

## TABLE OF CONTENTS

CHAPTER NO.	TITLE	PAGE NO.
	ABSTRACT	iii
	LIST OF TABLES	x
	LIST OF FIGURES	xiii
	LIST OF SYMBOLS AND ABBREVIATIONS	xv
1	INTRODUCTION	1
	1.1 DESCRIPTION OF FOURTH ORDER PROBLEMS	1
	1.2 PRELIMINARIES AND NOTATIONS	2
	1.3 OUTLINE OF THE THESIS	6
	1.4 LITERATURE SURVEY	9
2	QUADRATURE BASED MIXED PETROV-GALERKIN METHOD FOR A FOURTH ORDER PROBLEM IN DIVERGENCE FORM	15

CHAPTER NO.	TITLE	PAGE NO.
	2.1 INTRODUCTION	15
	2.2 ASSUMPTIONS	18
	2.3 CONVERGENCE ANALYSIS	18
	2.4 NUMERICAL EXPERIMENTS	31
<b>3</b>	<b>QUADRATURE BASED MIXED PETROV-GALERKIN METHOD FOR A GENERAL FOURTH ORDER BOUNDARY VALUE PROBLEM</b>	<b>35</b>
	3.1 INTRODUCTION	35
	3.2 ASSUMPTIONS AND REGULARITY	37
	3.3 CONVERGENCE ANALYSIS	38
	3.4 NUMERICAL EXPERIMENTS	54
<b>4</b>	<b><math>H^1</math>-GALERKIN MIXED FINITE ELEMENT METHOD FOR EXTENDED FISHER-KOLMOGOROV EQUATION</b>	<b>58</b>
	4.1 INTRODUCTION	58
	4.2 SEMI DISCRETE $H^1$ -GALERKIN FORMULATION	59

CHAPTER NO.	TITLE	PAGE NO.
	4.3 AUXILIARY PROJECTION	60
	4.4 ERROR ANALYSIS OF SEMI DISCRETE SCHEME	63
	4.5 ERROR ANALYSIS OF FULLY DISCRETE EULER BACKWARD SCHEME	71
	4.6 NUMERICAL EXPERIMENTS	84
<b>5</b>	<b><math>H^1</math>-GALERKIN MIXED FINITE ELEMENT METHOD FOR KURAMOTO -SIVASHINSKY EQUATION</b>	89
	5.1 INTRODUCTION	89
	5.2 SEMI DISCRETE $H^1$ -GALERKIN FORMULATION	89
	5.3 AUXILIARY PROJECTION	91
	5.4 ERROR ANALYSIS OF SEMI DISCRETE SCHEME	94

<b>CHAPTER NO.</b>	<b>TITLE</b>	<b>PAGE NO.</b>
	5.5 ERROR ANALYSIS OF FULLY DISCRETE EULER BACKWARD SCHEME	104
	5.6 NUMERICAL EXPERIMENTS	116
<b>6</b>	<b>SUMMARY AND CONCLUSION</b>	123
	<b>REFERENCES</b>	127
	<b>LIST OF PUBLICATIONS</b>	135
	<b>CURRICULUM VITAE</b>	136

## LIST OF TABLES

TABLE NO.	TITLE	PAGE NO.
2.1	$L_\infty$ errors in $u_h$ , $u_{hx}$ and $u_{hxx}$ of a fourth order Boundary Value Problem in divergence form	33
2.2	$L_\infty$ errors in $v_h$ , $v_{hx}$ and $v_{hxx}$ of a fourth order Boundary Value Problem in divergence form	33
2.3	$L_1$ errors in $u_h$ , $u_{hx}$ and $u_{hxx}$ of a fourth order Boundary Value Problem in divergence form	33
2.4	$L_1$ errors in $v_h$ , $v_{hx}$ and $v_{hxx}$ of a fourth order Boundary Value Problem in divergence form	33
2.5	$L_2$ errors in $u_h$ , $u_{hx}$ and $u_{hxx}$ of a fourth order Boundary Value Problem in divergence form	34
2.6	$L_2$ errors in $v_h$ , $v_{hx}$ and $v_{hxx}$ of a fourth order Boundary Value Problem in divergence form	34
3.1	$L_\infty$ errors in $u_h$ , $u_{hx}$ and $u_{hxx}$ of general fourth order Boundary Value Problem	56
3.2	$L_\infty$ errors in $v_h$ , $v_{hx}$ and $v_{hxx}$ of general fourth order Boundary Value Problem	56

TABLE NO.	TITLE	PAGE NO.
3.3	$L_1$ errors in $u_h$ , $u_{hx}$ and $u_{hxx}$ of general fourth order Boundary Value Problem	57
3.4	$L_1$ errors in $v_h$ , $v_{hx}$ and $v_{hxx}$ of general fourth order Boundary Value Problem	57
3.5	$L_2$ errors in $u_h$ , $u_{hx}$ and $u_{hxx}$ of general fourth order Boundary Value Problem	57
3.6	$L_2$ errors in $v_h$ , $v_{hx}$ and $v_{hxx}$ of general fourth order Boundary Value Problem	57
4.1	$L_2$ errors in $U$ and $W$ at one time level of extended Fisher-Kolmogorov equation	88
4.2	$L_2$ errors in $U$ and $W$ at another time level of extended Fisher-Kolmogorov equation	88
4.3	$L_\infty$ errors in $U$ and $W$ at one time level of extended Fisher-Kolmogorov equation	88
4.4	$L_\infty$ errors in $U$ and $W$ at another time level of extended Fisher-Kolmogorov equation	88

<b>TABLE NO.</b>	<b>TITLE</b>	<b>PAGE NO.</b>
5.1	$L_2$ errors in $U$ and $W$ at one time level of Kuramoto-Sivashinsky equation	121
5.2	$L_2$ errors in $U$ and $W$ at another time level of Kuramoto-Sivashinsky equation	121
5.3	$L_\infty$ errors in $U$ and $W$ at one time level of Kuramoto-Sivashinsky equation	122
5.4	$L_\infty$ errors in $U$ and $W$ at another time level of Kuramoto-Sivashinsky equation	122

## LIST OF FIGURES

TABLE NO.	TITLE	PAGE NO.
4.1	Approximate solution of $v(x) = u_{xx}$ for extended Fisher-Kolmogorov equation for $h = 1/10$ and $h = 1/20$ at different time levels	86
4.2	Approximate solution of $u(x)$ for extended Fisher-Kolmogorov equation for $h = 1/10$ and $h = 1/20$ at different time levels	86
5.1	Approximate solution of $v(x) = u_{xx}$ for Kuramoto-Sivashinsky equation for $h = 1/10$ and $h = 1/20$ at different different time levels	118
5.2	Approximate solution of $v(x) = u_{xx}$ for $h = 1/40$ and $u(x)$ for $h = 1/10$ for Kuramoto-Sivashinsky equation at different time levels	118
5.3	Approximate solution of $u(x)$ for Kuramoto-Sivashinsky equation for $h = 1/20$ and $h = 1/40$ at different time levels	119



<b>TABLE NO.</b>	<b>TITLE</b>	<b>PAGE NO.</b>
5.4	Approximate solution of $v(x) = u_{xx}$ for Kuramoto- Sivashinsky equation for $h = 1/20$ and $\gamma = 0.00001$ at different time levels	119
5.5	Approximate solution of $u(x)$ for Kuramoto- Sivashinsky equation for $h = 1/20$ and $\gamma = 0.00001$ at different time levels	120

## LIST OF SYMBOLS AND ABBREVIATIONS

- $I$  - Interval  $(0,1)$ .
- $I_k$  -  $k^{th}$  subinterval  $[x_{k-1}, x_k]$  of  $I$ .
- $W_p^m(I)$  - Sobolev space of order  $m$  for  $1 \leq p \leq \infty$  defined on  $I$ .
- $\|\cdot\|_{m,p}$  - Norm of the Sobolev space  $W_p^m(I)$  for  $1 \leq p \leq \infty$ .
- $\|\cdot\|_{m,p,k}$  - Sobolev norm in sub interval  $[x_{k-1}, x_k]$  for  $1 \leq p \leq \infty$ .
- $H^m(I)$  - Sobolev space of order  $m$  for  $p = 2$  on  $I$ .
- $H^1_0(I)$  - Sobolev space of order 1 for  $p = 2$  with zero boundary conditions on  $I$ .
- $S_{h,3}$  - Space of cubic splines.
- $S^0_{h,3}$  - Space of cubic splines with zero boundary conditions
- $S_{h,1}$  - Space of linear splines.
- $\mathcal{Q}_h(g)$  - Fourth order Gaussian quadrature rule.
- $E_h(g)$  - Error in fourth order Gaussian quadrature.
- $(\cdot, \cdot)$  -  $L_2$  inner product.
- $\langle \cdot, \cdot \rangle_h$  - Discrete inner product.