CHAPTER 6

INTERVAL INTEGER TRANSPORTATION PROBLEMS

6.1 INTRODUCTION

In this chapter a new method is proposed namely, separation method to find an optimal solution for fully interval integer transportation problems [FIITP] where transportation cost, supply and demand are intervals. Separation method was developed without using the midpoint and width of the interval in the objective function of the interval transportation problem which is a non-fuzzy method. The proposed method is based on zero suffix method Navaneetha kumar et al(2010). Various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter, and the penalty factors.

In real life problems, these conditions may not be satisfied always. To deal with inexact coefficients in transportation problems, many researchers Chanas et al (1996), Chinneck et al (200), Ishibuchi (1990), Moore (1979),Oliveira et al (2007) and Tong (1994) have proposed fuzzy and interval programming techniques for solving them. Das et al (1999) proposed a method, called fuzzy technique to solve interval transportation problem by considering the right bound and the midpoint of the interval. Sengupta et al(2003) proposed a new fuzzy orientated method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function.
6.2 FULLY INTERVAL INTEGER TRANSPORTATION PROBLEMS (FIITP)

Consider the following FIITP:

Minimize \[ z_1, z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}] \] 

(6.1)

Subject to \[ \sum_{j=1}^{n} [x_{ij}, y_{ij}] = [a_i, p_i], i=1,2,\ldots,m \] 

(6.2)

\[ \sum_{i=1}^{m} [x_{ij}, y_{ij}] = [b_j, q_j], j=1,2,\ldots,n \] 

(6.3)

\[ x_{ij} \geq 0, y_{ij} \geq 0, \ i=1,2,3,\ldots,m \text{ and } j=1,2,3,\ldots,n \] were integers

where \( c_{ij} \) and \( d_{ij} \) were positive real numbers for all \( i \) and \( j \), \( a_i \) and \( p_i \) were positive real numbers for all \( i \), \( b_j \) and \( q_j \) were positive real numbers for all \( j \).

**Theorem (6.2.1):** If the set \( \{ y_{ij} \} \text{ for all } i \text{ and } j \) is an optimal solution of the upper bound transportation problem (UBITP) of (FIITP) where,

(UBITP) Minimize \[ z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} \] 

(6.4)

Subject to

\[ \sum_{j=1}^{n} y_{ij} = p_i, i=1,2,\ldots,m \] 

(6.5)

\[ \sum_{i=1}^{m} y_{ij} = q_j, j=1,2,\ldots,n \] 

(6.6)

\( y_{ij} \geq 0, \ i=1,2,3,\ldots,m \text{ and } j=1,2,3,\ldots,n \) are integers
and the set \( \{ x_{ij}^* \text{ for all } i \text{ and } j \} \) is an optimal solution of the lower bound transportation problem (LBITP) of (FIITP) where,

\[
(\text{UBITP}) \text{ Minimize } z_l = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{6.7}
\]

Subject to

\[
\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m \tag{6.8}
\]

\[
\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n \tag{6.9}
\]

\( x_{ij} \geq 0 \quad i = 1, 2, 3, \ldots, m \text{ and } j = 1, 2, \ldots, n \) are integers,

then the set \( \{ [x_{ij}^*, y_{ij}^*] \text{ for all } i \text{ and } j \} \) is an optimal solution of the problem (FIITP) provided \( x_{ij}^* \leq y_{ij}^* \), for all \( i \text{ and } j \).

**Proof:** Let \( \{ [x_{ij}, y_{ij}] \text{ for all } i \text{ and } j \} \) be a feasible solution of the problem (FIITP). Therefore, \( \{ x_{ij}^* \text{ for all } i \text{ and } j \} \) and \( \{ y_{ij}^* \text{ for all } i \text{ and } j \} \) are feasible solution of the problems (UBITP) and (LBITP).

Now, \( \{ x_{ij}^* \text{ for all } i \text{ and } j \} \) and \( \{ y_{ij}^* \text{ for all } i \text{ and } j \} \) are feasible solution of the problems (UBITP) and (LBITP) as

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij}^* \leq \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij}, \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} y_{ij}^* \leq \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{6.10}
\]

and \( x_{ij}^* \leq y_{ij}^* \), for all \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

This implies that

\[
\left[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} \right] \leq \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \right] \leq \left[ \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} \right]
\]

That is,

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}^*, y_{ij}^*] \leq \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}],
\]
Now, since \{ x_{ij}^\ast \} for all \( i \) and \( j \) and \{ y_{ij}^\ast \} for all \( i \) and \( j \) satisfy (4) to (9) and \( x_{ij} \leq y_{ij}^\ast \), for all \( i \) and \( j \), it is concluded that, the set \{ [x_{ij}^\ast, y_{ij}^\ast] \} for all \( i \) and \( j \) is a feasible solution of the problem (FIITP). Thus, the set of intervals \{ [x_{ij}^\ast, y_{ij}^\ast] \} for all \( i \) and \( j \) is an optimal solution of the problem (FIITP). Hence the theorem is proved.

6.3 SEPARATION METHOD

The separation method proceeds as follows.

Step 1: Construct the UBITP of the given FIITP.

Step 2: Solve the UBITP by using the zero suffix method to get optimal solution. Let \{ y_{ij}^\ast \} for all \( i \) and \( j \) be an optimal solution of UBITP.

Step 3: Construct the LBITP of the given FIITP.

Step 4: Solve the LBITP with the upper bound constraints \( x_{ij} \leq y_{ij}^\ast \), for all \( i \) and \( j \) by using the zero suffix method. Let \{ x_{ij}^\ast \} for all \( i \) and \( j \) be an optimal solution of LBITP with \( x_{ij} \leq y_{ij}^\ast \), for all \( i \) and \( j \).

Step 5: The optimal solution of the given FIITP is \{ [x_{ij}^\ast, y_{ij}^\ast] \} for all \( i \) and \( j \) (by the Theorem (6.2.1))

6.4 NUMERICAL EXAMPLE

Consider the following FIITP

<table>
<thead>
<tr>
<th></th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1,2]</td>
</tr>
<tr>
<td></td>
<td>[1,2]</td>
</tr>
<tr>
<td></td>
<td>[7,9]</td>
</tr>
<tr>
<td>Demand</td>
<td>[10,12]</td>
</tr>
</tbody>
</table>
UBITP of the given problem is given below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Demand</td>
<td>12</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td></td>
<td>48</td>
</tr>
</tbody>
</table>

By applying the zero suffix method, the optimal solution to the UBITP is

\[ y_{11}^o = 5, y_{12}^o = 4, y_{21}^o = 7, y_{24}^o = 14, y_{33}^o = 15 \text{ and } y_{34}^o = 3 \]

The LBITP of the given problem with the upper bounded constraints are given below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Demand</td>
<td>10</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>

and also, \( x_{ij} \leq y_{ij}^o \), \( i=1,2,3,\ldots,m \) and \( j=1,2,3,\ldots,n \) are integers. Through the zero suffix method, the optimal solution to the LBITP with the upper bounded constraints are

\[ x_{11}^* = 5, x_{12}^* = 2, x_{21}^* = 5, x_{24}^* = 12, x_{33}^* = 13, x_{34}^* = 3 \]

Thus an optimal solution for the given FIITP is

\[ [x_{21}^*, y_{21}^*] = [5,7], [x_{24}^*, y_{24}^*] = [12,14], [x_{33}^*, y_{33}^*] = [13,15], [x_{34}^*, y_{34}^*] = [3,3] \] and also the minimum transportation cost is [102,202].
6.5 CONCLUSION

The separation method based on the zero suffix method provides an optimal value of an objective function for the fully interval transportation problem. This method is a systematic procedure, both easy to understand and to apply and also it is a non-fuzzy method. The proposed method provides more options and can be served as an important tool for the decision makers when they are handling various types of logistics problems having interval parameters.