CHAPTER I

INTRODUCTION

1.1 DESCRIPTION OF FOURTH ORDER PROBLEMS

Several problems in applied sciences and engineering are modelled using fourth order differential equations. A thorough understanding of the intricate processes involved in these problems often requires simulations based on the numerical solution of the equations. A large number of examples can be cited in this regard. The governing equation of Euler-Bernoulli beam theory is a fourth order ordinary differential equation, solution of which gives the transverse deflection of a cantilever beam under a uniform transverse load (Reddy 1993). The extended Fisher-Kolmogorov equation, a time-dependent fourth order partial differential equation, is used widely to model a variety of problems such as the physics of phase transitions and other bistable phenomena (Dee and Van Sarloos 1988) as well as instability in nematic liquid crystals (Zimmerman 1991). Another important time-dependent fourth order partial differential equation is Kuramoto-Sivashinsky equation, which is used to model a variety of engineering problems. These include problems related to the chaotic behaviour in a chemical reaction (Kuramoto 1978), flame front stability for stoichiometric composition of a combustible mixture (Sivashinsky 1980), instabilities in laminar flame fronts and thin hydrodynamic films (Nicolaenko *et al.* 1985).

In the thesis we study the following fourth order problems:

1. A fourth order two point boundary value problem in divergence form

$$\frac{d^2}{dx^2}\left[a(x)\frac{d^2u}{dx^2}\right] + b(x)u = f(x), \quad x \in I = (0,1); \tag{1.1}$$

subject to the boundary conditions

$$u(0) = 0,\ u(1) = 0;\ u_{xx}(0) = 0,\ u_{xx}(1) = 0, \tag{1.2}$$

where $a(x) \neq 0,\ \ x \in I$.

2. The general fourth order two point boundary value problem

$$u_{xxxx} + a(x)u_{xxx} + b(x)u_{xx} + c(x)u_x + d(x)u = f(x),\ x \in I = (0,1); \tag{1.3}$$

subject to the boundary conditions

$$u(0) = 0,\ u(1) = 0;\ u_{xx}(0) = 0,\ u_{xx}(1) = 0. \tag{1.4}$$

3. The extended Fisher-Kolmogorov equation

$$u_t + \gamma u_{xxxx} - u_{xx} + f(u) = 0, \quad 0 < t < T,\ \gamma > 0,\ \ x \in I = (0,1); \tag{1.5}$$

subject to the boundary and initial conditions

$$u(0,t) = 0,\ u(1,t) = 0,\ \ u_{xx}(0,t) = 0,\ u_{xx}(1,t) = 0; \tag{1.6}$$

$$u(x,0) = g(x); \tag{1.7}$$

where $f(u) = u^3 - u$.

4. The Kuramoto-Sivashinsky equation

$$u_t + \gamma u_{xxxx} + u_{xx} + uu_x = 0, \quad 0 < t < T,\ \gamma > 0,\ \ x \in I = (0,1); \tag{1.8}$$

subject to the boundary and initial conditions

$$u(0,t) = 0,\ u(1,t) = 0,\ \ u_{xx}(0,t) = 0,\ u_{xx}(1,t) = 0; \tag{1.9}$$

$$u(x,0) = g(x). \tag{1.10}$$

1.2 PRELIMINARIES AND NOTATIONS

Some of the results and notations, which we make use of in this work are given below.

Given an integer $n > 4$, let

$$\Pi_n : 0 = x_0 < x_1 < ... < x_n = 1,$$

be an arbitrary partition of the interval [0,1] with the property that $h \to 0$ as $n \to \infty$, where $h = \max\limits_{1 \leq k \leq n} h_k$ and $h_k = x_k - x_{k-1},\ \ k = 1, 2, ...n$.

For any two functions $u, v \in L_2(I)$, (u, v) represents the L_2 inner product and $\langle u, v\rangle_h$ represents the discrete inner product of u and v and are defined respectively as follows:

$$(u, v) = \int_I uv\ dx \ \text{ and } \ \langle u, v\rangle_h = Q_h(uv),$$

where $\mathcal{Q}_h$ is the fourth order Gaussian quadrature rule

$$\mathcal{Q}_h(g) := \frac{1}{2}\sum_{i=1}^{n} h_k[g(x_{k,1}) + g(x_{k,2})]. \tag{1.11}$$

Here $\mathrm{x}_{k,i} = x_{k-1} + \xi_i h_k$, $i = 1, 2$, are the two Gaussian points in the sub interval $[x_{k-1}, x_k]$ with $\xi_1 = \frac{1}{2}(1 - \frac{1}{\sqrt{3}})$, $\xi_2 = 1 - \xi_1$. Also,

$$(u, u) = \|u\|^2.$$

Further, it can also be proved that the quadrature rule in (1.11) has an error bound of the form

$$E_h(g) := |\mathcal{Q}_h(g) - \int_0^1 g| \leq C\sum_{k=1}^{n} h_k^4 \left(\int_{I_k} |g^{(4)}(x)|\right) dx, \tag{1.12}$$

where $\mathrm{I}_k = [x_{k-1}, x_k]$. This follows from the Peano's kernel Theorem (Davis and Rabinowitz 1975). Error analysis is discussed in the usual Sobolev spaces on the domain $I = (0, 1)$, which are defined as

$$W_p^m(I) = \{u \in L_p(I) : D^j u \in L_p(I) \ \text{ for } \ j = 0, 1, ..., m\}, \ \ 1 \leq p \leq \infty,$$

where $D^j u$ is the jth order distributional derivative of u. The Sobolev norms are given below: For an open interval E and a non negative integer m,

$$\begin{aligned} \|v\|_{W_p^m(E)} &= \left(\sum_{i=0}^{m} \left\|v^{(i)}\right\|_{L_p(E)}^p\right)^{\frac{1}{p}} \text{ if } 1 \leq p < \infty \\ &= \max_{0\leq i\leq m} \left\|v^{(i)}\right\|_{L_\infty(E)} \text{ if } p = \infty. \end{aligned}$$

We suppress the dependence of the norms on I when $E = I$. Throughout this thesis, for p and q with $1\leq p, q \leq \infty$, $p \neq 2$ and $p^{-1}+q^{-1} = 1$, we use the following notations:

$$\|v\|_{0,p} \ = \ \|v\|_{L_p}, \ \ \|v\|_{m,p} = \|v\|_{W_p^m}, \ \ \|v\|_{m,p,k} = \|v\|_{W_p^m(I_k)},$$

where I_k is a sub-interval of I.

We denote $W_2^m(I)$ by $H^m(I)$. The Sobolev norms on the space $H^m(I)$ are given below: For an open interval I and a non negative integer m,

$$\|v\|_m = \left(\sum_{i=0}^{m} \left\|v^{(i)}\right\|_{L_2(I)}^2\right)^{\frac{1}{2}}$$

and when $m = 0$, the corresponding norm is denoted by $\|v\|$. Let us denote $H^1(I)$ with zero Dirichlet boundary conditions by

$$\overset{0}{H^1}(I) = \left\{\varphi \in H^1(I) : \varphi(0) = \varphi(1) = 0\right\}.$$

Let us now consider the following cubic spline space defined over a partition on [0,1] as trial space

$$S_{h,3} = \left\{\varphi \in C^2(I) : \varphi\,|_{I_k} \in P_3(I_k),\ k = 1, 2, ...n\right\},$$

where $P_3(I_k)$ is the space of polynomials of degree 3 defined over the sub interval I_k. The corresponding space with zero Dirichlet boundary conditions is denoted by

$$\overset{0}{S}_{h,3} = \{\varphi \in S_{h,3} : \varphi(0) = \varphi(1) = 0\}.$$

Space of linear splines is defined as follows:

$$S_{h,1} = \{\varphi \in C(I) : \varphi\,|_{I_k} \in P_1(I_k),\ k = 1, 2, ...n\},$$

where $P_1(I_k)$ is the space of polynomials of degree 1 defined over the sub interval I_k. We use the following results in our error analysis.

Approximation property: For each fixed h and for each $v \in W_p^m(I)$, there exists a constant C independent of h and v, such that,

$$\inf_{\chi \in S_{h,3}} \|v - \chi\|_{j,p} \le Ch^{4-j}\|v\|_{4,p},\ \ 1 \le p \le \infty,\ \ j = 0, 1, 2.$$

Sobolev embedding theorem: We have the following inequality due to Sobolev embedding theorem, the proof of which can be found in (page 97, Adams 1975).

$$\|\phi\|_{0,\infty,k} \le \|\phi\|_{1,p,k},\ \ 1 \le p \le \infty,\ \ \phi \in W_p^1(I_k). \tag{1.13}$$

Hölder's inequality: For $1 \leq p, q \leq \infty$, we have

$$\sum_{i=1}^{n} |x_k y_k| \leq \left(\sum_{i=1}^{n} |x_k|^p \right)^{1/p} \left(\sum_{i=1}^{n} |y_k|^q \right)^{1/q}. \tag{1.14}$$

Inverse inequality: A family of finite dimensional sub spaces $S_{h,1}$ is said to possess an inverse property if there exists a positive constant C such that

$$\|\chi\|_{1,p,k} \leq C h_k^{-1} \|\chi\|_{0,p,k}, \;\; \chi \in S_{h,1}, \;\; 1 \leq p \leq \infty. \tag{1.15}$$

Young's inequality: For $a, b \geq 0$ and $\epsilon > 0$

$$ab \leq \frac{a^2}{2\epsilon} + \frac{\epsilon}{2} b^2. \tag{1.16}$$

Interpolation inequality: If $v \in W_p^m(E)$ with $p \in [1, \infty]$, then there exists a constant C depending only on m such that for any δ satisfying $0 < \delta \leq |E| \leq 1$,

$$\|v\|_{i,p,E} \leq C \left[\delta^{m-i} \|v\|_{m,p,E} + \delta^{-i} \|v\|_{0,p,E} \right] \qquad 0 \leq i \leq m-1, \tag{1.17}$$

where $|E|$ denotes the length of E. For a detailed proof, one may refer appendix of (Sloan *et al.* 1993) or Chapter 4 of (Adams 1975).

Gronwall's lemma (Continuous case): Let $f(t)$, $g(t)$ and $h(t)$ be piecewise continuous non negative functions defined on an interval $a \leq t \leq b$, $g(t)$ being non-decreasing and C a nonnegative constant. If for each $t \in [a, b]$,

$$f(t) + h(t) \leq g(t) + C \int_a^t f(\tau) d\tau,$$

then

$$f(t) + h(t) \leq g(t) e^{C(t-a)}.$$

Gronwall's lemma (Discrete case): Let $f(t)$, $g(t)$ and $h(t)$ be non negative functions defined on $\Gamma_\triangle = \{t \in [0, T] : t = j\triangle t, \; j = 0, 1, ...N \text{ with } \triangle t = T/N\}$. Further, let $g(t)$ be nondecreasing. If

$$f(t) + h(t) \leq g(t) + C \triangle t \sum_{\tau=0}^{t-\triangle t} f(\tau),$$

where C is a positive constant, then

$$f(t) + h(t) \leq g(t) e^{Ct}.$$

In primal based standard finite element methods, the dual variable, *i.e.*, the derivative is not computed as a fundamental unknown, but is obtained *a posteriori* by differentiation and hence the accuracy is reduced. But in mixed methods both the primal and dual variables are computed as fundamental unknowns and hence much better accuracy is expected than the primal based methods. The requirement of the desired smoothness of primal and dual solution will be reduced when we use mixed methods.

In the standard mixed finite element method, the approximating spaces should satisfy LBB consistency condition where as in the H^1-mixed finite element method it is not required. The error estimates are obtained without imposing the quasi uniformity on the finite element spaces. Even though more regularity is required on the solutions, the approximate solution to the dual variable also has the same order of convergence as the approximate solution to the primal variable. When H^1-mixed finite element method is applied to a second order time dependent partial differential equation, the C^1 continuity condition on the finite element space is relaxed to C^0. In chapters four and five, which deal with H^1-mixed finite element method, we seek approximate solutions in the cubic spline space for both the primal variable and the dual variable. Hence, the C^3 continuity condition on the finite element space is relaxed to C^2 in the H^1- mixed finite element method.

1.3 OUTLINE OF THE THESIS

In Chapter 1, introduction of mixed finite element methods, preliminaries, basic definitions and notations are discussed. In Chapter 2, we discuss a mixed discrete Petrov-Galerkin method for a particular type of fourth order boundary value problem in divergence form given by equation (1.1).
The formulation is as follows: Find $\{u_h, v_h\} \in \overset{0}{S}_{h,3}$ such that

$$\langle u_{hxx}, \phi_h \rangle_h \quad = \quad \langle \alpha v_h, \phi_h \rangle_h, \quad \phi_h \in S_{h,1}, \tag{1.18}$$

$$\langle v_{hxx} + bu_h, \phi_h\rangle_h \;=\; \langle f, \phi_h\rangle_h, \;\; \phi_h \in S_{h,1}. \tag{1.19}$$

For the above formulation, optimal order error *a priori* estimates in general p-norms $1 \leq p \leq \infty$ are obtained. The result is validated with a numerical example.

In Chapter 3, we discuss a mixed discrete Petrov-Galerkin method for a general fourth order boundary value problem given by equation (1.3).
The formulation is as follows: Find $\{u_h, v_h\} \in \overset{0}{S}_{h,3}$ such that

$$\langle u_{hxx}, \phi_h\rangle_h = \langle v_h, \phi_h\rangle_h, \;\; \phi_h \in S_{h,1}, \tag{1.20}$$

$$\langle v_{hxx} + av_{hx} + bv_h, \phi_h\rangle_h + \langle cu_{hx} + du_h, \phi_h\rangle_h = \langle f, \phi_h\rangle_h, \;\; \phi_h \in S_{h,1}. \tag{1.21}$$

For the above formulation, optimal order *a priori* error estimates in general p-norms for $1 \leq p \leq \infty$ are obtained. The result is validated with a numerical example. For the problems in Chapter 2 and Chapter 3, the mixed Petrov-Galerkin formulation leads to a coupled system of linear equations of size *(2n+6)* $\times$ *(2n+6)*, which can be solved directly.

In Chapter 4, we discuss a mixed H^1-Galerkin finite element method for extended Fisher-Kolmogorov equation (1.5). In semi discrete case, the formulation is as follows:
Find $U, V \in \overset{0}{S}_{h,3}$ such that

$$\begin{aligned} (U_{xx}, \phi_{h_{xx}}) &= (V, \phi_{h_{xx}}), \;\; \phi_h \in \overset{0}{S}_{h,3}, \\ U(x,0) &= g(x); \end{aligned} \tag{1.22}$$

$$\begin{aligned} (V_t, \phi_h) + A(\lambda : V, \phi_h) - \lambda(V, \phi_h) + (f(U), \phi_{hxx}) &= 0, \;\; \phi_h \in \overset{0}{S}_{h,3}, \\ V(x,0) &= g_{xx}(x). \end{aligned} \tag{1.23}$$

For the above formulation, optimal order *a priori* error estimates are obtained in the semi discrete case.
In the fully discrete case, the formulation is as follows:

Find $Z, W \in \overset{0}{S}_{h,3}$ such that

$$\begin{aligned}(d_t W^{n+1}, \phi_h) \quad + \quad & A(\lambda : W^{n+1}, \phi_h) - \lambda(W^{n+1}, \phi_h) + (f(Z^n), \phi_{hxx}) = 0, \\ & \phi_h \in \overset{0}{S}_{h,3}, \quad n = 0, 1, 2...M-1 \qquad (1.24)\end{aligned}$$

with $W^0 = g_{xx}$ and

$$(Z^{n+1}_{xx}, \phi_{hxx}) = (W^{n+1}, \phi_{hxx}), \quad \phi_h \in \overset{0}{S}_{h,3}, \quad n = 0, 1, 2...M-1 \qquad (1.25)$$

with $Z^0 = g$. In both the semi discrete and fully discrete schemes, $A(\lambda : v, \phi)$ is a coercive bilinear form for a suitably chosen λ.

For the above formulation, optimal order *a priori* error estimates are obtained in the fully discrete case . The result is validated with numerical examples.

In Chapter 5, we discuss a mixed H^1-Galerkin finite element method for KuramotoSivashinsky equation (1.8).

In semi discrete case, the formulation is as follows:

Find $U, V \in \overset{0}{S}_{h,3}$ such that

$$\begin{aligned}(U_{xx}, \phi_{h_{xx}}) &= (V, \phi_{h_{xx}}), \quad \phi_h \in \overset{0}{S}_{h,3}, \qquad (1.26) \\ U(x,0) &= g(x);\end{aligned}$$

$$\begin{aligned}(V_t, \phi_h) + A(\lambda : V, \phi_h) - \lambda(V, \phi_h) - \lambda(V_x, \phi_{hx}) + (UU_x, \phi_{hxx}) = 0, \qquad (1.27) \\ \phi_h \in \overset{0}{S}_{h,3}, V(x,0) = g_{xx}(x).\end{aligned}$$

For the above formulation, optimal order *a priori* error estimates are obtained in the semi discrete case .

In the fully discrete case, the formulation is as follows:

Find $Z, W \in \overset{0}{S}_{h,3}$ such that

$$\begin{aligned}(d_t W^{n+1}, \phi_h) \quad + \quad & A(\lambda : W^{n+1}, \phi_h) - \lambda(W^{n+1}, \phi_h) - \lambda(W^{n+1}_x, \phi_{hx}) \\ + \quad & (Z^n Z^n_x, \phi_{hxx}) = 0, \phi_h \in \overset{0}{S}_{h,3}, \quad n = 0, 1, 2...M-1 \quad (1.28)\end{aligned}$$

with $W^0 = g_{xx}$ and

$$(Z^{n+1}_{xx}, \phi_{hxx}) = (W^{n+1}, \phi_{hxx}), \quad \phi_h \in \overset{0}{S}_{h,3}, \quad n = 0, 1, 2...M-1 \qquad (1.29)$$

with $Z^0 = g$. In both the semi discrete and fully discrete schemes, $A(\lambda : v, \phi)$ is a coercive bilinear form for a suitably chosen λ.
For the above formulation, optimal order *a priori* error estimates are obtained in the fully discrete case . The result is validated with numerical examples.
For problems in Chapter 4 and Chapter 5, the size of the matrix containing unknowns in the decoupled system in the present method is *n+1* each.
Throughout this thesis, C is a generic positive constant, whose dependence on the smoothness of the exact solution can be easily determined from the proofs.

1.4 LITERATURE SURVEY

In Chapter 2 and Chapter 3, a discrete Petrov-Galerkin method is discussed for fourth order two point boundary value problems both in divergence and non divergence forms. It may also be viewed as a Petrov-Galerkin method with quadrature rule replacing integrals and having cubic splines and linear splines as trial and test spaces respectively. In general, for the computation of the H^1-Galerkin approximation, the integrals are rarely evaluated exactly. By replacing the integrals using a composite two point Gauss rule, the resulting method may be described as a "qualocation" approximation, which is a quadrature based modification of the collocation method. One practical advantage of this procedure over the orthogonal spline collocation method described by Douglas and Dupont (1973,1974) is that for a given partition there are only half the number of unknowns and therefore it reduces the size of the matrix. Earlier, the qualocation method was introduced and analyzed by Sloan (1988) for boundary integral equation on smooth curves. Subsequently, Sloan *et al.* (1993) extended the method to a class of linear second order two point boundary value problems and derived optimal error estimates without quasi-uniformity assumption on the finite element mesh. In Sloan *et al.* (1993), the discretization is achieved either by the two point composite Gauss rule or by the composite Simpson's quadrature formula. Jones and Pani (1998) discussed the qualocation method for a second order semi linear two point boundary value problem. Subsequently, Amiya K Pani (1999) expanded its scope by adapting the analysis to a semi linear parabolic initial and boundary value problem in a single space variable. Further, Jones and Pani (2005) extended this method to the free boundary problem, *i.e.*, one dimensional single phase Stefan problem for which part of the boundary has to be found out along with the solution process. Error estimates for iso parametric mixed finite element solution of 4^{th} order elliptic problems with variable coefficients have been developed by Bhattacharyya and Nataraj (2002). Kshitij Kulshreshtha *et al.* (2004) discussed about the performance of a parallel mixed finite element implementation for fourth order clamped anisotropic plate bending problems in distributed memory environments. Pulin Kumar Bhattacharyya and Neela Nataraj (1999) have developed error estimates for the mixed finite element solution of 4^{th} order elliptic problems with variable coefficients. An isoparametric mixed finite element approximation of eigen values and eigen vectors of 4^{th} order eigen value problems with variable coefficients have been developed by Pulin Kumar Bhattacharyya and Neela Nataraj (2002).

We now briefly state some results on the orthogonal spline collocation methods relevant to the present study. Earlier, Douglas and Dupont (1973,1974) studied orthogonal spline collocation methods for quasilinear parabolic equations in a single space variable using piecewise Hermite polynomials and derived optimal error estimates. For a comprehensive survey of spline collocation methods, one may refer to the literature (Bialecki and Fairweather 2001). A quadrature based Petrov-Galerkin method applied to higher dimensional boundary value problems is studied by Bialecki *et al.* (2004). The main idea in this work is that a quadrature based approximation for a fourth order problem is analyzed in mixed setting.

Related to fourth order evolution equations, Amiya K Pani and Chung (2001) analyzed a C^1-conforming finite element method for the Rosenau equation. Numerical studies of one dimensional and multi dimensional Cahn-Hilliard equation are discussed by Charles *et al.* (1987,1989), Danumjaya *et al.*

(2006) and Qiang *et al.* (1991). Existence and numerical approximations of periodic solutions of semi linear fourth order differential equations related to either extended Fisher-Kolmogorov equation or Swift-Hohenberg equation are discussed in the literature (Julia Chaparova 2002). Quarteroni (1980) discussed mixed methods for a biharmonic problem. Further, a quadrature Galerkin method is analyzed for biharmonic problem in the literature (Aitbayev 2008). A mixed finite element method for fourth order eigen value problem is given in the literature (Neela Nataraj 2009). Thirupathy Gudi *et al.* (2008) have discussed a mixed discontinuous Galerkin finite element method for the biharmonic equation.

One fourth order time dependent non linear differential equation considered in this dissertation is the extended Fisher-Kolmogorov equation

$$u_t + \gamma u_{xxxx} - u_{xx} + f(u) = 0,$$
$$0 < t < T,\ \gamma > 0,\ \ x \in I = (0,1);\ \ f(u) = u^3 - u.$$

This equation has got significance, as many problems in physics related to phase transition and other bistable phenomena are mathematically modelled as equation (1.5). For the case $\gamma > 0$, it was first proposed by Dee and Van Saarloos (1988) as a higher order model equation for physical systems that are bistable. Further, the extended Fisher-Kolmogorov equation (1.5) has a lot of applications in the theory of instability in nematic liquid crystals and travelling waves in reaction diffusion systems as discussed in the literatures (Zimmerman 1991) and (Aronson and Weinberger 1978). Van Saarloos (1987,1988) discussed the marginal stability for equation (1.5). Coullet *et al.* (1987) studied the chaotic characteristics of equation (1.5). Peletier and Troy (1995,1996) have analyzed the steady state equation corresponding to (1.5) by shooting methods. Peletier *et al.* (1997) and Stepan and Julia (2001) discussed the periodic solutions of the extended Fisher-Kolmogorov equation. Hong Luo (2011) studied about the global attractors of the extended Fisher-Kolmogorov equation in H^k spaces. Tlili and Khaled (2011) presented a Crank-Nicolson type finite difference scheme to approximate the extended Fisher-Kolmogorov equation and discussed the existence and uniqueness of the solution. Noomen and Khaled (2011) discussed the existence, uniqueness and convergence of Crank-Nicolson type finite difference solutions for the extended Fisher-Kolmogorov equation in two space dimension. In the literature (Danumjaya and Pani 2005), a second order splitting combined with orthogonal spline collocation method for equation (1.5) is formulated, analyzed and the error bounds are obtained for semi discrete scheme. Further, Danumjaya and Pani (2006) established optimal error estimates for both the semi discrete and fully discrete cases for the extended Fisher-Kolmogorov equation in two space dimension, using a C^1-conforming finite element method.

Another fourth order time dependent non linear partial differential equation considered in this dissertation is the Kuromoto-Sivashinsky equation

$$u_t + \gamma u_{xxxx} + u_{xx} + uu_x = 0, \;\; \gamma > 0, \;\;\; 0 < t < T, \;\; x \in I = (0, 1),$$

which also has significance in modeling many chemical and physical phenomena. The Kuramoto-Sivashinsky equation was derived independently by Kuramoto and Sivashinsky in the later 70's. They worked on problems related to chaotic behaviour in a distributed chemical reaction with respect to spatial pattern (Kuramoto 1978) and flame front stability for stoichiometric composition of a combustible mixture (Sivashinsky 1980). Many equations arising in reaction-diffusion systems, flame-propagation and viscous flow problems are modelled by the Kuramoto-Sivashinsky equation. Tadmor (1986) has shown that such equations are well posed. Akrivis (1992) has analyzed a Crank-Nicolson-type finite difference scheme for the Kuramoto-Sivashinsky equation in one space dimension with periodic boundary conditions and obtained second-order error estimates. Akrivis (1996) established optimal order error estimates for the Kuromoto-Sivashinsky equation with the periodic initial condition by discretization using implicit Runge-Kutta methods for time variable and Galerkin finite element method for the space variable. Akrivis *et al.* (2012) presented fully discrete schemes for a general class of dispersively modified Kuramoto-Sivashinsky equation in which linearly implicit schemes and spectral methods are used for the temporal and spatial discretizations respectively. Some dynamical properties of the Kuromoto-Sivashinsky equation, which model pattern formations on unstable flame fronts and thin hydrodynamic films are discussed by Nicolaenko (1985). Collet *et al.* (1993) studied the analyticity properties of solutions of the Kuramoto-Sivashinsky equation having periodic initial condition with period L and numerical experiments are presented showing the region of analyticity. Mittal and Geetha (2010) used quintic B-spline collocation scheme to find numerical solution to the Kuramoto-Sivashinsky equation. Yang and Shu (2006) proposed local discontinuous Galerkin methods for the Kuramoto-Sivashinsky equations and the Ito-type coupled KdV equations. In the literature (Lina Ye *et al.* 2011) proposed a Lattice Boltzmann model based on the higher order moment method for Kuramoto-Sivashinsky equation. Rebelo Paulo (2011) presented an approximate solution to the initial boundary value problem for the one dimensional Kuramoto-Sivashinsky equation using Fourier method combined with Adomian's decomposition method. In the literature (Manickam *et al.* 1998), a second order splitting combined with orthogonal spline collocation method for equation (1.8) is formulated, analyzed and the error bounds are obtained for semi discrete scheme.

Earlier in the literature (Amiya K Pani 1998), an H^1-Galerkin mixed finite element method is applied and error estimates are obtained for a parabolic partial differential equation. In this, the parabolic equation is split into two partial differential equations leading to a first order system after introducing an intermediate function. Ambit Kumar Pany *et al.* (2007) used an H^1-Galerkin mixed finite element method for Burgers' equation. Xianbiao *et al.* (2012) discussed an H^1-Galerkin mixed finite element method for the coupled Burgers' equation. Yang Liu and Hong Li (2009) discussed H^1-Galerkin mixed finite element method for a class of second order pseudo hyperbolic equation. H^1-Galerkin mixed finite element method for the regularized long wave equation is discussed by Guo and Chen (2006) and they have established optimal order error estimate. Madhusmitha and Rajen Sinha (2009,2012) have discussed the super convergence of H^1-Galerkin mixed finite element method for parabolic problems and second order elliptic equation over rectangular partitions in the respective literatures. An H^1-Galerkin mixed finite element method for linear and non linear parabolic problems is discussed by Neela Nataraj and Ambit Kumar (2006). Arul *et al.* (2004) have discussed higher order fully discrete scheme combined with H^1-Galerkin mixed finite element method for semi linear reaction-diffusion equations. H^1-Galerkin mixed finite element methods are analyzed for parabolic partial integro-differential equations by Amiya K pani and Graeme Fairweather (2002). In this, optimal order error estimates are obtained for both the semi discrete and fully discrete schemes in one space dimension. An H^1-Galerkin mixed finite element methods is analyzed for a special type of evolution equation in literature (Amiya K pani and Graeme Fairweather 2002). Zhaojie Zhou (2010) obtained optimal error estimates in L_2 and H^1 norms in both semi discrete and fully discrete cases for a class of heat transport equations in one dimensional case using an H^1-Galerkin mixed finite element method.