I. One Crop Model.

(1) In this type of models, only one crop is taken into account which is both produced and consumed. The works of Rajkrishna, (1) Nowshirvani, (2) and Krishnan (3) all into this category.

In Rajkrishna's model, as interpreted by A. Sen (4) we get the following chain of relationships between the relevant variables.

\[(4.1) \quad Q = Q(P)\]
\[(4.2) \quad C = C(Y, P)\]
\[(4.3) \quad M = Q - C\]
\[(4.4) \quad Y = Y^* = P.M + Y_o;\]

where \(Y_o\) is a constant indicating (relative) income from other sources. It can be represented in terms of a diagram.

![Diagram](image-url)
As, it can be seen from the diagram any change in $P$ will have an impact on $M$ through three distinct chains

1. $P \rightarrow Q \rightarrow M$
2. $P \rightarrow Y \rightarrow C \rightarrow M$
3. $P \rightarrow C \rightarrow M$

Besides these, there is a loop, namely, $M \rightarrow Y \rightarrow C \rightarrow M$ i.e., from $M$ to $Y$ and then through $C$ to $M$.

Differentiating $M$ with respect to $P$ we get,

$\frac{dM}{dP} = \frac{dQ}{dP} - \frac{\delta Q}{\delta P} \frac{\delta Q}{\delta Y} (M + P \cdot \frac{dM}{dP})$

Converting the relation in terms of elasticities, we get,

6. $e = rb - (r - l) g - (r - l) mhk - (r - l) mhke$
7. Therefore, $e = \frac{rb - (r - l) g - (r - l) mhk}{1 + (r - l) mhk}$

Where $r = \frac{O}{M}$ ratio of output to market supply

$e =$ price elasticity of marketable surplus

$b =$ price elasticity of total output $= \frac{dQ}{dP} \frac{P}{Q}$

$g =$ Price elasticity of home consumption.

$h =$ Income elasticity of home consumption.

$m = \frac{M}{Q} =$ the reciprocal of $r$.

$k = \frac{PQ}{Y} =$ ratio of total value of wheat production to total income earned.

(1) Raj Krishna's term for 'g' the elasticity of substitution effect unnecessarily creates a confusion with Slutsky's substitution effect, which 'g' is not.

(2) Raj Krishna has also called 'h' the elasticity of income effect, a term to be avoided for the same reason as above.
The first three terms on the numerator correspond to the above three effects. If we ignore the feedback of \( M \) on \( Y \) we get,

\[
e = rb - (r-1) (g^+mkr),
\]

which is the formula obtained by Rajkrishna. However, since \( k \) and \( (r-1) \) are positive and \( h \) is also expected to be positive (unless the commodity is an inferior good), the sign of \( e \) will not be changed if we allow for the feedback effect. On the other hand, Rajkrishna's formula will be an overestimate, as the denominator is greater than one.

In Nowshirvani Model (2) chain relations are slightly modified.

\[
\begin{align*}
Q &= Q (P) \\
C &= C (Y', P) \\
Y' &= PQ + Y_0 \\
M &= Q - C
\end{align*}
\]

They are shown in the following diagram.

Here, again any change in \( P \) will have an impact on \( M \) through four distinct chains.
In this model, there is no feedback from M on any of the relevant variables. The only change in this model from Rajkrishna's model lies in the definition of the income term. To avoid confusion, we have therefore used $Y'$ to denote Nowshirvani's income and $Y$ to denote Rajkrishna's income. According to Nowshirvani, however, the income definition that Rajkrishna had in mind is $Y'$. So, his model is the correct version of Rajkrishna's model. However, both Behrman (5) quite explicitly and Yotopoulos and Nungest (6) implicitly in their versions of Rajkrishna model, used the relation.

\[\frac{dY}{dP} = M.\]

Krishnan also notes that income term in Raj Krishna's model is a function of $P$ and $M$ and has criticised Rajkrishna on this point.\(^{(1)}\) Bardhan (7) also, has criticised Rajkrishna for using a narrower concept of income, but she interprets it as 'the cash income obtained from the sales of foodgrains. Actually, Rajkrishna has presented his model not for foodgrains but for a subsistence crop like wheat. Anyway, what

\(^{(1)}\) Krishnan's $P$, as it is going to be pointed out is, however, different from Rajkrishna's $P$. 

\[\text{(1)}\]
Bardhen seems to have in her mind is that

(11) \[ Y = PM \]

in Raj-Krishna's model. Behrman also notes that the relevant income in the determination of on-farm consumption of a subsistence crop is the cash income obtained from the sale of that crop. We have incorporated \( Y_0 \) in the above equation for the sake of symmetry with Nowshirvani's income term, \( Y' \). Note that since \( k \) is not to set equal to unity, we need a term like \( Y_0 \) in equation (9.3). Note that \( Y_0 \) must be dimensionally homogeneous to \( PQ \) or \( PM \) as the case may be. None of the above authors has, however, taken into

(2) Both \( Y \) and \( Y' \) are therefore real (or relative) income.

As a result, both \( k \) and \( \frac{dy}{dp} \) are independent of the price of numeraire, so long as they belong to 'other things remaining the same'. It is the failure to understand this that led Behrman to criticise Rajkrishna for confusing between a relative and an absolute price. So long as the price of numeraire remains the same, both equation (10) and the interpretation of '\( k \)' as given by us are correct. Rajkrishna's term for '\( k \)', the proportion of wheat revenue to total income is, however, confusing for it may suggest \( k = PM/Y \).
account the multiplier effect of $P$ on $e$ that results from a definition of income that corresponds to (4.3) though Behrman quite correctly noted that Rajkrishna's formula is only an approximation as it ignores the second round effects on $Y$ and $Q$. Only Sen has derived the equation (7) as the correct value of $e$ that is implied by Rajkrishna's model.

To be fair to Raj-krishna, seems to have withdrawn from his earlier position on the sign of the price-elasticity of the marketable surplus of wheat because of the absence of any reliable price-income and income-consumption relationships. (8)

Coming back to Nowshirvani's model, we get, differentiating $M$ with respect to $P$,

$$(12) \frac{dM}{dP} = \frac{\partial Q}{\partial P} - \frac{Q}{P} \frac{\partial Q}{\partial Y} = \frac{Q}{Y} - \frac{\partial Q}{\partial Y} \cdot P \cdot \frac{\partial Q}{\partial P}$$

Converting equation (12) in terms of elasticities, we get,

$$(13) e = xb - (x-1) g - (x-1) hk - (x-1) hkb$$

All the notations have their meanings, that we put on them, as before, and the four terms in the R.H. side of equation (13), correspond to four ways in which a change in $P$ has an impact on $M$, as shown in the diagram. Strictly speaking 'h' in Nowshirvani's model is $\frac{\partial Q}{\partial Y} \cdot \frac{Y'}{G}$ which is different from 'h' in Raj Krishna's model. But we have used
the same notation as both of them represent income elasticity of home consumption. Both Raj Krishna and Howshirvani have considered the marketable surplus function of a subsistence crop, wheat. Although Krishnan has addressed himself to the task of deriving the marketable (or marketed) surplus for good-grains, the formal mathematical structure of his model has many points of similarities with Nowshirvani's model. The major points of departure of his model are two:

1. He has considered \( Q \) to be pre-determined, (i.e., \( Q \); and

2. He has assumed no other source of income other than \( FQ \), i.e., \( Y_0 \), the term we have used for 'some constant' in Nowshirvani's income equation (9.3), should be set equal to zero.

The above two assumptions, if imposed on Nowshirvani's model imply \( b = 0 \) and \( k = 1 \). Putting these values in equation (13) we get

\[
(14) \quad e = - (r-1) (g^h)
\]

which is precisely the result Krishnan has obtained.

The following Table on notation may be helpful in translating Krishnan's result in Nowshirvani's terms.

<table>
<thead>
<tr>
<th>Nowshirvani</th>
<th>( b )</th>
<th>( k )</th>
<th>( g )</th>
<th>( h )</th>
<th>( m = 1/r )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Krishnan</td>
<td>0</td>
<td>1</td>
<td>( \lambda )</td>
<td>( \beta )</td>
<td>( 1-r )</td>
<td>( E_M )</td>
</tr>
</tbody>
</table>
There are, however, quite a few other minor points of difference in their approaches.

(1) Both Raj Krishna and Howshirvani have interpreted $P$ as a relative price. Krishnan seems to interpret $P$ as an absolute price.

(2) Krishnan starts from an aggregate demand function of the producers of foodgrains for foodgrains which has constant elasticities both with respect to price as well as income. In other words, $g$ and $h$ in our notations are assumed to be the same at all values of $P$ and $Q$, an assumption not really necessary to obtain equation (14). Such an assumption would be, however, necessary if for estimation purpose, we use a log-linear regression equation of consumption of the producers of foodgrains on price and income. In Chapter III, it would be shown that this is precisely the form used by all the studies that have been cited to set the plausible ranges of parameters $g$ and $h$ in Raj Krishna, Howshirvani and other models, though the variables (in per capita terms) are not always real price and real income (as in Howshirvani's model) or absolute price and money income (as in Krishnan's model).

One interesting point that is revealed by Krishna's approach is that the textbook theory of consumers' behaviour does not apply in this case. Suppose a consumer has two consumption alternatives, $C_1$ and $C_2$ and maximizes his utility
function \( u(C_1, C_2) \) subject to a budget constraint,

\[ P_1 C_1 + P_2 C_2 = K \]

where \( P_1, P_2 \) are the respective prices of the two commodities and \( M \) the given money income. Then it follows that,

1. The demand function for \( C_1 \) (say) would be homogeneous of degree zero in terms of \( P_1, P_2 \) and \( M \)

2. The sum of own price elasticity and income elasticity of demand for \( C_1 \) would be the negative value of the cross price elasticity of demand for \( C_1 \) with respect to \( P_2 \). Hence, if the cross price elasticity is positive, as it usually is in a two commodity framework, the sum of the own price elasticity and the income elasticity should be negative. (See Model IV on this point).

But we cannot use this property of demand curve to infer that \( g / h \), hence \( e \) must be negative. This is because income is not exogenously given, it is also a homogeneous function of degree one in price. As a result, the aggregate demand function in Krishnan's model is a homogeneous function of price of foodgrain only and of a degree equal to \( g^h \) in our notation.

It may be noted that Krishnan also uses the term marketed surplus for marketable surplus. But since his model, like Raj Krishna's and Nowshirvani's models, does not take into account other types of retentions of the producers besides that for consumption purposes, it would be more appropriate, by our logic, to call the relevant elasticity, the elasticity
of marketable surplus. Raj Krishna, however, uses marketed surplus, marketable surplus, sale and market supply synonymously because he abstracts from 'disposals other than consumptions and sale'.

Some of the points of difference between Krishnan's approach and Nowshirvani's approach could be explained by the fact that while Nowshirvani's intention was to present a correct version of Raj Krishna's model, retaining its behavioural assumptions, the primary objective of the Krishnan model is to lend a theoretical support to what may be called Mathur-Ezekiel phenomenon, namely an inverse relationship between 'sale' and price of foodgrains for less developed countries. Mathur and Ezekiel (9) have started by pointing out that the term 'marketable surplus' is a misnomer for a marginal farmer. To such a farmer, fixed cash obligations rather than 'consumption requirements' are the first charge on foodgrains produced. On the other hand well-to-do farmers (or in the case of a bumper crop, perhaps other farmers) save in-kind but even for them the demand for non-agricultural commodities is relatively rigid in comparison with the demand for foodgrains. Mathur-Ezekiel hypothesis suggests that

(16) \[ PM = K \]

where \( K \) is a constant and \( P \), as in Krishnan's model, is the absolute price of foodgrains. In equation (16) 'M' should be interpreted as marketed surplus, since according to Mathur-Ezekiel what is not sold is either consumed or retained in-kind.
as a safeguard against future contingency. It may be noted that they have discounted the possibility of speculative stock, which would, of course, be responsive to price. Obviously, the situation Mathur and Ezekiel have in mind is one where the output of foodgrains is already pre-determined.

Now equation (16) implies that the offer curve for foodgrains is a rectangular hyperbola and for stability in the Walrasian sense, we require that the price elasticity of demand for foodgrains on the part of non-producers of foodgrains should be more than unity. But as Nowshirvani (10) has pointed out that the demand for foodgrains is proverbially price-inelastic. Krishnan's model allows for an inverse relationship between P and M which is consistent with Walrasian stability condition.

The nature of the consumption function for a typical producer of foodgrains, according to Mathur-Ezekiel hypothesis would be something like

\begin{equation}
C_{it} = C_i^t (Q_{it}, P_t/K_{it}) \quad i = 1, 2, \ldots n,
\end{equation}

where \( Q_{it} \) is the (pre-determined) output of foodgrains for the \( i^{th} \) producer, \( K_{it} \) is his fixed cash obligations and \( C_{it} \) his consumption of foodgrains in \( t^{th} \) time period. Now according to Mathur Ezekiel hypothesis, \( C_{it} \) would very directly both with respect to price (which may be considered an exogenous variable for the producer, \( Q_{it} \) being very small in relation to total output), and \( Q_{it} \) as it is subjected to
parametric shifts over-time. So $C_{it}$ would also very directly with $Y_{it} = P_t Q_{it}$ of the $i^{th}$ producer. But income is a compound of two parameters in the above consumption function, and should be treated as such.

**Evaluation of One Commodity Models.**

Only one price is taken into account. With a perfect market and only one producing centre where there are a large number of producers so that no producer has any control over price, and many transport agencies, the price in any consumption point will be determined by the price ruling at the production centre plus the freight rate as between the two places, the latter, again would be competitively determined. Similarly, with one consumption centre, where all the consumers are located and many producers spread throughout the economy, the producers' price would be the price at the consumption centre minus the freight rate charged. In reality, there would be many production centres and many consumption centres and markets might be imperfect in the sense that the price one gets or receives would depend on one's relative bargaining strength. But still, it might be possible to find a regional pattern of the price structure which remains stable over time, provided the location of producers and consumers over time do not change in a significant manner. Only under such circumstances, one price may be taken as an index of the regional price structure.

In real life there are also many marketing stages in
between the producer and consumers and the price in the
different stages would not be the same. Once again, if the
marketing margin as between the different stages remain
the same either in absolute or in percentage terms, we
may represent the prices in different stages by one price,
(which, for convenience and ready availability is generally
the wholesale price).

(2) The price that will affect production decision
is the expected price at the time product will be sold. On
the other hand, the price that will determine home consump­
tion out of a given output is the ruling price at the time
of consumption decision. These two prices may not be equal.
The current price will, of course, affect production decision
in the next period by influencing the expected price rele­
vant for the next production decision.

The models of Raj Krishna and Nowshirvani are therefore
of a comparative static nature. In a comparative static
model, the effects of any once-for-all change in price is
determined by comparing two static equilibrium positions,
one before the price change, and the other, after a new sta­
tic equilibrium position has been established. Implicitly
this assumes that the dynamic process initiated by a change
in price is convergent and leads to a static equilibrium
where the actual price is equal to the expected price and
the actual output is equal to the desired output.
The variation of price over time ushers in other types of problems. Since the models are of a comparative static nature, sufficient time must be allowed for full adjustment and the price should remain stable within this period of adjustment. On the other hand, if price does not change at all, we cannot estimate the price-elasticity of marketable surplus from time-series data. So the price must change, discontinuously over-time, and we should be able to observe different snapshots of static equilibrium situations with alternative price configurations.

Most of the one-commodity models, interpret $P$ as the relative price. There are two reasons why a relative price may be a more appropriate variable than absolute price in explaining the behaviour of marketable surplus. In the first place, other things will not remain the same in a time-series. So by deflating the absolute price of the commodity we might, in a way, eliminate the disturbances in the stable relationship between the absolute price and marketable surplus of the commodity. It is, however, difficult to see how a deflation of the price by a cost of living index, say, will be able to neutralize all such nuisance variables. Alternatively, we may feel that both output and home-consumption are homogeneous functions of degree zero in all prices, so that, the marketable surplus is also a homogeneous function of degree zero in all prices. So if all prices increase
in the same proportion, neither $Q$ nor $C$, and hence $M$ would change.

Now, let us assume (anticipating some of the models to be presented later) that we have three commodities, the commodity under consideration, a consumption alternative and a production alternative. If the production alternative is taken as the numeraire, any change in the price of the consumption alternative, through its impact on $C$, will disturb the relationship between $M$ and $P$ (the relative price). On the other hand, if the consumption alternative is taken as the numeraire, any change in the price of the production alternative is going to disturb the relationship between $M$ and $P$, both through the direct impact on $Q$, as well as, through the indirect impact on $P$ through $Y$.

On the other hand if the price index is used as a deflator of $P$ is the cost of living index, as it usually is the case, the price of wheat (or foodgrains) is surely going to have an important weightage in such an index. So the relative price of wheat will depend on the absolute price of wheat in two ways, directly by changing the numerator and indirectly by changing the denominator. In the limiting case as the weightage given to the wheat price tends to one, the relative price will remain unchanged as the price of wheat changes. In such a predominantly wheat economy, the relative price of wheat is a constant.
All these may appear just mere fancy. But the problem is more serious if \( Y_o \) in the income term varies sympathetically with the price of wheat. Now hourly earning rate of labour may vary directly with the price of wheat. However, wage rate is expected to be sluggish in relation to the price of wheat and this is true even in the upward direction because the employers are often reluctant to raise wages apprehending that once raised, it would be very difficult to bring them down, as the equilibrium is restored.

(3) Consumption decisions are also assumed to be a function of gross income. Actually, the relevant variable should be net income, i.e., the gross income minus the production costs. However, if the production cost per unit of output is fixed, then gross income will be proportional to net income at a given price level. So gross income can be taken as an index of net income to indicate its change over time.

Three Commodity Models.

**Behrman Model**: We shall consider a simplified version of Behrman's model, assuming that \( Y \), gross income can be taken as a proxy for net income, \( I \); actual output is equal to desired output; and actual price is equal to expected price.

___

Behrman in his simplest model, assumes that gross income, actual price and actual output as approximations for net income, expected price and desired output respectively.
Behrman conducted his analysis in terms of two relative prices $\frac{P_1}{P_2}$ = price of $Q_1$ relative to price of a production alternative.

$\frac{P_1}{P_3}$ = price of $Q_1$ relative to the consumption alternative.

The equational structure of the model will look like:

\begin{align*}
(16.1) & \quad Y = P_1Q_1 + P_2Q_2 \quad \text{.......... (1)} \\
(16.2) & \quad Q_1 = Q_1(P_1/P_2, A) \quad \text{.......... (2)} \\
(16.3) & \quad Q_2 = Q_2(P_1/P_2, B) \quad \text{.......... (3)} \\
(16.4) & \quad C_1 = C_1(Y, P_1/P_3) \quad \text{.......... (4)} \\
(16.5) & \quad M_1 = M_1 = C_1 \quad \text{.......... (5)}
\end{align*}

Differentiating $M_1$ with respect to $P_1$ we get,

\begin{align*}
(17) \quad \frac{dM_1}{dP_1} &= \frac{1}{P_2} \frac{\partial C_1}{\partial Y} - \frac{1}{P_3} \frac{\partial C_1}{\partial Y} \\
&\quad - \frac{1}{P_2} \frac{\partial C_1}{\partial Y} \frac{Q_1}{P_2} - \frac{1}{P_3} \frac{\partial C_1}{\partial Y} \frac{Q_1}{P_3}
\end{align*}

Converting it in elasticity terms,

\begin{align*}
(18.1) & \quad e = rb_1 - (x-1) g - (x-1) hk - (x-1) hb_2 (1-k) \\
\text{or (18.2) } & \quad e = rb_1 - (x-1) g + hk (1+b_1) - (x-1) hb_2 (1-k),
\end{align*}

Where $r = Q_1/M_1 = \text{ratio of output to market supply in the case of the first commodity.}$

$b_1 = \text{elasticity of } Q_1 \text{ with respect to } P_1/P_2$ \\
$g = \text{elasticity of } C_1 \text{ with respect to } P_1/P_3$ \\
h = \text{elasticity of } C_1 \text{ with respect to } Y (\text{or } 1)$ \\
k = $\frac{P_1Q_1}{Y} = \text{Ratio of total value of production of } Q_1 \text{ to total income } (Y=1)$
\[ b_2 = \text{elasticity of } Q_2 \text{ with respect to } P_1/P_2 \]

\[ Y = (\text{net or gross}) \text{ income.} \]

The similarity of the first four terms in equation (18.1) with the Nowshirvani formula equation (13) is striking.

We now give the following diagrammatic interpretation of the Behrman model:

Here, again any change in \( P_1 \) will have an impact on \( M \) through five distinct chains as in the above diagram,

1. \( P_1 \rightarrow Y \rightarrow C_1 \rightarrow M_1 \)
2. \( P_1 \rightarrow Q_1 \rightarrow Y \rightarrow C_1 \rightarrow M_1 \)
3. \( P_1 \rightarrow Q_1 \rightarrow M_1 \)
4. \( P_1 \rightarrow Q_2 \rightarrow Y \rightarrow C_1 \rightarrow M_1 \)
5. \( P_1 \rightarrow C_1 \rightarrow M_1 \)
K. Bardhen's 1st Model:

The formal structure of Behrman model is almost identical with the first model of Bardhen's paper and the model presented in Bardhen and Bardhen's paper (12) (13). The following two tables may help to show this correspondence:

**TABLE - 2**

<table>
<thead>
<tr>
<th>Behrman</th>
<th>K. Bardhen</th>
<th>P. Bardhen &amp; K. Bardhen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1/P_3$</td>
<td>$P_f$</td>
<td>$P_x/P_y$</td>
</tr>
<tr>
<td>$P_2/P_3$</td>
<td>$P_c$</td>
<td>$P_z/P_y$</td>
</tr>
<tr>
<td>$P_3$</td>
<td>= 1</td>
<td>$P_y$</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>$0_f$</td>
<td>$0_x$</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>$0_c$</td>
<td>$0_z$</td>
</tr>
<tr>
<td>$Y/P_3=1/P_3$</td>
<td>0</td>
<td>$1/P_y$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$0_f$</td>
<td>$c_x$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$s$</td>
<td>$s$</td>
</tr>
<tr>
<td>$M_1/Q_1$</td>
<td>$s$</td>
<td>$s$</td>
</tr>
</tbody>
</table>

* Before $P_x$ is set equal to unity.

**TABLE - 3**

<table>
<thead>
<tr>
<th>Behrman</th>
<th>$k$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$s$</th>
<th>$h$</th>
<th>$r$</th>
<th>$r-l$</th>
<th>$l-k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bardhan</td>
<td>$P_f*0_f/0$</td>
<td>$\gamma_f$</td>
<td>$-\gamma_c$</td>
<td>$-\gamma_p$</td>
<td>$e_f$</td>
<td>$0_f/S$</td>
<td>$e_f/S$</td>
<td>$p_c\delta_c/0$</td>
</tr>
</tbody>
</table>
If we rearrange equation (18.1), we get

\begin{equation}
(18.3) \quad e = r b_1 - (r-1) h \sum b_1 k + b_2 (1-k) \int \tilde{\gamma} (x-1) (g+bk)
\end{equation}

Of the above three terms in the Right Hand side, the first would be operative only when sufficient time for adjustment is given to the producer and the only term effective in a short-term situation, when output is already predetermined, is the last term. The second term can, therefore, may be interpreted as an interaction of long-term response with the short-term response to a price change, Bardhan's equation (2) is exactly identical with (18.3). But somehow she fails to see that there are three terms, not two, in equation (18.3) and treat the first two terms as one term. There are, however, a few minor points of difference between her approach and Behrman's approach.

1. Both A and B, the shift parameters are dropped from both the production functions.
2. Own price elasticity of demand is defined with a negative sign attached, in the Marshallian tradition to keep it positive.
3. Relative price elasticity of the production alternative is also defined in a slightly different manner, namely as a function of the relative price of the production alternative.
4. The consumption alternative is taken as the numeraire, so that its price is set equal to unity.
(5) While Behrman uses his model to estimate price-elasticity of marketable surplus for wheat using Raj Krishna's data (and in a different context the price elasticity of marketable surplus for Rice in Thailand) Bardhan is interested in the marketed surplus of foodgrains.

As regards the equivalence of \( b_2 \) (in Behrman's notation) with \( -Y_c \) in Bardhan's notation, we may note that

\[
(19) \quad \frac{\partial 0_s (P_c/P_1)}{\partial P_1} = -\frac{P_2 Y_c}{Q_s} \quad \text{in Bardhan's notation}
\]

and

\[
(20) \quad \frac{\partial Q_2 (P_1/P_2)}{\partial P_1} = \frac{k_2 P_1}{Q_2} \quad \text{in Behrman's notation}.
\]

(6) In Bardhan's model, income is real income, in terms of the consumption alternative which is taken as the numeraire. But the sign and magnitude of the income-elasticity term is the same as in Behrman's model, as long as \( P_2 \) remains constant since

\[
(21) \quad \theta_p = \frac{c_1}{\delta (Y/P_2)} \times \frac{0}{0} \quad \text{(in Bardhan's notation)}
\]

\[
= \frac{c_1}{\delta (Y/P_2)} \times \frac{Y/P_2}{0} = \frac{c_1}{\delta Y} \times \frac{Y}{0} = \theta \quad \text{(in Behrman's notation)}.
\]

Bardhan and Bardhan's paper starts with an equation structure which is identical with Barman's model except for the fact that in both the production functions the same shift
parameter is used, implying thereby that both will go through an identical shift. They, however, address to the task of finding the signs of two partial derivatives of the proportion, \( s \), of cereal output that is sold by the agricultural population to (i) the price of cereals relative to price of other consumables and (ii) the price of noncereals output relative to the price of cereals.

It may be noted that they use sale as synonymous with 'marketed surplus', though in their theoretical model they have not taken any account of retentions by the agricultural population for purposes other than consumption.

To discuss the implications of their model for the price-elasticity of marketable surplus, let us translate it in Berkman's term.

\[
\begin{align*}
(22.1) \quad Y &= P_1 Q_1 + P_2 Q_2 \\
(22.2) \quad Q_1 &= Q_1 \left( \frac{P_1}{P_2}, A \right) \\
(22.3) \quad Q_2 &= Q_2 \left( \frac{P_2}{P_1}, A \right) \\
(22.4) \quad Q_1 &= Q_1 \left( Y_{1, P_1/P_2} \right) \\
(22.5) \quad s &= 1 - \frac{Q_1}{Q_1} \\
&= 1 - \frac{Q_1 \left( Y_{1, P_1/P_2} \right)}{Q_1 \left( P_1/P_2, A \right)}
\end{align*}
\]

Bardhan and Bardhan, now, sets \( P_1 \) equal to unity, by taking cereals as the numeraire and write,

\[
(26) \quad s = \left( \frac{1}{P_2}, P_{1}, A \right)
\]

and concludes \( s_1 > 0 \) and \( s_2 < 0 \).
The logic behind the first of above two assertions seem to like the following:

\[ \frac{1}{P_Y} \uparrow P_Y \downarrow C_1 \downarrow \ (\text{See Model IV in Chapter V}) \]

(Substitution effect of type two)

As regards the second assertion they start by noting that the slope of the production-possibility curve for cereals and noncereals in the agricultural sector in equilibrium should be equal to their price ratio.

Translating this in Behrman's notations we get

\[ (27) \quad P_1 \frac{dQ_1}{dP_1} + P_2 \frac{dQ_2}{dP_1} = 0 \]

and hence

\[ (28) \quad \frac{dy}{dP_1} = Q_1. \]

The equation (18.1), thus, reduces to

\[ (18.3) \quad e = rb_1 - (r-1) g - (r-1) h k \]

Note that if we set

\[ b_1 = b_2 = 0 \]

and \( k = 1 \)

in equation (18.1) we get once again

\[ (18.4) \quad e = - (r-1) (g + h) \]

which is identical with equation (14). So if we ignore the different interpretation of prices in their models, Krishnan's model can also be presented as a subcase of Behrman's model.
What is striking is that equation (27), which implies equation (28) is not sufficient to give us Krishnan's result, i.e., make e independent of terms involving long-term adjustments on the part of the farmers, though equation (28) is also implied by Krishnan's assumption that the output is pre-determined.

Bardhan and Bardhan, however, were not interested in the price elasticity of the marketable surplus and therefore did not arrive at equation (28). Since they have set $P_1 = 1$, the equation corresponding to (27) in their system is

\begin{align*}
(27)' & \quad \frac{dQ_1}{dP_2} + \frac{dQ_2}{dP_2} = 0 \\
(28)' & \quad \frac{dY}{dP_2} = Q_2
\end{align*}

(29) Hence, $sgn = sg(-Q_1 \frac{dQ_1}{dY} \frac{dY}{dP_2} + c_1 \frac{dQ_1}{dP_2})$

\begin{align*}
(30) \text{Now } \frac{dQ_1}{dP_2} \leq 0 \text{ (resource-transfer effect, if any).}
\end{align*}

and $\frac{dQ_1}{dY}$ = marginal propensity to consume the numeraire commodity.

So, if we assume, as it is usually done

(31) $\frac{dQ_1}{dY} > 0$, (i.e., the commodity is a non-inferior good)

we get $s_2 < 0$. 

Note that in the inequality (30), which corresponds to Bardhan and Bardhan's assumption that output of cereals is a non-increasing function of the price of non-cereals agricultural products, should be really a strict inequality if the production possibility curve is strictly concave to the origin, (see Model II in Chapter V on this point).

**K. Bardhan's second model:**

In her extended model for a cross section sample of farms, Bardhan has used an equation like (1) to define marketed surplus. She, however, interprets 'N' as net other disposals, meaning thereby payments in kind to others by the producers of Q (which as in her first model is foodgrains) minus their receipts in kind from others. She then considers the signs of the partial derivatives of s, the proportion of foodgrains produced that is marketed, to (a) the output of foodgrains (which is taken as predetermined and also the more relevant parameter, as we have already pointed out in connection with Krishnan's model); (b) the price of foodgrains to the price of other consumer goods purchased by the producers (the latter price being taken, once again, as a numeraire) and also (c) the proportion of net disposals to foodgrains output, which is n, in her notation.

In deriving these partial derivatives, Bardhan implicitly assumes, n to be independent of not only the (relative) price of foodgrains but also the output of foodgrains. Now, in a customridden society, N, may be conceivably be given,
so long as the output of foodgrains does not fall below a minimum level (because of a drought or a flood, say) but it is difficult to see why $n$, which is $N$ divided by the output of foodgrains could be independent of even foodgrains output.

Anyway, if we go by the above assumption, then, we get, by translating her results in Behrman's notation (except for $s$)

\[
\frac{d s}{d Q_1} = \frac{c_1}{Q_1} \left( -h k + 1 \right) \\
\frac{d s}{d P_1/P_3} = \frac{c_1}{y} \left( -h - g \right)
\]

Now equation (32) suggests that there is a strong possibility that the proportion of foodgrains output that is marketed will increase with an increase in foodgrains output, since the income elasticity for foodgrains is likely to be positive and less than one (foodgrains being a necessity not luxury and a non-inferior commodity). The possibility will further increased for a value of $k$ (which could be atmost equal to one for a purely commercial farm) further away from unity.

Unfortunately, equation (33) does not add any further insight in the behaviour of $s$, since, it's derivative with respect to $P_1/P_3$ can either be positive or negative, 'h' being usually positive and 'g' being usually negative.
Translating equation (33) in elasticity terms we get that the elasticity of $s$ with respect to the relative price of foodgrains, (setting $P_3 = 1$),

\[ e = (r-1) \left( -h - g \right), \]

This is, of course, what we should expect because the price elasticity of $s$ should be equal to the price elasticity of marketed surplus minus the price elasticity of the output of foodgrains and the latter, by assumption, is equal to zero.

**Evaluation of Three Commodity Models.**

Some of the limitations of the three commodity models that they share with one commodity models are (1) substitution of gross income for net income; and (2) substitution of actual price for an expected price.

One major point of departure of the three commodity models is to recognize and distinguish between two types of relative prices.

The most unsatisfactory feature of the three commodity models is that they suggest that the consumer-producer has a money illusion. This aspect is not explicit in Behrman's model. In Behrman's model, income is defined as

\[ (16.1) \quad Y = P_1 Q_1 \left( \frac{P_1}{P_2} A \right) + P_2 Q_2 \left( \frac{P_1}{P_2} B \right). \]

Let all the prices, $P_1$, $P_2$ and $P_3$ rise by the same proportions so that new prices, $P_1'$, are given by,
So all the prices and money income have risen by the same proportion. One would expect that, if there is no money illusion, $C_1$ would not change. But $C_1$ in Behrman's model is given by,

\begin{equation}
C_1 = \lambda \left( Y, \frac{P_1}{P_3} \right).
\end{equation}

Let us give it a specific form, say,

\begin{equation}
C_1 = a Y^h + b \left( \frac{P_1}{P_3} \right)^g + k
\end{equation}

where $a$, $b$, $g$, $h$ and $k$ are given positive constants.

At the new level of prices

\begin{equation}
C_1' = a Y^h + b \left( \frac{P_1}{P_3} \right)^g + k > C_1
\end{equation}

Alternatively, we may assume that

\begin{equation}
C_1 = a Y^h \left( \frac{P_1}{P_3} \right)^g
\end{equation}

The above form corresponds not only to Krishnan's aggregate home-consumption function of the producers of the commodity but also to the usual log-linear regression form of the equation for estimating $g$ and $h$.

Even now, at the new level of prices,

\begin{equation}
C_1' = \lambda \cdot C_1 > C_1
\end{equation}

In real life, money illusion often exists. But in real life, one has also a budget constraint which for simplicity, may be taken, as
At the new level of prices, if \( c_1 > c_4 \) and analogously, 
\( c_3' > c_3 \), the amount spent on the two commodities would be 
greater than \( y' \), at the new level of prices \( P_1' \) and \( P_3' \).

In Bardhan and Bardhan Model, both income and own-
consumption are defined in an identical manner and therefore 
we have the same difficulty.

Question may also be raised against the appropriateness 
of the choice of cereals as the numeraire and thus 
setting its price equal to unity.

Since taking one commodity as the numeraire is one 
of the standard practices of the economists, we may spend 
some time here to show why under certain circumstance it 
may be a hindrance rather than a help as an analytical device. 
In Bardhan and Bardhan model, we have three prices to start 
with, namely, \( P_x \), \( P_y \) and \( P_z \). Had the consumption function and 
marketed supply function been homogeneous of degree zero in 
all the prices we could have very easily taken only one price 
as the numeraire, since only relative prices matter, as in 
the Walrasian system.

Now it has been shown many times over that except 
under certain special situations, the sign of this price-elast-
ticity of marketable (or marketed) surplus is not definitely 
known. Bardhan and Bardhan have shown that the sign of the
price elasticity of the marketed proportion with respect to the relative prices of other two commodities could be unambiguously determined from theoretical considerations and both of them are negative (if the elasticity is positive with respect to \(1/P_y\) it must be negative with respect to \(P_y\)). This could have been called an advance on our state of knowledge if and only if the marketed proportion function were homogeneous of degree zero in all the prices. In that case change in the absolute price of \(P_x\) to \(P_x\) is equivalent to a simultaneous change in the relative prices \(P_y\) and \(P_z\) by the same proportion \(1/\lambda\). Hence, it follows that the own-absolute price elasticity of marketed proportion is positive. But the marketed proportion function is not homogeneous of degree zero.

Even then one can still take cereal as the numeraire provided that \(P_x\) is fixed, as money wages are in the Keynesian System. It may be noted that Keynes in discussing the consumption function in 'General Theory' has taken into account the possibility of a change in the wage-unit and its possible repercussions. It is the similar change in the absolute price of cereals, that has not been taken into account in Bardhan and Bardhan model. If all the prices increase in a given proportion and if the consumption function has the specific form given to it in equation (39) or (40) it could easily be shown that since \(Y\) will increase in the same proportion, \(s\) as well as \(M\) would decrease while \(C_1\) would increase even though none of the relative prices
would be affected by such an equi-proportionate change.

Moreover, if the absolute price of cereal is not fixed, the conclusion of the Bardhan and Bardhan model, that \( a_0 \) is suspect. To see it, let us revert once again to Behrman's notation and assume \( C_q \) to have the specific form given to it by equation (38).

\[
\begin{align*}
(43) \quad \frac{\partial C_1}{\partial P_3} &= \frac{b_1 \beta}{P_3} \left( \frac{P_1}{P_3} \right)^\beta, \\
(44) \quad \text{but} \quad \frac{\partial C_1}{\partial P_3} &= a_1 Q_1 - \frac{b_1 \beta}{P_3} \left( \frac{P_1}{P_3} \right)^{\beta - 1}
\end{align*}
\]

So, \( P_3 \) need no longer be true if the relative price of the consumption alternative decreases due to a rise in the price of cereals, the price of the consumption alternative remaining the same. (See Model IV on Chapter V on this point).

What is even more, even if we assume that \( \frac{\partial C_1}{\partial (P_3/P_1)} > 0 \), so that a decrease in the relative price of cereal in term of the consumption alternative is followed by a decrease in the consumption of cereals on the part of the producers of cereals, the output of cereals is also going to increase if the primary cause of the decrease in \( P_3/P_1 \) is an increase in the price of cereals and sufficient time for adjustment in \( Q_1 \) is given. So the marketed proportion, \( s \), may still increase in a comparative static equilibrium framework.

In Bardhan's model, income is relative income as in Raj Krishna's model and both the consumption function and the
production function, and as a consequence, marketable surplus function are homogeneous of degree zero in all the prices. But even if that had been not so, we could have still set the price of the consumption alternative equal to unity, impounding it in 'other things remaining the same'. In fact such an assumption is already implicit in our demonstration of the equivalence of the income elasticity terms of Behrman and Bardhan Models.

But in a model like the one presented by Bardhan and Bardhan which is constructed to find the price elasticity of marketed proportion of cereals, an assumption that the absolute price of cereals cannot change and only its relative price can change through changes in the price of the production and the consumption alternatives seems to be somewhat misplaced.

III. Two Recent One Commodity Models.

The model presented by Toquero and others (14) makes two significant departures from the other one-commodity models considered in this Chapter. In the first place there is no income term and secondly both consumption and marketable surplus are made functions of the price and the output level. The equational structure of the model is like the following:

\begin{align}
(45.1) \quad & Q = Q(P) \\
(45.2) \quad & M = M(P, Q) \\
(45.3) \quad & C = Q - M = C(P, Q).
\end{align}
The commodity considered is Rice and \( P \) is the price of rice deflated by the price index of non-rice commodities. As in Raj Krishna's model, they also abstract from other disposition of output other than consumption and sale.

Two results are given, one, expressing total price-elasticity of supply for the marketable surplus (\( e \)) in terms the partial price elasticity of supply for marketable surplus at a given output \( q_p \), and the elasticity of market supply with respect to output (eq) and the price elasticity of output (b). This can be derived, easily from (45.2).

\[
e = q_p + eq \cdot b
\]

The other result expresses total and partial price elasticities of demand for home consumption (\( Q \) and \( g \) respectively) and the output elasticity of demand for home consumption (\( h' \)) in terms of the elasticities mentioned above. For this relation, we obtain from (45.2) and (45.3)

\[
\frac{dQ}{dP} = \frac{\partial Q}{\partial P} + \frac{\partial G}{\partial Q} \cdot \frac{dQ}{dP} = \frac{dQ}{dP} - \frac{\partial M}{\partial P} \cdot \frac{dQ}{dP} - \frac{dQ}{dP}
\]

Converting in terms of elasticities, we now get,

\[
G = g + h' \cdot b = \frac{\partial P}{\frac{1}{x-1}} + b \left( \frac{1}{x-1} - \frac{1}{x-1} \cdot e_q \right)
\]

(49) \(Or - (x-1) \ (g \cdot h' \cdot b) = q_p + b \cdot e_q - rb\)

(50) \(Or e = rb - (x-1) \ (g \cdot h' \cdot b)\)
The following table is given for translating the notation of the above articles into our notation.

<table>
<thead>
<tr>
<th>Notation used</th>
<th>G</th>
<th>g</th>
<th>h'</th>
<th>e</th>
<th>ε_p</th>
<th>ε_q</th>
<th>b</th>
<th>r</th>
</tr>
</thead>
</table>

In this Paper

Toquero & others

The similarity of equation (50) with equation (23) is striking.

This model, however, is much closer to the Kathur and Ezekiel hypothesis than any of the other models presented in this Chapter. This becomes apparent from the chain of relationships of the model as presented in the following diagram.

![Diagram 4]

As can be seen from the above diagram, the marketable surplus rather than home-consumption is the first charge on output produced and it is determined directly by P and Q. However, their model does not allow 'saying in kind'
on the part of surplus farmers. Nor have they introduced
fixed cash obligations on the part of marginal farmers. The model also has an affinity with Bardhan's extended model except for the fact that net other disposals is explicitly assumed to be zero.

One-Commodity, Two-price Model.

The full model of Haessel (15) is a sequential model in the sense that there are really two prices involved. One price determines the (long-run) supply response on the part of the producers of the commodity. The other price is determined by the given supply and the demand for the commodity on the part of the entire commodity (i.e., that coming from non-producers plus that coming from the producer). No feedback effect of the latter price, which is endogenously determined, on the quantity supplied is incorporated in the model as it should have been done in a comparative static framework. Let us give our version of Haessel model in our notations.

\[(51.1) \quad Q = Q (P)\]
\[(51.2) \quad Q' = Q - N\]
\[(51.3) \quad Y'' = P' Q' + Y_0\]
\[(51.4) \quad C = C (P', Y'')\]
\[(51.5) \quad M = Q - N - C = Q' - C (P' Y'')\]
\[(51.6) \quad P' = P (Q)\]

Equation \((51.1)\) is identical to one used by Bardhan in her extended model, \(N\) being net disposals to non-cultivations.
Haessel, however, assumes that N consists of contractual payments and therefore it is independent of both price and output produced. It is a pure exogeneous variable.

Once again P or P' is relative price, the price of other commodities being taken as the numeraire. Income is defined as income in Nowshirvani's model, minus the value of net disposals to others.

The major departure of the model from the earlier models is in incorporating equation (51.6)

With this model as a background, Haessel derives his equation (10), the short-run (total) price-elasticity as

\[
(52) \ \epsilon = - (r-1) \ (g + hk) : k = P'Q'/Y',
\]

which is, of course, the result we would get if we put b = 0 in Nowshirvani's model. Haessel, however, notes in his equation (9), that the first term in the R - H side, \((r-1) g\) is the pure price elasticity of marketing and the second term \((r-1) hk\) is the k times pure income elasticity of marketing. Since output is predetermined and N is also given, the above result follows from the fact that \(dM = dQ\).

To find the long-run elasticity of marketable surplus with respect to price, first, we differentiate (51.5) with respect to Q, assuming \(\frac{dy}{dQ} = 0\), to obtain
Converting in terms of elasticities, we get,

\[ \frac{dN}{dq} = 1 - \frac{dG}{dP} \frac{dP'}{dq} - \frac{dG}{dY} \frac{dP'}{dq} - \frac{dG}{dP} \frac{dY}{dq} \]

\[ = 1 - \frac{dG}{dP} \frac{dP'}{dq} - \frac{dG}{dY} \frac{dP'}{dq} - \frac{dG}{dP} \frac{dY}{dq} \]

Haessel's formula for 'e', the long-run price elasticity of marketable surplus is,

\[ e = \frac{dN}{dq} \frac{Q}{M} = e_q = \frac{Q}{M} - (r-1) \left( gY^2 + hk + hky \right) \]

where \( K' = P'Q/Y \) and \( \frac{dP'}{dq} \frac{Q}{M} \)

The following diagram shows the chain of relationship in Haessel's model, as presented by us:

![Diagram](attachment:Diagram.png)

Diagram = 5

To translate our notation into Haessel's notation the following table is given below (after correcting for a printing error in the definition of \( d \) in Haessel's paper).
As it can be seen from the above table, Haessel does not make any distinction between $P$ and $P'$. This implies that $P$ is again mapped into $P$ through $Q$, (See the diagram). If there is a fixed point in this mapping, a solution for $P$ exists. If there are more than one such fixed points, the solution is not unique. But whatever may be the case, $P$ being now an endogenous variable, cannot be treated as a parameter. Hence the task of finding the price-elasticity of marketable surplus, as $P$ is subjected to parametric variation in a comparative static framework, cannot be undertaken. In the estimation part of his article, Haessel, however has not attempted to calculate the long-run price elasticity of marketing.