Fig. (1.1.19) The plots show the ideal (firm line) and the approximated (dotted line) amplitude transmittance of a five stage band suppression linear birefringent network.
The normalized intensity transmittance of a five-stage band suppression linear birefringent network.

Fig. (1.1.20) The normalized intensity transmittance of a five-stage band suppression linear birefringent network.
The normalized intensity transmittance of a thirteen stage band suppression linear birefringent network.

Fig.(1.1.21)
The intensity transmittance of the combination of a ten stage Solc filter and thirteen stage band suppression linear birefringent network.
The diagram shows a fan type Solc filter where a rotator is placed after each retarder and the retarder-rotator combination of the successive stages get double in thickness. $P_{\text{in}}(0)$ and $P_{\text{out}}(0)$ represent linear polarizers with transmission axes parallel to the $x$-direction of the reference coordinate system.
Fig. (1.2.2) T Vs. \( \lambda \) curve of a ten stage \((n=10)\) birefringent filter where the retarders and the rotators of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solec filter and the whole filter system is tuned at \( \lambda_{0p} = 509 \text{ nm} \) for narrow band pass purpose. The thickness of the thinnest rotator is obtained by \( m = 20 \).
Fig. (1.2.3) $T$ vs. $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_{op} = 509 \text{ nm}$ for narrow band pass purpose where the rotators and the retarders of the successive stages get double in thickness. The slow axis of each retarder is parallel to the transmission axis of the polarizers. The number of retarder is nine and the thickness of the thinnest rotator is obtained by $m = 20$. 
Fig. (1.2.4) $T$ Vs. $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_{op} = 509$ nm for narrow band pass purpose where the thickness of each retarder is same and the rotator of the successive stages get double in thickness. The thickness of the thinnest rotator is obtained by $m = 20$ and the azimuths of the retarders are arranged like a fan type Sole filter.
Fig. (1.2.5) $T$ Vs. $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_0 = 509$ nm for narrow band pass purpose where the thickness of each retarder is same and the rotator of the successive stages get double in thickness. The slow axis of each retarder is parallel to the transmission axes of the polarizers and the number of retarder is nine. The thickness of the thinnest rotator is obtained by $m = 30$. 
Fig. (1.2.6) $T$ Vs. $\lambda$ curve of a six stage Sole filter where the retarder of the successive stages get double in thickness.
Fig. (1.2.7) $T$ Vs. $\lambda$ curve of a ten stage Solc filter where the retarder of the successive stages get double in thickness.
and the retarder of the successive stages get 
The azimuths of the retarders are arranged
rotator is each rotator is same
double in thickness.

Fig. (1.2.8) $T$ Vs $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_0 = 509$ nm for narrow band pass purpose where the thickness of each rotator is same and the retarder of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solec filter and the thickness of each rotator is obtained by $m = 2$. 
T Vs. λ curve of a ten stage (n = 10) birefringent filter tuned at λQP = 453 nm for narrow band pass purpose where the thickness of each rotator is same and the retarder of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solc filter and the thickness of each rotator is obtained by m = 2.
Fig. (1.2.10) $T$ vs. $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_{op} = 640$ nm for narrow band pass purpose where the thickness of each rotator is same and the retarder of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solec filter and the thickness of each rotator is obtained by $m = 2$. 

$H_{co}^{m} E_U > c D$
T Vs. $\kappa$ curve of a six stage ($n=6$) birefringent filter tuned at $\lambda_0P = 509$ nm for narrow band pass purpose where the thickness of each rotator is same and the retarding of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solc filter and the thickness of each rotator is obtained by $m = 2$.

Fig. (1.2.11) T Vs. $\lambda$ curve of a six stage ($n=6$) birefringent filter tuned at $\lambda_0P = 509$ nm for narrow band pass purpose where the thickness of each rotator is same and the retarding of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solc filter and the thickness of each rotator is obtained by $m = 2$. 

$X (\text{in nm})$
T vs. \( \lambda \) curve of a ten stage \((n = 10)\) birefringent filter tuned at \( \lambda_{op} = 509 \text{ nm} \) for narrow band pass purpose where the thickness of each rotator is same and the retarder of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solc filter and the thickness of each rotator is obtained by \( m = 4 \).
The slow axis of each retarder is parallel to the transmission axes of the polarizers and the number of retarder is nine. The thickness of each rotator is obtained by $m = 2$. The transmission $T$ vs. $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_{op} = 509$ nm for narrow band pass purpose where the thickness of each rotator is same and the retarder of successive stages get double in thickness.
Fig. (1.2.14) $T$ vs. $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_{OR} = 509$ nm for band suppression purpose where the thickness of each rotator is same and the retarder of the successive stages get double in thickness. The azimuths of the retarders are arranged like a fan type Solc filter and the thickness of each rotator is obtained by $m = 1$. 
Fig. (1.2.15) T Vs. $\lambda$ curve of a ten stage ($n = 10$) birefringent filter tuned at $\lambda_0 = 509$ nm for band suppression purpose where the thickness of each rotator is same and the retarder of the successive stages get double in thickness. The slow axis of each retarder is parallel to the transmission axes of the polarizers and the number of retarder is nine. The thickness of each rotator is obtained by $m = l$. 
Fig. (1.2.16) Schematic diagram of a five stage linear birefringent network. \( C(\phi, \theta) \) represents the Jones matrix of a linear retarder at an azimuth \( \theta \). \( P(0) \) and \( P(\theta_p) \) represent the Jones matrices of the linear polarizers with transmission axes parallel to the x-direction and making an angle \( \theta \) to the positive x-direction of a reference coordinate system.
Fig. (1.2.17) The curve shows the variation of the intensity transmission $T$ with the variation of the wavelength $\lambda$ of a five stage linear birefringent network with a periodic triangular wave amplitude transmittance.
Fig.(1.2.12) Schematic diagram of a birefringent network where the slow axis of each retarder is parallel to the transmission axis of the input polarizer \( P(0) \) and the thickness of the interstage rotators are determined by the azimuths of the retarders of the Fig.(1.2.16). \( P(\theta_p) \) represents the Jones matrix of a linear polarizer at an azimuth \( \theta_p \).
Fig. (1.2.19) T Vs. λ curve of a birefringent network tuned at λ = 509 nm. Here the slow axis of each retarder is parallel to the transmission axis of the input polarizer $P(0)$ and the thickness of the inter-stage rotators are determined by the azimuths of the linear birefringent network having a periodic triangular wave amplitude transmittance.
Fig. (1.2.20) $T$ Vs. $\lambda$ curve of a birefringent network tuned at $\lambda = 546$ nm. Here the slow axis of each retarder is parallel to the transmission axis of the input polarizer $P(0)$ and the thickness of the interstage rotators are determined by the azimuths of the linear birefringent network having a periodic triangular wave amplitude transmittance.
Fig.(1.2.21) T Vs. λ curve of a birefringent network tuned at λ = 432 nm. Here the slow axis of each retarder is parallel to the transmission axis of P(0) and the thickness of the interstage rotators are determined by the azimuths of the linear birefringent network having a periodic triangular wave amplitude transmittance.
Fig. (1.2.22) T vs. \( \lambda \) curve of the same birefringent network as used to obtain the transmission curve of the fig. (1.2.19) with the only difference that the first right handed rotator is substituted by a left handed rotator.