CHAPTER 2

MHD EFFECTS ON A MOVING SEMI-INFINITE VERTICAL CYLINDER WITH HEAT AND MASS TRANSFER

2.1 INTRODUCTION

Unsteady free convection flows with combined buoyancy effects are practically important in many areas. Mixed types of boundary conditions are commonly met in practice. A study of the temperature and mass distribution around the intrusive and associated mass surface change is important in geothermal resources during geophysical exploration. The time required for the intrusive magma to set, is very crucial. These types of problems also arise in many technological applications, where the devices are cooled by natural convection as in the case of electrical heaters and in transformers. The main concern in such flows is the possibility of flux boundary conditions.

These types of problems are commonly met in the start-up of a chemical reactor, and the emergency cooling of a nuclear fuel element. In the case of power or pump failure, similar conditions may arise for devices cooled by forced circulation, as in the core of a nuclear reactor. In the glass and polymer industries, hot filaments that are considered as vertical cylinders are cooled, as they pass through the surrounding environment.
Goldstein and Briggs (1964) carried out a one-dimensional study of transient natural convection in cylinders. They presented conduction analytical solutions for infinite cylinders. Subsequently, Dring and Gebhart (1966) presented the experimental results for the transient average temperature of Nichrome wires in silicone oils and in air. They also compared their experimental results with those of pure conduction, and with a simplified quasistatic theory that yields a simple exponential solution for the temperature response. But, the quasistatic theory failed, however, for silicone fluids. Even for air, the conduction solution was found to be better than that predicted by this theory.

Some steady state analyses for a cylinder also exist. Amongst them, Minkowycz and Sparrow (1974) obtained the steady boundary layer velocity and temperature profiles for isothermal cylinders of various radii placed in air, using the local non-similarity method. Nagendra et al (1970) carried out a numerical study of the steady boundary layer equations for cylinders subjected to uniform heat flux, and compared their predictions with the experimental results of their earlier study on water. They found that the difference of the heat transfer coefficients between the cases of isothermal and constant heat flux cylinders is about 6%.

In all the above cases, the boundary can be considered as an isothermal along with heat and mass transfer. The local and average heat and mass transfer rates and the flow characteristics are very important. Due to the numerous applications of this problem, it has been taken up for analysis. The study of magnetohydrodynamics incompressible viscous flow has many important engineering applications in devices, such as the MHD power generator, the cooling of nuclear reactors, geothermal systems, aerodynamic processes, and heat exchange designs. In the field of power generation, MHD
is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Acharya et al (2000) have considered the steady two-dimensional free convection and mass transfer flow of a viscous incompressible electrically conducting fluid through a porous medium over vertical infinite surface with constant suction velocity and constant heat flux in the presence of a magnetic field. Pop et al (1994) studied the boundary layer solution for the conjugate forced convection flow of an electrically conducting fluid over a semi-infinite plate in the presence of a transverse magnetic field.

Shankar and Kishan (1997) presented the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate, when subjected to a variable temperature and constant heat flux. The problem has been solved by the explicit (Schmidt) finite difference method. It was found that an increase in the Schmidt number leads to an increase in the skin friction. Yih (2000) presented the effect of uniform blowing/suction on MHD natural convection over a horizontal cylinder. A laminar boundary layer analysis is used to investigate this flow. The non-similar governing equations are obtained by using a suitable transformation, and they are solved by the Keller box method.

From the forgoing survey, it is evident that there is no research considering the problem of MHD natural convection boundary layer flow over a moving vertical cylinder with heat and mass transfer. Flows arising from differences in concentration or material differences have great significance, not only because of their own interest, but also because of their applications in chemical engineering, geophysics and aeronautics. Many interesting aspects of such flow have been studied in recent years.
The purpose of the present work is to study the free convective flow of a viscous incompressible and electrically conducting fluid past a moving semi-infinite vertical cylinder with heat and mass transfer under the action of a uniform magnetic field applied normal to the direction of the flow.

2.2 MATHEMATICAL ANALYSIS

The magnetohydrodynamics flow of a viscous incompressible fluid past an impulsively started a moving semi-infinite vertical cylinder of radius $r_0$ with heat and mass transfer is considered. The x-axis is measured along the axis of the cylinder in the vertically upward direction, and the radial co-ordinate $r$ is taken along the direction of the magnetic field, and $x$ and $r$ are mutually perpendicular.

Initially, at time $t' = 0$, both the cylinder and the fluid are stationary at the same temperature $T'_\infty$ and also at the same species concentration level $C'_\infty$. At a time $t' > 0$, the cylinder starts moving in the vertical direction with velocity $u_0$. The temperature and the concentration on the surface of the cylinder are also raised to $T'_w$ and $C'_w$. Under these assumptions, the governing boundary layer equations of continuity, momentum, energy and species concentration with the Boussinesq’s approximation are as follows.

**Equation of continuity**

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0 \quad (2.1)$$

**Equation of momentum with MHD**

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = g \beta (T' - T'_\infty) + g \beta' (C' - C'_\infty)$$

$$+ \frac{v}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{\sigma B_0^2}{\rho} u \quad (2.2)$$
Energy equation

\[
\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial r} = \frac{k}{\rho c_p} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T'}{\partial r} \right)
\]  
(2.3)

Mass diffusion equation

\[
\frac{\partial C'}{\partial t'} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial r} = \frac{D}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C'}{\partial r} \right)
\]  
(2.4)

The initial and boundary conditions are:

\[ t' \leq 0 : u = 0, v = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } x \geq 0 \text{ and } r \geq 0 \]

\[ t' > 0 : u = u_0, v = 0, T' = T'_w, C' = C'_w \text{ at } r = r_0 \]

\[ u = 0, T' = T'_\infty, C' = C'_\infty \text{ at } x = 0 \text{ and } r \geq r_0 \]

\[ u \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } r \rightarrow \infty \]

Naturally, the terms \( g\beta(T' - T'_\infty) \) and \( g\beta'(C' - C'_\infty) \) in the
momentum equations represent the buoyancy effect. An idea of the physical
geometry may be acquired by noting the schematic diagram in Figure 2.1. It is
convenient to introduce the following dimensionless variables and
parameters:

\[
\begin{align*}
X &= \frac{x}{u_0 r_0^2}, R &= \frac{r}{r_0}, U &= \frac{u}{u_0}, V = \frac{vr_0}{u}, t = \frac{ut'}{r_0^2}, T = \frac{T' - T'_w}{T'_w - T'_\infty}, \\
C &= \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gr &= \frac{g\beta r_0^2 (T'_w - T'_\infty)}{u u_0}, Gc &= \frac{g\beta r_0^2 (C'_w - C'_\infty)}{u u_0}, \\
Sc &= \frac{\nu}{D}, Pr &= \frac{\nu}{\alpha}, M &= \frac{\sigma B_0^2 r_0^2}{\rho u_0}
\end{align*}
\]  
(2.6)
Figure 2.1 The physical model and co-ordinate system

Equations (2.1), (2.2), (2.3) and (2.4) are reduced to the following non-dimensional form:

\[
\frac{\partial (RU)}{\partial X} + \frac{\partial (RV)}{\partial R} = 0 \tag{2.7}
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial R} + \text{Gr}T + \text{Gec}C + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) - MU \tag{2.8}
\]

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial R} = \frac{1}{\text{Pr}} \frac{\partial}{\partial R} \left( R \frac{\partial T}{\partial R} \right) \tag{2.9}
\]

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial R} = \frac{1}{\text{Sc}} \frac{\partial}{\partial R} \left( R \frac{\partial C}{\partial R} \right) \tag{2.10}
\]

The corresponding initial and boundary conditions in non-dimensional quantities are:

\[ t \leq 0 : U = 0, V = 0, T = 0, C = 0 \text{ for all } X \text{ and } R \]

\[ t > 0 : U = 1, V = 0, T = 1, C = 1 \text{ at } R = 1 \]
\[ U = 0, T = 0, C = 0 \text{ at } X = 0 \]  \hspace{1cm} (2.11)

\[ U \to 0, T \to 0, C \to 0 \text{ as } R \to \infty \]

### 2.3 FINITE DIFFERENCE METHOD

In order to solve these unsteady, non-linear and coupled equations (2.7), (2.8), (2.9) and (2.10) under the conditions (2.11), an implicit finite difference scheme of the Crank-Nicolson type has been employed. The finite difference equations corresponding to equations (2.7), (2.8), (2.9) and (2.10) are given by

\[
\begin{align*}
\frac{U_{i,j}^{k+1} - U_{i,j}^k + U_{i+1,j}^{k+1} + U_{i,j+1}^{k+1}}{4\Delta X} + \frac{U_{i,j}^k - U_{i-1,j}^k + U_{i,j}^{k+1} - U_{i,j+1}^k}{4\Delta X} & = 0 \\
\frac{V_{i,j}^{k+1} - V_{i,j}^k + V_{i,j+1}^{k+1} - V_{i,j-1}^k}{2\Delta R} + \frac{V_{i,j}^k}{1 + (j - 1)\Delta R} & = 0
\end{align*}
\]  \hspace{1cm} (2.12)

\[
\frac{U_{i,j}^{k+1} - U_{i,j}^k + U_{i+1,j}^{k+1} + U_{i,j+1}^{k+1}}{\Delta t} + \frac{U_{i,j}^k - U_{i-1,j}^k + U_{i,j}^{k+1} - U_{i,j+1}^k}{2\Delta X} + \frac{V_{i,j}^{k+1} - V_{i,j}^k + V_{i,j+1}^{k+1} - V_{i,j-1}^k}{4\Delta R} & = 0 \\
\frac{T_{i,j}^{k+1} - T_{i,j}^k + T_{i+1,j}^{k+1} + T_{i,j+1}^{k+1}}{\Delta t} + \frac{T_{i,j}^k - T_{i-1,j}^k + T_{i,j}^{k+1} - T_{i,j+1}^k}{2\Delta X} + \frac{C_{i,j}^{k+1} - C_{i,j}^k + C_{i,j+1}^{k+1} - C_{i,j-1}^k}{4\Delta R} & = 0 \\
\frac{T_{i,j}^{k+1} - T_{i,j}^k + T_{i+1,j}^{k+1} + T_{i,j+1}^{k+1}}{2\Delta X} + \frac{T_{i,j}^k - T_{i-1,j}^k + T_{i,j}^{k+1} - T_{i,j+1}^k}{4\Delta R} & = 0
\]  \hspace{1cm} (2.13)

\[
\frac{T_{i,j}^{k+1} - T_{i,j}^k + T_{i+1,j}^{k+1} + T_{i,j+1}^{k+1}}{\Delta t} + \frac{T_{i,j}^k - T_{i-1,j}^k + T_{i,j}^{k+1} - T_{i,j+1}^k}{2\Delta X} + \frac{T_{i,j}^{k+1} - T_{i,j}^k + T_{i+1,j}^{k+1} + T_{i,j+1}^{k+1}}{4\Delta R} & = 0 \\
\frac{T_{i,j}^{k+1} - T_{i,j}^k + T_{i+1,j}^{k+1} + T_{i,j+1}^{k+1}}{2\text{Pr}(\Delta R)^2} + \frac{T_{i,j}^k - T_{i-1,j}^k + T_{i,j}^{k+1} - T_{i,j+1}^k}{4\text{Pr}[1 + (j - 1)\Delta R]\Delta R} & = 0
\]  \hspace{1cm} (2.14)
\[
\frac{C_{i,j}^{k+1} - C_{i,j}^k}{\Delta t} + U_{i,j}^k \frac{C_{i,j}^{k+1} - C_{i-1,j}^k + C_{i,j}^k - C_{i,j}^{k+1}}{2\Delta X} + V_{i,j}^k \frac{C_{i,j+1}^{k+1} - C_{i,j-1}^k + C_{i,j+1}^k - C_{i,j}^{k+1}}{4\Delta R}
\]

\[
= \frac{C_{i,j}^{k+1} - 2C_{i,j}^{k+1} + C_{i,j+1}^{k+1} + C_{i,j-1}^k - 2C_{i,j}^k + C_{i,j}^k}{2Sc(\Delta r)^2} + \frac{C_{i,j+1}^{k+1} - C_{i,j-1}^{k+1} + C_{i,j+1}^{k+1} - C_{i,j}^{k+1}}{4Sc[1 + (j - 1)\Delta R]\Delta R}
\]

(2.15)

The region of integration is considered as a rectangle with sides \(X_{\text{max}}(=1.0)\) and \(R_{\text{max}}(=15.0)\) where \(R_{\text{max}}\) corresponds to \(R = \infty\) which lies well outside the momentum, and the energy and concentration boundary layers. The maximum of \(R\) was chosen as 15 after some preliminary investigations, so that the last two of the boundary conditions (2.11) are satisfied. Here, the subscript \(i\)-designates the grid point along the \(X\)-direction, \(j\) along the \(R\)-direction and the superscript \(k\) along the \(t\) direction. At any one time step, the coefficients \(U_{i,j}^k\) and \(V_{i,j}^k\) appearing in the finite difference equations are treated as constants. The values of \(U, V, T\) and \(C\) are known at all grid points at \(t = 0\) from the initial conditions.

The computations of \(C, T, V\) and \(U\) at time level \((k+1)\) using the values at the previous time level \((k)\) are carried out as follows: The finite difference equation (2.15) at every internal nodal point on a particular \(i\)-level constitutes a tridiagonal system of equations. Such a system of equations is solved by using the Thomas algorithm as discussed in Carnahan et al (1969). Thus, the values of \(C\) are found at every nodal point for a particular \(i\) at the \((k+1)^{\text{th}}\) time level. Similarly, the values of \(T\) are calculated from equation (2.14). Using the values of \(C\) and \(T\) at the \((k+1)^{\text{th}}\) time level in the equation (2.13), the values of \(U\) at the \((k+1)^{\text{th}}\) time level are found in a similar manner. Thus, the values of \(C, T\) and \(U\) are known on a particular \(i\)-level. Finally, the values of \(V\) are calculated explicitly using the equation (2.12) at every nodal point on a particular \(i\)-level at the \((k+1)^{\text{th}}\) time level. This process is repeated for various \(i\)-levels. Thus, the values of \(C, T, U\) and \(V\) are known at all the
grid points in the rectangular region at the \((k+1)^{th}\) time level. In a similar manner, computations are carried out by moving along \(i\)-direction. After computing the values, corresponding to each \(i\) at a time level, the values at the next time level are determined in a similar manner. Computations are repeated until the steady state is reached. The steady state solution is assumed to have been reached, when the absolute difference between the values of velocity \(U\), temperature \(T\) as well as concentration \(C\) at two consecutive time steps are less than \(10^{-5}\) at all the grid points.

After experimenting with a few sets of mesh sizes, they have been fixed at the level \(\Delta X = 0.02, \Delta R = 0.2\), with a time step \(\Delta t = 0.01\). In this case, the spatial mesh sizes are reduced by 50\% in one direction, and later in both the directions, and the results are compared. It is observed that, when the mesh size is reduced by 50\% in the R-direction, the results differ in the fifth decimal place, while if the mesh sizes are reduced by 50\% in the X-direction or in both directions, the results are correct to five decimal places. Hence, the above mesh has been considered as appropriate for calculation.

### 2.4 STABILITY OF THE SCHEME

The stability criterion of the finite difference scheme for constant mesh sizes is examined, using the Von-Neumann technique as given in Carnahan et al (1969). The general term of the Fourier expansion for \(U\), \(T\) and \(C\) at a time arbitrarily called \(t = 0\) is assumed to be of the form \(e^{iaX}e^{iR}e^{i\beta R}\) (here \(i = \sqrt{-1}\)).
At a later time $t$, these terms will become

$$U = F(t)e^{i\alpha X}e^{i\beta R}$$
$$T = G(t)e^{i\alpha X}e^{i\beta R}$$
$$C = H(t)e^{i\alpha X}e^{i\beta R}$$

(2.16)

Substituting (2.16) in equations (2.13), (2.14) and (2.15) under the assumption that the coefficients $U$, $T$ and $C$ are constants over any one time step, and denoting the values after one time step by $F', G'$ and $H'$, after simplification, we obtain

$$\begin{align*}
\frac{F' - F}{\Delta t} + \frac{U(F' + F)(1 - e^{i\alpha X})}{2\Delta X} + \frac{V(F' + F)i\sin(\beta R)}{2\Delta R} &= \frac{G' + G}{2} + \frac{H' + H}{2} + \frac{(F' + F)(\cos(\beta R) - 1) + (F' + F)i\sin(\beta R)}{(\Delta R)^2} + \frac{M(F' + F)}{2[1 + (j - 1)\Delta R]\Delta R} \\
&= G' + G + \frac{U(G' + G)(1 - e^{i\alpha X})}{2\Delta X} + \frac{V(G' + G)i\sin(\beta R)}{2\Delta R} + \frac{(G' + G)(\cos(\beta R) - 1) + (G' + G)i\sin(\beta R)}{\Pr(\Delta R)^2} + \frac{2\Pr[1 + (j - 1)\Delta R]\Delta R}{2} \\
&= \frac{(H' + H)(\cos(\beta R) - 1) + (H' + H)i\sin(\beta R)}{Sc(\Delta R)^2} + \frac{2Sc[1 + (j - 1)\Delta R]\Delta R}{2} \\
\end{align*}$$

(2.17)

(2.18)

(2.19)

The above equations can be written as follows:

$$(1 + A)F' = (1 - A)F + \left[ \frac{G' + G}{2} + \frac{H' + H}{2} \right] \Delta t$$

(2.20)
(1 + B)G \quad (1 - B)G \quad (2.21)

(1 + E)H \quad (1 - E)H \quad (2.22)

where

\[
A = \frac{U}{2} \left(1 - e^{-a\Delta X}\right) \frac{\Delta t}{\Delta X} + \frac{V}{2} \sin(\beta \Delta R) \frac{\Delta t}{\Delta R} - (\cos(\beta \Delta R) - 1) \frac{\Delta t}{\Delta R}^2 - \frac{1}{2[1 + (j - 1)\Delta R] \Delta R} \sin(\beta \Delta R) \frac{\Delta t}{\Delta R} + M \frac{\Delta t}{2}
\]

\[
B = \frac{U}{2} \left(1 - e^{-a\Delta X}\right) \frac{\Delta t}{\Delta X} + \frac{V}{2} \sin(\beta \Delta R) \frac{\Delta t}{\Delta R} - (\cos(\beta \Delta R) - 1) \frac{\Delta t}{Pr(\Delta R)^2} - \frac{1}{2Pr[1 + (j - 1)\Delta R] \Delta R} \sin(\beta \Delta R) \frac{\Delta t}{\Delta R}
\]

\[
E = \frac{U}{2} \left(1 - e^{-a\Delta X}\right) \frac{\Delta t}{\Delta X} + \frac{V}{2} \sin(\beta \Delta R) \frac{\Delta t}{\Delta R} - (\cos(\beta \Delta R) - 1) \frac{\Delta t}{Sc(\Delta R)^2} - \frac{1}{2Sc[1 + (j - 1)\Delta R] \Delta R} \sin(\beta \Delta R) \frac{\Delta t}{\Delta R}
\]

After eliminating \( F^\prime, G^\prime \) and \( H^\prime \) in equation (2.20) using equation (2.21) and (2.22), the resultant equation can be written in the matrix form as

\[
\begin{pmatrix}
F^\prime \\
G^\prime \\
H^\prime
\end{pmatrix} = \begin{pmatrix}
\frac{1 - A}{1 + A} & D_1 & D_2 \\
0 & 1 - B & 0 \\
0 & 0 & 1 - E
\end{pmatrix} \begin{pmatrix}
F \\
G \\
H
\end{pmatrix}
\quad (2.23)
\]
where \( D_1 = \frac{\Delta t}{(1 + A)(1 + B)} \), \( D_2 = \frac{\Delta t}{(1 + A)(1 + E)} \).

Now, for the stability of the finite difference scheme, the modulus of each eigen value of the amplification matrix does not exceed unity. Since the matrix equation (2.23) is upper triangular, the eigen values are its diagonal elements. The eigen values of the amplification matrix are \((1 - A)/(1 + A), (1 - B)/(1 + B)\) and \((1 - E)/(1 + E)\). Assuming that \( U \) is everywhere non-negative, and \( V \) is everywhere non-positive, we obtain

\[
A = 2a \sin^2 \left( \frac{\alpha \Delta X}{2} \right) + 2d \sin^2 \left( \frac{\beta \Delta R}{2} \right) + i a \sin(\alpha \Delta X) \\
+ i(b - c) \sin(\beta \Delta R) + M \frac{\Delta t}{2}
\]

where \( a = \frac{U \Delta t}{2 \Delta X}, b = \frac{V \Delta t}{2 \Delta R}, c = \frac{\Delta t}{2[1 + (J - 1) \Delta R] \Delta R} \) and \( d = \frac{\Delta t}{(\Delta R)^2} \).

Since the real part of \( A \) is greater than or equal to zero, \(|(1 - A)/(1 + A)| \leq 1\) always. Similarly \(|(1 - B)/(1 + B)| \leq 1\) and \(|(1 - E)/(1 + E)| \leq 1\).

Hence, the finite difference scheme is unconditionally stable. The local truncation error is \( O(\Delta t^2 + \Delta R^2 + \Delta X) \) and it tends to zero as \( \Delta t, \Delta X \) and \( \Delta R \) tend to zero. Hence the scheme is compatible. The stability and the compatibility ensure the convergence.
2.5 RESULTS AND DISCUSSION

In order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The velocity profiles for the stationary vertical cylinder (i.e. $u = 0$) with $M = 0$, $Pr = 0.71$, $Sc = 0.16$ and $Gr = Gc = 2$ are compared with the results of the Chen and Yuh (1980) in Figure 2.2, and they are found to be in good agreement.

![Figure 2.2 Comparison of the velocity profiles](image)
The effect of the transient and steady state velocity profiles for different values of the magnetic parameter and the Prandtl number (for air and water) are shown in Figure 2.3. It is observed that the magnetic field modifies the velocity gradients near the cylinder. It is noted that the presence as well as the increase of the magnetic field leads to a decrease in the velocity field; that is, it retards the flow field. In the absence of a magnetic field, the velocity overshoots near the cylinder. The lower value of the Prandtl number of air exhibits a greater velocity gradient. The velocity profiles for different values of the magnetic parameter $M$, $Gr = Gc = 5.0$ and $Sc = 0.3$ in the presence of Prandtl numbers are shown in Tables 2.1 and 2.2. This trend shows that the velocity profile decreases with increasing values of the Prandtl numbers.

**Figure 2.3** Velocity profiles at $X=1.0$ for different values of $M$ and $Pr$ (*- Transient state)
Table 2.1 Velocity profiles for different values of M with Pr = 0.71

<table>
<thead>
<tr>
<th>R</th>
<th>M = 0</th>
<th>M = 1</th>
<th>M = 3</th>
<th>M = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>1.6891</td>
<td>1.5311</td>
<td>1.2940</td>
<td>0.8826</td>
</tr>
<tr>
<td>1.4</td>
<td>1.9897</td>
<td>1.7434</td>
<td>1.3785</td>
<td>0.7840</td>
</tr>
<tr>
<td>1.6</td>
<td>2.0460</td>
<td>1.7638</td>
<td>1.3486</td>
<td>0.6998</td>
</tr>
<tr>
<td>1.8</td>
<td>1.9595</td>
<td>1.6760</td>
<td>1.2600</td>
<td>0.6270</td>
</tr>
<tr>
<td>2</td>
<td>1.7997</td>
<td>1.5342</td>
<td>1.1460</td>
<td>0.5637</td>
</tr>
</tbody>
</table>

Table 2.2 Velocity profiles for different values of M with Pr = 7.0

<table>
<thead>
<tr>
<th>R</th>
<th>M = 0</th>
<th>M = 1</th>
<th>M = 3</th>
<th>M = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>1.5128</td>
<td>1.3653</td>
<td>1.1559</td>
<td>0.8153</td>
</tr>
<tr>
<td>1.4</td>
<td>1.7032</td>
<td>1.4695</td>
<td>1.1456</td>
<td>0.6678</td>
</tr>
<tr>
<td>1.6</td>
<td>1.7205</td>
<td>1.4439</td>
<td>1.0669</td>
<td>0.5533</td>
</tr>
<tr>
<td>1.8</td>
<td>1.6504</td>
<td>1.3602</td>
<td>0.9685</td>
<td>0.4661</td>
</tr>
<tr>
<td>2</td>
<td>1.5387</td>
<td>1.2542</td>
<td>0.8715</td>
<td>0.4005</td>
</tr>
</tbody>
</table>
Figure 2.4 represents the temperature distribution for different values of M and Pr. The presence as well as the increase in the magnetic field leads to a rise in the temperature distribution in the flow field. It is observed that the time taken to reach the steady state increases with an increasing value of M. Lower temperature profiles are observed for higher Pr. This is due to the fact that fluids with higher Pr give rise to less heat transfer. But the thermal boundary layer thickness increases with an increasing value of the magnetic parameter M.

![Graph showing temperature profile](image_url)

**Figure 2.4**  Temperature profiles at X=1.0 for different values of M and Pr (*-Transient state)
The effect of the transient and steady state concentration profiles for different values of the magnetic parameter and the Prandtl number are shown in Figure 2.5. The effect of M plays an important role in the concentration field. It is observed that the concentration profile increases with an increasing value of the magnetic parameter M. The time taken to reach the steady state increases as M increases.

![Figure 2.5](image-url)  
**Figure 2.5**  Concentration profiles at X=1.0 for different values of M and Pr (*-Transient state)
Knowing the numerical values of the velocity, temperature and concentration, calculate the local and average skin-friction, the rate of heat transfer and mass transfer. The local as well as the average skin-friction, Nusselt number and Sherwood number in terms of dimensionless quantities are given by

\[ \tau_x = -\left( \frac{\partial U}{\partial R} \right)_{R=1} \]  \hspace{1cm} (2.24)

\[ \bar{\tau} = -\int_0^L \left[ \left( \frac{\partial U}{\partial R} \right)_{R=1} \right] dX \]  \hspace{1cm} (2.25)

\[ \text{Nu}_x = -X \left( \frac{\partial T}{\partial R} \right)_{R=1} \]  \hspace{1cm} (2.26)

\[ \bar{\text{Nu}} = -\int_0^L \left[ \left( \frac{\partial T}{\partial R} \right)_{R=1} \right] dX \]  \hspace{1cm} (2.27)

\[ \text{Sh}_x = -X \left( \frac{\partial C}{\partial R} \right)_{R=1} \]  \hspace{1cm} (2.28)

\[ \bar{\text{Sh}} = -\int_0^L \left[ \left( \frac{\partial C}{\partial R} \right)_{R=1} \right] dX \]  \hspace{1cm} (2.29)

The derivatives involved in the equations from (2.24) to (2.29) are evaluated, using the five-point approximation formula, and the integrals are evaluated using the Newton cotes formula.
The steady state local skin friction values are shown in Figure 2.6, against the axial co-ordinate X. It is observed that the local skin friction decreases as X increases. The local shear stress increases when M increases. This behavior is consistent with the velocity profiles shown in Figure 2.3. The shear stress increases as Pr increases, since the velocity gradient is more for fluids with a smaller Pr (= 0.71 such as air) than for fluids with a larger Pr (=7.0 such as water).

![Figure 2.6 Local skin-friction](image-url)
Figure 2.7 shows the dimensionless steady state local Nusselt number for different values of the magnetic parameter and Prandtl number. It is noted that the Nusselt number increases with decreasing M. The local heat transfer rate increases with an increasing value of Pr.

![Figure 2.7 Local Nusselt number](image-url)
The dimensionless steady state local Sherwood number is shown in Figure 2.8 for different values of the magnetic parameter and Prandtl number. It is observed that the Sherwood number increases with a decreasing M. It is also observed that the local mass transfer rate increases with a decreasing value of Pr.

Figure 2.8  Local Sherwood number
The average shear stress, the rate of heat transfer and the rate of mass transfer are shown in Figures 2.9, 2.10 and 2.11 respectively, as functions of time at X=1.0 for different values of the magnetic parameter and Prandtl number. It is observed that the rate of shear stress decreases with time and attains the steady state after a lapse of time. The effects of M and Pr on the average shear stress are similar to their effects on the local skin friction.

![Figure 2.9 Average skin-friction](image-url)
Initially, higher values of the average Nusselt number and Sherwood number are observed, and they decrease with time. It can be seen that there is no change in the average Nusselt number and Sherwood number in the initial period with respect to $M$ and $Pr$. This shows that the initial heat and mass transfer is due to conduction only. In Figure 2.10, it is observed that the average Nusselt number increases as $Pr$ increases. It is also observed that both the average Nusselt number and Sherwood number increase as $M$ decreases.
Figure 2.11 Average Sherwood number

Gr = 5.0, Gc = 5.0, Sc = 0.6

Pr

--- 0.71

--- 7.0
2.6 CONCLUSION

A numerical analysis is performed to study the effect of the magnetic field on the flow due to an impulsively moving semi-infinite vertical cylinder with heat and mass transfer. The dimensionless governing equations are solved by the finite difference scheme of the Crank-Nicolson type. The stability of the finite difference scheme is analyzed using the Von-Neumann technique. The effect of the Prandtl number, Grashof number and magnetic parameters are studied. The conclusions of the study are as follows:

i) The study shows that the number of time steps to reach the steady state depends strongly on the magnetic field parameter.

ii) An increase in the magnetic field leads to a decrease in the velocity field. In the absence of a magnetic field, the velocity overshoots near the cylinder.

iii) The thermal boundary layer thickness increases as \( M \) increases. An increase in the magnetic field leads to a rise in the temperature and concentration distribution.

iv) The local and average skin-friction and the Nusselt number increase as \( M \) increases, and the local and average Sherwood number increase with decreasing \( M \).