CHAPTER 2

THERMOPOPHORESIS EFFECTS ON NON-DARCY MIXED
CONVECTIVE HEAT AND MASS TRANSFER PAST A
POROUS WEDGE IN THE PRESENCE OF
SUCTION/INJECTION

2.1 INTRODUCTION

The phenomenon by which micron sized particles suspended in a non-isothermal fluid acquire a velocity relative to the fluid as in the direction of decreasing temperature is termed as thermophoresis. The velocity acquired by the particles is termed as thermophoretic velocity and the force experienced by the suspended particles due to the temperature gradient is termed as thermophoretic force. The magnitude of thermophoretic velocity and thermophoretic force are proportional to the temperature gradient and depend on thermal conductivity of aerosol particles, the carrier fluid, heat capacity of the fluid, thermophoretic coefficient and Knudsen number. Due to thermophoresis, small micron sized particles are deposited on cold surfaces. In this process, the repulsion of particles from hot objects also takes place and a particle-free layer is observed around hot bodies. Thermophoresis causes small particles to be driven away from a hot surface and toward a cold one. This phenomenon has gained importance for engineering applications; include particle deposition onto wafers in the microelectronics industry, particle surfaces produced by condensing vapor–gas mixtures, particles impacting the
blade surface of gas turbines, and others such as filtration in gas-cleaning and nuclear reactor safety.

The use of thermophoretic heaters has led to a reduction in chip failures. In the same way there is the potential application of thermophoresis to remove radioactive aerosols from containment domes in the event of a nuclear reactor accident. Thermophoresis principle is utilized to manufacture graded index silicon dioxide and germanium dioxide optical fiber performs used in the field of communications. In view of these various applications, England and Emery (1969) studied the thermal radiation effect of an optically thin gray gas bounded by a stationary vertical plate. Raptis (1998) studied radiation effect on the flow of a micro polar fluid past a continuously moving plate. Hossain and Takhar (1996) analyzed the effect of radiation using the Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Duwairi and Damesh (2004a, 2004b), Duwairi (2005), Damesh et al (2006) studied the effect of radiation and heat transfer in different geometry for various flow conditions.

In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic and/or endothermic chemical reactions and dissociating fluids in packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important. In the application of pigments, or chemical coating of metals, or removal of particles from a gas stream by filtration, there can be distinct advantages in exploiting deposition mechanisms to improve efficiency. Goldsmith and May (1966) first studied the thermophoretic transport involved in a simple one-dimensional flow for the measurement of the thermophoretic velocity. Thermophoresis in laminar flow over a horizontal flat plate has been studied
theoretically by Goren (1977). Thermophoresis in natural convection with variable properties for a laminar flow over a cold vertical flat plate has been studied by Jayaraj et al (1999). Selim et al (2001) studied the effect of surface mass flux on mixed convective flow past a heated vertical flat permeable plate with thermophoresis. The first analysis of thermophoretic deposition in geometry of engineering interest appears to be that of Hales et al (1972). They have solved the laminar boundary layer equations for simultaneous aerosol and steam transport to an isothermal vertical surface. Recently, Chamkha and Pop (2004) studied the effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium.

2.2 OBJECTIVE AND SCOPE

An analysis is presented to investigate the effect of thermophoresis particle deposition and variable viscosity on non-Darcy MHD mixed convective heat and mass transfer of a viscous, incompressible fluid past a porous wedge in the presence of suction/injection. The wall of the wedge is embedded in a uniform non-Darcy porous medium in order to allow for possible fluid wall suction or injection. The governing partial differential equations of the problem, subjected to their boundary conditions, are solved numerically by applying an efficient solution scheme for local non-similarity boundary layer analysis. Numerical calculations are carried out for different values of dimensionless parameter in the problem and an analysis of the results obtained show that the flow field is influenced appreciably by thermophoresis particle deposition.

Thermophoresis occurs because of kinetic theory in which high energy molecules in a warmer region of liquid impinge on the molecules with greater momentum than molecules from a cold region. This leads to a migration of particles in the direction opposite the temperature gradient, from
warmer areas to cooler areas. Thermophoresis is of practical importance in a variety of industrial and engineering applications including aerosol collection (thermal precipitators), nuclear reactor safety, corrosion of heat exchangers, and micro contamination control. It has important current applications in the production of optical fibers. A modified chemical vapor deposition process is used to build up layers of glass (GeO$_2$ and SiO$_2$) by deposition of particles on the tube wall. Thermophoresis can be used in clean rooms to inhibit the deposition of small particles on electronic chips. As the dimensions of the circuit elements are reduced, the potential failures due to micro contamination by particle deposition increase.

2.3 FORMULATION OF THE PROBLEM

Let us consider a steady, laminar coupled heat and mass transfer by mixed convection flow in front of a stagnation point on a wedge embedded in porous medium. The fluid is assumed to be Newtonian, electrically conducting and its property variations due to temperature are limited to density and viscosity. The density variation and the effects of the buoyancy are taken into account in the momentum equation (Boussinesq's approximation) and the concentration of species far from the wall, $C_\infty$, is infinitesimally small. Let the x-axis be taken along the direction of the wedge and y-axis normal to it. A uniform transverse magnetic field of strength $B_0$ is applied parallel to the y-axis.

The chemical reaction is takes place in the flow and the effect of thermophoresis is being taken into account to help in the understanding of the mass deposition variation on the surface. Fluid suction or injection is imposed at the wedge surface, as shown in Figure 2.1. The effect of viscous dissipation and Joule heat are neglected on account of the fluid is finitely conducting. It
is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible.

Figure 2.1 Flow analysis along the wall of the wedge

Under these conditions, the basic governing boundary layer equation of momentum, energy and diffusion for mixed convection flow neglecting Joule’s heating and viscous dissipation under Boussinesq’s approximation including variable viscosity, Landau and Lifshitz (1962) can be simplified to the following equations:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{u}{\rho} \frac{\partial u}{\partial x} + \frac{v}{\rho} \frac{\partial u}{\partial y} &= \frac{1}{\rho} \frac{\partial}{\partial y} (\mu \frac{\partial u}{\partial y}) + u \frac{\partial U}{\partial y} - \left(\frac{\sigma B_0^2}{\rho} + \frac{v}{K}\right) (u - U) - \frac{F}{\sqrt{K}} (u^2 - U^2) \\
&+ (g \beta (T - T_\infty) + g \beta^* (C - C_\infty)) \sin \frac{\Omega}{2} \\
\frac{u}{\rho} \frac{\partial T}{\partial x} + \frac{v}{\rho} \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} \\
\frac{u}{\rho} \frac{\partial C}{\partial x} + \frac{v}{\rho} \frac{\partial C}{\partial y} &= D \frac{\partial^2 C}{\partial y^2} - \frac{\partial (V_T C)}{\partial y}
\end{align*}
\]
The boundary conditions are,

\[
\begin{align*}
  u &= 0, v = -v_0, T = T_w, C = C_w \text{ at } y = 0 \\
  u &= U(x), T = T_\infty, C = C_\infty \text{ at } y \to \infty
\end{align*}
\] (2.5) (2.6)

where \( D \) is the effective diffusion coefficient; \( K \) is the permeability of the porous medium. \( V_T = -k \frac{\nu \delta T}{T \delta y} \) is the thermophoretic velocity, where \( k \) is the thermophoretic coefficient and \( F \) is the empirical constant in the second order resistance. As neglecting the convective term, viscous term and setting \( F = 0 \) in equation (2.2) is reduced to the Darcy law, Bejan (1995). The fourth and fifth terms on the right-hand side of equation (2.2) stand for the first-order (Darcy) resistance and second-order (porous inertia) resistance, respectively.

The first step is to predict the pressure and velocity within the porous medium. The approach is commonly used for laminar flows and was first described by Darcy, who postulated that the pressure drop within the medium is due to viscous stress and is proportional to the velocity. However Darcy’s law is not valid for high velocity flows and a correction term must be included to take account of inertial effects. This term, known as the Forchheimer term, is a quadratic function of the velocity.

As in Kafoussias and Nanousis (1997), the following change of variables is introduced:

\[
\eta = \left( \frac{(1 + m)U}{2\nu x} \right) y, \psi = \left( \frac{2U\nu x}{1 + m} \right) f(x, \eta), \theta = \frac{T - T_\infty}{T_w - T_\infty} \text{ and}
\]

\[
\phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{2.7}
\]

Under this consideration, the potential flow velocity can be written as
\[ U(x) = A x^n, \quad \beta_1 = \frac{2m}{1+m} \] (2.8)

where \( A \) is a constant and \( \beta_1 \) is the Hartree pressure gradient parameter that corresponds to \( \beta_1 = \frac{\Omega}{\pi} \) for a total angle \( \Omega \) of the wedge.

The continuity equation (2.1) is satisfied by the stream function \( \psi(x, y) \) defined by

\[
u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \] (2.9)

The Equations (2.2) to (2.4) become

\[
\frac{\partial^3 f}{\partial \eta^3} = -f \frac{\partial^2 f}{\partial \eta^2} - \frac{2m}{1+m} (1 - \frac{\partial f}{\partial \eta})^2 - \frac{2}{1+m} \gamma_1 (\theta + N \phi) \sin \frac{\Omega}{2} + \frac{2x}{1+m} \left( \frac{\partial f}{\partial \xi} \frac{\partial^2 f}{\partial \eta \partial \xi} - \frac{\partial f}{\partial \xi} \frac{\partial^2 f}{\partial \eta^2} \right) + \frac{2x}{m+1} \left( \frac{\partial B^2}{\partial \eta} - 1 \right) + \frac{2}{m+1} \lambda \left( \frac{\partial f}{\partial \eta} - 1 \right) (2.10)
\]

\[
\frac{\partial^2 \theta}{\partial \eta^2} = -\Pr \frac{\partial \theta}{\partial \eta} + \frac{2 \Pr}{1+m} \frac{\partial f}{\partial \eta} \phi + \frac{2x}{1+m} \left( \frac{\partial f}{\partial \xi} \frac{\partial \theta}{\partial \eta} - \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \xi} \right) (2.11)
\]

\[
\frac{\partial^2 \phi}{\partial \eta^2} = -\Sc \left( \frac{f - \tau}{\partial \eta} \right) \frac{\partial \phi}{\partial \eta} + \frac{2 \Sc}{1+m} \frac{\partial f}{\partial \eta} \phi + \frac{2x \Sc}{1+m} \left( \frac{\partial f}{\partial \xi} \frac{\partial \phi}{\partial \eta} - \frac{\partial f}{\partial \eta} \frac{\partial \phi}{\partial \xi} \right) + \Sc \tau \frac{\partial^2 \theta}{\partial \eta^2} \phi (2.12)
\]

where the Local Grashof number \( (\text{Gr}_x) \), Local buoyancy parameter \( (\gamma_1) \), Sustentation parameter \( (N) \), Local Reynolds number \( (\text{Re}_x) \), Modified local Reynolds number \( (\text{Re}_k) \), Prandtl number \( (\text{Pr}) \), Forchheimer number \( (\text{F}_x) \), Schmidt number \( (\text{Sc}) \), Magnetic parameter \( (M^2) \), suction/injection
parameter ($S$), thermophoresis particle deposition parameter ($\tau$) and porous medium parameter ($\lambda$) are defined as

$$\text{Gr}_x = \frac{g\beta x^3 (T_w - T_\infty)}{v^2}, \gamma_1 = \frac{\text{Gr}_x}{\text{Re}_x^2}, \text{Re}_x = \frac{U x}{v}, \text{Re}_k = \frac{U \sqrt{K}}{v}, \text{Pr} = \frac{\nu}{\alpha}$$ (where $\alpha$ is the effective thermal diffusivity of the porous medium).

$$F_n = \frac{FU \sqrt{K}}{v},$$

$$\text{Sc} = \frac{v}{D}, \quad S = \nu_0 \sqrt{\frac{(1 + m)x}{2vU}}, \quad \tau = -\frac{k(T_w - T_\infty)}{T_s}$$ and $$\lambda = \frac{\alpha}{KA} \quad (2.13)$$

The boundary conditions can be written as

$$\eta = 0 : \frac{\partial f}{\partial \eta} = 0, \quad -\frac{f}{2} \left( 1 + \frac{x}{U} \frac{dU}{dx} \right) + x \frac{\partial f}{\partial x} = -\nu_0 \sqrt{\frac{(1 + m)x}{2vU}}, \quad \theta = 1, \phi = 1$$

$$\eta \rightarrow \infty : \frac{\partial f}{\partial \eta} = 1, \quad \theta = 0, \phi = 0 \quad (2.14)$$

where $v_0$ is the velocity of suction if $v_0 < 0$. $F_n$ is the dimensionless inertial parameter (Forchheimer number) and $\xi = k x^{-\frac{1}{2}}$. Kafoussias and Nanousis (1997), is the dimensionless distance along the wedge ($\xi > 0$).

The Equations (2.10) to (2.12) and boundary conditions (2.14) can be written as

$$\frac{\partial^3 f}{\partial \eta^3} + f \frac{\partial^2 f}{\partial \eta^2} - \frac{2x}{m+1} M^2 \xi^2 \left( \frac{\partial f}{\partial \eta} - 1 \right) + \frac{2}{1 + m} \gamma_1 (\theta + N \phi) \sin \frac{\Omega}{2}$$

$$- \frac{2}{m+1} \xi^2 \lambda \text{Pr} \left( \frac{\partial f}{\partial \eta} - 1 \right) - \frac{2}{m+1} \left( \frac{\partial f}{\partial \eta} - 1 \right) \left( \frac{\text{Re}_x}{\text{Re}_k^2} F_n + m \right) - \frac{2}{1 + m} \frac{\partial \theta}{\partial \eta} \frac{\partial^2 f}{\partial \eta^2}$$

$$= - \frac{1-m}{1+m} f' \left( \xi \frac{\partial f}{\partial \xi} \right) - f' \left( \xi \frac{\partial f}{\partial \xi} \right) \right)$$

$$\quad (2.15)$$
\[
\frac{\partial^2 \theta}{\partial \eta^2} + \text{Pr} f \frac{\partial \theta}{\partial \eta} - 2 \text{Pr} \frac{\partial f}{\partial \eta} = - \text{Pr} \frac{1-m}{1+m} \left[ \theta' \left( \xi \frac{\partial f}{\partial \xi} \right) - f' \left( \xi \frac{\partial \theta}{\partial \xi} \right) \right] \quad (2.16)
\]

\[
\frac{\partial^2 \phi}{\partial \eta^2} + \text{Sc} \left( f - \tau \frac{\partial \theta}{\partial \eta} \right) \frac{\partial \phi}{\partial \eta} - 2 \text{Sc} \frac{\partial f}{\partial \eta} - \text{Sc} \frac{\partial^2 \theta}{\partial \eta^2} \phi
= - \text{Sc} \frac{1-m}{1+m} \left[ \phi' \left( \xi \frac{\partial \phi}{\partial \xi} \right) - f' \left( \xi \frac{\partial \phi}{\partial \xi} \right) \right] \quad (2.17)
\]

\[\eta = 0: \quad \frac{\partial f (\xi, \eta)}{\partial \eta} = 0, \quad f(\xi, \eta) + \frac{1-m}{2} \xi \frac{\partial f (\xi, \eta)}{\partial \xi} = -S, \quad \theta(\xi, \eta) = 1, \phi(\xi, \eta) = 1 \]

\[\eta \to \infty: \quad \frac{\partial f (\xi, \eta)}{\partial \eta} = 1, \quad \theta(\xi, \eta) = 0, \quad \phi(\xi, \eta) = 0 \quad (2.18)\]

where the prime denotes partial differentiation with respect to \( \eta \), and the boundary conditions (2.18) remain the same. This form of the system is the most suitable for the application of the numerical scheme described below.

In this system of equations, it is obvious that the non-similarity aspects of the problem are embodied in the terms containing partial derivatives with respect to \( \xi \). This problem does not admit similarity solutions. Thus, with \( \xi \)-derivative terms retained in the system of equations, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. Formulation of the system of equations for the local non-similarity model with reference to the present problem will be discussed subsequently.

At the first level of truncation, the terms accompanied by \( \xi \frac{\partial}{\partial \xi} \) are small. This is particularly true when \( (\xi \ll 1) \), thus the terms with \( \xi \frac{\partial}{\partial \xi} \) on the
right-hand sides of Equations (2.15) – (2.17) are deleted to get the following systems of equations:

\[ f'' + ff'' = \frac{2}{m+1} M^2 \xi^2 (f' - 1) + \frac{2}{1+m} \gamma_1 (\theta + N\phi) \sin \frac{\Omega}{2} \frac{2}{m+1} \xi^2 \alpha \Pr (f' - 1) \]
\[ - \frac{2}{m+1} (f'' - 1)(\frac{\Re_x}{\Re_k} F_n + m) - \frac{2}{1+m} \theta' f f'' = 0 \]  \hspace{2cm} (2.19)

\[ \theta'' + \Pr \theta' - \frac{2\Pr}{1+m} f \theta' = 0 \]  \hspace{2cm} (2.20)

\[ \phi'' + \Sc (f - \tau \theta') \phi - \frac{2Sc}{1+m} \phi f' - Sc \tau \phi' = 0 \]  \hspace{2cm} (2.21)

where \( f' = \frac{\partial f}{\partial \eta}, f'' = \frac{\partial^2 f}{\partial \eta^2} \) and \( f''' = \frac{\partial^3 f}{\partial \eta^3} \)

with boundary conditions

\[ f'(\xi,0) = 0, \frac{(1+m)}{2} f(\xi,0) + \frac{1-m}{2} \xi \frac{\partial f(\xi,0)}{\partial \xi} = -S, \theta(\xi,0) = 1, \phi(\xi,0) = 1 \]
\[ f'(\xi,\infty) = 1, \theta(\xi,\infty) = 0, \phi(\xi,\infty) = 0 \]  \hspace{2cm} (2.22)

Equations (2.19) to (2.21) can be regarded as a system of ordinary differential equations for the functions \( f, \theta \) and \( \phi \) with \( \xi \) as a parameter for given pertinent parameters. The major physical quantities of interest are the local skin friction coefficient; the local Nusselt number and the local Sherwood number are defined by

\[ C_f = \frac{f''(\xi,0)}{Re_x^{\frac{1}{2}}} ; N_u = -\frac{\theta'(\xi,0)}{Re_x^{\frac{1}{2}}} \quad \text{and} \quad S_h = -\frac{\phi'(\xi,0)}{Re_x^{\frac{1}{2}}} \]  \hspace{2cm} (2.23)
This form of the system is the most suitable for the application of the numerical scheme described below.

### 2.4 Numerical Solution

The boundary layer over the wedge, subjected to a velocity of suction or injection, is described by the system of partial differential Equations (2.15) - (2.17), and its boundary conditions (2.18). In this system of equations $f(\xi, \eta)$ is the dimensionless stream function; $\theta(\xi, \eta)$ be the dimensionless temperature; $\phi(\xi, \eta)$ be the dimensionless concentration. It is obvious that the non-similarity aspects of the problem are embodied in the terms containing partial derivatives with respect to $\xi$. Thus, with $\xi$ derivative terms retained in the system of Equations (2.19) - (2.21), it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent stream wise locations through the $\xi$ derivatives, a locally autonomous solution, at any given stream wise location, cannot be obtained. In such a case, an implicit marching numerical solution scheme is usually applied preceding the solution in the $\xi$ direction, i.e., calculating unknown profiles at $\xi_{t+1}$ when the same profiles at $\xi_t$ are known. The process starts at $\xi = 0$ and the solution proceeds from $\xi_{t+1}$ to $\xi_t$ but such a procedure is time consuming.

However, when the terms involving $\frac{\partial f}{\partial \xi}, \frac{\partial \theta}{\partial \xi}, \frac{\partial \phi}{\partial \xi}$ and their $\eta$ derivatives are deleted, the resulting system of equations resembles, in effect, a system of ordinary differential equations for the functions $f$, $\theta$ and $\phi$ with $\xi$ as a parameter and the computational task is simplified. Furthermore a locally autonomous solution for any given $\xi$ can be obtained because the stream wise
coupling is severed. So, in this work, a modified and improved numerical solution scheme for local non-similarity boundary layer analysis is used. The scheme is similar to that of Minkowycz et al (1988), but it deals with the differential equations in lieu of integral equations. In this, each level of truncation, the governing coupled and nonlinear system of differential equations is solved by applying the common finite difference method, with central differencing, a tri-diagonal matrix manipulation, and an iterative procedure. The whole numerical scheme can be programmed and applied easily and has distinct advantages compared to that in Minkowycz et al (1988) with respect to stability, accuracy, and convergence speed. The details of this scheme are described in Kafoussias and Karabis (1996) and Kafoussias and Williams (1993).

To examine the behavior of the boundary layer over the wedge, numerical calculations were carried out for different values of the dimensionless parameters. The numerical results are shown in Figures 2.2 – 2.7 for the velocity, temperature, and concentration of the fluid along the wall of the wedge.

2.5 RESULTS AND DISCUSSION

Numerical computations are carried out for $0 \leq \tau, \ M^2 \leq 5; \ 0.01 \leq F_n \leq 0.2; 0.1 \leq \lambda \leq 3.0; -3.0 \leq \gamma_1 \leq 1.0; -1.5 \leq S \leq 3.0$. Typical velocity, temperature and concentration profiles are shown in following Figures for $Pr = 0.71$ and some values for the parameters $\gamma, M^2, \tau, \gamma_1, Sc, m, N$ and $\lambda$. The case $\gamma_1 \gg 1.0$ corresponds to pure free convection, $\gamma_1 = 1.0$ corresponds to
mixed convection and $\gamma_1 << 1.0$ corresponds to pure forced convection. Throughout this calculation consider $\gamma_1 = 1.0$ unless otherwise specified. The velocity, temperature and concentration profiles obtained in the dimensionless form are presented in the following Figures for $Pr = 0.71$ which represents air at temperature $20^0C$ and $Sc = 0.62$ which corresponds to water vapor that represents a diffusion chemical species of most common interest in air. Grashof number for heat transfer is chosen to be $Gr_x = 9$, since these values corresponds to a cooling problem, and Reynolds number $Re_x = 3.0$.

The computations have been carried out for various values of Magnetic parameter ($M^2$), Forchheimer number ($F_n$), Thermophoresis particle deposition parameter ($\tau$) and porous medium ($\lambda$). In the absence of diffusion equation, in order to validate our method, it is compared with steady state results of skin friction $f''(\xi,0)$ and rate of heat transfer $-\theta'(\xi,0)$ for various values of $\xi$ (Table 2.1) with those of Minkowycz et al (1988) and found them in excellent agreement.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$Minkowycz$ et al (1988)</th>
<th>Present works</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f''(\xi,0)$</td>
<td>$-\theta(\xi,0)$</td>
</tr>
<tr>
<td>0</td>
<td>0.33206</td>
<td>0.29268</td>
</tr>
<tr>
<td>0.2</td>
<td>0.55713</td>
<td>0.33213</td>
</tr>
<tr>
<td>0.4</td>
<td>0.75041</td>
<td>0.35879</td>
</tr>
<tr>
<td>0.6</td>
<td>0.92525</td>
<td>0.37937</td>
</tr>
<tr>
<td>0.8</td>
<td>1.08792</td>
<td>0.39640</td>
</tr>
<tr>
<td>1.0</td>
<td>1.24170</td>
<td>0.41106</td>
</tr>
<tr>
<td>2.0</td>
<td>1.92815</td>
<td>0.46524</td>
</tr>
<tr>
<td>10.0</td>
<td>5.93727</td>
<td>0.64956</td>
</tr>
</tbody>
</table>
The velocity and temperature profiles are shown in Figure 2.2. It is observed that the absence of diffusion equations, in order to ascertain the accuracy of our numerical results, the present study is compared with the available exact solution in the literature. The velocity profiles for $\xi$ are compared with the available exact solution of Minkowycz et al (1988), is shown in Figure 2.2. This shows that the agreements with the theoretical solution of velocity and temperature profiles are excellent.

![Graphs showing velocity and temperature profiles](image)

Minkowycz et al (1988)  
Present work

**Figure 2.2 Comparison of the effect of $\xi$ on velocity and temperature profiles**

Figure 2.3 depicts the dimensionless velocity, temperature and concentration profiles for different values of the Prandtl number (Pr). As the velocity of the fluid within boundary layer increases, the concentration remains constant and the temperature of the fluid inside the boundary layer decreases as Pr increases. All these physical behaviors are due to the combined effects of the uniform strength of the magnetic field, chemical reaction and thermophoresis.
Figure 2.3 Effect of Prandtl number on dimensionless velocity, temperature and concentration profiles

Figure 2.4 illustrates the dimensionless velocity, temperature and concentration profiles for different values of the Schmidt number. In the presence of uniform magnetic effects, viscosity and heat source, it is clear that the velocity and the temperature of fluid inside the boundary layer remain constant, whereas the concentration of fluid decreases with an increase of the Schmidt number. All these physical behaviors are due to the combined effect of the strength of magnetic field and viscosity.
The effects of thermophoretic parameter $\tau$ on velocity, temperature and concentration field are shown in Figure 2.5. It is observed that the velocity, temperature and concentration of the fluid decrease with increase of thermophoretic parameter. In particular, the effect of increasing the thermophoretic parameter is limited to increasing slightly the wall slope of the concentration profiles but decreasing the concentration. This is true only for small values of Schmidt number for which the Brownian diffusion effect is large compared to the convection effect. However, for large values of Schmidt number ($Sc > 100$) the diffusion effect is minimal compared to the convection effect and, therefore, the thermophoretic parameter is expected to alter the concentration boundary layer significantly. This is consistent with the work of Goren (1997) on thermophoresis of aerosol particles in flat plate boundary layer.
Figure 2.5 Thermophoretic effect on velocity, temperature and concentration profiles

Figure 2.6 presents typical profiles for velocity, temperature and concentration for different values of magnetic parameter. Due to the uniform suction effects, it is clearly shown that the velocity of the fluid increases and the temperature and concentration of the fluid slightly decrease with increase of the strength of magnetic field. The effects of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid and to reduce its temperature and concentration profiles. This result qualitatively agrees with the expectations, since magnetic field exerts retarding force on the mixed convection flow. Application of a magnetic field moving with the free stream has the tendency to induce a motive force which decreases the motion of the fluid and increases its boundary layer. This is accompanied by a decrease in the fluid temperature and concentration.
Figure 2.6  Magnetic effect on velocity, temperature and concentration profiles

Figure 2.7 shows the influence of the inertial parameter ($F_a$) on the dimensionless velocity, temperature and concentration profiles, respectively. It is observed that, the velocity increases as the inertial parameter (Forchheimer number) increases. The reason for this behavior is that the inertia of the porous medium provides an additional resistance to the fluid flow mechanism, which causes the fluid to move at a retarded rate with reduced temperature and concentration. These behaviors are shown in Figure 2.7. The decreasing of thickness of the concentration layer is caused by the direct action of suction at the wall of the surface. All these physical behaviors are the combined effect of thermophoresis particle deposition with suction at the wall of the surface.
Figure 2.8 shows the effect of the porosity parameter on the dimensionless velocity, temperature and concentration profiles, respectively. It is observed that the velocity increases as the porosity increases. The reason for this is that the suction of the wall of the wedge provides an additional effect to the fluid flow mechanism, which causes the fluid to move at a retarded rate with reduced temperature. These behaviors are shown in Figure 2.8. Also, it is observed that the concentration of the fluid is almost not affected with increase of the porosity parameter.
Figure 2.8  Porosity effects on velocity, temperature and concentration profiles

Figure 2.9 depicts the dimensionless velocity, temperature and concentration profiles for different values of buoyancy parameter. In the presence of uniform magnetic effect, it is seen that the velocity for free convection is more dominant to compare with the forced and mixed convection flow and there is no significant effects on temperature and concentration boundary layer.
Figure 2.9  Buoyancy effects on velocity, temperature and concentration profiles

Figure 2.10 illustrates the influence of the suction/injection parameter $S$ on the velocity, temperature and concentration profiles, respectively. The imposition of wall fluid suction for this problem has the effect of increasing the entire hydrodynamic and reduces the thermal and concentration boundary layers causing the fluid velocity to increase while decreasing its temperature and concentration for suction/injection. It is observed that the velocity of the fluid decreases with increase of injection ($S<0$). The decreasing of thickness of the concentration layer is caused by two effects; (i) the direct action of suction, and (ii) the indirect action of suction causing a thicker thermal boundary layer, which corresponds to lower temperature gradient, a consequent increase in the thermophoretic force and higher concentration gradient.
Figure 2.10 Suction/injection effects on velocity, temperature and concentration profiles

Table 2.2 Analysis for skin friction and rate of heat and mass transfer

<table>
<thead>
<tr>
<th>$f' (0)$</th>
<th>$\theta' (0)$</th>
<th>$\phi' (0)$</th>
<th>Parameter</th>
<th>Thermophoretic</th>
<th>Magnetic</th>
<th>Suction</th>
<th>Forced convection</th>
<th>Mixed convection</th>
<th>Free convection</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.034338</td>
<td>-2.580389</td>
<td>-5.260410</td>
<td>$\tau = 1.0$</td>
<td>$\tau = 2.0$</td>
<td>$\tau = 3.0$</td>
<td>$M^2 = 0.1$</td>
<td>$M^2 = 5.0$</td>
<td>$M^2 = 10.0$</td>
<td></td>
</tr>
<tr>
<td>6.034184</td>
<td>-2.580390</td>
<td>-6.804620</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.034067</td>
<td>-2.580391</td>
<td>-8.366967</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.176755</td>
<td>-5.772941</td>
<td>-3.716062</td>
<td>$S = -1.5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.335917</td>
<td>-5.791495</td>
<td>-3.732215</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.267194</td>
<td>-5.804222</td>
<td>-3.744466</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.899134</td>
<td>-7.543495</td>
<td>-4.788675</td>
<td>$S = -1.0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.665340</td>
<td>-9.336595</td>
<td>-5.882326</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.459151</td>
<td>-11.141771</td>
<td>-6.988321</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.284197</td>
<td>-5.789842</td>
<td>-3.757063</td>
<td>$\gamma_1 = 0.1$</td>
<td>$\gamma_1 = 1.0$</td>
<td>$\gamma_1 = 5.0$</td>
<td>Forc</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.357326</td>
<td>-5.791082</td>
<td>-3.758139</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.681770</td>
<td>-5.796572</td>
<td>-3.762913</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From the Table 2.2, it is observed that the skin friction increases and the rate of heat and mass transfer decrease with increase of magnetic and suction parameters respectively, whereas the skin friction and the rate of mass transfer decrease and the rate of heat transfer increases with increase of thermophoretic parameter. It is noted that the skin friction and the rate of mass transfer of forced convection flow is more significant to compare with free and mixed convection whereas the rate of heat transfer of free convection flow is faster when compare with the other two convection flows.

2.6 CONCLUSION

In this chapter, the effect of thermophoresis particle deposition on non-Darcy mixed convection boundary layer flow over a porous wedge in the presence of suction or injection has been studied numerically. In mixed convection regime, the concentration boundary layer thickness decreases with increase of the thermophoretic parameter. So, the thermophoretic effects in the presence of magnetic field have a substantial effect on the flow field and, thus, on the heat and mass transfer rate from the sheet to the fluid.

Thermophoresis is an important mechanism of micro-particle transport due to a temperature gradient in the surrounding medium and has found numerous applications, especially in the field of aerosol technology. It is expected that this research may prove to be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes.