ABSTRACT

Introduction:

Chaos and Fractals are parts of a grander subject 'Dynamics'. Similarly, Dynamics was originally a branch of physics.

Dynamics started in the mid 1600s. At that time Newton invented differential equations, discovered his laws of motion and of universal gravitation and combined them to explain Kepler's laws of planetary motion. He also solved the problem of calculating the motion of the Earth around the Sun, and gave the inverse square law of gravitational attraction between them. The problem of calculating the motion of the Earth around the Sun is known as two-body problem. With the help of Newton's analytical methods, the subsequent generations of mathematicians and physicists tried to solve the three-body problem (e.g. Sun, Earth, and Moon) but they were not successful in this respect. They tried many decades and it was eventually realized that the three-body problem was essentially impossible to solve, in the sense of obtaining explicit formulas for the motions of the three bodies. In this purpose, one breakthrough came up in the late 1800. It was the work of Poincare. Through his work, Poincare emphasized qualitative rather than quantities questions. Poincare asked "Is the solar system stable forever, or will some planets eventually fly off to infinity instead of asking for the exact positions of the planets at all times? " For analyzing such type of questions, he developed a powerful geometric approach which had flowered into the modern object of dynamics with applicants reaching far beyond celestial mechanics. In the field of chaos, Poincare was the first person who predicted the glimpse of possibility. He introduced the motion of sensitivity to initial conditions and long term unpredicted
behavior of process. Geometric methods of Poincare were much deeper than any other methods of that time for understanding of classical mechanics.

A new era also started in the history of dynamics from 1950’s, after the invention of high speed computer. The works (i.e. mainly experiment with equations) which were impossible to do before, computer made them easy and thereby to develop some intuition about nonlinear systems. In 1963, such works led to Lorenz’s discovery of chaotic motion on a strange attractor. Until 1970’s, Lorenz’s work had a little impact. This time is known as the boom years for chaos. In 1971, on the basis of abstract considerations, about strange attractors, Ruelle & Takens proposed a new theory for the onset of turbulence in fluids. Then after few years in 1976 Robert May found examples of chaos in iterated mappings arising in population biology, and wrote influential review article. In this article he stressed the pedagogical importance of studying simple nonlinear systems to counter balance the often misleading linear intuition fostered by traditional education. Next came probably the most beautiful, surprising, important discovery of all was the route from order into chaos i.e. the Feigenbaum universality. Mitchell J. Feigenbaum, a renowned American particle theorist was known as the founder of the period doubling bifurcation that may be described as a universal route to chaos an exciting discovery in nonlinear dynamical system. Benoits B. Mandelbrot's fractal geometry & Geometric method of Winfree on Non linear oscillators in Biology were two other major developments in dynamics in 1970’s.

Now a days there has been a remarkable research interest in chaos, fractals & dynamical systems. In recent years, deterministic chaos has been observed when applying simple models to cardiology, chemical reactions, electronic circuits, laser
technology, population dynamics, turbulence and weather forecasting. In the past, scientists attempted to remove the chaos when applying the theory to physical models. It is only the last decade that they have come to realize the potential uses for system displaying chaotic phenomena. There are tremendous fascinating results with chaos & fractals today. In 1987, James Gleick's book 'Chaos-Making a New Science' was a bestseller for months an amazing accomplishment for a book about mathematics & Science. In that book about the chaos James Gleick said, "Over the last decade physicists, biologists, astronomers and economists have created a new way of understanding the growth of complexity in nature. This new science, called chaos, offers a way of seeing order and pattern where formerly only the random, erratic, the unpredictable in short the chaotic - had been observed."

A dynamical system has two parts (i) a state vector, (ii) a function (i.e. a rule)

A state vector describes exactly the state of some real or hypothetical systems and a function tells as, given the current state, that the state of the system will be in the next instant of time.

Again, there are two main types of dynamical systems. The main types are (i) Differential equations (ii) iterated maps.

Differential equations describe the evolution of a system in continuous time. These equations are used much more widely in science and engineering. On the other hand iterated maps arise problems where time is discrete. This maps can also be very useful, both for providing simple examples of chaos and also as tools for analyzing periodic or chaotic solution of differential equations.
Methodology:

To carry out our study and investigation we apply both analytical and computation methods. In our exploration following tools will be used. The tools are nonlinear maps, fractal geometry, renormalization group method (used in theoretical physics and statistical mechanics), many technical results in numerical analysis, mathematical models.

Chapter Overview:

Here we give a synopsis of the contents of the various chapters before proceeding to the principal goals.

Chapter 1 introduces some basic concepts and results which are closely related to the research work. These are drawn from physics, biology, numerical mathematics, different literature, monographs and research papers.

Chapter 2 deals with a one dimensional nonlinear cubic map to find out a few inherent attributes i.e. fixed points, periodic points, bifurcation values of periods $2^n, n = 0,1,2,3,4 \ldots \ldots \ldots \ldots$. We use suitable numerical methods and have shown how the period doubling bifurcation points ultimately converge to the Feigenbaum constant. We have calculated Feigenbaum $\delta$ value also. We have further verified our findings with the help of bifurcation diagram, Lyapunav exponent, time series analysis of the map.

In Chapter 3 we highlight the Feigenbaum renormalization theory through the non linear discrete map $f(x) = \mu x(1-x^4)$ with control parameter $\mu \in [0,3]$ and $x \in [-1,1]$. Also determine the scaling ratio (the Feigenbaum number $\alpha$) as the period doubling sequence proceeds towards chaos.
Chapter 4 describes the study of chaos control. Here we have applied N.P. Chau’s periodic proportional pulses technique to stabilize unstable periodic orbits embedded in the chaotic attractors of one dimensional nonlinear discrete maps, which follow a period-doubling route to chaos beyond the accumulation point. For controlling chaos, there are several methods for different models.

In Chapter 5 we focus on a two dimensional nonlinear discrete map to find out a few important characteristics viz. fixed points, periodic points, bifurcation values of periods $2^n, n = 0,1,2,3,4,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ We use suitable numerical methods and have shown how the period doubling bifurcation points ultimately converge to the Feigenbaum constant. We have established the Feigenbaum universe route of period doubling bifurcation tending to chaos. Moreover our results are verified with the help of bifurcation diagram, Lyapunav exponent of the map.

The principal goals of Chapter 6 are to find out various fractal dimensions viz. box counting, information, correlation dimension. Here the suitable numerical methods are used.

In Chapter 7, same technique is demonstrated to control chaos on the two dimensional nonlinear chaotic model. A chaotic attractor consists of infinitely many unstable periodic orbits, which coexist with the strange attractor. It plays an important role in the system. Here we apply periodic proportional pulses technique which is proposed by N.P. Chou, an interesting one out of several techniques for controlling chaos, to stabilize unstable periodic orbits embedded in the chaotic attractor of the two dimensional nonlinear dynamics.
Chapter 8 is concentrated with some open problems which may be fruitful in the development of theory as well as applications in future research work of dynamical systems.

Finally, a suitable Bibliography and an Appendix are included. The bibliography is put forward containing a list of books, monographs and research papers closely related to our field of research. On the other hand appendix contains some C-programs and Mathematica programs which help in our findings.

*****