4.1 Introduction

As we have seen in the previous Chapters, electromagnetically-induced transparency (EIT) [1, 2, 3] exploiting quantum coherence has proved to be a powerful tool to control the propagation of light through a medium interacting with another strong coherent field. This has opened a very promising approach to quantum information processing by allowing a procedure for the storage and retrieval of light pulses in a medium by a simple control of the excitation in the medium, as proposed by Fleischhauer et al. [4]. The method has now been experimentally demonstrated in ultracold Na, in hot Rb vapor, and more recently, a storage time of several seconds has been obtained in solid Pr$^{3+}$:Y$_2$SiO$_5$ at liquid helium temperature [5, 6, 7]. This procedure provides, in principle, for a mechanism to transfer quantum state information from the light pulse to an atomic medium and vice versa, and thus enables communication of quantum information between different units of a quantum network. Thus, understanding the precise conditions, the fidelity and the experimental limitations of this procedure are of prime importance. Some recent papers [8, 9, 10, 11] have examined these issues, and this Chapter is a contribution [12] in this direction.

The basic idea of quantum storage and retrieval can be understood in terms of interaction of light with three-level $A$ type atoms. Strong "writing" (or control) and weak "signal" (or probe) light pulses propagate in the atomic medium and excite a spatial profile of a long-lived coherence between the lower states of the. Under the perfect EIT conditions of two-photon resonance, a stationary eigenstate exists for the atom plus resonant light field, where the atom is in a dark, coherent superposition of the lower states. The signal pulse enters the medium, adiabatically slows down with decreasing writing field, and disappears, transferring all its properties to the medium. During the storage time, information about the amplitude of the signal field is contained in the population amplitudes defining the atomic dark states.
Information about the mode vector of the signal field is contained in the relative phase between different atoms in the medium. Sending a strong reading pulse into the medium results in its Raman scattering off the atomic coherence and generation of a retrieved pulse with the same carrying frequency, the same profile and quantum statistics, and propagating in the same direction as the signal pulse.

In order to describe the precise questions tackled in our work, we present a brief review of the basic ideas. One considers the propagation of light in a medium consisting of three-level A-type atoms as shown in Fig. 4.1, in the presence of an additional laser exciting the atoms. The light pulses to be stored and retrieved, termed ‘signal’ or ‘probe’, are nearly resonant to the energy difference between levels $|a\rangle$ and $|b\rangle$ (see Fig. 4.1), and thus the medium is opaque to the signal under normal conditions. Another laser nearly resonant between levels $|a\rangle$ and $|c\rangle$, of intensity usually much larger than the signal, can coherently excite the system. This is termed as the ‘control’ or ‘coupling’ laser, which in coherent action with the signal on the atomic states induces a quantum interference effect that allows the signal pulse to enter the medium. This phenomenon observed under the condition of two-photon resonance is termed as EIT mentioned above [13]. The signal propagates in the medium as a polariton, which is a coupled mode of the electromagnetic wave and the atomic polarization in the so-called ‘dark state’. In the simplest terms, one considers a single atom in interaction with a signal photon and a time-dependent control field. Then the dark-state polariton (DSP) can be regarded as in a coherent superposition of two states: the atom in level $|b\rangle$ with one ‘signal photon’ and the atom in level $|c\rangle$ with zero signal photon. The ratio of the electric fields of the two lasers determines the mixing angle $\theta$ between the two states. The speed $v_g$ of the signal inside the medium, determined from the atomic polarization, can be made orders of magnitude lower than its speed outside the medium. If the intensity of the control laser is reduced when the pulse is in the medium, the pulse slows down and can get arrested as the control field is made zero. In this process the electromagnetic energy is completely transferred to the atomic ‘spin’ excitation from level $|b\rangle$ to level $|c\rangle$. During the storage time, information about the amplitude of the signal field is contained in the population amplitudes defining the atomic dark states. Information about the mode vector of the signal field is contained in the relative phase between different atoms in the medium. Amazingly, the signal pulse can be
retrieved by switching on the control laser, a process which makes the medium transparent to the signal again and transfers all the mapped information from the atoms in state $|c\rangle$ back to the electromagnetic field.

![Diagram of a three-level A scheme for EIT](image)

Figure 4.1: Three-level A scheme for EIT

In order to explain the process of storage and retrieval of a light pulse in terms of the DSP [4], we write the one atom-one photon wavefunction in the dark state,

$$|\Psi_{DSP}\rangle = \cos \theta(t)|b, 1\rangle - \sin \theta(t)|c, 0\rangle,$$

(4.1)

with

$$\tan \theta(t) = \frac{g_0}{|\Omega_C(t)|}, \quad \dot{\theta} = \frac{-g_0\dot{\Omega}_C}{g_0^2 + |\Omega_C|^2},$$

(4.2)

where $g_0$ denotes (half of) the Rabi frequency of a signal photon, and $\Omega_C(t)$ is (half of) the Rabi frequency of the control laser. In the state $|x, n\rangle$, $x$ denotes the atomic level and $n$ denotes the signal photon number in $|x\rangle$. The control field is treated classically.

On incorporating the bulk polarization associated with the above state in the calculation of the dielectric constant for the medium, one obtains the group velocity of the signal photon as [14]

$$v_g \approx \frac{c}{|\Omega_C|^2 + g_0^2 N},$$

(4.3)

where $c$ is the speed of light in vacuum, and $N$ is the number of atoms in the interaction volume. Under the assumption of adiabaticity, one can now easily see that as the control
field is varied, say, from some value $\Omega_0 (\gg g_0)$ to zero, the DSP becomes purely $|c, 0\rangle$, corresponding to storage, and the group velocity goes to zero. Similarly, when the control field is varied from zero to $\Omega_0$, the state goes back to $|b, 1\rangle$ with the energy getting transferred to the photon of the signal field. However, the $\Lambda$-system has two other eigenstates which involve the upper level $|a, 0\rangle$ as well. As the parameters of the Hamiltonian are varied, the quantum mechanical principles require that the system gets in a superposition of all the allowed states. The general wisdom is that the propagation of the signal field inside the medium can be controlled by varying the control field smoothly [4, 5, 8]. This ensures an adiabatic change within the dark state, without allowing it to mix with the other eigenstates to which the transitions are allowed. The rate of change of the control Rabi frequency $\Omega_C$ or the mixing angle $\theta$ should obey the adiabatic condition

$$|\dot{\theta}| \ll |\omega^\pm - \omega^0|,$$  \hspace{1cm} (4.4)

where $\dot{\theta}$ is the same as in Eq. (4.2), and

$$\omega^\pm = \left(\frac{\Delta \pm \sqrt{\Delta^2 + \Omega_{\text{eff}}^2}}{2}\right),$$  \hspace{1cm} (4.5)

$$\omega^0 = \Delta$$  \hspace{1cm} (4.6)

are the eigenvalues of the Hamiltonian in the dressed-state basis, with $\Omega_{\text{eff}} = \sqrt{g_0^2 + |\Omega_C|^2}$, and the detuning of the two fields is $\Delta_P = \Delta_C \equiv \Delta$. For resonant fields, with $\Delta = 0$, Eq. (4.4) simplifies to

$$2 \frac{g_0 \dot{\Omega}_C}{\Omega_{\text{eff}}^3} \ll 1.$$  \hspace{1cm} (4.7)

This Chapter deals with the analysis of this adiabatic transfer process, which is obviously a key to the scheme.

A detailed study of the adiabatic transfer described above has been done by Fleischhauer and Lukin [8]. Their analysis focuses on a collective polariton, which is a coupled mode of the electromagnetic excitation and a collective atomic ‘spin’ excitation [15], corresponding to the many-atom DSP state. The mixing angle $\theta_N(t)$ in this analysis is given by the relation

$$\tan \theta_N(t) = g_0 \sqrt{N}/\Omega_C(t),$$

$N$ being the number of atoms in the interaction volume. This permits only a small range of variation of the mixing angle – a value of $\theta_N \rightarrow 0$, requiring $\Omega_C^2 \gg g_0^2 N$, puts an unrealistically high demand on the control laser power, not found
necessary in any of the successful experiments. We argue that the adiabatic transfer issue arises at the level of a single atom, and the mixing angle $\theta(t)$ we have introduced in Eq. (4.2) (or its variant for an $n$-photon signal pulse) is appropriate for such a treatment. The group velocity of the signal, of course, emerges from the bulk response of the medium. Fleischhauer and Lukin further assume that for a weak signal field, the atomic populations in levels $|a\rangle$ and $|c\rangle$ are negligible. With this assumption, one needs to consider only the three off-diagonal elements of the atomic density matrix, and the system becomes analytically tractable. The non-adiabatic effects in this analysis come from the transfer of the many-atom DSP state to other states involving only levels $|b\rangle$ and $|c\rangle$. The neglect of excitation to level $|a\rangle$ does not seem justified as may be seen by perusal of the equations for the amplitudes, and hence this assumption is a drawback of the analysis in [8].

Matsko et al. [9] have also worked on the same problem. They have numerically solved the full set of density matrix equations for the single-atom case. Their solutions indicate that there is no need for a slow variation of the control field for adiabatic passage in the storage and retrieval of the signal pulse in an optically thick vapor. An abrupt change of the control field gives almost the same result as that for a slow change. In this calculation, the physical factors responsible for the adiabatic transfer are not identified. In a recent paper, Shakhmuratov et al. [10] have argued that this happens because an instantaneous change of the control field does not affect the atomic excitation amplitude. On the other hand, any change of the amplitude of the atomic state — for example, by rf pulses — causes a proportional change of the signal field amplitude, without changing its group velocity and duration. But in [10], this conclusion is arrived at using the effective atom-signal coupling $g_0\sqrt{N}$ which is orders of magnitude larger than $\Omega_C$. Under this condition of $g_0\sqrt{N} \gg \Omega_C$, it is easy to see also in our analysis that the non-adiabatic effects would be negligible.

For realistic quantum information storage, the storage time is bounded by the lower-state (Raman) coherence lifetime, and thus one must complete at least one storage and retrieval operation within this time. This puts a practical lower bound on how slowly the control field can be turned off and on. In the experiments, the control field is not strictly changed adiabatically. Liu et al. [5] have noted that the adiabaticity criteria such as in Eq. (4.4) can be considerably relaxed.
In view of these results, we feel that a re-examination of the issues involved in adiabatic transfer is called for. In this Chapter we adopt a wavefunction approach to the problem. Most of the analysis is done by considering a single three-level atom in interaction with a single-mode signal laser field, treated quantum mechanically, and a classical control field. Such an analysis with dissipative effects is parallel to the density matrix approach adopted by Matsko et al. [9]. In the wavefunction approach, the diagonal and the off-diagonal elements are put on the same footing. We find that this approach provides a fresh insight on the key issues of adiabaticity and dissipation, and allows a quantum treatment of the signal field.

For the consideration of propagation of pulses in a medium, one clearly needs an analysis with many electromagnetic modes and many atoms. However, since here one is interested in the propagation of the signal field in the linear regime, one only needs the linear response of the medium to the signal field [14]. For this purpose, the above analysis suffices and one can deal with an arbitrary laser pulse by simple superposition using the frequency-dependent refractive index. It should be noted that these approaches, being based on the response of a single atom, do not carry over to the many-atom case, as was pointed out by Fleischhauer and Lukin [8]. The enlarged Hilbert space of many atoms allows for a hierarchy of dark states for different number of photons and makes the adiabaticity issue a more complex one.

The analysis of non-adiabatic effects on a single atom is of direct interest to other propositions made in the context of atoms trapped in or passing through electromagnetic cavities. These are generation of Fock states in the cavity [16], implementation of two qubit quantum gates [17] and quantum state transfer among nodes of a network [18]. All these schemes require an adiabatic transfer through the atomic dark state of the kind described above. In [16], for example, two atoms inside a cavity are considered to be coupled to a single mode of the cavity, which entangles them and allows for transfer of quantum states between them. Two lasers with Rabi frequencies $\Omega_1$ and $\Omega_2$ shine individually on the two atoms. The dark state is the superposition $[\Omega_1 g_0 |b, c, 0\rangle + \Omega_2 g_0 |c, b, 0\rangle - \Omega_1 \Omega_2 |b, b, 1\rangle]$. By changing the ratio $\Omega_1 / \Omega_2$ from large to small, one can go from $|b, c, 0\rangle$ to $|c, b, 0\rangle$ adiabatically.

This Chapter is organized as follows. In Sec. 4.2, we first study the problem of an atomic system in a $\Lambda$-configuration interacting with a signal field and a time-dependent control field, ignoring any dissipative effects. The non-adiabatic effects are large for small signal
fields though the situation becomes better as the signal strength is increased. The reason is that the time-dependent effects come through $\dot{\theta} = d\theta/dt$, the rate of variation of the mixing angle defined in Eq. (4.2). It is easily seen that $|\dot{\theta}|$ becomes large as $\Omega_C(t)$ becomes small, especially when $g_0$ or the signal strength is small. Since during storage as well as retrieval, $\Omega_C(t)$ is made zero, the non-adiabatic effects are large, and the problem cannot be treated perturbatively. Accordingly, we study the problem numerically.

Next, in Sec. 4.3, we analyze the effect of dissipation by allowing for the possibility of spontaneous emission, which is inevitable in any realistic system. Since spontaneous decay sends the system to the lower levels, one might expect that dissipation would mitigate the non-adiabatic effects. We study the impact of dissipation using again the wavefunction formalism. In this method, the quantum evolution of the wavefunction is interrupted by spontaneous transitions of the state to the lower levels $|b\rangle$ and $|c\rangle$. Spontaneous decays are governed by a stochastic Poissonian process with a rate which we take to be the width of the level $|a\rangle$. Though dissipation in quantum systems is naturally incorporated in the density matrix formalism, we believe that the wavefunction treatment is a fair approximation, and it has two distinct advantages. First, the physical picture of the system evolution and the role of dissipation is rather transparent. Second, there is a numerical simplification of solving only three coupled time-dependent differential equations.

In Sec. 4.4, we present our results on the dynamics of the fidelity of the storage and the retrieval process. Different rates of variation of the control field at different signal strengths are explored, each in the absence and the presence of dissipation. Finally, in Sec. 4.5, we present our conclusions. Appendix 4.A, we compare the wavefunction approach with the density matrix approach and point out the nature of the approximation made in our analysis. In Appendix 4.B, we discuss the many atom-many photon case and make a few remarks on the adiabaticity issue, pointing out specifically the limitations of neglecting excitation to level $|a\rangle$. The results of this Chapter are given in Ref. [12].

## 4.2 Formulation with a Single Isolated Atom

We now consider the $\Lambda$ atomic system as shown in Fig. 4.1, which interacts with the fields of the signal and the control lasers. The Hamiltonian of the isolated atom-signal field system
is $H_0 + H_I$, with the non-interacting part as:

$$H_0 = \hbar \omega_p \left( a_p^+ a_p + \frac{1}{2} \right) + \sum_x E_x |x \rangle \langle x|, \quad (4.8)$$

where $a_p$ denotes the annihilation operator for the signal field in a particular mode of frequency $\omega_p$ of the signal, and $x = a, b, c$. In the rotating-wave approximation, the time-dependent interaction Hamiltonian of the laser fields with the atomic system (in one dimension) is

$$H_I = \hbar g_0 \left[ |a \rangle \langle b| a_p + a_p^+ |b \rangle \langle a| \right] - \hbar \left[ \Omega_C(t) e^{-i\omega_C t} |a \rangle \langle c| + \Omega_C^*(t) e^{i\omega_C t} |c \rangle \langle a| \right], \quad (4.9)$$

where $\omega_C$ is the frequency of the control field, $g_0$ is half of the Rabi frequency for a signal photon as before,

$$\hbar g_0 = \mu_{ab} \sqrt{\frac{2 \hbar \omega_p}{\epsilon_0 V_0}}, \quad (4.10)$$

with $\mu_{ab}$ being the dipole moment between levels $|a \rangle$ and $|b \rangle$, $V_0$ the interaction volume, and $\epsilon_0$ the permittivity of vacuum. It is assumed that only transitions $|a \rangle \rightarrow |b \rangle$ and $|a \rangle \rightarrow |c \rangle$ are dipole-allowed. As before, the control field with a half-Rabi frequency $\Omega_C$ is treated classically.

Consider an $n$-photon quantum field as the signal to be stored and retrieved. The wavefunction of the one atom+$n$-photon system can be expressed in general as

$$|\Psi(t)\rangle = \sum_n \left[ A_n(t) e^{-i\omega_{xn} t} |a, n \rangle + B_n(t) e^{-i\omega_{bn} t} |b, n \rangle + C_n(t) e^{-i\omega_{cn} t} |c, n \rangle \right]. \quad (4.11)$$

Here

$$\hbar \omega_{xn} = E_x + \left( n + \frac{1}{2} \right) \hbar \omega_p. \quad (4.12)$$

For this one-atom case, the Schrödinger equation gives us the time evolution of the wavefunction:

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = (H_0 + H_I) |\Psi(t)\rangle \quad (4.13)$$

which yields

$$i\hbar \sum_n |n \rangle \left[ \frac{dA_n}{dt} e^{-i\omega_{xn} t} |a \rangle + \frac{dB_n}{dt} e^{-i\omega_{bn} t} |b \rangle + \frac{dC_n}{dt} e^{-i\omega_{cn} t} |c \rangle \right]$$

$$= \sum_n \left[ A_n(t) e^{-i\omega_{xn} t} (\hbar g \sqrt{n+1}|1, n \rangle + \hbar \Omega_C^*(t) e^{i\omega_C t} |n \rangle |c \rangle \right)$$

$$+ B_n(t) e^{-i\omega_{bn} t} \hbar g \sqrt{n} |n-1 \rangle |a \rangle + C_n e^{-i\omega_{cn} t} (\hbar \Omega_C^*(t) e^{i\omega_C t} |n \rangle |a \rangle). \quad (4.14)$$
Equating coefficients of $|n⟩⟨x|$ with $x = a, b, c$ from both sides, we obtain the closed set of equations of motion for the coefficients as

$$\frac{idA_n}{dt} = g_n e^{-iΔst}B_{n+1}(t) - Ω_C(t)e^{-iΔc't}C_n(t);$$

$$\frac{idB_{n+1}}{dt} = g_n e^{iΔst}A_n(t),$$

$$\frac{idC_n}{dt} = -Ω_C^*(t)e^{iΔc't}A_n(t).$$

Here $g_n = g_0 \sqrt{n + 1}$ denotes the (half of) the Rabi frequency of the $n$-photon signal field, and $Δ_S ≡ ω_p - ω_{ab}$ and $Δ_C ≡ ω_c - ω_{ac}$ denote, respectively, the detunings of the two fields.

For the analysis of the non-adiabatic effects, it is convenient to work with the vector $X$ with components

$$a_n = A_n,$$

$$b_n = e^{-iΔst}B_{n+1},$$

$$c_n = e^{-iΔc't}C_n.$$  \hfill (4.16)

Then $X$ obeys the equation

$$\frac{idX}{dt} = H(t) X(t),$$ \hfill (4.17)

where

$$H(t) = \begin{pmatrix} 0 & g_n & -Ω_C(t) \\ g_n & Δ_S & 0 \\ -Ω_C^*(t) & 0 & Δ_C \end{pmatrix}. \hfill (4.18)$$

We write the solution of the above equation in terms of the instantaneous eigenvalues and eigenvectors:

$$H(t) |u_k(t)⟩ = ℏλ_k(t) |u_k(t)⟩, \quad k = 1, 2, 3. \hfill (4.19)$$

At two-photon resonance, i.e., with $Δ_S = Δ_C = Δ$, the instantaneous eigenvalues are

$$λ_1 = Δ, \quad λ_2 = \frac{Δ}{2} + Ω_R(t), \quad λ_3 = \frac{Δ}{2} - Ω_R(t),$$ \hfill (4.20)

where

$$Ω_R(t) = \frac{\sqrt{Δ^2 + 4Ω_{eff}^2(t)}}{2},$$

$$Ω_{eff}(t) = \sqrt{g_n^2 + |Ω_C(t)|^2}. \hfill (4.21)$$
The instantaneous eigenvectors of the atomic-field system are

\[
|u_1(t)\rangle = \cos \theta(t)e^{i\phi} |b, n + 1\rangle + \sin \theta(t) |c, n\rangle,
\]
\[
|u_2(t)\rangle = \cos \frac{\psi(t)}{2} |a, n\rangle + \sin \frac{\psi(t)}{2} \left[ \sin \theta(t) |b, n + 1\rangle - \cos \theta(t)e^{-i\phi} |c, n\rangle \right],
\]
\[
|u_3(t)\rangle = -\sin \frac{\psi(t)}{2} |a, n\rangle + \cos \frac{\psi(t)}{2} \left[ \sin \theta(t) |b, n + 1\rangle - \cos \theta(t)e^{-i\phi} |c, n\rangle \right].
\] (4.22)

Here \(\phi\) is some arbitrary constant phase of the control field, and

\[
\tan \theta(t) = \frac{g_n}{\Omega_{\text{C}}(t)},
\]
(4.23)

\[
\tan \psi(t) = \frac{\Omega_{\text{eff}}(t)}{\Delta/2}.
\]
(4.24)

The eigenstate \(|u_1(t)\rangle\) is termed the 'dark state' as it does not contain the \(|a, n\rangle\) level. Now we can expand the solution in terms of these eigenvectors as

\[
|\Psi(t)\rangle = \sum_k D_k(t)e^{-i \int_0^t \lambda_k(t')dt'} |u_k(t)\rangle.
\] (4.25)

The time-dependent coefficients \(D_m(t)\) obey the following equation:

\[
\frac{dD_m(t)}{dt} + D_m(t)\langle u_m(t)|\dot{u}_m(t)\rangle = -\sum_{k \neq m} D_k(t)\langle u_m(t)|\dot{u}_k(t)\rangle e^{-i \int_0^t (\lambda_k(t') - \lambda_m(t'))dt'}.
\] (4.26)

By making a change of variable to

\[
V_m(t) = D_m(t)e^{i \int_0^t \beta_m(t')dt'}, \quad i\beta_m(t') = \langle u_m(t)|\dot{u}_m(t)\rangle,
\] (4.27)

the evolution of \(V_m(t)\) is obtained as

\[
\frac{dV_m(t)}{dt} = -\sum_{k \neq m} V_k(t)\langle u_m(t)|\dot{u}_k(t)\rangle e^{-i \int_0^t \lambda_{km}(t')dt'},
\] (4.28)

where

\[
\lambda_{km} = \lambda_{km} + \beta_{km},
\]
\[
\lambda_{km} = \lambda_k - \lambda_m,
\]
\[
\beta_{km} = \beta_k - \beta_m.
\] (4.29)

The above Eqs. 4.28 can be expressed in matrix form as

\[
\begin{pmatrix}
\dot{V}_1(t) \\
\dot{V}_2(t) \\
\dot{V}_3(t)
\end{pmatrix} = -\begin{pmatrix}
0 & \langle u_1|\dot{u}_2\rangle e^{-i \int_0^t \lambda_{21}dt'} & \langle u_1|\dot{u}_3\rangle e^{-i \int_0^t \lambda_{31}dt'} \\
\langle u_2|\dot{u}_1\rangle e^{-i \int_0^t \lambda_{12}dt'} & 0 & \langle u_2|\dot{u}_3\rangle e^{-i \int_0^t \lambda_{32}dt'} \\
\langle u_3|\dot{u}_1\rangle e^{-i \int_0^t \lambda_{13}dt'} & \langle u_3|\dot{u}_2\rangle e^{-i \int_0^t \lambda_{23}dt'} & 0
\end{pmatrix} \begin{pmatrix}
V_1(t) \\
V_2(t) \\
V_3(t)
\end{pmatrix}.
\]
Then an exact solution of $V_m(t)$ would be

$$V_m(t) = V_m(0) - \sum_{k \neq m} \int_0^t \langle u_m(t') | \dot{u}_k(t') \rangle e^{-i \int_0^{t'} \lambda_{km}(t'') dt''} \, dt' \, V_k(t'). \quad (4.30)$$

Note that the assumption of adiabaticity implies that the coefficients $D_{ms}$ or $V_{ms}$ are independent of time. The time-variation of these coefficients is governed by the terms $\langle u_m(t) | \dot{u}_k(t) \rangle$, which essentially give rise to non-adiabatic effects. From the eigenvectors in Eqs. (4.22), we derive the following:

$$\langle u_1 | \dot{u}_2 \rangle = -\langle u_2 | \dot{u}_1 \rangle^* = \dot{\theta}(t) \sin \frac{\psi(t)}{2} e^{-i \psi},$$

$$\langle u_1 | \dot{u}_3 \rangle = -\langle u_3 | \dot{u}_1 \rangle^* = \dot{\theta}(t) \cos \frac{\psi(t)}{2} e^{-i \psi},$$

$$\langle u_2 | \dot{u}_3 \rangle = -\langle u_3 | \dot{u}_2 \rangle^* = -\frac{\dot{\psi}(t)}{2}, \quad (4.31)$$

where

$$\dot{\theta}(t) = -\frac{g_n}{g_n^2 + |\Omega_C(t)|^2} \frac{d\Omega_C}{dt}, \quad (4.32)$$

$$\dot{\psi}(t) = \frac{4 \Delta \Omega_C(t)}{\sqrt{g_n^2 + |\Omega_C(t)|^2} \left[ \Delta^2 + 4(g_n^2 + |\Omega_C(t)|^2) \right]} \frac{d\Omega_C}{dt}. \quad (4.33)$$

All $\beta_k$s are zero, and therefore

$$\beta_{km} = \beta_k - \beta_m = 0,$$

$$\lambda_{km} = \lambda_{km} = \lambda_k - \lambda_m. \quad (4.34)$$

Then, finally we can express the coefficients $A_n(t), B_{n+1}(t), C_n(t)$ of our initial basis of the bare states in (4.11) in terms of these solutions as

$$A_n(t) = \frac{1}{\sqrt{2}} \left( V_2(t) e^{-i \int_0^t \lambda_1(t') dt'} - V_3(t) e^{-i \int_0^t \lambda_2(t') dt'} \right), \quad (4.35)$$

$$B_{n+1}(t) = V_1(t) e^{-i \int_0^t \lambda_1(t') dt'} \cos \theta(t) + \frac{\sin \theta(t)}{\sqrt{2}} \left( V_2(t) e^{-i \int_0^t \lambda_2(t') dt'} + V_3(t) e^{-i \int_0^t \lambda_3(t') dt'} \right), \quad (4.36)$$

$$C_n(t) = V_1(t) e^{-i \int_0^t \lambda_1(t') dt'} \sin \theta(t) - \frac{\cos \theta(t)}{\sqrt{2}} \left( V_2(t) e^{-i \int_0^t \lambda_2(t') dt'} + V_3(t) e^{-i \int_0^t \lambda_3(t') dt'} \right). \quad (4.37)$$

From the above equations, it is quite evident that the non-adiabatic perturbation, which is proportional to $\dot{\theta}$, becomes very large in the storage and retrieval process as $\Omega_C(t)$ becomes zero. Thus we solve Eqs. (4.28) for $V_m$s numerically.
4.2-1 Relevant equations for $\phi = 0$ and $\Delta = 0$

For simplicity, we take the phase of the control field, $\phi = 0$ and the optical detuning, $\Delta = 0$. The latter would imply that

$$\psi = \frac{\pi}{2},$$

(4.38)

and thus

$$\dot{\psi} = 0.$$ 

(4.39)

Also, $\Omega_R(t) = \Omega_{\text{eff}}(t) = \sqrt{g_n^2 + |\Omega_C(t)|^2}$. As a result, the eigenvalues in Eq. (4.20) become

$$\lambda_1 = 0,$$

$$\lambda_2 = \Omega_{\text{eff}},$$

$$\lambda_3 = -\Omega_{\text{eff}},$$

(4.40)

and the energy differences between the corresponding states relate to

$$\lambda_{12} = -\lambda_{21} = -\Omega_{\text{eff}},$$

$$\lambda_{13} = -\lambda_{31} = \Omega_{\text{eff}},$$

$$\lambda_{23} = -\lambda_{32} = 2\Omega_{\text{eff}}.$$ 

(4.41)

Incorporating these conditions in Eq. (4.29), we obtain the following differential equations for the non-adiabatic coefficients of the instantaneous eigenvectors:

$$\frac{dV_1(t)}{dt} = -\frac{\dot{\theta}(t)}{\sqrt{2}} \left( V_2(t) e^{-i\int_0^t \Omega_{\text{eff}}(t') dt'} + V_3(t) e^{i\int_0^t \Omega_{\text{eff}}(t') dt'} \right),$$ 

(4.42)

$$\frac{dV_2(t)}{dt} = \frac{\dot{\theta}(t)}{\sqrt{2}} V_1(t) e^{i\int_0^t \Omega_{\text{eff}}(t') dt'},$$

(4.43)

$$\frac{dV_3(t)}{dt} = \frac{\dot{\theta}(t)}{\sqrt{2}} V_1(t) e^{-i\int_0^t \Omega_{\text{eff}}(t') dt'}.$$ 

(4.44)

This leads to considerable numerical simplification.

4.2-2 Storage of a photon

We first consider the storage of a signal pulse that is in the medium due to an initial non-zero control of $\Omega_C(0) = \Omega_0$. The initial state of the atom-signal system is taken to be the dark state (with zero eigenvalue),

$$|\Psi(0)\rangle = \cos \theta(0) |b, n + 1\rangle + \sin \theta(0) |c, n\rangle = |u_1(0)\rangle,$$

(4.45)
where
\[
\theta(0) = \tan^{-1}\left( \frac{g_0}{\Omega_0} \right).
\] (4.46)

The signal pulse is to be stored by making the control field \( \Omega_C(t) \) zero. A suitable form for the control pulse (see Fig. 4.2), also used in Ref. [8], is
\[
\Omega_C(t) = \Omega_0 \left[ 1 - \tanh(rt) \right].
\] (4.47)

Here \( r \) basically decides the rate at which \( \Omega_C(t) \) falls off to zero. As seen from Fig. 4.2, for different values of \( r \) in the range 0.1 to 0.5, \( \Omega_C(t) \) falls off smoothly for low values and then more sharply for higher values of \( r \).

From Eq. (4.32), we can derive
\[
\dot{\theta}(t) = -\frac{g_0 \Omega_0 r}{g_0^2 \cos^2(rt) + \Omega_0^2 [\cosh(rt) - \sinh(rt)]^2}.
\] (4.48)

The adiabatic evolution of the above state leads to just \(|u_1(t)\rangle\) at time \( t \), i.e.,
\[
|\Psi_{ad}(t)\rangle = \cos \theta(t) |b\rangle + \sin \theta(t) |c\rangle = |u_1(t)\rangle.
\] (4.49)

In a time of the order \( 1/r \), we expect the following evolutions:
\[
\begin{align*}
\Omega_C(t) & : \Omega_0 \to 0, \\
\theta(t) & : \tan^{-1}\left( \frac{g_0}{\Omega_0} \right) \to \frac{\pi}{2}, \\
|\Psi(t)\rangle & : |b, 0\rangle \to |c, 0\rangle, \text{ provided } g_0 \ll \Omega_0.
\end{align*}
\]
It is interesting to place our analysis in the context of a known general result on adiabaticity for three-level systems obtained by Oreg et al. [24]. These authors analyzed the density-matrix equations as SU(3) rotations of an 8-component vector around two axes determined by detunings $\Delta_S$, $\Delta_C$ and Rabi frequencies of $g_n$ and $\Omega_C$. They showed that there is an adiabatic passage under change of $g_n$ and $\Omega_C$ for states whose corresponding 8-component vector is nearly aligned to the two axes of rotations, provided the following are satisfied [24]: (a) Cook-Shore pulses are used, which implies that both $g_n$ and $\Omega_C$ should have identical time variation, and (b) the detunings $\Delta_S$ and $\Delta_C$ are time-varying and non-zero. Since the situation considered in the present work violates both these conditions, one does not expect the adiabatic theorem to apply here. Specifically, in the wavefunction language for $\Delta = 0$, the Cook-Shore condition implies a constant $\theta$ so that the adiabaticity condition is trivially obeyed.

As a measure of any departure of our solution $|\Psi(t)\rangle$ from the adiabatic answer, we compute the fidelity $F(t)$ of the process given by

$$F(t) = \langle u_1(t)|\Psi(t)\rangle,$$

for different values of $g_n$ and $r$. We also compute $|A_n(t)|^2$, $|B_{n+1}(t)|^2$ and $|C_n(t)|^2$ to portray the actual evolution of the state.

**4.2-3 Retrieval of a photon**

For the retrieval of a stored pulse, we need to increase the control field from zero to $\Omega_0$. For this, we take the control pulse to be of the form [8] (see Fig. 4.3)

$$\Omega_C(t) = \Omega_0 \operatorname{tanh}(rt).$$

(4.51)

Here $r$ basically decides the rate at which $\Omega_C(t)$ rises to the maximum value of $\Omega_0$. As seen from Fig. 4.3, for different values of $r$ in the range 0.1 to 0.5, $\Omega_C(t)$ rises smoothly for low values and then more sharply for higher values of $r$.

From Eq. (4.32), we can derive

$$\dot{\theta}(t) = \frac{g_0 \Omega_0 r}{g_0^2 \cos^2(rt) + \Omega_0^2 [\cosh (rt) - \sinh (rt)]^2}.$$  

(4.52)

The initial state of the system is the dark state (4.45), now with $\theta(0) = \frac{\pi}{2}$. In a considerable
time $t$ (of the order $1/r$), we expect the following evolutions:

$$
\begin{align*}
\Omega_C(t) &: 0 \rightarrow \Omega_0, \\
\theta(t) &: \frac{\pi}{2} \rightarrow \tan^{-1} \left( \frac{g_0}{\Omega_0} \right), \\
|\Psi(t)\rangle &: |c, 0\rangle \rightarrow |b, 0\rangle, \text{ provided } g_0 \ll \Omega_0.
\end{align*}
$$

Again, the fidelity (4.50) of the process records the deviation from the adiabatic evolution.

### 4.3 Wavefunction Formulation in the Presence of Dissipation

There are standard ways of incorporating dissipation in quantum systems. A comprehensive account of those which are of particular use in quantum optics can be found in the textbook by Scully and Zubairy [14]. For a three-level system, the density matrix equations incorporating dissipation have been investigated in the literature in a somewhat different context [19, 20]. Here we adopt an approach which we believe to be quite transparent from a physical point of view, as supported by our results in the next Section. We work directly with the wavefunction [21] and this formulation can be regarded as an approximation to the full set of density matrix equations. We present a discussion of this approximation with regard to density-matrix treatment in Appendix 4.A. Here we remark that our approach is similar in
spirit to the formulation of Barchielli and Belavkin for a continuously measured system \[23\],
and has also been used in a similar context for \(\Lambda\)-systems in interaction with cavity fields
\[17, 18\].

Recall that under unitary evolution (without dissipation), the atomic state is
\[
|\Psi(t)\rangle = U(t) |\Psi(0)\rangle,
\]
where
\[
U(t) = \sum_k |u_k(t)\rangle \langle u_k(t)| e^{-i \int_0^t \lambda_k(t') dt'}.
\]
Again, in terms of the coefficients of the instantaneous eigenvectors, one can express
\[
|\Psi(t)\rangle = \sum_k V_k(t) e^{-i \int_0^t \lambda_k(t') dt'} |u_k(t)\rangle,
\]
where \(V_k(t) = \langle u_k(t)|\Psi(0)\rangle\).

In our model, the unitary evolution of the system of a three-level atom interacting with
the signal and control fields is interrupted by spontaneous decays. We assume that in the
presence of spontaneous decay, which occurs over a negligible time, the system collapses to
either level \(|b\rangle\) or level \(|c\rangle\) with equal probabilities. The decays occur in time according to a
Poissonian distribution. To write down the wavefunction in this model, we use the following
notation. \(U(t)\) denotes the evolution operator for the isolated system, which is computed
in the last section through the computation of \(V_m(t)\)s in (4.28). The probability that a
spontaneous decay occurs in the time interval \(dt\) is denoted by \(\Gamma dt\), where \(\Gamma\) is the rate of
spontaneous emission decay. The probability \(P_0(t)\) that a decay has not occurred for time \(t\),
after preparation of the system at \(t = 0\) is
\[
P_0(t) = e^{-\Gamma t}.
\]
Note that
\[
P_0(t + dt) = P_0(t) [1 - \Gamma dt],
\]
such that
\[
\frac{dP_0}{dt} = -\Gamma P_0(t).
\]

The operators that cause spontaneous decays to states \(|b\rangle\) and \(|c\rangle\) are denoted by \(\zeta_b\) and
\(\zeta_c\), respectively. In writing down the wavefunction at time \(t\), we have to include the possibili-
ties of 0, 1, 2, \ldots, \(l\), \ldots decays, with each of these weighted by the probability distribution
mentioned above, and thus
\[
|\Psi(t)\rangle = \frac{1}{Z(t)} \sum_1^\infty Q_l |\Psi(0)\rangle, \tag{4.59}
\]
where \(Q_l\)s denote the possibility in which \(l\) spontaneous decays have occurred over the interval \(t\). These are given as

\[
Q_0 = e^{-\Gamma t} U(t),
\]
\[
Q_1 = \int_0^t dt_1 e^{-\Gamma (t-t_1)} U(t-t_1) \Gamma \zeta e^{-\Gamma t_1} U(t_1),
\]
\[
Q_l = \int_0^t \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_{l-1}} dt_1 dt_2 \cdots dt_l e^{-\Gamma (t-t_1)} U(t-t_1) \Gamma \zeta e^{-\Gamma t_1} U(t_1) e^{-\Gamma t_2} U(t_2) \cdots e^{-\Gamma (t_{l-1}-t_l)} U(t_{l-1}-t_l) \Gamma \zeta e^{-\Gamma t_l} U(t_l), \tag{4.60}
\]

where \(\zeta\) denotes either \(\zeta_b\) or \(\zeta_c\). We shall also average over these stochastic histories by assuming the decays to be independent. In (4.59), \(Z(t) = \sqrt{\langle \Psi(t)|\Psi(t)\rangle}\) is the normalization of the wavefunction, which is necessitated as the evolution is no longer unitary. The summation over the series (4.59) in the present case is very easy, as the quantum evolution after the last collapse is what matters. The evolution from that state is either from level \(|b\rangle\) or level \(|c\rangle\). Suppose that the last \(l\)th collapse occurred to \(|b\rangle\) level. Then

\[
Q_l |\Psi(0)\rangle = \int_0^t dt_1 e^{-\Gamma (t-t_1)} \Gamma U(t-t_1) |b\rangle P_l(t_1), \tag{4.61}
\]

where \(P_l(t_1)\) denotes the probability that \(l\) collapses have occurred in the interval \(0\) to \(t_1\). This is given by

\[
P_l(t_1) = \frac{(\Gamma t_1)^l}{l!} e^{-\Gamma t_1}. \tag{4.62}
\]

The summation over \(l\) now yields

\[
\sum_1^\infty Q_l |\Psi(0)\rangle = \Gamma \int_0^t dt_1 (1 - e^{-\Gamma t_1}) e^{-\Gamma (t-t_1)} U(t-t_1) |b\rangle. \tag{4.63}
\]

Here we have used the result:

\[
\sum_1^\infty P_n(t_1) = 1,
\]
\[
\Rightarrow \sum_1^\infty P_n(t_1) = 1 - P_0(t_1) = 1 - e^{-\Gamma t_1}. \tag{4.64}
\]
One can write a similar expression if the last collapse occurred to level $|c\rangle$, by replacing $|b\rangle$ with $|c\rangle$. Combining these two possibilities with equal probabilities, we write the wavefunction as

$$|\Psi(t)\rangle = \frac{1}{Z(t)} \left[ e^{-\Gamma t} U(t) |\Psi(0)\rangle + \frac{\Gamma}{2} \int_0^t dt_1 \left( 1 - e^{-\Gamma t_1} \right) e^{-\Gamma (t-t_1)} U(t-t_1) (|b\rangle + |c\rangle) \right]. \quad (4.65)$$

This is the final expression for the wavefunction in this model.

We can simplify the second term in Eq. (4.65) by writing $t - t_1 = t_2$. Thus, Eq. (4.65) becomes

$$|\Psi(t)\rangle = \frac{1}{Z(t)} \left[ e^{-\Gamma t} U(t) |\Psi(0)\rangle - \frac{\Gamma}{2} \int_0^t dt_2 \left( 1 - e^{-\Gamma (t-t_2)} \right) e^{-\Gamma t_2} U(t_2) (|b\rangle + |c\rangle) \right]$$

$$= \frac{1}{Z(t)} \left[ e^{-\Gamma t} U(t) |\Psi(0)\rangle + \frac{\Gamma}{2} \int_0^t dt_2 \left( 1 - e^{-\Gamma (t-t_2)} \right) e^{-\Gamma t_2} U(t_2) |\langle \Psi(0) | (|b\rangle + |c\rangle)\rangle\rangle \right].$$

By using the adiabatic definition of $U(t)$ from Eq. (4.54) we obtain

$$|\Psi(t)\rangle = \frac{1}{Z(t)} \left[ e^{-\Gamma t} \sum_k e^{-i \int_0^t \lambda_k(t') dt'} |u_k(t)\rangle \langle u_k(t) | \langle \Psi(0)\rangle + \frac{\Gamma}{2} \int_0^t dt_2 \left( 1 - e^{-\Gamma (t-t_2)} \right) e^{-\Gamma t_2} \right.$$

$$\times \sum_k e^{-i \int_0^{t_2} \lambda_k(t'') dt''} |u_k(t_2)\rangle \langle u_k(t_2) | \langle \Psi(0)\rangle \langle \Psi(0) | (|b\rangle + |c\rangle)\rangle\rangle\right]. \quad (4.66)$$

Substituting $V_k(t_2) = \langle u_k(t_2) | \langle \Psi(0)\rangle$, the state $|\Psi\rangle$ can be given in terms of the instantaneous eigenvectors

$$|\Psi(t)\rangle = \frac{1}{Z(t)} \left[ e^{-\Gamma t} \sum_k V_k(t) e^{-i \int_0^t \lambda_k(t') dt'} |u_k(t)\rangle + \frac{\Gamma}{2} \int_0^t dt_2 \left( 1 - e^{-\Gamma (t-t_2)} \right) e^{-\Gamma t_2} \right.$$

$$\times \sum_k V_k(t_2) e^{-i \int_0^{t_2} \lambda_k(t'') dt''} |u_k(t_2)\rangle \langle \Psi(0)\rangle \langle \Psi(0) | (|b\rangle + |c\rangle)\rangle\rangle\right]. \quad (4.67)$$

To compute the wavefunction, we again resolve it in terms of instantaneous eigenfunctions:

$$|\Psi(t)\rangle = \sum_k W_k(t) e^{-i \int_0^t \lambda_k(t') dt'} |u_k(t)\rangle. \quad (4.68)$$

By comparison with Eq. (4.67), the coefficients with dissipation $W_k(t)$ can be straightforwardly expressed in terms of $V_k(t)$s as

$$W_k(t) = \frac{1}{Z(t)} \left[ e^{-\Gamma t} V_k(t) + \frac{\Gamma}{2} \int_0^t dt_1 \left( e^{-\Gamma t_1} - e^{-\Gamma t} \right) \right.$$  

$$\times \sum_i V_i(t_1) e^{-i \int_0^{t_1} \lambda_i(t') dt'} |u_k(t)\rangle \langle u_i(t_1) | \langle \Psi(0) | (|b\rangle + \langle \Psi(0) | c\rangle)\rangle\right]. \quad (4.69)$$
where $X_k(t) = \int_0^t \lambda_k(t') dt'$. In this notation, $Z(t) = \sqrt{\sum_k |W_k(t)|^2}$. We solve these equations numerically and compute the fidelities (4.50) and other relevant quantities in the storage and retrieval processes, as before, generalizing the coefficients $V_k$s in (4.35)-(4.37) to the above $W_k(t)$s.

### 4.4 Results and Discussions

We now present our results for a single atom coupled to the signal and control fields, using the half-Rabi frequency $g_n$, corresponding to the coupling of a single atom to the signal pulse. As mentioned before, for the one photon-many atom dark states discussed by Fleischhauer and Lukin [8, 15], the 'collective coupling strength' of the signal $g_0\sqrt{N}$ enters the mixing angle. However, the adiabatic transfer problem basically arises at the level of a single atom, and we wish to stick to the core issue. We adopt the standard approach [14] of calculating the polarization of the excited medium from the polarizability of a single atom and obtain the refractive index of the medium which naturally involves the atomic density. Thus, the factor of $N$ naturally enters the group velocity as in Eq. (4.3) and not the mixing angle in Eq. (4.2). The issue is much more complex for the many-atom case with dissipation (see Appendix 4.B).

For the storage process with the control field of the form (4.47), Fig. 4.4 shows the fidelity (4.50) as a function of time (in units of $\Omega_0^{-1}$) for a signal field $g_n/\Omega_0 = 0.05$ without dissipation ($\Gamma = 0$) at (a), and with dissipation at (b) $\Gamma/\Omega_0 = 0.1$, (c) $\Gamma/\Omega_0 = 0.5$, and (d) $\Gamma/\Omega_0 = 1$, for different rates of variation $r$ (in units of $\Omega_0$) = 0.1, 0.2, 0.5 and 0.8 of the control field in each case. Figures 4.5 and 4.6 show the same set of results for higher signal field strengths, $g_n/\Omega_0 = 0.1$ and 0.2, respectively. Note that the variation of the control field ends at a time of the order of $3/r$, and indeed one finds that there is no change in fidelity values after this time. This is also true for the plots shown with dissipation.

We first comment on the results without dissipation. In Fig. 4.6(a), one sees that the fidelities saturate to higher values as $r$ decreases, for example, $F \sim 0.39$ for $r = 0.1$ and $F \sim 0.07$ for $r = 0.8$. Thus fidelity of storage is better with a slow variation of the control field, as expected from the adiabatic theory. Next we comment on the effect of dissipation.

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Figure 4.4: Fidelity of the storage problem versus time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $\eta_m/\Omega_0 = 0.05$: (a) without dissipation ($\Gamma = 0$), (b) with dissipation at $\Gamma/\Omega_0 = 0.1$, (c) with dissipation at $\Gamma/\Omega_0 = 0.5$, and (d) with dissipation at $\Gamma/\Omega_0 = 1$. Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).

Figure 4.5: Fidelity of the storage problem versus time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $\eta_m/\Omega_0 = 0.1$: (a) without dissipation ($\Gamma = 0$), (b) with dissipation at $\Gamma/\Omega_0 = 0.1$, (c) with dissipation at $\Gamma/\Omega_0 = 0.5$, and (d) with dissipation at $\Gamma/\Omega_0 = 1$. Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).

Figure 4.6: Fidelity of the storage problem versus time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $\eta_m/\Omega_0 = 0.2$: (a) without dissipation ($\Gamma = 0$), (b) with dissipation at $\Gamma/\Omega_0 = 0.1$, (c) with dissipation at $\Gamma/\Omega_0 = 0.5$, and (d) with dissipation at $\Gamma/\Omega_0 = 1$. Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).
We see that with dissipation, the fidelities are no better than that for an isolated system, especially for slower rates of variation, \( r = 0.1 \) and \( 0.2 \). The adiabatic behavior still exists, though the difference becomes marginal as \( \Gamma \) increases. Note that for a given system, \( \Gamma \) is a fixed parameter; however, here the rates are scaled by \( \Omega_0 \) and hence its variation physically implies the inverse variation of the control laser power. Thus, the higher the control power, the better is the adiabatic following. But for low control powers, the variation rate has a marginal effect.

Next we examine how fidelity depends on the signal strength. This variation is shown for a range of values of \( g_n/\Omega_0 \) from 0.05 to 0.2, in Figs. 4.4 to 4.6. For zero dissipation, Figs. 4.4(a) to 4.6(a) show that the fidelity increases with an increase of \( g_n/\Omega_0 \), for example, \( F \approx 0.92 \) at \( g_n/\Omega_0 = 0.2 \). This can be understood easily. The fidelity decreases as the non-adiabatic perturbation (proportional to \( \theta \)) becomes large when \( \Omega_C(t) \rightarrow 0 \), and it is larger, smaller the value of \( g_n/\Omega_0 \).

Next we turn to the retrieval process with the control field of the form (4.51). Figure 4.7 shows the fidelity (4.50) of the retrieval process as a function of time (in units of \( \Omega_0^{-1} \)) for a relative signal field \( g_n/\Omega_0 = 0.05 \) without dissipation (\( \Gamma = 0 \)) at (a), and with dissipation at (b) \( \Gamma/\Omega_0 = 0.1 \), (c) \( \Gamma/\Omega_0 = 0.5 \), and (d) \( \Gamma/\Omega_0 = 1 \), for different rates of variation \( r = 0.1, 0.2, 0.5 \) and 0.8 of the control field in each case. Figures 4.8 and 4.9 show the same set of results for higher signal field strengths, \( g_n/\Omega_0 = 0.1 \) and 0.2, respectively. We again note that, in general, the fidelity at a particular \( r \) saturates after a time \( t > 3/r \) when the control field variation saturates to \( \Omega_0 \).

As before, we first comment on the results without dissipation. We note from Fig. 4.7(a) that the fidelities are much lower than that in the storage process, though they do decrease with increasing \( r \), and this is only a marginal effect. Thus the adiabaticity expectations are not quite fulfilled. This difference in the behavior of fidelity between storage and retrieval is easily understood by recognizing that for retrieval the non-adiabatic perturbation is the largest at the beginning of the procedure whereas for storage it is the largest toward the end of the procedure. As seen from Figs. 4.7(b)-(d), with dissipation the behavior is complex. For low dissipation rates (or high control power), the fidelity improves considerably but the behavior is non-monotonic. The best results are achieved with \( \Gamma/\Omega_0 = 0.1 \) and hereafter
Figure 4.7: Fidelity of the retrieval process versus time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $\Omega_0/\Omega_0 = 0.05$: (a) without dissipation ($\Gamma = 0$), (b) with dissipation at $\Gamma/\Omega_0 = 0.1$, (c) with dissipation at $\Gamma/\Omega_0 = 0.5$, and (d) with dissipation at $\Gamma/\Omega_0 = 1$. Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).

Figure 4.8: Fidelity of the retrieval process versus time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $\Omega_0/\Omega_0 = 0.1$: (a) without dissipation ($\Gamma = 0$), (b) with dissipation at $\Gamma/\Omega_0 = 0.1$, (c) with dissipation at $\Gamma/\Omega_0 = 0.5$, and (d) with dissipation at $\Gamma/\Omega_0 = 1$. Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).

Figure 4.9: Fidelity of the retrieval process versus time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $\Omega_0/\Omega_0 = 0.2$: (a) without dissipation ($\Gamma = 0$), (b) with dissipation at $\Gamma/\Omega_0 = 0.1$, (c) with dissipation at $\Gamma/\Omega_0 = 0.5$, and (d) with dissipation at $\Gamma/\Omega_0 = 1$. Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).
the results deteriorate with decreasing control power. At the optimum value of $\Gamma/\Omega_0 = 0.1$, initially there are some oscillations before the fidelities saturate to their steady values. Surprisingly, for retrieval we find that in the presence of dissipation, the general wisdom of adiabaticity is not followed; instead, the fidelity is better as $r$ increases (except for very weak signal intensity, when $g_n/\Omega_0 \sim 0.01$).

In Figs. 4.8 and 4.9, we mark the effect of the signal strength. It is seen that with the increase of the signal strength, the fidelity of retrieval also improves in the range of $g_n/\Omega_0$ from 0.05 to 0.2, shown in Figs. 4.7-4.9. For each signal strength, there is an optimum value of $\Gamma$ (or control power) at which the best fidelities are achieved. The behavior with respect to $r$ shows the same unexpected trend.

**Plots of $|V_1(t)|$, $|V_2(t)|$, $|V_3(t)|$ and $|W_1(t)|$, $|W_2(t)|$, $|W_3(t)|$**

In support of the results on fidelity, Figs. 4.10 and 4.11 show the plots of the coefficients of the instantaneous eigenvectors for the storage and the retrieval problem, respectively. On the left-hand side of each figure are the plots of $|V_1(t)|$, $|V_2(t)|$, $|V_3(t)|$ (i.e., without dissipation) at $g_n = 0.1$, and on the right-hand side are the plots of $|W_1(t)|$, $|W_2(t)|$, $|W_3(t)|$ (i.e., in the presence of dissipation) at $g_n = 0.1$ and $\Gamma = 0.1$. Comparison of these plots shows that the occupation of level $|a\rangle$ is suppressed while those of levels $|b\rangle$ and $|c\rangle$ are considerably enhanced with optimal dissipation.

**Plots of $|A_n(t)|^2$, $|B_{n+1}(t)|^2$, $|C_n(t)|^2$**

To give a detailed picture of the evolution of our solution for $|\Psi(t)\rangle$, we plot $|A_n(t)|^2$, $|B_{n+1}(t)|^2$, $|C_n(t)|^2$, given in (4.35)-(4.37) but with the coefficients $V_k$s generalized to the $W_k(t)s$ in (4.69).

These are shown for the storage process in Fig. 4.12 in case of $g_n/\Omega_0 = 0.1$. Similar plots are shown for retrieval in Fig. 4.13. On the left-hand side are the plots for no dissipation while on the right-hand side are those with dissipation at $\Gamma/\Omega_0 = 0.1$. In storage, as $\Omega_C \to 0$, the dark state approaches $|c,n\rangle$. Thus $|C_n(t)|^2$ should be large at the end of the process. However, without dissipation, this is marginally fulfilled for the smallest rate $r = 0.1$, and in the presence of dissipation, it is worse. For retrieval, as $\Omega_C \to \Omega_0$, the dark state approaches...
Figure 4.10: Plots of $|V_1(t)|$, $|V_2(t)|$, $|V_3(t)|$, $|W_1(t)|$, $|W_2(t)|$, and $|W_3(t)|$ for the storage process, as a function of time (scaled with $\Omega_0^{-1}$) with a signal half-Rabi frequency of $g_0/\Omega_0 = 0.1$ without dissipation, $\Gamma = 0$ (left-hand side) and with dissipation at $\Gamma/\Omega_0 = 0.1$ (right-hand side). Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).
Figure 4.11: Plots of $|V_1(t)|$, $|V_2(t)|$, $|V_3(t)|$ and $|W_1(t)|$, $|W_2(t)|$, $|W_3(t)|$ for the retrieval process, as a function of time (scaled with $\Omega_0^{-1}$) with a signal half-Rabi frequency of $g_n/\Omega_0 = 0.1$ without dissipation, $\Gamma = 0$ (left-hand side) and with dissipation at $\Gamma/\Omega_0 = 0.1$ (right-hand side). Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).
Figure 4.12: Plots of $|A_n(t)|^2$, $|B_{n+1}(t)|^2$, $|C_n(t)|^2$ for the storage process, as a function of time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $\gamma_n/\Omega_0 = 0.1$ without dissipation, $\Gamma = 0$ (left-hand side), and with dissipation at $\Gamma/\Omega_0 = 0.1$ (right-hand side). Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).
Figure 4.13: Plots of $|A_n(t)|^2$, $|B_{n+1}(t)|^2$, $|C_n(t)|^2$ for the retrieval process, as a function of time (in units of $\Omega_0^{-1}$) with a signal half-Rabi frequency of $g_0/\Omega_0 = 0.1$ without dissipation, $\Gamma = 0$ (left-hand side), and with dissipation at $\Gamma/\Omega_0 = 0.1$ (right-hand side). Each graph shows the effect of the variation of $r$ (in units of $\Omega_0$) with different symbols: $r = 0.1$ (dashed), $r = 0.2$ (continuous), $r = 0.5$ (dotted), and $r = 0.8$ (dot-dashed).
\( |b, n+1\rangle \) (for \( \Omega_0 \gg g_n \)). So, \( |B_{n+1}(t)|^2 \) should be large at the end of the process. However, as we find in Fig. 4.13, \( |B_{n+1}(t)|^2 \) is quite small without dissipation, but improves on inclusion of dissipation and it is better for a fast variation of the control field.

4.5 Summary

In this Chapter, we have addressed the core problem of adiabatic transfer at the level of a single atom coupled to the signal and control fields. It may be pointed out that the existing density matrix approaches [8, 9, 10] which treat an ensemble of atoms are essentially based on the single-atom dynamics. These treatments use an implicit approximation that the multi-atom density matrix can be factored into single-atom density matrices, even though the common laser fields induce an effective interaction between atoms. Our results [12] for an isolated system (without dissipation) concur with the adiabatic idea of increased fidelity for slower variation of the control field, in case of both storage and the retrieval of a photon, though the effect is marginal for small signal powers. The fidelities of both the processes are better for higher signal powers, at any rate of variation of the control field.

Within the approximations of our formalism, the fidelity of storage in the presence of dissipation still follows the adiabatic property; however, it gets worse with dissipation for slow variations of the control field. We also find that given a particular system, the fidelity of the storage process is better for higher control powers with respect to the spontaneous decay rate \( \Gamma \). On the other hand, in the case of retrieval, the behavior with dissipation is non-monotonic. For an optimal control power, which depends on the spontaneous decay rate of the system, the fidelity with dissipation is, in fact, better than that without dissipation, the best being at \( \Gamma/\Omega_0 \approx 0.1 \). Moreover, contrary to the adiabatic idea and unlike the behavior of an isolated system, in presence of dissipation, retrieval fidelities are higher for faster rates of variation of the control field.

In general, we conclude that dissipation definitely aids the retrieval process but not so much the storage. Further, we assert that for storing the signal, the control field should be switched off as slowly as possible, but while retrieving it, no such restriction is required; in fact the faster we switch on the control field, the better the signal is retrieved. Our results have partial support from reported experimental observations by Liu et al. [5].
Appendix

4.A Density-Matrix Approach for Dissipative Systems

We describe here briefly the dissipation model, used in Sec. 4.3, for the evolution of the density matrix $\rho(t)$. For a pure isolated system, the wavefunction at time $t$ is:

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle,$$

where $U(t) = e^{-iHt}$. The corresponding density matrix of the system is

$$\rho(t) = |\Psi(t)\rangle \langle \Psi(t)|$$

$$= U(t) |\Psi(0)\rangle \langle \Psi(0)| U^\dagger(t)$$

$$= U(t) \rho(0) U^\dagger(t)$$

$$= U^x(t) \rho(0)$$

$$= e^{-iH^x t} \rho(0).$$

(4.A2)

$U^x(t)$ is a linear operator and $H^x A = [H, A]$.

For a dissipative system, we introduce decay due to spontaneous emission through the collapse model [22]. We assume that some impulses are given to the system which collapses the wavefunction to states $|b\rangle$ or $|c\rangle$. This is described by the impulse operator $\zeta_b$ or $\zeta_c$, respectively, such that

$$\zeta_b |\Psi(t)\rangle = |b\rangle,$$

$$\zeta_c |\Psi(t)\rangle = |c\rangle.$$

In general, let us denote it as $\zeta$. In a similar notation as above for the density-matrix, we have

$$\zeta^x \rho(t) = \zeta \rho(t) \zeta^\dagger,$$

$$\zeta^x_b |\Psi(t)\rangle \langle \Psi(t)| = |b\rangle \langle b|.$$

(4.A3)

As the system evolves unitarily according to its Hamiltonian, the evolution is interrupted by spontaneous decays. Following the same approach as in Sec. 4.3, over a period of time $t$, we include the possibilities of 1, 2, 3, ..., $l$, ... decays, with each of these possibilities weighted
by their probability distribution. To allow for all stochastic histories and averaging over them, one must look at the density-matrix equation. Consider the cases when

(a) no impulse/decay has occurred in time $t$:

$$
\rho_0(t) = e^{-\Gamma t} U^\times(t) \rho(0),
$$

(4.4)

where $e^{-\Gamma t}$ is the probability of no impulse during time $t$ in a system that is open to dissipation/spontaneous emission at the rate $\Gamma$;

(b) one impulse has occurred in time $t$:

$$
\rho_1(t) = \int_0^t e^{-\Gamma(t-t_1)} U^\times(t-t_1) \Gamma dt_1 \zeta^\times e^{-\Gamma t_1} U^\times(t_1) \rho(0),
$$

(4.5)

where the term $e^{-\Gamma t_1} U^\times(t_1) \rho(0)$ denotes dissipationless evolution between time 0 to $t_1$ when no impulse has occurred and therefore the system evolves like $\rho_0(t_1)$; the term $\Gamma dt_1 \zeta^\times$ denotes that $\zeta^\times$ operates on the system to cause an impulse at $t_1$ that occurs with a probability of $\Gamma dt_1$; and finally, the term $e^{-\Gamma(t-t_1)} U^\times(t-t_1)$ denotes the unitary evolution of the system after time $t_1$ till $t$;

(c) two decay events have occurred in time $t$:

$$
\rho_2(t) = \int_0^t \int_0^{t_1} e^{-\Gamma(t-t_1)} U^\times(t-t_1) \Gamma dt_1 \zeta^\times e^{-\Gamma(t_1-t_2)} U^\times(t_1-t_2) \Gamma dt_2 \zeta^\times e^{-\Gamma t_2} U^\times(t_2) \rho(0);
$$

(4.6)

(d) after $m$ impulses, the density-matrix is

$$
\rho_m(t) = \int_0^t \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_{m-1}} e^{-\Gamma(t-t_1)} e^{-iH^\times(t-t_1)} \Gamma dt_1 \zeta^\times \times e^{-\Gamma(t_1-t_2)} e^{-iH^\times(t_1-t_2)} \Gamma dt_2 \zeta^\times \cdots
\times e^{-\Gamma(t_{m-1}-t_m)} e^{-iH^\times(t_{m-1}-t_m)} \Gamma dt_{m-1} \zeta^\times e^{-\Gamma t_m} e^{-iH^\times t_m} \rho(0).
$$

(4.7)

Then we have

$$
\rho(t) = \sum_{l=0}^{\infty} \rho_l(t),
$$

(4.8)
To find $\frac{dp}{dt}$, we evaluate separately the following:

$$
\frac{d\rho_0}{dt} = -\Gamma e^{-\Gamma t} U^x(t) \rho(0) + e^{-\Gamma t} \frac{d}{dt} \left( U^x(t) \rho(0) \right)
= -\Gamma \rho_0(t) + e^{-\Gamma t} \left( \frac{d}{dt} U^x(t) \rho(0) + U^x(t) \frac{d\rho_0}{dt} \right)
= -\Gamma \rho_0(t) + e^{-\Gamma t} \frac{dU^x(t)}{dt} \rho(0), \quad [\text{since } \frac{d\rho_0}{dt} = 0]. \quad (4.A9)
$$

Note that

$$
U(t) = e^{-iHt}
\Rightarrow \frac{dU}{dt} = -iH e^{-iHt} = -iH U.
$$

Similarly,

$$
U^x(t) = e^{-iH^xt}
\Rightarrow \frac{dU^x}{dt} = -iH^x U^x = -i \left( HU^x - U^x H \right)
= -i \left[ H, U^x \right]. \quad (4.A10)
$$

Thus,

$$
\frac{d\rho_0}{dt} = -\Gamma \rho_0(t) + e^{-\Gamma t} \frac{dU^x(t)}{dt} \rho(0)
= -\Gamma \rho_0(t) + e^{-\Gamma t} \left( -iH^x U^x \right) \rho(0)
= -\Gamma \rho_0(t) - iH^x e^{-\Gamma t} U^x \rho(0)
= -\Gamma \rho_0(t) - iH^x \rho_0(t)
= -\left( \Gamma + iH^x \right) \rho_0(t). \quad (4.A11)
$$

Similarly,

$$
\frac{d\rho_1}{dt} = \Gamma\zeta^x \rho_0(t) - \left( \Gamma + iH^x \right) \rho_1(t), \quad (4.A12)
\frac{d\rho_2}{dt} = \Gamma\zeta^x \rho_1(t) - \left( \Gamma + iH^x \right) \rho_2(t). \quad (4.A13)
$$

Adding all these contributions, we can derive

$$
\frac{d\rho}{dt} = \Gamma\zeta^x \left[ \rho_0(t) + \rho_1(t) + \rho_2(t) + ... \right] - \left( \Gamma + iH^x \right) \left[ \rho_0(t) + \rho_1(t) + \rho_2(t) + ... \right]
+ i \left( \Gamma\zeta^x - \Gamma - iH^x \right) \rho
= -i \left[ H^x + i \Gamma \left( \zeta^x - 1 \right) \right] \rho
= -i \mathcal{L} \rho, \quad (4.A14)
$$

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where $\mathcal{L}$ is the new Liouville operator. If we define a ‘no-decay’ operator as $W = \zeta - 1, 1$ being the unit operator, we get

$$
\frac{d\rho}{dt} = -iH^x + \Gamma [\zeta \rho \zeta^\dagger - \rho]
$$

$$
= -iH^x + \Gamma [(W + 1) \rho (W^\dagger + 1) - \rho]
$$

$$
= -iH^x \rho + \Gamma (W \rho + \rho W^\dagger) + \Gamma W \rho W^\dagger.
$$

(4.A15)

To find the closest equivalent equation for the wavefunction corresponding to Eq. (4.A14), we write it in an expanded form:

$$
i \frac{d}{dt} (|\Psi(t)\rangle \langle \Psi(t)|) = \left[ H^x + i \Gamma \left( \zeta^\dagger - 1 \right) \right] |\Psi(t)\rangle \langle \Psi(t)|,
$$

or,

$$
i \frac{d}{dt} |\Psi(t)\rangle \langle \Psi(t)| + i |\Psi(t)\rangle \frac{d}{dt} |\Psi(t)\rangle
= H |\Psi(t)\rangle \langle \Psi(t)| - |\Psi(t)\rangle \langle \Psi(t)| H
$$

$$
+i \Gamma \zeta |\Psi(t)\rangle \langle \Psi(t)| \zeta^\dagger - i \Gamma 1 |\Psi(t)\rangle \langle \Psi(t)| 1.
$$

(4.A16)

This may be broken down to yield

$$
i \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle + i \Gamma \zeta |\Psi(t)\rangle - i \Gamma 1 |\Psi(t)\rangle,
$$

(4.A17)

$$
-i \frac{d}{dt} |\Psi(t)\rangle = \langle \Psi(t)| H - i \Gamma \langle \Psi(t)| \zeta^\dagger + i \Gamma \langle \Psi(t)| 1.
$$

(4.A18)

Thus, for the wavefunction the evolution equation emerges as:

$$
\frac{i}{\hbar} \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle + i \Gamma (\zeta - 1) |\Psi(t)\rangle.
$$

(4.A19)

This equation (with dissipation) in the wavefunction language may be converted back in the density-matrix form for comparison. Multiplying Eq.(4.A17) with $|\Psi(t)\rangle$ on the right and Eq.(4.A18) with $|\Psi(t)\rangle$ on the left, and subtracting, we get

$$
i \frac{d}{dt} (|\Psi(t)\rangle \langle \Psi(t)|) = H |\Psi(t)\rangle \langle \Psi(t)| - |\Psi(t)\rangle \langle \Psi(t)| H
$$

$$
+ i \Gamma [\zeta |\Psi(t)\rangle \langle \Psi(t)| + |\Psi(t)\rangle \langle \Psi(t)| \zeta^\dagger - 1 |\Psi(t)\rangle \langle \Psi(t)| - |\Psi(t)\rangle \langle \Psi(t)| 1],
$$

or,

$$
\frac{d\rho}{dt} = (H \rho - \rho H) + i \Gamma \left[ (\zeta - 1) \rho + \rho \left( \zeta^\dagger - 1 \right) \right],
$$

or,

$$
\frac{d\rho}{dt} = -iH^x \rho + \Gamma (W \rho + \rho W^\dagger).
$$

(4.A20)

A comparison of (4.A15) and (4.A20) shows that the wavefunction approach agrees with the density matrix approach only up to first order in the ‘no-decay’ operator $W$. Now, if we
put $\zeta = 1$, the above equation reduces to the case of the dissipationless system that evolves unitarily under the Hamiltonian as

$$i\frac{d\rho}{dt} = H^s \rho.$$ 

4.B $N$-Atom Dark States

For $N$ atoms interacting with a single-mode signal laser and a control laser, the Hamiltonian (4.8) + (4.9) for a single atom can be easily generalized to

$$H = \sum_i \sum_x E_x |x_i\rangle \langle x_i| + \hbar g_0 \sum_i [\langle a_i|b_i|a_P + a_P^\dagger|b_i\rangle \langle a_i|]$$

$$- \hbar \sum_i \Omega_C(t)e^{-i\omega_C t|a_i\rangle \langle c_i|} + \Omega_C^*(t)e^{i\omega_C t|c_i\rangle \langle a_i|}, \quad (4.B1)$$

where $|x_i\rangle$ denotes the $x$-state of the $i^{th}$ atom, $x = a, b, c$.

For the case of a single photon, the wavefunction can be written in the following basis:

$$|0, 1\rangle = |b_1, b_2, \ldots, b_N, 1\rangle,$$

$$|a, 0\rangle = \frac{1}{\sqrt{N}} \sum_i |b_1, \ldots, a_i, \ldots, b_N, 0\rangle,$$

$$|c, 0\rangle = \frac{1}{\sqrt{N}} \sum_i |b_1, \ldots, c_i, \ldots, b_N, 0\rangle. \quad (4.B2)$$

Writing the wavefunction as

$$|\Psi(t)\rangle = A(t) e^{-iE_a t}|a, 0\rangle + B(t) e^{-i(E_b + \hbar \omega_p) t}|0, 1\rangle + C(t) e^{-iE_c t}|c, 0\rangle, \quad (4.B3)$$

one notes that the coefficients $A$, $B$, $C$ obey the same equations as Eq. (4.15) for the one-photon case with one difference that $g_0$ is replaced by $g_0 \sqrt{N}$. With this change, the rest of the analysis is exactly parallel to the one-atom case as discussed in Secs. 4.2 and 4.3 with $\theta(t)$ replaced by $\theta_N(t) = \tan^{-1}[g_0 \sqrt{N}/\Omega_C(t)]$.

However, this analysis cannot be extended to the $n$-photon case. As shown by Fleischhauer and Lukin [8], the number of states involved for $n \ll N$ is $(n + 1)(n + 2)/2$, and the dark state is given as

$$|\Psi_{DSP}\rangle = \sum_{k=0}^{n} \frac{n!}{k! (n-k)!} (\cos \theta_N)^{n-k} (-\sin \theta_N)^k |c^k, n-k\rangle, \quad (4.B4)$$

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where
\[ |c^k, m\rangle = \sqrt{\frac{k!}{N^k}} \sum_{i_1, i_2, \ldots, i_k} |b_1, b_2, \ldots, c_{i_1}, c_{i_2}, c_{i_3}, \ldots, c_{i_k}, \ldots, b_N, m\rangle. \] (4.B5)

The adiabatic analysis with the variation of \( \theta_N \) is far more complex as the number of states to which transfers can happen is far more numerous. The method employed by Fleischhauer and Lukin [8] suggests that the transfers are predominantly into other linear combinations of \( |c^k, n - k\rangle \). However, an examination of the equations of motion shows that all other instantaneous eigenstates involve higher levels with superpositions from level \( |a\rangle \). So, ignoring excitations to \( |a\rangle \) is not justified for the adiabatic analysis.

References


Part B

Quantum Coherence Properties of a Two-Mode Laser Source in a Three-Level Model