CHAPTER 5
MULTIOBJECTIVE TWO STAGE FUZZY TRANSPORTATION PROBLEM

5.1 INTRODUCTION

In general the real life problems were modeled with multi-objective which are measured in different scales and at the same time in conflict. In some situations due to storage constraints destinations are unable to receive the quantity in excess of their minimum demand. After consuming the parts of whole quantity of this initial shipment they are prepared to receive the excess quantity in the second stage. According to Sonia et al(2003), in such situations the product transported to destination has two stages. Just enough of the product is shipped in stage 1 so that the minimum requirements of the destinations were satisfied and having done this, the surplus quantities (if any) at these sources were shipped to destinations according to cost consideration in stage 2. In both stages, the transportations of the product from sources to destinations were done in parallel.

Lot of efficient algorithms had been developed for solving the transportation problems when the cost coefficients, supply and demand quantities are known exactly. However, there are cases that these parameters may not be presented in an exact manner. The unit shipping cost may vary in a time frame accordingly. The supplies and demands may be uncertain due to some uncontrollable factors. To deal quantitatively with not exact information in making decisions, Bellman et al(1970) and Zadeh(1965) introduced the
notion of fuzziness. Since, the transportation problem is almost a linear program one straightforward idea is to apply the existing fuzzy linear programming techniques, the fuzzy transportation problems.

Unfortunately, most of the existing techniques provide only crisp solutions. The methods of Julien(1994) and Parra et al(1999) were able to find the possibility distribution of the objective values provided all the inequality constraints are of “≤” type or “≥” type. However due to the structure of the transportation problem in some cases, their method requires the refinement of the problem parameters to be able to derive in bouncing the objective value. There are also some studies discussing the fuzzy transportation problem. Chanas et al(1984) investigated the transportation problems with fuzzy supplies and demands. This was solved by them via the parametric programming techniques in terms of Bellman-Zadhe criterion.

Their method is to derive the solution which simultaneously satisfies the constraints and the goal to reach a maximal degree. Kuchta(1984) discussed the type of the transportation problems with fuzzy cost coefficients and transformed the problem to bicriterial transportation with crisp objective function. Their method was able to determine the efficient solutions of the transformed problem, unless only crisp solutions were provided. Verma et al(1997) applied the fuzzy programming technique with hyperbolic and exponential membership function to solve a multi-objective transportation problem and the solution derived was a compromise solution. Finally crisp solution was provided. Obviously when the cost coefficients, supply and demand quantities were fuzzy numbers, the total transportation cost will be fuzzy as usual.

This chapter identified the best compromise solution among the set of feasible solution for the multi-objective two stage transportation problem
using zero suffix method. To illustrate the proposed method, an example is used. Finally some conclusion is obtained from the discussion.

5.2 COMPROMISE SOLUTION

A feasible vector \( x^* \in S \), called a compromise solution of \( P_i \) if and only if \( x^* \in E \) and \( F(x^*) \leq \wedge_{x \in S} F(x) \) where \( \wedge \) stands for ‘minimum’ and \( E \) defines set of feasible solutions. If \( E \) is not necessary practically, a procedure is needed to determine a compromise solution. The purpose of this chapter is to present a fuzzy programming approach to find an optimal compromise solution of a transportation problem with several objectives in which the quantities are transported in two stages. A numerical example is given to illustrate the approach.

5.3 THEORETICAL DEVELOPMENT

Let \( \tilde{b}_j \) be the minimum fuzzy requirement of a homogeneous product at the destination \( j \) and \( \tilde{a}_i \) the fuzzy availability of the same at source \( i \). \( F^k(x) = \{ F^1(x), F^2(x), \ldots, F^n(x) \} \) is a vector of \( K \) objectives and the superscript of both \( F^k(x) \) and \( C^k_{ij} \) were used to identify the number of objective functions, without loss of generality it will be assumed in the whole chapter that \( a_i > 0 \ \forall i, b_j > 0 \ \forall j, C^k_{ij} > 0 \ \forall i, j \) and \( \sum_i \tilde{a}_i = \sum_j \tilde{b}_j \). The Multi-Objective Two stage Fuzzy Cost Minimization Transportation Problem (MOTSFTP) deals with supplying the destinations to their minimum requirement is state 1 and the quantity \( \sum_i \tilde{a}_i - \sum_j \tilde{b}_j \) is supplied to the destination in stage 2 from the sources which have surplus quantity left after the completion of stage 1, mathematically stated as,
\[ \min_{x \in \mathcal{S}_1} \left[ F^k(x) \right] = \min_{x \in \mathcal{S}_1} \left[ \max_{i \in [m]} x(C_i^k(x)) \right] \]

(5.1)

Where the set \( \mathcal{S}_1 \) is given by

\[
\mathcal{S}_1 = \left\{ \sum_{j=1}^{m} x_{ij} \leq \bar{a}_i, i = 1,2, \ldots, m \right\} \]

\[
\sum_{i=1}^{m} x_{ij} = \bar{b}_j, j = 1,2, \ldots, n \}
\]

(5.2)

\( x_{ij} \geq 0 \forall (i,j) \) Corresponding to a feasible solution \( X=(x_{ij}) \) of the stage 1 problem, the set \( \mathcal{S}_2 = \{ \bar{X}=(\bar{x}_{ij}) \} \) of feasible solutions of the stage 2 problem was given by

\[
\mathcal{S}_2 = \left\{ \sum_{j=1}^{m} x_{ij} \leq \overline{\bar{a}}_i, i = 1,2, \ldots, m \right\} \]

\[
\sum_{i=1}^{m} x_{ij} \geq \overline{\bar{b}}_j, j = 1,2, \ldots, n \}
\]

(5.3)

\( \bar{x}_{ij} \geq 0, \forall (i,j) \) where \( \bar{a}_i \) is the quantity available at the \( i^{th} \) source on completion so the stage 1, that is

\( \overline{\bar{a}}_j = \bar{a}_i - \sum_j x_{ij} \). Clearly \( \sum_i \overline{\bar{a}}'_i = \sum_i \bar{a}_i - \sum_j \bar{b}_j \). Thus, the stage 2 problem would be mathematically formulated as:

\[ \min_{x \in \mathcal{S}_2} \left[ F^k(x) \right] = \min_{x \in \mathcal{S}_2} \left[ \max_{i \in [m]} x(C_i^k(x)) \right] \]

(5.4)

It aims at finding the schedule \( x=(x_{ij}) \) of the stage 1 problem corresponding to which the optimal cost for stage 2 was such that the sum of
the shipment was the least. The Multi-objective two stage fuzzy cost minimizing transportation problem can be stated as,

\[ \text{Min} F^k(x) = \text{Min} \left[ C^k_1(x) + \text{Min}\{C^k_2(x)\} \right] \]  \hspace{1cm} (5.5)

Also from a feasible solution, of the problem can be obtained. Further the problem can be solved by following fuzzy cost minimizing transportation problem.

\[ \text{Min} \left[ F^k(x') \right] = \text{Min} \left[ \text{Min} F^k(x') = \text{Min} \left[ \text{Min} M_a(x (C^k_0(x'))) \right] \right] \]  \hspace{1cm} (5.4)

where \( S_2 \)

\[ s_2 = \left\{ \sum_{j=1}^{n} x'_j = \overline{a}_i, i = 1, 2, \ldots, m \right\} \]
\[ \left\{ \sum_{i=1}^{m} x'_j = \overline{b}_j, j = 1, 2, \ldots, n \right\} \]  \hspace{1cm} (5.5)

\( x'_j \geq 0 \forall (i, j), \) where \( \overline{a}_i \) and \( \overline{b}_j \) represent fuzzy parameters involved in the constraints with their membership functions for \( \mu_\alpha \) a certain degree \( \alpha \) together with the concept of \( \alpha \) level set of the fuzzy numbers \( \overline{a}_i, \overline{b}_j \). Therefore MOFCMTP can be understood as following non fuzzy \( \alpha \)-general two stage transportation problem (\( \alpha \)-two stage MOFCMTP).

\[ s = \left\{ \sum_{i=1}^{n} x'_j = \overline{a}_i, i = 1, 2, \ldots, m \right\} \]
\[ \left\{ \sum_{i=1}^{m} x'_j = \overline{b}_j, j = 1, 2, \ldots, n \right\} \]  \hspace{1cm} (5.6)

\( a_i, b_j \in L\alpha(\overline{a}_i, \overline{b}_j) \)
Where \( a_i, b_j \in L_{\alpha}(\overline{a}_i, \overline{b}_j) \) were the \( \alpha \)-level set of the fuzzy number \( \overline{a}_i, \overline{b}_j \), let \( x(\overline{a}_i, \overline{b}_j) \) denotes the constraint set of the problem and supposed to be non empty. On the basis of the \( \alpha \)-level set of the fuzzy numbers gives the concept of \( \alpha \)-optimal solution in the following definition.

A point \( x^* \in x(\overline{a}_i, \overline{b}_j) \) is said to be \( \alpha \)-optimal solution (\( \alpha \)-two stage FCMTP), if and only if there does not exist another \( x, y \in x(a, b), a, b \in L_{\alpha}(\overline{a}_i, \overline{b}_j) \), such that \( c_j x_j \leq c_j x_j^* \) with strict inequality holding for atleast one \( c_j \) where the corresponding values of parameters \( (\overline{a}_i, \overline{b}_j) \) are called \( \alpha \)-level optimal parameters.

The problem (\( \alpha \)-two stage MOFCMTP) can be rewritten in the following equivalent form (\( \alpha' \)-two stage MOFCMTP)

\[
\begin{align*}
\min \left\{ \sum_{i=1}^{n} x_{ij} = \overline{a}_i, i = 1,2,\ldots,m \right\} \\
\sum_{i=1}^{m} x_{ij} = \overline{b}_j, j = 1,2,\ldots,n \right\}
\end{align*}
\]  
(5.7)

\[
h^i \leq a_i \leq H^i, h^j \leq b_j \leq H^j
\]

\( x_{ij} \geq 0 \forall i, j \)

It should be noted that the constraint \( a_i, b_j \in L_{\alpha}(\overline{a}_i, \overline{b}_j) \) has been replaced by the constraint \( h^i \leq a_i \leq H^i \) and \( h^j \leq b_j \leq H^j \), where \( h^i, H^i \) and \( h^j, H^j \), \( H^j \) are lower and upper bounds and \( a_i, b_j \) are constants.

The parametric study of the problem (\( \alpha' \)-two stage MOFCMTP) where \( h^i, H^i \) and \( h^j, H^j \) are assumed to be parameters rather than constants and ( renamed \( h_i, H_i \) and \( h_j, H_j \) ) can be understood as follows.
Let $X(h, H)$ denote the decision space of problem ($\alpha'$-two stage FCMTP) defined by

$$X(h, H) = (X_{ij}, a_{ij}, b_{ij}) \in \mathbb{R}^{n+1}
\begin{align*}
a_i - \sum_j x_{ij} &\geq 0, b_j - \sum_i x_{ij} \geq 0 \\
H_i - a_i &\geq 0, H_j - b_j \geq 0 \\
a_i - h_i &\geq 0, b_j - h_j \geq 0, x_{ij} \geq 0, i \in I, j \in J
\end{align*} \quad (5.8)$$

5.4 SOLUTION ALGORITHM

**Step 1** : Construct the Fuzzy Transportation problem

**Step 2** : Supplies and Demands are fuzzy numbers $(a_1, a_2, a_3, a_4)$ and $(b_1, b_2, b_3, b_4)$ in the formulation Problem (two stage MOFCMTP).

**Step 3** : Convert the problem $(\alpha$-two stage MOFCMTP) in the form of the $(\alpha'$-two stage MOFCMTP)

**Step 4** : Formulate the $(\alpha'$-two stage FCMP) in the parametric form.

**Step 5** : Apply the zero suffix method to get optimal value in stage 1 and Stage 2.

**Step 6** : The optimal value of the objective function of the problem is $\text{Min}(C_1 + C_2)$

5.5 NUMERICAL EXAMPLE

Consider the following two stage cost minimizing transportation problem. Here supplies and demands are trapezoidal fuzzy numbers.
Consider $\alpha$ level set to be $\alpha = 0.75$, then

i. $a_1 = 4, a_2 = 5, a_3 = 7, a_4 = 8$

$$1 - \left( \frac{a - 5}{4 - 5} \right)^2 = 0.75$$

$$1 - (a - 5)^2 = 0.75$$

$$1 - (a^2 + 25 - 10a) = 0.75$$

$$1 - a^2 - 25 + 10a = 0.75$$

$$a^2 - 10a + 24.75 = 0$$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{10 \pm \sqrt{100 - 4(24.75)}}{2}$$

$$a = \frac{10 \pm \sqrt{36}}{2}$$

$$a = \frac{10 \pm 6}{2}$$

$$a = 5.5 \text{ (or) } 4.5$$

$$1 - \left( \frac{a - 7}{8 - 7} \right)^2 = 0.75$$

$$1 - (a - 7)^2 = 0.75$$

$$a^2 - 14a + 48.75 = 0$$

$$a = \frac{14 \pm \sqrt{196 - 4(48.75)}}{2}$$

$$a = \frac{14 \pm \sqrt{16}}{2}$$

$$a = \frac{14 \pm 4}{2}$$

$$a = 7.5 \text{ (or) } 6.5$$

$\therefore 4.5 \leq a_1 \leq 7.5$
ii. $a_1 = 6, a_2 = 7, a_3 = 8, a_4 = 9$

$1 - \left( \frac{a - 7}{6 - 7} \right)^2 = 0.75 \Rightarrow 1 - a^2 - 49 + 14a = 0.75$

$a = 7.5$ (or) $6.5$

$1 - \left( \frac{a - 8}{9 - 8} \right)^2 = 0.75$

$a^2 - 16a + 63.75 = 0$

$a = \frac{16 \pm \sqrt{16^2 - 4(63.75)}}{2}$

$= \frac{16 \pm 1}{2}$

$= 8.5$ (or) $7.5$

$\therefore 6.5 \leq a_2 \leq 8.5$

iii. $a_1 = 5, a_2 = 6, a_3 = 7, a_4 = 8$

$1 - \left( \frac{a - 6}{5 - 6} \right)^2 = 0.75$

$a^2 - 12a + 35.75 = 0$

$a = \frac{12 \pm \sqrt{144 - 4(35.75)}}{2}$

$= \frac{12 \pm 1}{2}$

$= 6.5$ (or) $5.5$

$1 - \left( \frac{a - 7}{8 - 7} \right)^2 = 0.75$

$a^2 - 14a + 48.75 = 0$

$a = \frac{14 \pm \sqrt{14^2 - 4(48.75)}}{2}$

$= \frac{14 \pm 1}{2}$

$= 7.5$ (or) $6.5$

$\therefore 5.5 \leq a_3 \leq 7.5$
iv. \( a_1 = 4, a_2 = 6, a_3 = 8, a_4 = 9 \)

\[
1 - \left( \frac{a - 6}{5 - 6} \right)^2 = 0.75
\]

\[
a^2 - 12a + 35 = 0
\]

\[
a = \frac{12 \pm \sqrt{144 - 4(35)}}{2}
\]

\[
= \frac{12 \pm 2}{2}
\]

\[
= 7 \text{ (or) } 5
\]

\[
1 - \left( \frac{a - 8}{9 - 8} \right)^2 = 0.75
\]

\[
a^2 - 16a + 63.75 = 0
\]

\[
a = 8.5 \text{ (or) } 7.5
\]

\[
\therefore 5.0 \leq a_4 \leq 8.5
\]

v. \( b_1 = 1, b_2 = 2, b_3 = 4, b_4 = 5 \)

\[
1 - \left( \frac{b - 2}{1 - 2} \right)^2 = 0.75
\]

\[
b^2 - 4b + 3.75 = 0
\]

\[
b = \frac{4 \pm \sqrt{16 - 4(3.75)}}{2}
\]

\[
= \frac{4 \pm 1}{2}
\]

\[
= 2.5 \text{ (or) } 1.5
\]

\[
1 - \left( \frac{b - 4}{5 - 4} \right)^2 = 0.75
\]

\[
b^2 - 8b + 15.75 = 0
\]

\[
b = \frac{8 \pm \sqrt{64 - 4(15.75)}}{2}
\]

\[
= \frac{8 \pm 1}{2}
\]

\[
= 4.5 \text{ (or) } 3.5
\]

\[
\therefore 1.5 \leq b_4 \leq 4.5
\]
vi. $b_1 = 4, b_2 = 5, b_3 = 6, b_4 = 7$

\[
1 - \left( \frac{b - 5}{4 - 5} \right)^2 = 0.75
\]

\[
b^2 - 10b + 24.75 = 0
\]

\[
b = 5.5 \text{ (or) } 4.5
\]

\[
1 - \left( \frac{b - 6}{7 - 6} \right)^2 = 0.75
\]

Therefore $b = 6.5 \text{ (or) } 5.5$

\[
\therefore 4.5 \leq b_2 \leq 6.5
\]

vii. $b_1 = 3, b_2 = 4, b_3 = 5, b_4 = 7$

\[
1 - \left( \frac{b - 4}{3 - 4} \right)^2 = 0.75 \quad \text{Therefore } b = 4.5 \text{ (or) } 3.5
\]

\[
1 - \left( \frac{b - 5}{7 - 5} \right)^2 = 0.75 \Rightarrow b^2 - 10b + 24 = 0
\]

\[
b = \frac{10 \pm \sqrt{100 - 4(24)}}{2}
\]

\[
b = 6 \text{ (or) } 4
\]

\[
\therefore 3.5 \leq b_3 \leq 6.0
\]

viii. $b_1 = 4, b_2 = 5, b_3 = 6, b_4 = 7$

\[
1 - \left( \frac{b - 5}{4 - 5} \right)^2 = 0.75 \quad \text{Therefore } b = 5.5 \text{ (or) } 4.5
\]

\[
1 - \left( \frac{b - 6}{7 - 6} \right)^2 = 0.75 \Rightarrow b^2 - 12b + 35.75 = 0 \quad \text{Therefore }
\]

\[
b = 6.5 \text{ (or) } 5.5
\]

\[
\therefore 4.5 \leq b_3 \leq 6.5
\]

ix. $b_1 = 2, b_2 = 3, b_3 = 4, b_4 = 5$
\[ 1 - \left( \frac{b - 3}{2 - 3} \right)^2 = 0.75 \Rightarrow b^2 - 6b + 8.75 = 0 \]

\[ b = \frac{6 \pm \sqrt{36 - 4(8.75)}}{2} \]

\[ b = 3.5 \text{ (or) } 2.5 \]

\[ 1 - \left( \frac{b - 4}{5 - 4} \right)^2 = 0.75 \quad \text{Therefore } b = 4.5 \text{ (or) } 3.5 \]

\[ \therefore 2.5 \leq b_3 \leq 4.5 \]

x. \quad b_1 = 3, b_2 = 4, b_3 = 5, b_4 = 6

\[ 1 - \left( \frac{b - 4}{3 - 4} \right)^2 = 0.75 \quad \text{Therefore } b = 4.5 \text{ (or) } 3.5 \]

\[ 1 - \left( \frac{b - 5}{6 - 5} \right)^2 = 0.75 \quad \text{Therefore } b = 5.5 \text{ (or) } 4.5 \]

\[ \therefore 3.5 \leq b_6 \leq 5.5 \]

We get

\[ 4.5 \leq a_1 \leq 7.5, \quad 6.5 \leq a_2 \leq 8.5, \quad 5.5 \leq a_3 \leq 7.5, \quad 5.0 \leq a_4 \leq 8.5 \]

\[ 1.5 \leq b_1 \leq 4.5, \quad 4.5 \leq b_2 \leq 6.5, \quad 3.5 \leq b_3 \leq 6.0, \quad 4.5 \leq b_4 \leq 6.5 \]

\[ 2.5 \leq b_5 \leq 4.5, \quad 3.5 \leq b_6 \leq 5.5 \]

(5.10)

The \( \alpha \)-optimal parameters are

\[ a_1 = 6, a_2 = 8, a_3 = 7, a_4 = 7 \]

\[ b_1 = 3, b_2 = 5, b_3 = 5, b_4 = 6, b_5 = 4, b_6 = 5 \]

Penalties:

\[
C^1 = \begin{bmatrix}
2 & 3 & 5 & 11 & 4 & 2 \\
4 & 7 & 9 & 5 & 10 & 4 \\
12 & 25 & 9 & 6 & 26 & 12 \\
8 & 7 & 9 & 24 & 10 & 8
\end{bmatrix}
\]

and
C^2 =
\[
\begin{bmatrix}
1 & 2 & 7 & 7 & 4 & 2 \\
1 & 9 & 3 & 4 & 5 & 8 \\
8 & 9 & 4 & 6 & 6 & 2 \\
3 & 4 & 9 & 10 & 5 & 1
\end{bmatrix}
\]

**Stage 1**

The values assigned are,
\[a_1 = 3, a_2 = 4, a_3 = 3, a_4 = 3, b_1 = 1, b_2 = 2, b_3 = 3, b_4 = 3, b_5 = 2, b_6 = 2\]

With respect to \(C_1\), apply the zero suffix method which gives following allocation
\[X_{12} = 1, X_{15} = 2, X_{21} = 1, X_{22} = 1, X_{26} = 2, X_{34} = 3, X_{43} = 3\] and Minimum \(Z = 75\)

With respect to \(C_2\), apply the zero suffix method which gives following allocation
\[X_{12} = 2, X_{15} = 1, X_{21} = 1, X_{24} = 3, X_{33} = 3, X_{45} = 1, X_{46} = 2\] and Minimum \(Z = 40\)

**Stage 2**

Assign the values as,
\[a_1 = 3, a_2 = 4, a_3 = 4, a_4 = 4, b_1 = 2, b_2 = 3, b_3 = 2, b_4 = 3, b_5 = 2, b_6 = 3\]

With respect to \(C_1\), apply the zero suffix method which gives following allocation
\[X_{12} = 1, X_{15} = 2, X_{21} = 2, X_{22} = 2, X_{33} = 1, X_{34} = 3, X_{43} = 1, X_{46} = 3\] and Minimum \(Z = 83\)
With respect to $C^2$, apply the zero suffix method which gives following allocation

\[ X_{12} = 3, \ X_{21} = 2, \ X_{24} = 2, \ X_{33} = 2, \ X_{34} = 1, \ X_{35} = 1, \ X_{45} = 1, \ X_{46} = 3 \] and Minimum $Z = 44$

The optimal value of the objective function is obtained by combining stage 1 and stage 2, therefore Minimum $F^1(x) = 75 + 83 = 158$ and Minimum $F^2(x) = 40 + 44 = 88$.

### 5.7 CONCLUSION

Transportation models have wide applications in logistics and supply chain management for reducing the cost. Some previous studies have devised solution procedures for fuzzy transportation problems. In this chapter zero suffix method was used to determine the optimal compromise solution for a multi-objective two stage fuzzy transportation problem, in which supplies and demands were trapezoidal fuzzy numbers and fuzzy membership of the objective function is defined. In real world applications, the parameters in transportation problem may not be known precisely due to some uncontrollable factors. If the obtained results were crisp values, then it might have lost some helpful information. Since the objective value was expressed by membership function rather than by a crisp value, more information was provided for making effective decisions.