The results established in this chapter have been published as detailed below:

7.1 INTRODUCTION

Lambert’s play a very important role in investigation of the range of absorption of light with the thickness of medium. He stated that “when a beam of light is allowed to pass through a Transparent medium, the rate of decrease of intensity with the thickness of medium is directly proportional to the intensity of the light”.

Later on Beer applied Lambert’s concept to solution of different concentrations. From the Beer’s law it follows that if the absorption remains constant, then the concentration (c) should be related to the thickness of absorbing layer (t) in a reciprocal manner. Beer-Lambert’s law can be used for the determination of an unknown concentration by comparison with a solution of known concentration by using colorimeter or spectrophotometer.

This chapter is divided into two sections ‘A’ and ‘B’.

In the section A, we give an introduction to the Lambert’s law and Aleph-function.

In section B, we express the Lambert’s law in terms of Aleph-function.
SECTION A

7.2 MATHEMATICAL DERIVATION OF LAMBERT'S LAW

When light is incident upon a homogeneous medium, a part of the incident light is reflected, a part is absorbed by the medium and the remainder is allowed to transmit as such. If $I_0$ denote the incident light, $I_r$ is the intensity of reflected light, $I_a$ is the intensity of the absorbed light and $I_t$ is the transmitted light, then one can write

$$I_0 = I_a + I_t + I_r,$$  \hspace{1cm} \text{...(7.2.1)}

If a comparison cell is used, the value of $I_r$, which is very small (about 4 percent) can be eliminated for air-glass interfaces under the condition, equation (7.2.1) becomes as

$$I_0 = I_a + I_t.$$  \hspace{1cm} \text{...(7.2.2)}

Mathematically the Lambert’s law may be stated as follows

$$-\frac{dI}{dt} \propto I \quad \text{or} \quad -\frac{dI}{dt} = kI,$$  \hspace{1cm} \text{...(7.2.3)}

where $I$ denotes the intensity of incident light of wavelength $\lambda$, $t$ denote the thickness of the medium and $k$ denote the proportional factor. On integrating equation (7.2.3) and putting $I = I_0$ when $t = 0$, we get

$$\frac{I_0}{I_t} = kt \quad \text{or} \quad I_t = I_0 e^{-kt},$$  \hspace{1cm} \text{...(7.2.4)}
where \( I_0 \) denotes the intensity of the incident light, \( I_t \) denotes the intensity of the transmitted light and \( k \) is a constant which depends upon the wavelength and absorbing medium used.

On changing equation (7.2.4), from natural to common logarithms, we get

\[
I_t = I_0 \left( 10^{-0.4343kt} = I_0 10^{-Kt}, \right. 
\]

where

\[
K = \frac{k}{2.3026}, 
\]

where in equation (7.2.6), \( K \) is the absorption coefficient which is defined as

“It is the reciprocal of the thickness which is required to reduce the light to \( \frac{1}{10} \) of its intensity”.

The above definition follows from equation (7.2.5),

\[
\frac{I_t}{I_0} = 0.1 = 10^{-Kt} \quad \text{or} \quad Kt = 1 \quad \text{or} \quad K \propto \frac{1}{t}. 
\]

The ratio \( I_t/I_0 \) is termed as the transmittance.

Srivastava [7] determine the Lambert’s law in terms of Fox’s \( H \)-function. Further, Chaurasia [8] obtained the Lambert’s law in terms of \( \overline{H} \)-function. In this chapter we derive the Lambert’s law by using Aleph-function which is discussed in Chapter 1 by equation (1.2.1).
7.3 MAIN THEOREM

With $\phi_\ell$ and $\xi_\ell$ given by (1.2.5) and (1.2.6), let $I$ be the intensity of incident light of wavelength $\lambda$, $t$ be the thickness of the medium and

(i) $I > I_1$, $t > t_1$, $k$ is proportional constant

(ii) $\phi_\ell > 0$, $|\arg(x)| < \frac{\pi}{2} \phi_\ell$, $\ell = 1, \ldots, r'$

(iii) $\phi_\ell \geq 0$, $|\arg(x)| < \frac{\pi}{2} \phi_\ell$ and $R \{\xi_\ell\} + 1 < 0$.

Then

$$
\int \mathcal{R}^{m,n+1}_{p_1+1,q_1+1,\tau_1'} \left[ \left(1-L_{11}\right) \left(b_{ji}^{\prime} B_{ji}^{\prime}\right)_{1,n\prime} \left[\tau_{ji}^{\prime} \left(b_{ji}^{\prime} B_{ji}^{\prime}\right)\right]_{n+1,p_1'} \right] \left[\left(v_{ji}^{\prime} V_{ji}^{\prime}\right)_{1,m\prime} \left[\tau_{ji}^{\prime} \left(v_{ji}^{\prime} V_{ji}^{\prime}\right)\right]_{m+1,q_1'} \right] \ dI
$$

$$
= C \mathcal{R}^{m,n}_{p_1,q_1,\tau_1} \left[ b_{ji}^{\prime} B_{ji}^{\prime} \left[\tau_{ji}^{\prime} \left(b_{ji}^{\prime} B_{ji}^{\prime}\right)\right]_{n+1,p_1'} \right] \left[\left(v_{ji}^{\prime} V_{ji}^{\prime}\right)_{1,m\prime} \left[\tau_{ji}^{\prime} \left(v_{ji}^{\prime} V_{ji}^{\prime}\right)\right]_{m+1,q_1'} \right]
$$

$$
- k \mathcal{R}^{m+1,n'}_{p_1+1,q_1+1,\tau_1'} \left[ b_{ji}^{\prime} B_{ji}^{\prime} \left[\tau_{ji}^{\prime} \left(b_{ji}^{\prime} B_{ji}^{\prime}\right)\right]_{n+1,p_1'} \right] \left[\left(t_{ji}^{\prime} V_{ji}^{\prime}\right)_{1,m\prime} \left[\tau_{ji}^{\prime} \left(t_{ji}^{\prime} V_{ji}^{\prime}\right)\right]_{m+1,q_1'} \right]. \quad \ldots(7.3.1)
$$

**Proof.** To prove (7.3.1), let $I$ be the intensity of incident light of wavelength $\lambda$, $t$ be the thickness of the medium and $k$ denote the proportional factor, then the Lambert’s law mathematically may be stated as follows [[4], p.744-745].

$$
\frac{dI}{dt} = -kI \quad \ldots(7.3.2)
$$
or
\[
\frac{dI}{I} = -k \, dt
\]

On integrating, we have
\[
\int \frac{dI}{I} = -k \int dt + C \quad \text{...(7.3.3)}
\]
or
\[
\int \frac{\Gamma(I)}{\Gamma(I+1)} \, dI = -kt + C
\]
\[
\Rightarrow \int \frac{\Gamma(I)}{\Gamma(I+1)} \, dI = -k \frac{\Gamma(t+1)}{\Gamma(t)} + C, \quad \text{...(7.3.4)}
\]

where \(C\) is integral constant.

Replacing \(I = I - I_1\), \(t = t + t_1\) (as thickness of medium increases, intensity of light will decrease) in (7.3.4) and multiplying both sides by \(\Omega_{m,r,p,q}^{i,j,x} (s)x^{-s}\), further integrating with respect to \(s\) in the direction of contour from \(\gamma - \omega_{\infty}\) to \(\gamma + \omega_{\infty}\) and with the help of (1.2.1), we get (7.3.1).

### 7.4 SPECIAL CASES

(i) If we take \(\tau_1 = \tau_2 = \ldots = \tau_r = 1\) in (7.3.1), then the Aleph-function reduces to an I-function [9] and we get Lambert’s law in terms of I-function.

(ii) If we set \(\tau_1 = \tau_2 = \ldots = \tau_r = 1\) and \(r' = 1\) in (7.3.1), then the Aleph-function reduces to Fox’s H-function and we get known results obtained by Srivastava [7].
REFERENCES

Chaurasia, V.B.L.


Chhatwal, G.R.


Fox, C.


Mathai, A.M. and Saxena, R.K.


Mathai, A.M., Saxena, R.K. and Haubold, H.J.


Saxena, V.P.


Srivastava, R.

Südland, N., Baumann, B. and Nonnenmacher, T.F.
