Chapter 6

Zakharov-Kuznetsov equation for ion acoustic waves in a magnetized plasma in the presence of electron beam

6.1 Introduction

Nonlinear structures viz. solitons, shocks, double layers etc. are the beautiful manifestations of a balance between various effects (nonlinearity, dispersion and dissipation) in plasma environments. Among these nonlinear structures ion acoustic wave is the main focus in the present study. Ion acoustic waves (IAWs) are electrostatic plasma modes in which the heavier ions oscillate due to inertia on a background of electrons providing the restoring force (Krall & Trivelpiece, 1973). Over the last many years, wave dynamics has been the key issue for plasma physics community to study the propagation properties of nonlinear solitary structures under the influence of various plasma parameters. The propagation of nonlinear solitary waves in multi-component plasmas has a considerable importance in understanding the behavior of space/astrophysical environments (e.g. ionosphere, Saturn’s rings, magnetosphere etc.). IA solitary waves can be analyzed theoretically either in the frame work of Korteweg-deVries (KdV) or Zakharov-Kuznetsov (ZK) equation (Tajiri & Nishihara, 1984; Washimi & Taniuti, 1966) or in the frame work of Sagdeev potential formalism, (Buti, 1980; Ghosh et al., 1996; Nishihara & Tajiri, 1981; Sagdeev, 1966) and have also been confirmed experi-
mentally (Ikezi et al., 1970).

It is believed that kappa distribution is more appropriate than Maxwellian to model the data in various space/astrophysical environments. The investigations reported by numerous researchers have confirmed that the particles distribution in velocity space depart considerably from a Maxwellian distribution and presented a theory of excess superthermal particles. Such distributions called “Lorentzian distribution” (or kappa distribution) (Vasyliunas, 1968) have been used to interpret spacecraft data on the Earth’s magnetospheric plasma sheet (Lui & Krimigis, 1981), the solar wind (Gosling et al., 1981), Jupiter (Leubner, 1982) and Saturn (Armstrong et al., 1983). The occurrence of superthermal particles and kappa distribution have been already discussed in detail in earlier investigations.

Various satellite observations in space plasma indicate that ion-acoustic solitary waves generally appear in association with ion beams or electron beams (Omura et al., 1994, 2001). The numerical simulation results have shown the formulation of ion acoustic solitary structures driven by electron and ion beams in the auroral acceleration region using two dimensional electrostatic particle simulation (Marchenko & Hudson, 1995). Witt & Lotko (1983) presented a theory for IASWs in a magnetized plasma excited in the presence of an electron beam. Several investigations in upper layer of magnetosphere where two different electron populations (say, cold, i.e., “inertial” and warm i.e., “energetic” ones) coexist and have been reported by various satellite missions viz., S3-3 (Temerin et al., 1982), Viking (Bostrom et al., 1988), the FAST (Ergun et al., 1998; Mcfadden et al., 2003), GEOTAIL (Mcfadden et al., 2003), POLAR (Matsumoto et al., 1994) and in laboratory experiments have confirmed that localized electrostatic (ES) modes in a plasma may be excited by impinging an electron beam (Giulietti et al., 2001; Takeda & Yamagiwa, 2003, 2004; Yamagiwa et al., 1997). Such investigations have been theoretically predicted and analyzed by researchers (Lakhina et al., 2008; Nejoh & Sanuki, 1995; Salau & Roychoudhury, 2004; Saini & Kourakis, 2010). Lakhina et al. (2008) analyzed the propagation properties of IA and electron acoustic (EA) waves in an unmagnetized multicomponent plasma system consisting of cold background electrons, ions, hot electrons as well as ions, hot electron beam and hot ion beam. The characteristics of three types of solitary wave viz., slow IA, IA and EA solitons were studied. Using Sagdeev pseudopotential method, Saini & Kourakis (2010) studied the propagation properties of IASW’s in an superthermal plasma in the presence of an electron beam. It was observed that various parameters including beam density and velocity have profound effect on the amplitude and width of IA solitary structures.
Three decades ago Zakharov-Kuznetsov (ZK) model was proposed to study three-dimensional problems of ion acoustic waves in different kinds of plasma system (Zakharov & Kuznetsov, 1974). The ZK equation for ion acoustic solitary waves using reductive perturbation method was derived to describe the behavior of nonlinear solitary structure under the influence of uniform magnetic field. Employing this ZK model, numerous authors reported the investigation of solitary structures in a variety of plasma systems (Bains et al., 2011; Cairns et al., 1996; El-Taibany et al., 2011; Lee & Kan, 1981; Mace & Hellberg, 2001; Mahmood et al., 2011; Mushtaq, 2008; Mushtaq & Shah, 2005; Saini et al., 2014; Sultana et al., 2010; Xue & Lang, 2004; Zhang & Xue, 2007). Mushtaq & Shah (2005) reported the ZK equation in a weakly relativistic electron-positron-ion plasma for ion-acoustic waves propagating obliquely to an external magnetic field. They observed that magnetic field strength, obliqueness, relativistic effects etc. have a profound effect on the characteristics of ion acoustic solitons. Bains et al. (2011) studied the nonlinear behavior of IAWs in a magnetized electron-ion plasma in the presence of hot nonextensive electrons. It was noticed that compressive and rarefactive solitary structures are significantly modified with combined effects of various physical parameters. Mahmood et al. (2011) derived the ZK equation for nonlinear acoustic waves in dense magnetized electron-positron plasmas using reductive perturbation method. From the solution of ZK equation, it was seen that ion temperature, magnetic field, concentration of positron density and rotation of plasma play very crucial role to modify the nonlinear structures. A nonlinear ZK equation for ion acoustic solitary waves in a two temperature superthermal plasma in the presence of uniform magnetic field was presented by Saini et al. (2014). Employing small-k perturbation technique stability analysis was performed to see the presence of stable and unstable solitons. It was also illustrated by numerical computation that the profile of IASWs is greatly modified with variation in superthermality of electrons, concentration of cold electrons and the strength of magnetic field. Very recently, Devanandhan et al. (2015) have analyzed the propagation of EASWs in a magnetized plasma consisting of fluid cold electrons, superthermal hot electrons and an electron beam. A KdV-ZK equation has been derived to study the characteristics of EASWs.

Owing to the importance of kappa type distribution and significant role of electron beam for the formation of solitary structures, we have derived the nonlinear ZK equation governing the IA solitary waves using well known reductive perturbation technique. From the solutions of ZK equation, we have numerically analyzed the combined effects of superthermality of electrons, magnetic field, obliqueness and electron beam parameters on the characteristics of IASWs.
6.2 Basic fluid equations

We consider a collisionless three-component plasma embedded in an ambient magnetic field \( \mathbf{B}_0 = B_0 \hat{z} \) directed along \( z \)-axis, composed of cold ions (charge \( q_i = e \), mass \( m_i \)), electrons assumed to obey kappa velocity distribution and an electron beam (charge \( q_e = -e \), mass \( m_e \)). The fluid continuity and momentum equations are given as follows:

For ions,
\[
\frac{\partial \tilde{n}_i}{\partial t} + \nabla \cdot (\tilde{n}_i \tilde{u}_i) = 0, \tag{6.1}
\]
\[
\frac{\partial \tilde{u}_i}{\partial t} + (\tilde{u}_i, \nabla) = -\frac{e}{m_i} \nabla \tilde{\phi} + \frac{e}{m_i} (\tilde{u}_i \times B_0 \hat{z}), \tag{6.2}
\]

For electron beam,
\[
\frac{\partial \tilde{n}_b}{\partial t} + \nabla \cdot (\tilde{n}_b \tilde{u}_b) = 0, \tag{6.3}
\]
\[
\frac{\partial \tilde{u}_b}{\partial t} + (\tilde{u}_b, \nabla) = \frac{e}{m_e} \nabla \tilde{\phi} + \frac{e}{m_e} (\tilde{u}_b \times B_0 \hat{z}), \tag{6.4}
\]

and Poisson’s equation takes the form
\[
\nabla^2 \tilde{\phi} = -4\pi e (\tilde{n}_i - \tilde{n}_e - \tilde{n}_b), \tag{6.5}
\]

where
\[
\tilde{n}_e = n_{e0} \left[ 1 - \frac{e \tilde{\phi}}{(\kappa - 3/2) k_B T_e} \right]^{-\kappa+1/2}, \tag{6.6}
\]

After normalization Eqs. (6.1-6.6) lead to following equations
\[
\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{u}_j) = 0, \tag{6.7}
\]
\[
\frac{\partial \mathbf{u}_j}{\partial t} + (\mathbf{u}_j, \nabla) = -\delta_j \nabla \phi + \Omega_j (\mathbf{u}_j \times \hat{z}), \tag{6.8}
\]
\[
\nabla^2 \phi = -(n_i - n_e - n_b), \tag{6.9}
\]
\[
n_e = \mu \left[ 1 - \frac{\phi}{(\kappa - 3/2)} \right]^{-\kappa+1/2}, \tag{6.10}
\]

where the number density \( n_j (j = i, b) \), fluid velocity \( \mathbf{u}_j (= u_j, v_j, w_j) \) and electrostatic potential \( \phi \) are scaled as \( n_j = \tilde{n}_j / n_{i0} \), \( \mathbf{u}_j = \tilde{\mathbf{u}}_j / C_s \) \( (C_s = \sqrt{k_B T_e / m_i}) \) and \( \phi = \)
6.3 Derivation of the ZK equation

c\dot{\phi}/k_BT_e$ respectively. Space and time variables are scaled by the Debye length $\lambda_D (=k_BT/e\pi e^2n\eta_0)^{1/2}$, and the inverse ion plasma frequency $\omega_p^{-1} (=4\pi n\eta_0e^2/m_i)^{-1/2}$, respectively. $\Omega_j = eB_0/\omega_p m_j$ is the cyclotron frequency, $\delta_j = 1(-\frac{1}{2})$ for ions (beam) and the mass ratio $\gamma = m_e/m_i$ ($\approx 1/1836$). The assumption of charge neutrality at equilibrium is $n_{i0} = n_{e0} + n_{b0}$ which implies that $1 = \mu + \nu$ ($\mu = n_{e0}/n_{i0}$ and $\nu = n_{b0}/n_{i0}$).

Using the Taylor’s expansion of Eq.(6.10) and substituting in the right hand side of Eq. (6.9), we have

$$\nabla^2 \phi = 1 + \epsilon_1 \phi + \epsilon_2 \phi^2 + \epsilon_3 \phi^3 - n_i + n_b,$$

(6.11)

where

$$\epsilon_1 = \mu \frac{(\kappa - 1/2)}{\kappa - 3/2}, \quad \epsilon_2 = \mu \frac{(\kappa^2 - 1/4)}{2(\kappa - 3/2)^2}, \quad \epsilon_3 = \mu \frac{(\kappa^2 - 1/4)(\kappa + 3/2)}{6(\kappa - 3/2)^3},$$

where the real parameter $\kappa$ measures the deviation from Maxwellian equilibrium.

6.3 Derivation of the ZK equation

The independent stretching variables used to study the characteristics of small amplitude ion acoustic solitary waves (IASWs), are as:

$$\xi = \epsilon^{1/2}x, \quad \eta = \epsilon^{1/2}y, \quad \zeta = \epsilon^{1/2}(z - \lambda_0 t), \quad \text{and} \quad \tau = \epsilon^{3/2}t,$$

$\lambda_0$ represents phase velocity of IASWs. The dependent variables can be expanded in the following form:

$$
\begin{pmatrix}
    n_i \\
    w_i \\
    n_b \\
    w_b \\
    \phi
\end{pmatrix} = 
\begin{pmatrix}
    1 \\
    0 \\
    \nu \\
    w_0 \\
    0
\end{pmatrix} + \sum_{m=1}^{\infty} \epsilon^m 
\begin{pmatrix}
    n_i^{(m)} \\
    w_i^{(m)} \\
    n_b^{(m)} \\
    w_b^{(m)} \\
    \phi^{(m)}
\end{pmatrix},
$$

$$
\begin{pmatrix}
    u_i \\
    v_i \\
    u_b \\
    v_b
\end{pmatrix} = \sum_{m=1}^{\infty} \epsilon^{1+m/2} 
\begin{pmatrix}
    u_i^{(m)} \\
    v_i^{(m)} \\
    u_b^{(m)} \\
    v_b^{(m)}
\end{pmatrix},
$$

(6.12)

Using stretching coordinates $\xi$, $\eta$, $\zeta$, $\tau$ and the expanded variables in Eqs. (6.7-6.11). By collecting the terms to lowest powers of $\epsilon$ in evolution equations, we obtain
the following first order relations.

\[
\lambda_0 n_i^{(1)} = w_i^{(1)}, \quad n_i^{(1)} = \frac{\phi_i^{(1)}}{\lambda_0}, \quad w_i^{(1)} = \frac{\phi_i^{(1)}}{\lambda_0},
\]

\[
v_i^{(1)} = \frac{\omega_{pi} \partial \phi_i^{(1)}}{\Omega_i \partial \xi}, \quad u_i^{(1)} = -\frac{\omega_{pi} \partial \phi_i^{(1)}}{\Omega_i \partial \eta},
\] (6.13)

\[
(\lambda_0 - w_{b0}) n_b^{(1)} = \nu w_b^{(1)}, \quad n_b^{(1)} = \frac{\nu \phi_1}{\gamma (\lambda_0 - w_{b0})^2}, \quad w_b^{(1)} = -\frac{\phi^{(1)}}{\gamma (\lambda_0 - w_{b0})},
\]

\[
v_b^{(1)} = \frac{\omega_{pb} \partial \phi^{(1)}}{\Omega_b \partial \xi}, \quad u_b^{(1)} = -\frac{\omega_{pb} \partial \phi^{(1)}}{\Omega_b \partial \eta},
\] (6.14)

By eliminating the first order quantities from Eq. (6.14), the phase velocity of ion acoustic solitary waves \(\lambda_0\) is obtained as

\[
c_1 = \frac{1}{\lambda_0^2} + \frac{\nu}{\gamma (\lambda_0 - w_{b0})^2},
\] (6.15)

which is quartic equation, giving four values of \(\lambda_0\). Comparing next order of \(\epsilon\), the second order relations are obtained as

\[
v_i^{(2)} = -\frac{\omega_{pi}}{\Omega_i} \lambda_0 \frac{\partial u_i^{(1)}}{\partial \zeta}, \quad u_i^{(2)} = \frac{\omega_{pi}}{\Omega_i} \lambda_0 \frac{\partial v_i^{(1)}}{\partial \zeta},
\] (6.16)

\[
v_b^{(2)} = -\frac{\omega_{pb}}{\Omega_b} (\lambda_0 - w_{b0}) \frac{\partial u_b^{(1)}}{\partial \zeta}, \quad u_b^{(2)} = \frac{\omega_{pb}}{\Omega_b} (\lambda_0 - w_{b0}) \frac{\partial v_b^{(1)}}{\partial \zeta},
\] (6.17)

Comparing higher orders of \(\epsilon\), we get

\[
\frac{\partial n_i^{(1)}}{\partial \tau} - \lambda_0 \frac{\partial n_i^{(2)}}{\partial \zeta} + \frac{\partial u_i^{(2)}}{\partial \zeta} + \frac{\partial v_i^{(2)}}{\partial \zeta} + \frac{\partial w_i^{(2)}}{\partial \zeta} + \frac{\partial (n_i^{(1)} w^{(1)})}{\partial \zeta} = 0,
\] (6.18)

\[
\frac{\partial w_i^{(1)}}{\partial \tau} - \lambda_0 \frac{\partial w_i^{(2)}}{\partial \zeta} + \frac{\partial v_i^{(1)}}{\partial \zeta} = -\frac{\partial \phi^{(2)}}{\partial \zeta},
\] (6.19)

\[
\frac{\partial n_b^{(1)}}{\partial \tau} - (\lambda_0 - w_{b0}) \frac{\partial n_b^{(2)}}{\partial \zeta} + \nu \frac{\partial u_b^{(2)}}{\partial \zeta} + \nu \frac{\partial v_b^{(2)}}{\partial \zeta} + \nu \frac{\partial w_b^{(2)}}{\partial \zeta} + \frac{\partial (n_b^{(1)} w_b^{(1)})}{\partial \zeta} = 0,
\] (6.20)

\[
\frac{\partial w_b^{(1)}}{\partial \tau} - (\lambda_0 - w_{b0}) \frac{\partial w_b^{(2)}}{\partial \zeta} + w_b^{(1)} \frac{\partial w_b^{(1)}}{\partial \zeta} = \frac{1}{\gamma} \frac{\partial \phi^{(2)}}{\partial \zeta},
\] (6.21)
6.4 Solitary wave solution of the ZK equation

From Poisson’s equation, after comparing second order $\epsilon$ coefficients, we obtain

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = e_1 \phi^{(2)} + e_2 (\phi^{(1)})^2 - n_1^{(2)} + n_b^{(2)}, \quad (6.22)$$

Eliminating the second order quantities from Eqs. (6.13)-(6.22), we obtain the following ZK equation:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + B \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} + C \frac{\partial}{\partial \zeta} \left( \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} \right) = 0, \quad (6.23)$$

The nonlinear coefficient $A$ is

$$A = \frac{3/2\lambda_0^4 - 3\nu/2\gamma(\lambda_0 - w_{b0})^4 - e_2}{1/\lambda_0^3 + \nu/\gamma(\lambda_0 - w_{b0})^3}, \quad (6.24)$$

the dispersive coefficient $B$ is

$$B = \frac{1}{2/\lambda_0^3 + 2\nu/\gamma(\lambda_0 - w_{b0})^3}, \quad (6.25)$$

and the higher order coefficient $C$ is

$$C = \frac{1 + (\omega_{pi}/\Omega_i)^2 - \nu\omega_{ph}/\Omega_b^2}{2/\lambda_0^3 + 2\nu/\gamma(\lambda_0 - w_{b0})^3}, \quad (6.26)$$

6.4 Solitary wave solution of the ZK equation

To study the general solution of Eq. (6.23), we have introduced transformations of the form $X = \xi \sin \theta + \zeta \cos \theta$, $Y = \eta$, $Z = \xi \cos \theta - \zeta \sin \theta$, $T = \tau$, where $\theta$ is the angle between the direction of external magnetic field and the $\zeta$-axis (Shukla & Yu, 1978; Washimi & Taniuti, 1966). Keeping $\eta$-axis fixed and rotating the coordinate axes $(\xi, \zeta)$ through an angle $\theta$, Eq. (6.23) may be reformulated in time and space coordinates in the following form

$$\frac{\partial \phi^{(1)}}{\partial T} + \Delta_1 \frac{\partial \phi^{(1)}}{\partial X} + \Delta_2 \frac{\partial^3 \phi^{(1)}}{\partial X^3} + \Delta_4 \frac{\partial^2 \phi^{(1)}}{\partial Z^2} + \Delta_5 \frac{\partial^3 \phi^{(1)}}{\partial X^2 \partial Z} + \Delta_6 \frac{\partial^3 \phi^{(1)}}{\partial X \partial Z^2} + \Delta_7 \frac{\partial^3 \phi^{(1)}}{\partial X \partial Y^2} + \Delta_8 \frac{\partial^3 \phi^{(1)}}{\partial Z \partial Y^2} = 0, \quad (6.27)$$

where

$$\Delta_1 = A \cos \theta, \quad \Delta_2 = B \cos^3 \theta + C \cos \theta \sin^2 \theta,$$
6. ZAKHAROV-KUZNETSOV EQUATION FOR ION ACOUSTIC WAVES IN A MAGNETIZED PLASMA IN THE PRESENCE OF ELECTRON BEAM

\[ \Delta_3 = -A \sin \theta, \quad \Delta_4 = -B \sin^3 \theta - C \sin \theta \cos^2 \theta, \]
\[ \Delta_5 = -3B \sin \theta \cos^2 \theta + C(-\sin^2 \theta + 2\sin \theta \cos^2 \theta), \]
\[ \Delta_6 = 3B \sin^2 \theta \cos \theta + C(\cos^3 \theta - 2\sin^2 \theta \cos \theta), \quad \Delta_7 = C \cos \theta, \Delta_8 = -C \sin \theta, \quad (6.28) \]

Using the single variable transformation, \( \chi = X - U_0 T \) \((U_0 \text{ is a constant velocity normalized to } C_s)\) and \( \phi^{(l)} = \phi(\chi) \) in Eq. (6.27), we obtain
\[ -U_0 \frac{d\phi}{d\chi} + \Delta_1 \frac{d\phi}{d\chi} + \Delta_2 \frac{d^3 \phi}{d\chi^3} = 0. \quad (6.29) \]
The solution of Eq. (6.29) is determined as
\[ \phi(\chi) = \phi_m \text{sech}^2(\chi/\Lambda), \quad (6.30) \]
where peak amplitude and width of solitons respectively, are given as
\[ \phi_m = 3U_0/\Delta_1 = 3U_0/(A \cos \theta), \quad (6.31) \]
and
\[ \Lambda = \sqrt{4\Delta_2/U_0} = 2 \sqrt{\frac{B \cos^2 \theta + C \sin^2 \theta}{U_0/\cos \theta}}. \quad (6.32) \]
To find this solution, we have used the boundary conditions, as \(|\chi| \to \infty, (\phi, \phi', \phi'') \to 0\). The amplitude and width of IASWs are dependent on physical parameters \( \kappa, \omega_{\text{bi}}, \mu, \nu, \theta \) and \( \Omega_i \). Therefore, it is of paramount importance to study their influence on the propagation properties of IASWs.

6.5 Formation and parametric analysis of ion acoustic solitary waves

From Eqs. (6.24-6.26), it is seen that nonlinear coefficient \( A \), dispersion coefficient \( B \) and higher order coefficient \( C \) are functions of numerous physical parameters. We have performed numerical analysis in this investigation to see the change in solitary profile of IASWs under the influence of superthermality of electrons, strength of magnetic field and electron beam parameters (velocity and density). The solution of ZK equation (6.23) in the form of Eq. (6.30) is determined to study the characteristics of IASWs. Further, we have also studied the influence of ion plasma frequency \((\omega_{\text{pi}})\) and ion-gyrofrequency (via \( \Omega_i \)) on the profile of IASWs. The change in nonlinear coefficient \( A \)
6.5 Formation and parametric analysis of ion acoustic solitary waves

leads to the variation in nature and amplitude of IA solitary structures. A is always negative and only negative potential ion acoustic solitary structures exist for the present case. The higher order coefficient $C$ (via $\Omega_i$) illustrates the effect of the magnetic field on IASWs. From Eqs. (6.31) and (6.32), it is very clear that with increase in solitary speed ($U$), the amplitude (width) of solitary waves enhances (shrinks). Hence, IA solitary structures become taller and narrower.

The dispersion relation Eq. (6.15) is a fourth-order polynomial in $\lambda_0$ and hence have four roots. The dispersion relation is split into four modes: two of which are only due to the presence of beam. Furthermore, two of the modes are imaginary and we have limited our discussion to real modes, out of which only positive one has been taken for analysis. It is noted that, the familiar ion acoustic waves dispersion relation is recovered in the absence of beam i.e., upon setting $\nu = 0$ in Eq. (6.15) which agrees with the finding of Zakharov & Kuznetsov (1974). Fig. 6.1 illustrates the variation of phase velocity ($\lambda_0$) with superthermality of electrons (via $\kappa$) for different values of beam velocity ($w_{0b}$). The phase velocity is enhanced with increase in electron beam velocity ($w_{0b}$), increase in $\kappa$ (i.e. decrease in superthermality), increase in beam density ($\nu$) and is reduced with increase in electron concentration (via $\mu$). This shows that IASWs move faster in the presence of beam in a magnetized plasma having small number of superthermal electrons.

**Effect of superthermality (via $\kappa$):**

To see the effect of superthermality of electrons (via $\kappa$) on the potential profile, we have plotted the solitary wave profile with different values of $\kappa$ as shown in Fig. 6.2. The superthermality of electrons significantly modifies the pulse profile of negative polarity IASWs by varying the values of $\kappa$. The amplitude and width of solitons increase with increase in $\kappa$ (i.e., decrease in superthermality of electrons). These results are compared with the findings of Sultana *et al.* (2010) for $e - i$ plasma, where only positive polarity solitons exist. Hence, in the presence of electron beam, e-i plasma yields modified IA solitary structures.

**Effect of beam parameters (via $w_{0b}$ and $\nu$):**

The effects of beam velocity and density ($w_{0b}$ and $\nu$) are studied on the IASWs and illustrated in Fig. 6.3. The amplitude and width are decreased with increase in the value of beam velocity. The solitons become shorter and narrower. The results are compared with Saini & Kourakis (2010) (keeping magnetic field parameters zero) and has shown the similar behavior. Fig. 6.4 depicts the influence of beam density (via $\nu$)
**Figure 6.1:** Phase velocity ($\lambda_0$) versus beam velocity ($w_{b0}$) with $\kappa, \nu, \mu = 3.25, 0.0025, 0.1$ (solid); $\kappa, \nu, \mu = 3.5, 0.0025, 0.1$ (dashed); $\kappa, \nu, \mu = 3.25, 0.0035, 0.1$ (dotted); $\kappa, \nu, \mu = 3.25, 0.0035, 0.11$ (dot-dashed).

**Figure 6.2:** Effect of superthermality of electrons (via $\kappa$) on IASWs for $\gamma = 0.01$, $U = 0.06$, $w_{b0} = 0.5$, $\omega_{pe} = 1.4$, $\Omega_i = 0.4$, $\omega_{pi} = 1.1$, $\Omega_b = 0.5$, $\theta = 30^\circ$, $\nu = 0.0025$, $\mu = 0.1$ with different values of $\kappa = 3.25$ (solid), 5.25 (dashed) and 7.25 (dotted).

on the characteristics of IASWs potential profile. An increase in beam density reduces the nonlinearity coefficient $A$ which leads to decrease in steepening of IASWs. Subsequently, both amplitude and width have shown a decrease for an increase in beam
6.5 Formation and parametric analysis of ion acoustic solitary waves

density. This further illustrates that the presence of electron beam makes solitary structures to shrink in size. The influence of beam parameters (via $w_{b0}$ and $\nu$) have profound effect on the IA soliton profiles.

Figure 6.3: Effect of beam velocity (via $w_{b0}$) on IASWs for $\gamma = 0.01$, $U = 0.06$, $\kappa = 3.25$, $\omega_{pi} = 1.4$, $\Omega_i = 0.4$, $\omega_{pb} = 1.1$, $\Omega_b = 0.5$, $\theta = 30^\circ$, $\nu = 0.0025$, $\mu = 0.1$ with different values of $w_{b0} = 0.5$ (solid), 0.7 (dashed) and 0.9 (dotted).

Figure 6.4: Effect of beam density ratio (via $\nu$) on IASWs for $\gamma = 0.01$, $U = 0.06$, $w_{b0} = 0.5$, $\omega_{pi} = 1.4$, $\Omega_i = 0.4$, $\omega_{pb} = 1.1$, $\Omega_b = 0.5$, $\theta = 30^\circ$, $\kappa = 3.25$, $\mu = 0.1$ with different values of $\nu = 0.0025$ (solid), 0.0045 (dashed) and 0.0075 (dotted).
**Effect of obliqueness (via $\theta$):**
The effect of obliqueness (via $\theta$) on negative potential profile solitons is depicted in Fig. 6.5. As the obliqueness ($\theta$) increases, the amplitude and width are enhanced. In the absence of beam parameters (i.e., $w_{b0}, \nu \to 0$), the findings of the present study (for $\kappa \to \infty$) are in agreement with the results of Cairns et al. (1996) (for $\beta \to 0$).

**Figure 6.5:** Effect of obliqueness (via $\theta$) on IASWs for $\gamma = 0.01$, $U = 0.06$, $w_{b0} = 0.5$, $\omega_{pi} = 1.4$, $\Omega_i = 0.4$, $\omega_{pb} = 1.1$, $\Omega_b = 0.5$, $\kappa = 3.25$, $\nu = 0.0025$, $\mu = 0.1$ with different values of $\theta = 30^\circ$ (solid), $40^\circ$ (dashed) and $50^\circ$ (dotted).

**Effect of magnetic field (via $\Omega_i$):**
The gyro-frequency of ions (via $\Omega_i$) makes the influence on IASWs via the nonlinear coefficient $C$, so it is of paramount importance to study the impact of magnetic field (via $\Omega_i$) on IA solitary structures. The width of IA solitons increases with increase in the magnetic field (via $\Omega_i$). This means that stronger magnetic field leads to the formation of more steeper and narrower soliton profiles (see Fig. 6.6). For Maxwellian case ($\kappa \to \infty$) and under the same conditions, results of present study agree with the earlier findings of Bains et al. (2011) in nonextensive plasma for $q \to 1$.

### 6.6 Conclusions

In this present investigation, we have derived a nonlinear ZK equation for IASWs in a multicomponent magnetized plasma traversed by an electron beam. The influence
of various plasma parameters, viz., strength of magnetic field, superthermality of electrons, electron beam parameters (density and velocity) have been studied numerically on IASWs. The main findings are illustrated as under.

- Only negative potential ion acoustic solitary structures are observed.
- The phase velocity is enhanced in the presence of electron beam and reduced with increase in electron concentration.
- The characteristics of IASWs (amplitude and width) are significantly modified with increase in various plasma parameters. Both amplitude and width are reduced with increase in beam parameters (i.e., increase in both beam velocity $w_{bo}$ and density $\nu$), while enhanced with increase in $\kappa$ (i.e., decrease in superthermality) and obliqueness.
- The variation of the magnetic field has been realized through the higher order coefficient ($C$) (via $\Omega_i$), which modifies the solitary profile.
- In the limiting case, for $\kappa \to \infty$ and in the absence of electron beam results agree with e-i Maxwellian plasma.

The findings of the present investigation may be significant in understanding the properties of ion acoustic solitary waves under the influence of external magnetic field as
6. ZAKHAROV-KUZNETSOV EQUATION FOR ION ACOUSTIC WAVES IN A MAGNETIZED PLASMA IN THE PRESENCE OF ELECTRON BEAM

well as traversed by an electron beam in space observations and plasma laboratory experiments.