2. LITERATURE SURVEY

After the world War second, there was a very rapid development in the study of thermodynamics, stimulated by various engineering sciences. Thermoelasticity contains the generalized theory of heat conductions, the generalized theory of thermal stresses. A considerable progress in the field of air-craft and machine structures, mainly with gas and steam turbines and the emergence of new topics in chemical engineering have given rise to numerous problems in which thermal stresses play an important role and frequently even a primary role.

The first paper on Thermoelasticity by Duhamel [1] was read before the French academy of sciences in Paris in 1835 and published in the Journal de l’Ecole Polytechnique in 1837. In this paper, Duhamel formulated some boundary value problems and also derived the equations for the coupling of temperature and deformation fields.

Cinelli [4] have introduced the new finite Hankel transforms and the corresponding infinite series, which bring the solution of Bessel’s equation with asymmetric endpoint conditions within the realm of integral transform theory. A general solution for the transient temperatures produced in a finite, hollow cylinder by an asymmetrical internal heat source, when radiation takes place on all four surfaces, is accomplished. It is shown how this solution contains Carslaw and Jaeger’s results on hollow cylinders as a special case.

Bahar and Hetnarski [6] have obtained the Laplace transforms with respect to time of the governing equations of one-dimensional thermoelasticity, with the displacement and temperature fields coupled. Elimination of the Laplace transform of either field variable between the resulting linear, simultaneous, and coupled ordinary differential equations results in ordinary differential equations in the other field variable. The characteristic equations are identical for both variables. Depending on the sequence in which the elimination is carried out, the solutions are obtained in two different forms. The equivalence of these forms is established by using the properties of the characteristic roots. This process provides an alternative to the state approach recently developed by the
authors. Applications to problems pertaining to a half-space and a layer of finite thickness are presented.

Wu and Tauchert [7] analyzed the thermally induced deformations and stress resultants in symmetric laminated plates. The method of M. Levy is used to study the transverse bending of an especially orthotropic laminate having two simply supported edges and subjected to temperature distribution that does not vary in a direction parallel to the simple supports. A solution is also obtained for a problem of in-plane stretching of the plate middle surface caused by a general three dimensional temperature fields. As an illustrative example, the thermoelastic response of a unidirectional fiber-reinforced plate to a temperature variation that is linear in the thickness direction is computed.

Reddy and Hsu [8] presented a finite element formulation of equations governing layered anisotropic composite plated subjected to thermal and mechanical loading. An exact closed-form solution is also presented for simply supported rectangular cross-ply laminated plates under sinusoidal loading to validate the finite element developed. The finite element results are in good agreement with closed form results and with the results of others. Material properties typical of advanced fiber-reinforced composites are used to show the parametric effects of the plate aspect ratio, side-to-thickness ratio, orientation of layers and edge conditions on the deflections and stresses.

Tauchert [9] have discussed and investigated the stationary two-dimensional temperature, stress, and displacement distributions are investigated for simply supported slab consisting of bonded orthotropic layers. The top and bottom faces of the slab are subjected to arbitrary variations of temperatures, heat flux, or convection and arbitrary distributions of tractions. Displacement potentials are used to obtain the exact thermoelastic solution for the temperature-induced stresses and middle surface displacements is presented. As an illustrative example, the thermoelastic response to a sinusoidal distribution of temperature rise over the top face of the slab is computed; numerical results are given for laminates of various geometric configurations.

Wu and Tauchert [10] have discussed and investigated the thermal deformations and stress resultants in rectangular, antisymmetric cross-ply and angle-ply laminates are
investigated. Exact solution is obtained for the response of simply supported plates to generate three-dimensional temperature variations. To illustrate the thermoelastic behavior, deflections are computed for fiber-reinforced laminates undergoing constant and linearly varying temperature fields. Thermal membrane-bending coupling is found to be significant as the number of layers within a plate of a given thickness increase.

Hata [11] have concerned with a method for calculating the thermal-stress distribution in a nonhomogeneous medium whose shear modulus and coefficient of thermal expansion are assumed to be functions of $z$, the solution of the problem is determined by using displacement functions. A solution is then derived for the thermal-stress distribution in a nonhomogeneous, thick elastic plate under steady distribution of the surface temperature.

Rogers and Spencer [12] have extended the plane stress theory of Michell for a moderately thick homogeneous elastic plate and that of Kaprielian for a laminated plate to include stretching and bending solutions for an inhomogeneous thermoelastic plate. The inhomogeneities both in the elastic properties and thermal expansion coefficients, can vary arbitrarily through the thickness of the plate though for simplicity the analysis is restricted to plates with geometric and material properties symmetric with respect to the mid-plane. The deformation is produced by a temperature field which can also vary arbitrarily through the thickness. The solutions are expressed in terms of the solution of the approximate two-dimensional. Thin plate equations governing an “equivalence” homogeneous plate and are exact solutions of the full equations of three-dimensional thermo-elasticity by considering laminated plates to be a special case of inhomogeneous plates. They derive an “exact” laminate theory for plates consisting of different, homogeneous and isotropic layers which are perfectly bonded to each neighboring layer.

Tanigawa et al. [13] have discussed thermal bending analysis of a laminated composite rectangular plate due to a partially distributed heat supply. For the theoretical development, they have introduced the methods of finite cosine transformation and Laplace transformation to the temperature field and adapted the classical plate theory based on Kirchhoff-Love’s hypothesis to the thermoelastic field. Thereafter, they have applied the theoretical development proposed in the present article to the analysis of a nonhomogeneous rectangular plate of the thermal stress relaxation type such as
functionally gradient material. Then they have evaluated temperature change, thermal stress, and thermal deflection of simply supported plates and clamped ones. And they have examined the effect of relaxation on distributions of the thermal stress and thermal deflection for the nonhomogeneous rectangular plate.

Wan-Lee Yin [14] have used a variational method involving stress functions is used to determine the interlaminar stresses and the free-edge effects in a laminated beam under a temperature loading. The stress function in each layer is approximated by a cubic polynomial function of the thickness coordinate. The equilibrium equations, the traction boundary conditions, and the continuity conditions of the interlaminar stresses are exactly satisfied in his analysis, while the compatibility equations are interfacial continuity of the tangential strains are enforced in an average sense by applying the principle of the complementary virtual work.

Sherief and Hamza [15] have considered the two-dimensional problem of a thick plate whose lower and upper surfaces are traction free and subjected to a given axisymmetric temperature distribution within the context of the theory of generalized thermoelasticity with one relaxation time. Potential functions together with Laplace and Hankel transform techniques are used to derive the solution in the transformed domain. The Hankel transforms are inverted analytically. The inversion of the Laplace transforms is carried out using the inversion formula of the transform together with Fourier expansion techniques. Numerical methods are used to accelerate the convergence of the resulting series and to evaluate the improper integrals involved to obtain the temperature and stress distributions in the physical domain. Analysis of wave propagation in the medium is presented. Numerical results are represented graphically and discussed. A comparison is made with the solution of the corresponding coupled problem.

Ochiai et al. [16] have discussed the boundary element method (BEM), which does not require the domain integral in steady thermoelastic problems without heat generation; but in problems with heat generation, the domain integral is necessary in previous formulation. They also shows that the problems of steady thermoelasticity under nonuniform heat generation over the region can easily be solved without the region integral by means of the boundary element method in the authors' formulation. However,
for the case of general complicated heat generation the domain must be divided into small areas, where distributions of heat generation satisfy the Laplace equations.

Tungikar and Rao [17] have derived 3D exact solutions for the temperature and thermal stress distribution in simply supported rectangular orthotropic laminated cross-ply subjected to thermomechanical loads.

Ootao et al. [18] have developed a theoretical analysis of a three-dimensional transient thermal stress problem for a nonhomogeneous hollow circular cylinder due to a moving heat source in the axial direction from the inner and/or outer surfaces. Assuming that the hollow circular cylinder has nonhomogeneous thermal and mechanical material properties in the radial direction, the heat conduction problem and the associated thermoelastic behaviors for such nonhomogeneous medium are developed by introducing the theory of laminated composites as one of theoretical approximation. The transient heat conduction problem is treated with the help of the methods of Fourier cosine transformation and Laplace transformation, and the associated thermoelastic field is analyzed making use of the thermoelastic displacement potential, Michell's function, and there Boussinesq's function. Some numerical results for the temperature change and the stress distributions are shown in figures, and the effect of relating the thermal stress in the nonhomogeneous hollow circular cylinder and the influence of the velocity of a moving heat source are briefly discussed.

J and Fan He [19] have used discrete-layer shear deformation laminated plate theory to analyze steady-state thermal stresses in laminated plates. Both the in-plane displacements and the temperature rise are assumed to be piecewise linear across the thickness. By the assumption that the transverse shear strains across any two layers are linearly dependent on each other, the theory contains the same dependent variables as first-order shear deformation theory. The set of governing differential equations is of the twelfth order which does not depend on the number of layers. No shear correction factors are required. The thermal bending of simply supported, symmetric and antisymmetric cross-ply plates is calculated. Numerical results of the present theory for thermal stresses and deflections are compared with those obtained using the classical, first-order and higher-order shear deformation plate theories.
Savoia and Reddy [20] have presented the results of the stress analysis of multilayered plates subject to thermal and mechanical loads in the context of the three-dimensional quasi-static theory of thermoelasticity. The governing equations for a plate composed of monoclinic layers and subject to any given temperature and mechanical load distributions are derived in terms of displacements. Making use of a Navier-like approach, exact three-dimensional solutions are obtained for crossply and antisymmetric angle-ply laminated rectangular plates subject to thermomechanical loads. Polynomial and exponential temperature distributions through the thickness are considered. Numerical results for plates with the simply supported boundary conditions are presented. They showed that the inplane shear stresses at the corners are unbounded when plate faces are subject to uniform heating.

Bhaskar et al. [21] have presented the Solutions, within the framework of linear uncoupled thermoelasticity, for certain problems of flexure of composite laminates. Benchmark numerical results, useful for the validation or otherwise of approximate laminate models, are tabulated. Finally, these results are used to examine the accuracy of the classical lamination theory based on Kirchhoff’s hypothesis.

Tanigawa and Komatsubara [22] have examined a transient thermal stress problem of a rectangular plate due to a nonuniform heat supply is treated theoretically and, thereafter, fracture behaviors of the plate with a crack for compressive stress states. Assuming that a crack located on an arbitrary position, with an arbitrary direction, is sufficiently small and is closed because of the compressive stress field, a temperature field, in a transient state, is analyzed by taking into account the effect of relative heat transfer on both surfaces of the plate. Thereafter, the corresponding thermal stress analysis is developed on the basis of the two-dimensional plane stress problem using Airy's stress function method, and the stress intensity factor is analyzed for the biaxial stress state. As an analytical model; they consider mechanical boundary conditions of prescribed displacement and estimate the stress intensity factor of a crack tip using parameters of the crack configuration such as the location, direction, length, and coefficient of friction.

Hui-Shen Shen [23] has discussed the thermal post-buckling analysis for simply supported, composite laminated plate subjected to uniform or non-uniform temperature
loadings. The initial geometrical imperfection of the plate is taken into account. The formulations are based on Reddy’s higher order shear deformation plate theories, and include thermal effects. The analysis uses mixed Galerkin-perturbation techniques to determine thermal buckling loads and post buckling equilibrium paths. A numerical example covers performances of perfect and imperfect, antisymmetrically angle-ply and symmetrically cross-ply laminated plates. The effects played by transverse shear deformation, thermal load ratio, plate aspect ratio, total number of piles, fiber orientation and initial geometrical imperfections are studied. Hui-Shen Shen has discussed the typical result by presenting results in dimensionless graphical form.

Hui-Shen Shen [24] presented and discussed the thermal post-buckling analysis for shear-deformable rectangular plate subjected to uniform or non-uniform temperature loadings and resting on two-parameter elastic foundation. The initial geometrical imperfection of the plate is taken into account. The formulations are based on Raissner-Mindlin plate theory considering first order shear deformation effect and including thermal effects. The analysis uses deflection type perturbation techniques to determine thermal buckling loads and post buckling equilibrium paths. A numerical example covers performances of perfect and imperfect, shear deformable plates resting on Winkler or Pasternak-type elastic foundations.

Shang-sheng Wu [25] have discussed the investigated the transient thermal stresses in an annular fin with its base subjected to a heat flux of a decayed exponential function of time.

Locke [26] developed a numerical Fourier series solution for the classical thin laminated plate equations as applied to inhomogeneous antisymmetric cross-ply laminates. He has presented the results for laminates subjected to a combined thermal-mechanical load. Inhomogeneous constitutive terms are due to temperature-dependent material properties. Out-of-plane boundary conditions are simply supported and clamped; in-plane boundary conditions are stress free. The solution is exact in the sense that it satisfies both the governing equations and the boundary conditions.
Reddy and Chin [27] studied the dynamic thermoelastic response of functionally graded cylinders and plates are studied. Thermomechanical coupling is included in the formulation and a finite element model of the formulation is developed. The heat conduction and thermoelastic equations are solved for a functionally graded axisymmetric cylinder subjected to thermal loading. In addition a thermoelastic boundary value problem using the first order shear deformation plate theory (FSDT) that accounts for the transverse shear strains and the rotations, coupled with three-dimensional heat conduction equation, is formulated for a functionally graded plate. Both problems are studied by varying the volume fraction of ceramic and a metal using power law distribution.

Rolfes et al. [29] have discussed and presented a postprocessing procedure for the evaluation of the transverse thermal stresses in laminated plates. The analytical formulation is based on the first order shear deformation theory and the plate is discretized by using single-field displacement finite element model. The procedure is based on neglecting the derivatives of the in-plane forces and the twisting moments, as well as the mixed derivatives of the bending moments, with respect to the in-plane coordinates. The calculated transverse shear stiffness reflects the actual stacking sequence of the composite plate. The distributions of the transverse stresses through the thickness are evaluated by using only the transverse shear forces and the thermal effects resulting from the finite element analysis. The procedure is implemented into a post-processing routine which can be easily incorporated into existing commercial finite element codes. And also numerical results are presented for four and ten layer cross-ply laminates subjected to mechanical and thermal loads.

Noda [30] discussed the thermal stress problems of functionally graded material (FGM), as one of the advanced high-temperature materials capable of withstanding the extreme temperature environments. The functionally graded materials (FGMs) consist of the continuously changing composition of two different materials. For example one is an engineering ceramic to resist the fever thermal loading from the high-temperature environments, and other is light metal to maintain the structural rigidity. When the FGMs are subjected to extremely sever thermal loading, large thermal stresses are produced in
FGMs. Therefore the one of the most important problem of FGMs is how to decrease the thermal stresses and how to increase the heat resistance. The optimal composition profile problems of the FGMs in decreasing thermal stresses are discussed in detail by Noda. When FGMs are subjected to extremely sever loadings, the FGMs are damaged. The crack initiates on the ceramic surface and propagates in the FGMs. It is important to discuss the thermal stresses in the FGMs with various types of cracks. The thermal stress intensity factors in the FGMs with various types of crack are treated analytically and numerically. The optimal composition profile problems of the FGMs in decreasing thermal stress intensity factor are studied. Finally, the crack propagation paths due to thermal shock are discussed by Noda.

The non-Fourier heat conduction in a finite medium subjected to a periodic heat flux is modeled using the finite integral transform technique and an analytic solution is obtained by Hamid [31]. An analogy between thermal oscillation and oscillation of mechanical and electrical systems is drawn. A transition criterion from the non-Fourier heat conduction formulation to the Fourier formulation is obtained and a simple analytical expression of the phase and amplitude of thermal oscillation is derived by Bishri Abdel-Hamid.

Ali et al. [32] has presented a new displacement-based higher order theory. The theory employs realistic displacement variations and is shown to be extremely accurate for even thick laminates and for any combination of mechanical and thermal loading. The importance of various higher-order terms in the proposed theory is discussed with reference to a specific numerical example.

Cheng and Batra [33] have discussed and obtained a new solution in closed form is obtained for the thermomechanical deformations of an isotropic linear thermoelastic functionally graded elliptic plate rigidly clamped at the edges. The through-thickness variation of the volume fraction of the ceramic phase in a metal–ceramic plate is assumed to be given by a power-law type function. The effective material properties at a point are computed by the Mori–Tanaka scheme Cheng and Batra have found that the through-thickness distributions of the in-plane displacements and transverse shear stresses in a
functionally graded plate do not agree with those assumed in classical and shear deformation plate theories.

Dawe and Ge [34] have given a description of the spline strip method for predicting the critical buckling temperatures of rectangular composite laminated plates. The analysis takes place in two distinct parts, namely an in-plane thermal stress analysis in the pre-buckling stage followed by a buckling analysis using the determined in-plane stress distribution. The buckling analysis takes place in the context of first order shear deformation theory. The permitted lay-up of the laminates is quite general, within the constraint that the plate remains flat prior to buckling, and the boundary conditions are versatile. The distribution of temperature can be non-uniform in the plane of the plate. A range of applications of the developed procedures is presented and numerous comparisons are made with the results of previous studies. The spline finite strip method is shown to be versatile and accurate, with good convergence characteristics.

Carrera [35] compares theories formulated on the basis of the classical Principle of Virtual Displacements (PVD) to mixed theories formulated on the basis of the Reissner Mixed Variational Theorem (RMVT) to evaluate the thermal response of orthotropic laminated plates. So-called layer-wise (LW) and equivalent single-layer (ESL) modelings have been developed for both classical and mixed approaches (each layer is considered as a single plate for LW analysis, while the unknown variables are independent of the number of the constitutive layers for the ESL cases). Linear up-to fourth-order displacement and stress field cases have been implemented to derive thermomechanical governing equations, consistent with the used variational statements. All the theories have been presented in a unified manner by referring to findings recently presented by the author. The numerical investigation has been restricted to a simply supported plate loaded by harmonic distribution of in-plane temperature fields. Constant and linear through-the-thickness temperature distributions have been considered for which exact three-dimensional solutions are available. The results of in-plane and out-of-plane displacement and stress components are given in the form of tables and diagrams. The superiority of the mixed approach to the PVD formulation has been confirmed. Higher order, layer-wise expansions for the unknown variables are required for an
accurate description of the thermomechanical response of thick multilayered plates. It has been found by Carrera that the thickness temperature distributions $T(z)$ have a significant influence on the accuracy of the considered theories.

Makhecha et al. [36] have discussed and deal with the application of a new higher-order theory that accounts for the realistic variation of in-plane and transverse displacements through the thickness for the dynamic response analysis of thick multi-layered composite plates. The solutions are obtained employing the finite element procedure based on a $C^0$ eight-noded serendipity quadrilateral element. The importance of various higher-order terms in the present model is highlighted through the numerical study of mechanical and thermal loads. A detailed study is also carried out considering the influences of ply angle, aspect ratio, number of layers and thermal coefficients on the global response of thick laminates.

Javaheri and Eslami [37] states that the Equilibrium and stability equations of rectangular plate made of functionally graded material (FGM) under thermal loads are derived based on higher order shear deformation plate theory. Assuming that the material properties vary as the power form thickness coordinate variable $z$ and using the variationally method, the system of fundamental partial differential equation is established. The derived equilibrium and stability equations for functionally graded plates (FGPs) are identical to the equations for laminated composite plates. The buckling analysis of functionally grades plates under four types of thermal load is carried out and results in closed-form solutions. The critical buckling temperature relations are reduced to the respective relations for functionally graded plated with linear composition of constituent materials and homogeneous plates. The results are compared with the critical buckling temperatures obtained for functionally graded plates based on classical plate theory. The study concludes that higher order shear deformation theory accurately predicts the behavior of functionally graded plates, where as the classical plate theory overestimates buckling temperature.

Yang and Shen [38] has presented free and forced vibration analysis for initially stressed functionally graded plates in thermal environment. Material properties are assumed to be temperature dependent, and graded in thickness direction according to a simple power
law distribution in terms of the volume fractions of the constituents. Theoretical formulations are based on Reddy’s higher order shear deformation plate theory and include the thermal effects due to uniform temperature variation. The plate is assumed to be clamped on two opposite edges with the remaining two others either free, simply supported or clamped. One-dimensional differential quadrature technique, Gelerkin approach, and the model superposition method are used to determine the transient response of the plate subjected to lateral dynamic loads. Comprehensive numerical results for silicon nitride/stainless-steel rectangular plates are presented in dimensionless tabular and graphical forms. The roles played by the constituents volume fraction index, temperature rise, shape and duration of dynamic loads, initial membrane stresses as well as the character of boundary conditions are studied. The results reveal that, when thermal effects are considered, functionally graded plates with material properties intermediate to those of isotropic ones do not necessarily have intermediate natural frequencies and dynamic responses.

A stress-field in an annular fin of temperature-dependent conductivity under a periodic heat transfer boundary condition is analyzed by the Adomian’s decomposition method by Chiu and Chen [39]. The distribution of the transient thermal stress is obtained by direct integration of the temperature distribution. The heat transfer process is governed by the parameters of the convectonal in parameter N, the thermal conductivity parameter e, the frequency parameter B, and the amplitude parameter s. for e < 0, the mean temperature is decreased, which in turn increases the thermal stress and enhances the oscillation of the time wise thermal stresses. The opposite effects occur for e > 0. The maximum radial stress appears at R = 1:3, and the maximum tangential stress occurs at the inner base of the fins. Detailed results showing the effects of various parameters on temperature and thermal stresses are presented and discussed.

Mukhopadhyay [40] have discussed and studied the thermoelastic interactions on the basis of the generalized theory of thermoelasticity and with a comparison of the generalized theory of thermoelasticity with the coupled dynamical theory of thermoelasticity for the case of a transversely isotropic elastic medium with a cylindrical cavity when the surface of the cavity is subjected to a ramp-type increase in temperature
or load. The solution for the distributions of temperature and stresses are obtained with the help of the Laplace transform technique. Inversions of Laplace transforms are carried out analytically. Discontinuities of the field variables are analyzed.

Carrera [41] have conducted a study on the influence of the through-the-thickness temperature profile $T(z)$ on the thermomechanical response of multilayered anisotropic thick and thin plates. The heat conduction problem is solved, and the temperature variation $Tc(z)$ is then calculated. The governing thermomechanical equations of multilayered plates are written considering a large variety of classical and advanced or zigzag theories into account. The principle of virtual displacement and the Reissner mixed variational theorem are employed. Linear, up to fourth-order expansions in $z$ are retained for the assumed transverse stress and displacement fields. As a result, more than 20 plate theories are compared. The numerical investigation is restricted to orthotropic layered plates with harmonic in-plane distribution of both thermal loadings and unknown variables. Four sample plate problems are treated that are related to plates made of isotropic and/or orthotropic layers that are loaded by different top–bottom plate surface temperature conditions. Comparison is made to results related to a linear profile $Ta(z)$, which is usually assumed in open literature. The following is concluded: Thick plates could exhibit a layer wise form temperature profile $Tc(z)$. $Ta(z)$ case is approached for thin plate geometries. The use of linear temperature profile leads to large errors in tracing the response of thick plate geometries. The accuracy of plate theories is affected to great extent by the form of temperature variation $T(z)$. Refinements of classical plate theories can be meaningless unless the calculated $Tc(z)$ is introduced. The layerwise form of $Tc(z)$ would require layerwise assumptions for stresses and/or displacements. Plate theories that neglect transverse normal strains lead to very inaccurate results in both thick and thin plate analysis. Carrera concluded that at least a parabolic expansion for transverse displacement is required to capture transverse normal thermal strains that vary linearly along the plate thickness.

Vel and Batra [43] have discussed and presented an analytical solution is for three-dimensional thermomechanical deformations of a simply supported functionally graded (FG) rectangular plate subjected to time-dependent thermal loads on its top and/or bottom
surfaces. Material properties are taken to be analytical functions of the thickness coordinate. The uncoupled quasi-static linear thermoelasticity theory is adopted in which the change in temperature, if any, due to deformations is neglected. A temperature function that identically satisfies thermal boundary conditions at the edges and the Laplace transformation technique are used to reduce equations governing the transient heat conduction to an ordinary differential equation (ODE) in the thickness coordinate which is solved by the power series method. Next, the elasticity problem for the simply supported plate for each instantaneous temperature distribution is analyzed by using displacement functions that identically satisfy boundary conditions at the edges. The resulting coupled ODEs with variable coefficients are also solved by the power series method. The analytical solution is applicable to a plate of arbitrary thickness. Results are given for two-constituent metal-ceramic FG rectangular plates with a power-law through-the-thickness variation of the volume fraction of the constituents. The effective elastic moduli at a point are determined by either the Mori–Tanaka or the self-consistent scheme. The transient temperature, displacements, and thermal stresses at several critical locations are presented for plates subjected to either time-dependent temperature or heat flux prescribed on the top surface. Results are also given for various volume fractions of the two constituents, volume fraction profiles and the two homogenization schemes.

Mead [44] have given a detailed consideration to the mode and frequencies of a free rectangular Kirchoff plate subjected to in-plane stresses generated by prescribed non-uniform surface temperature distributions which are doubly symmetrical about the plate central axes. Physical understanding is sought of phenomena observed by previous investigators. Stress distribution corresponding to three different temperature distributions have first been studied an incorporated in Rayleigh Ritz analysis to find the natural frequencies and modes. All frequencies change as the temperature changes, some much more than others. All eventually vanish, one after the other, as the temperature reaches certain critical positive and negative values at which the plate goes into statically unstable buckling modes. Whether the frequencies rise or fall with rising temperature at the plate center depends on the relative magnitudes of pairs of positive and negative critical temperatures. The modes of buckling at each pair of critical temperatures may differ greatly from one another and also from the vibration modes at zero temperature.
The relations between the square of the frequency and the temperature is then no longer approximately linear, although it is exactly so for certain simple in-plane stress distributions. Conditions have nevertheless been identified under which it is a very good approximation to the actual frequencies of the heated plate over wide temperature ranges.

Carrera [45] have gives a historical review of the theories that have been developed for the analysis of multilayered structures. Attention has been restricted to the so-called Zig-Zag theories, which describe a piecewise continuous displacement field in the plate thickness direction and fulfill interlaminar continuity of transverse stresses at each layer interface. Basically, plate and shell geometries are addressed, even though beams are also considered in some cases. Models in which the number of displacement variables is kept independent of the number of constitutive layers are discussed to the greatest extent. Attention has been restricted to those plate and shell theories which are based on the so-called method of hypotheses or axiomatic approach in which assumptions are introduced for displacements and/or transverse stresses.

Sherief et al. [46] have considered the one-dimensional problem for an infinitely long hollow cylinder in the context of the theory of generalized thermoelasticity with one relaxation time. The Laplace transform technique is used to solve the problem. The solution in the transformed domain is obtained by a direct approach. The inverse transforms are obtained in an approximate analytical manner using asymptotic expansions valid for small values of time. The temperature, displacement, and stress are computed and represented graphically.

El-Maghraby [47] has solved a two-dimensional problem for a half-space. The problem is in the context of the theory of generalized thermoelasticity with one relaxation time. The surface of the half-space is taken to be traction free and the temperature on it is specified. Heat sources permeate the medium. Laplace and exponential Fourier transform techniques are used. The solution in the transformed domain is obtained by a direct approach. The inverse double transform is evaluated numerically.

Qian and Batra [48] have study transient thermoelastic deformations of a thick functionally graded plate with edges held at a uniform temperature and either simply
supported or clamped. Either the temperature or the heat flux is prescribed on the top surface of the plate with the bottom surface of the plate kept at either a uniform temperature or thermally insulated. Stresses and deformations induced due to the simultaneous application of the transient thermal and mechanical loads are also computed. The problem is solved by using a higher order shear and normal deformable plate theory and a meshless local Petrov–Galerkin method. Only nodal coordinates are needed, and neither nodal connectivity nor a background mesh is employed. The validity of the method and of the computer code is established by comparing computed results with the analytical solution of the threedimensional thermoelasticity equations for a simply supported plate. Results are then computed for clamped plates. It is found that the centroidal deflection and the axial stress induced at the centroid of the top surface of the plate are significantly influenced by boundary conditions at the plate edges.

Sharma et al. [49] have discussed and developed a mathematical model has been developed and solved for predicting the response of a thick thermoelastic axisymmetric solid plate subjected to sudden lateral mechanical and thermal loads. The governing equations of motion and heat conduction have been solved by using Laplace and Fourier transform methods to predict the response of the plate in the physical time domain. A modified Bessel function solution with complex argument is directly used. The model is also formulated and solved with the help of the finite element method (FEM). The results for radial and axial displacements and temperature change have been computed numerically and illustrated graphically for different theories of generalized thermoelasticity. The comparison of exact analysis with that of FEM is also discussed. Excellent agreement is found between the finite element analysis and analytical and classical solutions.

Ma and Chang [50] have presented a analytical exact solutions of a fundamental heat conduction problem in anisotropic multi-layered media. The steady-state temperature and heat flux fields in multi-layered media with anisotropic properties in each layer subjected to prescribed temperature on the surfaces are analyzed in detail. Investigations on anisotropic heat conduction problems are tedious due to the presence of many material constants and the complex form of the governing partial differential equation. It is
desirable to reduce the dependence on material constants in advance of the analysis of a given boundary value problem. One of the objectives of this study is to develop an effective analytical method to construct full-field solutions in anisotropic multi-layered media. A linear coordinate transformation is introduced to simplify the problem. The linear coordinate transformation reduces the anisotropic multi-layered heat conduction problem to equivalent isotropic ones without complicating the geometry and boundary conditions of the problem.

Kim and Kim [51] have analyzed a simply supported thin plate with temperature distribution varying over the thickness. They used the classical Von Karman equations for the geometrically nonlinear, large deflection behavior of the plate.

Sherief and Saleh [52] have considered the problem of a thermoelastic half-space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. The bounding surface of the half-space is taken to be traction free and subjected to a time dependent thermal shock. The chemical potential is also assumed to be a known function of time on the bounding plane. Laplace transform techniques are used. The solution is obtained in the Laplace transform domain by using a direct approach. The solution of the problem in the physical domain is obtained numerically using a numerical method for the inversion of the Laplace transform based on Fourier expansion techniques.

Hasebe and Wangthe [53] have considered the basic equations of complex variable functions in conjunction with a rational mapping function. The formulation of the rational mapping function and its accuracy are indicated. The general solutions for the external force, displacement, and mixed boundary value problems under uniform heat flux and under a point heat source are stated. The Green’s functions for a point heat source and sink are suggested for the three kinds of boundary value problems. The equation to obtain the stress intensity factor for the crack problems and the stress intensity of debonding for a partially debonded rigid inclusion are shown. The interaction problem between a hole and a line crack under uniform heat flux is analyzed.
Khobragade and Deshmukh [54, 55] have discussed and prescribed the inverse axially symmetric quasi-static problem of thermoelasticity for a thin clamped circular plate in which a heat flux is on an internal cylindrical surface of the plate and suitable heat exchange conditions are met on the upper and lower surfaces of the plate is solved with the help of a generalized integral transform technique.

Khobragade and Deshmukh [56] have developed an integral transform to determine temperature distribution in a thin circular plate, subjected to a partially distributed and axisymmetric heat supply on the curved surface, and study the thermal deformation.

El-Maghraby [57] has solved a two-dimensional problem for a thick plate. The upper surface of the plate is traction free and subjected to a known temperature distribution, while the lower surface is laid on a rigid foundation and thermally insulated. Heat sources permeate the medium. The problem is in the context of the theory of generalized thermoelasticity with one and two relaxation times. Laplace and exponential Fourier transform techniques are used. The solution in the transformed domain is obtained by a direct approach. The inverse double transform is evaluated numerically.

Ruhi et al. [58] have presented the semianalytical thermoelasticity solution for thick-walled finite-length cylinders made of functionally graded (FG) materials. The governing partial differential equations are reduced to ordinary differential equations using Fourier expansion series in the axial coordinate. The radial domain is divided into some virtual subdomains in which the power-law distribution is used for the thermomechanical properties of the constituent components. Imposing the necessary continuity conditions between adjacent subdomains, together with the global boundary conditions, a set of linear algebraic equations are obtained. Solution of the linear algebraic equations yields the thermoelastic responses for each subdomain as exponential functions of the radial coordinate. Some results for the stress, strain, and displacement components through the thickness and along the length are presented due to uniform internal pressure and thermal loading. Based on the results, the gradation of the constitutive components is a significant parameter in the Thermomechanical responses of FG cylinders.
Carrera [59] has presented a study of the transverse normal strain effect on the static thermoelastic response of homogeneous and multilayered plates. Numerical evaluations have been given for classical, refined, and advanced zig–zag plate theories. Constant, linear, and higher-order forms of temperature profile in the plate thickness direction have been accounted for. Basic assumptions of the considered theories are quoted. The related governing equations are not given because these were derived from a unified formulation that was described elsewhere. Closed-form solutions are discussed by addressing three plate problems: a homogeneous plate made of an isotropic layer; a two-layered plate consisting of two layers made of different isotropic materials; a multilayered composite plate made of three cross-ply layers. It has been confirmed that any refinements of classical models are generally meaningless, unless the effects of transverse shear and normal strains are both taken into account in a plate theory. Furthermore, it has been found that transverse normal strains cannot be discarded even though thin plates are considered and the accurate description of the temperature profile in the plate thickness direction could result to be meaningless, unless transverse normal strains are taken into account, and vice versa.

Shidfar and Zakeri [60] have considered an inverse heat conduction problem in one-dimensional space with two unknown terms. In this problem, the initial temperature and histories of temperature in the finite interval of time may be determined. Then, by using a semi-implicit finite difference method, an estimation of these unknown terms will be found. They concluded that by applying a suitable parameter, we can obtain the best of these estimations.

Tikhe and Deshmukh [61] have discussed and solved an inverse problem of heat conduction in a thin circular plate with the given temperature distribution on the interior surface of a circular plate being a function of both time and position with the help of integral transform technique and also determine the thermal deflection on the outer curved surface of a thin circular plate as \(0 \leq r \leq a, \ 0 \leq z \leq h\).

Khobragade and Deshmukh [62] have developed an integral transform to determine temperature distribution in a thin circular plate, subjected to partially distributed and axisymmetric heat supply on the curved surface, and study the thermal deformation.
Robaldo [63] presented various finite plate elements for the thermal analysis of multilayered anisotropic structures, plates for various thickness ratio values. Robaldo investigated the influence of the temperature description through the thickness of laminated plates. Assumptions made for the displacement fields in thickness direction and the principle of virtual displacements (PVD) is employed to derive finite element matrices. A unified formulation is employed so the FE matrices have been derived in terms of few fundamental nuclei which form is not affected by: the number of the nodes of the elements and the order of the expansion of the assumptions made for the displacement-variable. Two different profiles of temperature have been compared on cross-ply laminates: (1) the linear one; (2) the actual profile obtained by solving heat conduction problem. Comparison to closed form solutions as well as to available 3D exact analysis have shown the effectiveness of the proposed elements and their capability to trace quasi 3D description of thermal stresses in layered plates.

Shairiat and Eslami [64] have presented the thermal buckling analysis of rectangular functionally graded plates (FGPs) with geometrical imperfections. The equilibrium, stability, and compatibility equations of imperfect functionally graded plate are derived using the classical plate theory. It is assumed that the nonhomogeneous mechanical properties of the plate, graded through thickness, are described by a power function of the thickness variable. The plate is assumed to be fewer than three types of thermal loading as uniform temperature rise, nonlinear temperature through the thickness, and axial temperature rise. Resulting equations are employed to obtain the closed-form solutions for the critical buckling temperature and imperfect FGP. The results are reduced and compared with the results of perfect functionally graded and imperfect isotropic plates.

Ohmichi and Noda [65] have discussed and presents plane thermal stresses in a functionally graded plate (FGPs) subjected to a partial heating. The heat conductivity, Young’s modulus and the coefficient of the linear thermal expansion are expressed by exponential functions of the position. The analytical solution for the FGP with two-dimensional temperature distribution is obtained by use of the stress function method. General solution of the governing equation of the stress function is derived in the functionally graded materials (FGMs). The numerical calculations are carried out for
ZrO2/Ti-6 Al-4V and ZrO2/ stainless (SUS304) functionally graded plates, when the ceramic surface is partially heated. Even though the FGM, suitable selection of the compositional materials does not produce thermal stresses in the FGP.

Naji et al. [67] have applied the hyperbolic energy model for determining transient temperature variation across a unidirectional composite plat subjected to different temperature changes at the boundaries. Thermal stresses developed are then analyzes for different material properties. Governing equations are solved numerically using implicit methods. The results are presented over a wide range of variables commonly found in most composite materials. The transient thermal stresses generated inside the plate ware found to fluctuate between compressive and tensile quantities, a result that was not predicted using the classical heat model. Consequently, this will lead to an earlier crack initiation and failure of the material.

Chattopadhyay and Biswas [68] have been developed generalized thermoelasticity theories to eliminate the paradox of infinite velocity of thermal wave propagation and to correct the classical coupled thermoelasticity theory on the assumption that thermal wave propagates with a finite speed. Lord and Shulman, as well as Green and Lindsay, formulated independently generalized thermoelasticity theories by introducing one or two relaxation times in the process with a view to eliminate the paradox of infinite speed. In the last decade, relevant theoretical developments on this subject are due to Green and Naghdi, which provide sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems. The present paper deals with a boundary value problem of an isotropic elastic half-space using generalized thermoelasticity of type II developed by Green and Naghdi. The plane boundary of the material mediums either held rigidly fixed or considered as traction-free and subjected to ramp-type heating. Introducing potential function in the process and applying Laplace transform technique, the closed-form solutions for the displacement, temperature and stress fields are obtained. Numerical estimates are also obtained for a particular example and are represented graphically.

Kulkarni and Deshmukh [69] have discussed the determination of a quasi-static thermal stresses in a thick circular plate subjected to arbitrary initial temperature on the upper
face with lower face at zero temperature and the fixed circular edge thermally insulated. The results are obtained in series form in terms of Bessel’s functions and they are illustrated numerically.

Taniguchi [70] et al. have discussed and studied numerically the stationary heat conduction in a one-dimensional hard point gas with molecular dynamics simulations and theoretically on the basis of extended thermodynamics. Temperature profile with jumps at the boundaries is analyzed consistently. Heat conductivity seems to converge as the system size tends to infinity.

Robaldo and Carrera [71] have presented some finite elements for the thermoelastic analysis of multilayered plates which are based on unified formulation and the Reissner mixed variational theorem (RMVT). The pure mechanical statement of the theorem is herein extended to thermoelastic analysis and the related mixed constitutive equations have been derived. Assumptions are made for the displacement fields in thickness direction and the RMVT is employed to obtain finite element (FE) matrices. The unified formulation allows to easily deriving the FE matrices in terms of a few fundamental nuclei whose form is not affected by: number of the nodes of the elements; order of the expansion for the displacements-variable description (Layer-Wise, LW, and Equivalent-Single Layer, ESL, cases are both addressed). The Murakami Zig-Zag function is used to introduce Zig-Zag effects in the framework of ESL descriptions. Comparisons to exact 3D solutions as well as to classical finite elements have proved the accuracy of the proposed elements and their capability to accurately describe the stress field of layered plates subjected to thermal loadings.

Kulkarni and Deshmukh [72] have determined the transient thermal stresses in annular disk. A thick annular disk is considered having zero initial temperature and subjected to arbitrary heat flux on the upper and lower surface where as the fixed circular edges are at zero temperature. They solved the governing heat conduction equation by using integral transformation technique.

Brischetto et al. [73] have discussed and analyzed the deformation of simply supported, functionally graded, rectangular plate subjected to thermo mechanical loading, extending
unified formulation by Carrere. The governing equations are derived from the principle of virtual displacement accounting for the temperature as an external load only. The required temperature field is not assumed a priori but determined separately by solving Fourier equations. Numerical results for temperature, displacement and stress distribution are provided for different volume fraction of the metallic ceramic constituent as well as for different plate thickness ratios. They correlate very well with three-dimensional solutions given in the literature.

Kidawa-Kukla [74] has discussed the heat conduction problem in a circular thin plate subjected to the activity of a heat source which changes its place on the plate surface with time. The heat source moves along a circular trajectory around centre of the plate with constant angular velocity. The solution of the problem is obtained in an analytical form by using the Green’s function method.

Kidawa-Kukla [75] have discussed and presented a solution to the problem of heat conduction in a rectangular plate subjected to the activity of a moving heat source. The temperature of the plate changes because a limited area on the plate surface is heated by a heat source. The heat source moves along an elliptical trajectory which always remains within the plate area. An exact solution to the problem in an analytical form is obtained by applying the Green’s function method. Exemplary results of numerical calculations to determine the temperature distribution in the plate are presented.

A two-dimensional (2D) higher-order deformation theory is discussed and presented by Matsunaga [77] for the evaluation of displacements and stresses in functionally graded (FG) plates subjected to thermal and mechanical loadings. The modulus of elasticity of plates is assumed to vary according to a power law distribution in terms of the volume fractions of the constituents. By using the method of power series expansion of continuous displacement components, a set of fundamental governing equations which can take into account the effects of both transverse shear and normal stresses is derived through the principle of virtual work. Several sets of truncated Mth order approximate theories are applied to solve the static boundary value problems of a simply supported FG plate. In order to assure the accuracy of the present theory, convergence properties of the displacements and stress components are examined in detail. A comparison of the present
results of isotropic plates is also made with previously published results. The transverse stresses have been obtained by integrating the three-dimensional (3D) equations of equilibrium in the thickness direction starting from the top or bottom surface of a plate. The distributions of transverse shear and normal stresses in the thickness direction are obtained accurately by satisfying the surface boundary conditions of a plate. The present numerical results are also verified by satisfying the energy balance of external and internal works are considered to be sufficient with respect to the accuracy of solutions. Matsunaga concluded that the present 2D higher-order approximate theories can predict accurately the stresses and displacements of simply supported FG plates subjected to thermal and mechanical loadings.

Woo et al. [78] have presented an analytical solution for nonlinear free behavior of the plate made of functionally graded materials. The material properties of the functionally graded plates are assumed to vary continuously through the thickness, according to a power-law distribution of the volume fraction of the constituents. The fundamental equations for thin rectangular plates of functionally graded materials are obtained using von Karman theory for large transverse deflection, and the solution is obtained in terms of mixed Fourier series. The effect of material properties, boundary conditions and thermal loading on the dynamic behavior of the plates in determined and discussed. The results reveal that nonlinear coupling effects play a major role in dictating the fundamental frequency of functionally graded plates.

Deshmukh et al. [79, 80, 81, 82, and 83] have discussed the determination of the displacement and thermal stresses in a thin circular plate defined as $0 \leq r \leq b$, $0 \leq z \leq h$ under a steady temperature field, due to a constant rate of heat generation within it. A thin circular plate is insulated at the fixed circular boundary $(r = b)$, and the remaining boundary surfaces $(z = 0, z = h)$ are kept at zero temperature. The governing heat conduction equation has been solved by using an integral transform technique. The results are obtained in series form in terms of modified Bessel functions.

Carrera and Nail [84] have discussed and deals with advanced finite element (FE) formulations for the analysis of multilayered structures in the case of multi field problems. The following four fields are considered: mechanical, thermal, electrical and
magnetic. Constitutive equations, in terms of coupled mechanical – thermal – electrical – magnetic field variables, are obtained on the basis of a thermodynamic approach. The four-field principle of virtual displacements is employed to derive FE matrices. Three-fields, two-fields as well as pure mechanical problems have been discussed as relevant particular cases. A condensed notation, known as Carrera unified formulation, has been employed to establish a comprehensive two-dimensional modeling with variable kinematic features. Layer-wise / equivalent single layers plate elements have been developed according to linear up to fourth-order expansion in the layer/plate thickness directions. FE matrices have been obtained in terms of a few fundamental nuclei whose dimension is 6X6 for the full four fields case. Numerical results show the effectiveness of the proposed implementation by encompassing various static

Deshmukh et al. [85] have discussed the determination of the quasi-static thermal stressed in a semi-infinite thin rectangular plate define as $0 \leq x \leq \infty; \ 0 \leq y \leq b$ subjected to the heat generation within the solid at the rate of $g(x, y, t)$. The initial edges $x = 0$ and $y = 0$ are thermally insulated while the boundary $y = b$ is kept at temperature $\phi (x, t)$. The governing heat conduction equation has been solved by using integral transformation technique.

Raju and Kumar [86] have developed an analytical procedure to investigate the Thermal characteristics of laminated composite plates under thermal loading based on higher-order displacement model with zig-zag function, without enforcing zero transverse shear stresses on the top and bottom faces of the laminated plates. This function improves slope discontinuities at the interfaces of laminated composite plates. The related functions are obtained using the dynamic version of principle of virtual work or Hamilton’s principle. The solutions are obtained using Navier’s and numerical methods for anti-symmetric cross-ply and angle-ply laminates with a specific type of simply supported boundary conditions SS-1 and SS-2. The Numerical results are presented for anti-symmetric cross-ply and angle-ply laminated plates. All the solutions presented are close agreement with the theory of elasticity and available literature.

Liu and Zhong [87] have presented the three-dimensional thermoelastic analysis for an orthotropic functionally graded rectangular plate, which is simply supported and
isothermal on its four lateral edges. With the assumption that material properties have arbitrary dependence on the thickness-coordinate, a Peano-Baker series solution is obtained for the thermoelastic fields of the functionally graded plate subjected to mechanical and thermal loads on its upper and lower surfaces by means of state space method. The correctness of the obtained series solution is validated through numerical examples. Wuxiang Liu and Zheng Zhong have also studied the influence of different material properties distributions on the structural response of the plate.

Khdeir [88] investigated the thermal deformations on symmetrical and antisymmetric cross-ply arches. The state-space approach has been used to generate exact solutions for the thermoelastic response of cross-ply arches for arbitrary boundary conditions and subjected to general temperature field. A rigorous first-order arch theory is used in the analysis. Deflections are computed for arches with various lamination schemes and boundary conditions undergoing uniform and linearly varying temperature through the thickness. The effect shallowness of the arch on maximum deflection is studied.

Kiani et al. [89] studied the buckling analysis of functionally graded material (FGM) beams with surface-bonded piezoelectric layers which are subjected to both thermal loading and constant voltage is studied. The material homogeneous properties are assumed to vary smoothly by distribution of power law through the beam thickness. The Euler-Bernoulli beam theory and nonlinear strain-displacement relation are used to obtain the governing equations of piezoelectric FGM beam. Beam is assumed under three type of thermal loading and various types of boundary conditions. For each case of thermal loading and boundary conditions, closed-form solutions are obtained. The effect of the applied actuator voltage, beam geometry, boundary conditions, and power law index of functionally graded material on the buckling temperature are investigate.

Pi et al. [90] have discussed and investigates the interval thermoelastic analysis for the responses of elastically restrained steel beams under a linear temperature gradient. Previous studies of thermal responses of such beams have been based on the assumptions that the properties of the materials and the geometry of the beam and elastic restraints, and the thermal expansion coefficient of the material are predetermined, and no uncertainties in these properties have been taken into account. However, the material and
thermal properties of steel, the elastic stiffnesses of the restraints, and dimensions of such beams are often subjected to a certain amount of scatter in their values and so the uncertainties of these parameters have to be considered in the thermal responses analysis. In practice, it is difficult to estimate experimentally the autocorrelation function, or spectral density function of the stochastic variation of these properties. Hence, in this paper, uncertainties of these parameters are accounted for by use of interval modeling of uncertainties. Both upper and lower thermoelastic responses are calculated using an interval valued extension of real solutions so that designers may have a better understanding of the actual responses of elastically restrained steel beams under the linear temperature gradient.

Zenkour and Sobhy [91] investigated the thermal buckling analysis of functionally graded material (FGM) plates resting on two-parameter Pastenak’s foundations. Equilibrium and stability equations of FGM plates are derived based on trigonometric shear deformation plate theory and includes the plate foundation interaction and thermal effects. The material properties vary according to a power law form through the thickness coordinate. The governing equations are solved analytically for a plate with simply supported boundary conditions and subjected to uniform temperature rise and gradient through the thickness. Resulting equations are employed to obtain the close-form solution for the critical buckling load for each loading case. The influences of the plate aspect ratio, side to thickness ratio, gradient index, and elastic foundation stiffnesses on the buckling temperature are discussed.

The thermal expansions and rotations of a fixed slender beam that result from a linear temperature gradient field are fully restrained at the beam ends. These restrained expansions and rotations will produce internal bending and compressive actions in the beam, and these actions increase with an increase of the temperature differential and average temperature of the linear temperature gradient field. When these actions reach critical values, the fixed beam may bifurcate from its primary equilibrium configuration to a buckled equilibrium configuration. In previously-reported studies of the thermoelastic buckling of a member or structure, the material and geometric properties of the structure are predetermined. However, in practice, these properties are always
subjected to a certain amount of uncertainty due to inaccurate measurement, and manufacture and construction errors. For example, a material manufactured by the same process may demonstrate differences in its elastic properties. Pi et al. [92] have uses convex sets to model these uncertainties and derive the upper and lower critical temperatures for the thermoelastic in-plane buckling of fixed slender beams.

Varghese and Khalsa [93] have apply the integral transformation techniques to study thermoelastic response of a thick annular disc, in general in which sources are generated according to the linear function of the temperature, with boundary conditions of the radiation type. They have obtained the result as a series of Bessel functions.

Mohamed et al. [94] have represented a four-Variable refined plate theory for buckling analysis of functionally graded plates. The theory, which has strong similarity with classical plate theory in many aspects, accounts for a quadric variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. A power law distribution is used to describe the variation of volume fraction of material compositions. Equilibrium and stability equations are derived based on the present theory. The non-linear governing equations are solved for plates subjected to simply supported boundary conditions. The thermal loads are assumed to be uniform, linear and non-linear distribution through the thickness. The influences of many plate parameters on buckling temperature differences will be investigated. It is noticed that the present refined plate theory can predict accurately the critical temperature of simply supported graded plates.

Ghugal and Kulkarni [95] have discussed and presented the thermal stresses and displacements for orthotropic, two-layer anti symmetric, and three-layer symmetric square cross-ply laminated plates subjected to nonlinear thermal load through the thickness of laminated plates by using trigonometric shear deformation theory. The in-plane displacement field uses sinusoidal function in terms of thickness co-ordinate to include the shear deformation effect. The theory satisfies the shear stress free boundary conditions on the top and bottom surfaces of the plate. The present theory obviates the need of shear correction factor. Governing equations and boundary conditions of the
theory are obtained using the principle of virtual work. The validity of present theory is verified by comparing the results with those of classical plate theory and first order shear deformation theory and higher order shear deformation theory is studied.

Sayyad A.S. et al. [96 and 97] have presented a hyperbolic shear deformation theory taking into account transverse shear deformation effects for the bending analysis of thick isotropic plates subjected to linear thermal load. The displacement field of the theory contains three variables. The hyperbolic sine and cosine function is used in the displacement field in terms of thickness coordinate to represent the effect of shear deformation. The most important feature of the theory is that the transverse shear stresses can be obtained directly from the use of constitutive relations, satisfying the stress free boundary conditions at top and bottom surfaces of the plate. Hence, the theory eliminates the need of shear correction factor. Governing differential equations and boundary conditions of the theory are obtained using the principle of virtual work. Shinde et al. have compared the obtained results for bending analysis of isotropic plates subjected to linear thermal load with those of other higher order theories, lower order theories to validate the accuracy of the present theory.

Kant and Shiyekar [98] discussed and assessed a complete analytical model, which incorporates shear deformation as well as transverse normal thermal strains for the thermal stress analysis of cross-ply laminates subjected to linear or gradient thermal profile across thickness of the laminate. Primary displacement field is expanded in the thickness direction using twelve degrees of freedom. Equilibrium equations in the present higher order shear and normal deformation theory (HOSNT12) are variationally consistent and obtained using principle of virtual work. Numerical results of displacements and stresses are compared with three dimensional (3D) elasticity solution and other two dimensional (2D) models.

Ghugal and Kulkarni [99] states that the flexural response of symmetric cross-ply laminated plates subjected to uniformly distributed linear and non-linear thermo-mechanical loads is presented using trigonometric shear deformation theory. The in-plane displacement field uses sinusoidal function in terms of thickness coordinate to include the shear deformation effect. The theory satisfies the shear stress-free boundary conditions on
the top and bottom surface of the plate. The present theory obviates the need of shear correction factor. Governing equations and boundary conditions of the theory are obtained using the principle of the virtual work. Thermal stresses and displacements for three-layer symmetrical square cross-ply laminated plates subjected to uniform linear and non-linear and thermo-mechanical loads are obtained. The results of present theory have compared with those of classical theory, first order shear deformation theory and higher order shear deformation theory.

Zhen et al. [100] have proposed an enhanced Reddy’s theory for sandwich plates subjected to varying temperature fields. Based on the enhanced Reddy’s theory, analytical formulations and solutions are obtained by means of the principle of virtual displacement and Navier’s technique. Typical examples are considered to present the effect of transverse normal strain and temperature profile on the stress distributions of sandwiches. A numerical results show that transverse normal strain as well as temperature profile across the thickness direction has significant influence on the thermal stresses of sandwich plates.

Tounsi et al. [101] have presented the thermoelastic bending analysis of functionally graded sandwich plates by using a refined trigonometric shear deformation theory (TSDT) by taking in to account the transverse shear deformation effect. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories. The theory presented the variationally consistent, does not require shear correction factor, the displacement components are expressed by trigonometric series presented through the plate thickness to develop a two-dimensional theory and gives rise to transverse shear stress variation such that the transverse shear stress vary parabolically across the thickness satisfying shear stress free surface conditions. The sandwich with homogenous facesheet and FGM core is considered. Material properties of the present FGM cores are assumed to vary according to a power law distribution in terms of the volume fractions of the constituents. The influence played by the transverse shear deformation, thermal load, plate aspect ratio and volume fraction distribution are studied by the Tounsi et al. Numerical results for deflection and stresses of functionally graded metal-ceramic plates are investigated. They have concluded that
the presented theory is accurate and simple in solving the thermoelastic bending behavior of functionally graded plates.

Motoyamaa [102] has developed the restraint exerted on a casting by a furan sand mold on the casting and the contraction of the casting during cooling was dynamically and simultaneously measured using a device. The measurements were compared during cooling with thermal stress analyses. The thermal stress analyses were based on the representative mechanical models for the furan sand mold, i.e., the elastic and elasto-plastic models used in previous studies. The comparison demonstrated that the elasto-plastic model simulates the restraint force more accurately than the elastic model. In the thermal stress analysis, it was important to describe the development of inelastic deformation and the fracture of the sand mold. However, the simulated restraint force was still twice as large as the measured force even in the elasto-plastic model. This error is most likely attributable to using the temperature-independent mechanical properties of the furan sand mold and the mechanical model of the casting alloy, which neglected the viscoplasticity at high temperature in the thermal stress analysis.

Mathews and Shabna [103] have analyzed Static and thermal analysis of isotropic rectangular plate made up of various materials with various types of load applications and different boundary conditions is in this research. Both numerical and by finite element formulation are carried out in the analysis and the FEM formulation is done in the analysis section of the ANSYS package. In this study, flat rectangular plates are analysed. Finally comparison has been done between the results obtained from numerical analysis and ANSYS results for isotropic rectangular plate without cut-out. Study on the structural without cut-out which can be assumed as prefect plates and plates with a central rectangular cut-out that is considered as imperfect plates which are made up of steel, aluminum and invar materials properties such as deflections and thermal behavior of different materials has been conducted by Mervin Ealiyas Mathews and Shabna M.S. during the analysis which will be helpful in various fields of engineering.

Pachauri and Jatav [104] have discussed the analysis of thermally induced stresses of a composite material are a very important and necessary aspect with respect to design and modeling of composite structures in order to save them from failure. In this paper,
thermal deformations and residual stresses of thin isotropic composite plate were analyzed using Classical Laminated Plate Theory. The proposed model (3 x 2 x 1.4) was supposed to be an isotropic composite plate having an isotropic matrix with longitudinal isotropic fibers embedded in it and was thermally loaded. Transverse shear stress is not considered. MATLAB (R2012b) is also used for finding the more accurate results. The stiffness matrix used for evaluating thermal stresses are developed and applied to analyze various plates. Boundary conditions and governing equations of the plate are obtained by using principle of minimum potential energy and principle of virtual work. Results are compared with the existing solutions. Pachauri and Jatav concludes that this is the best method for providing accurate and efficient analysis of stresses and displacements under thermal loading.

Mhaske et al. [105] have discussed and presented two variable plate theories for thermal analysis of isotropic plates which takes into account effect of transverse shear deformations. Unlike any other theory, the theory presented gives rise to only two governing equations, which are completely uncoupled for static analysis. Number of unknown functions involved is only two, as against three in case of simple shear deformation theories of Mindlin and Reissner. The theory presented in variationally consistent, has strong similarity with classical plate theory in many aspects, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying the zero traction boundary conditions on the top and bottom surface of the plate. Comparison studies are performed by Mhaske et al. to verify the validity of the present results. Finally the effects of aspect ratios on the deflection and stress of isotropic plates subjected to uniformly distributed thermal load are investigated and discussed.

Mackiewicz [106] has concluded that daily changing temperature causes significant thermal stress and deformations in concrete pavement with laterally fixed planes. These stresses are important parameters included in dimensioning of pavement and are significantly depended on thermal difference of plane (i.e. difference in temperature between upper and lower surface plane). In this article, thermal stresses were analyzed in relationship to different diameters of dowel bars. First of all, the studies were focused on
stresses distributed around the dowel bar and caused by thermal differences. The analyses were carried out using finite element method (FEM). Based on this model, temperature distribution in concrete slab was studied and next calculations for different types of dowel joints and various climatic conditions in Central Europe (Poland) were performed. Results showed that application of dowel bars with small diameters increases stresses around them. Especially harmful is concentration of stretching stresses in concrete on two sides of dowel bar. These stresses can exceed permissible values in the case of incidental strong thermal differences.

Kulikov and Plotnikova [107] has focuses on the application of the method of sampling surfaces (SaS) to three-dimensional (3D) steady-state thermo elasticity problems for orthotropic and anisotropic laminated plates subjected to thermal loading. This method is based on selecting inside the nth layer in not equally spaced SaS parallel to the middle surface of the plate in order to choose temperatures and displacements of these surfaces as basic plate variables. Such an idea permits the presentation of the proposed thermoelastic laminated plate formulation in a very compact form. It is worth noting that the SaS are located inside each layer at Chebyshev polynomial nodes that leads to a uniform convergence of the SaS method. As a result, Kulikov and Plotnikova concluded that the SaS method can be applied efficiently to the 3D stress analysis of cross-ply and angle-ply composite plates with a specified accuracy utilizing the sufficient number of SaS.

Li and Qiao [108] have discussed and presented thermal postbuckling analysis of shear deformable anisotropic laminated composite beams with temperature-dependent material properties subjected to uniform temperature distribution through the thickness and resting on a two-parameter elastic foundation. The material of each layer of the beam is assumed to be linearly elastic and fiber-reinforced. The governing equations are based on Reddy's high order shear deformation beam theory with a von Karman-type of kinematic nonlinearity. Composite beams with clamped-clamped, clamped-hinged, and hinged-hinged boundary conditions are considered. A numerical solution for the nonlinear partial-integral differential form in terms of the transverse deflection using Galerkin's method is employed to determine the buckling temperatures and postbuckling
equilibrium paths of anisotropic laminated beams with uniform temperature distribution through the thickness. The numerical illustrations on the thermal postbuckling response of laminated beams with different types of boundary conditions, ply arrangements (lay-ups), geometric and physical properties are also presented, and the results reveal that the geometric and physical properties, temperature dependent properties, boundary conditions, and elastic foundation all have a significant effect on thermal postbuckling behavior of anisotropic laminated composite beams.

2.1 Comments on Literature Reviewed

From the literature studied it is found that thermomechanical problems are solved by using assumed thermal profile. Also exact solution of heat conduction equation in three dimensional problems considering conduction and convention are not available in the literature. In the present work thermal profile are determined by solving heat conduction equations in thermodynamics which gives exact solution of thermoelastic problems. The results presented here will be useful in thermal analysis of high speed vehicles, aircraft, space craft etc. made up of isometric and composite material.