ABSTRACT

Graph theory, the study of graphs and networks, is often considered as a part of combinatorics, but has grown large enough and distinct enough, with its own kind of problems, to be regarded as a subject in its own right. Graphs are one of the prime objects of study in Discrete Mathematics.

The study of graceful graphs and graceful labeling methods were introduced by Rosa (1967). However, the term graceful labeling was not used until Bloom & Golomb (1977) studied such labeling. In the same manner, Graham & Sloane (1980) introduced harmonious labeling in connection with their study on additive bases problem stemming from error-correcting codes.

In the subsequent year, Chang et al (1981) introduced elegant labeling as a variation of harmonious labeling and proved that $C_n$ is elegant when $n \equiv 0$ or $3$ (mod 4) and not elegant when $n \equiv 1$ (mod 4). In a novel way, Lee et al (1991) introduced the concept of felicitous labeling.

Cahit (1987) defined the cordial labeling as a weaker version of both harmonious and graceful labeling. He proved that every tree is cordial and $K_n$ is cordial if and only if $n \leq 3$.

Scope of the present investigation deals with five chapters, which discuss the problems in different graph labelings.

Chapter 1 is of introductory nature which gives a glimpse of the work embodied in the thesis and it is aimed to provide basic terminology and preliminaries, which are needed for the subsequent chapters.
In Chapter 2, a new class of graph called a cycle with zigzag chords is defined. It is proved that every $n$-cycle ($C_n$) with zigzag chords is graceful and cordial. Further, a new graph namely $kZ_n$-graph is also introduced. It is proved that, the $kZ_n$-graph is graceful and felicitous whereas the $kZ_n$-graph with zigzag origin is graceful and the $kZ_n$-graph with gear origin is graceful and felicitous.

In Chapter 3, a new class of graph called a cycle with chord hamiltonian path is defined. It is proved that every $n$-cycle ($C_n$) with chord hamiltonian path is harmonious, elegant and cordial for all $n \geq 6$.

Sethuraman & Selvaraju (2001) have introduced a new method of construction called Supersubdivision of a graph as a possible way to generate graceful graphs from any connected graph.

In Chapter 4, it is proved that, an arbitrary supersubdivision of grid graph is graceful and felicitous and some supersubdivision of cyclic snake also proved to be graceful and felicitous. Further, a new method of graph construction called arbitrary contract supersubdivision of paths and cycles are introduced and it is proved that an arbitrary contract supersubdivision of paths and cycles are graceful and felicitous.

In Chapter 5, Stem-Lotus graph is introduced. It is proved that a Stem-Lotus graph with $n$-pair of petals is graceful and felicitous for all $n \geq 1$. Furthermore, a new method of graph construction called the complete $m$-point projection on some projected vertices of a graph is defined. It is proved that the complete $m$-point projection on some projected vertices of $kC_4$-snake with string-2 is graceful and felicitous.