CHAPTER 1

INTRODUCTION

1.1 FUZZY LOGIC

The term fuzzy means shades of grey between 0% and 100%. Almost all concepts fall into domain of fuzzy, because they are endowed with inexact boundaries. Fuzzy logic resembles human reasoning in its use of vague information to generate decisions. Unlike conventional logic which needs a deep understanding of a system, exact equations, and accurate numeric values, fuzzy logic incorporates an alternative way of thinking, which allows modelling complex systems using a higher level of abstraction originating from our knowledge and experience. Fuzzy logic allows expressing this knowledge with subjective concepts such as very large, very small and short time which are mapped into exact numeric ranges. Since knowledge can be expressed in more natural way by using fuzzy sets, many decision problems can be greatly simplified (Robert Fuller 1998).

Fuzzy logic is defined as a form of mathematical logic in which truth can assume a continuum of values between 0 and 1 (including 0 and 1). The notion that every proposition must be either true or false is known as bivalent logic. The fundamental idea of fuzzy logic is that every proposition, in addition of being true or false, can also be partially true and partially false at the same time (Zadeh 1965). It is the simplest way of expressing partial truth mathematically (Zadeh 1978).

Fuzzy logic can be explained by the following examples. Take the statement, “St James is a large college”. Bivalent logic allows this statement
to be either true or false. Fuzzy logic, on the other hand, contends that this statement is 100% true if student enrolment is 5,000 or more, but only 50% true if enrolment is 2,500 and 0% true if enrolment is less than 500.

In 1965, Fuzzy logic was introduced by LoftiZadeh, as a mathematical tool for dealing with uncertainties. It was designed to acquire mathematical strength to capture the uncertainties associated with human cognitive processes, such as thinking and reasoning. The philosophical foundation of Fuzzy logic can be divided into four major contributions.

(i) The first contribution belongs to Aristotle, who in 200 B.C. proposed the “Law of the Excluded Middle”, which held that every proposition must be either true or false. Plato was among the first to propose that this dichotomy did not fully describe reality. He theorized that there was a state between true and false.

(ii) In 1920, the logician Jan Lukasiewicz proposed the mathematics for a tri-valued logic which included the concept of fractional truth (Lejewshi et al 1967). He referred to this third logic value (beyond true and false) as “Possible”.

(iii) Lukasiewicz’s ideas formed the basis for a wide body of research into what eventually became many-valued logic.

(iv) The notion of an infinite-valued logic was introduced in Zadeh’s seminal work ‘Fuzzy Sets’ where he described the mathematics of fuzzy set theory, and by extension fuzzy logic. This theory proposed making the membership function operate over the range of real numbers $[0, 1]$. 

1.2 FUZZY SET THEORY

Most of our conventional tools for formal modelling, reasoning and computing are crisp, deterministic and precise in character. By crisp it is meant dichotomous that is, yes-or-no type rather than more-or-less type.

Fuzzy set theory provides a mean for representing uncertainties. Historically, probability theory has been the primary tool for representing the uncertainty in mathematical models. Because of this, the uncertainty was assumed to follow the characteristics of random uncertainty.

A random process is one in which the outcome is strictly a matter of chance and the prediction of sequence of events is not possible. Hence, a precise description of the statistics of the long run averages is necessary. However, all the random processes are not uncertain. Yet, some forms of uncertainties may happen in random process. Hence, they are not suitable for implementation or they may not be taken as a model for probability theory. Fuzzy set theory is a marvellous tool which is used to handle uncertainty and vagueness. It is endowed with all information and precision.

The crisp set is defined in such a way to dichotomize the individuals in some given area of discourse; members (those that certainly belong to the set) and non-members (those that certainly do not). For instance, by stating conditions for membership of the set \( A = \{ x / x < 5 \} \), \( \chi(x) = 1 \) indicates that \( x \) is a member and \( \chi(x) = 0 \) indicates that \( x \) is a non-member.

Thus \( \chi(3) = 1 \), but \( \chi(5) = 0 \). For a fuzzy set, the membership function \( \mu: X \to [0, 1] \) allows various degree of membership between 0 and 1 for the elements of a given set.
The characteristic function of a crisp set assigns a value 1 and 0 to each individual in the universal set. So, discrimination between members and non-members of the crisp set may be taken into consideration. The function can be generalized, in such a way that the values assigned to the elements of the universal set fall within the specified range and indicate the membership grade of these elements in the set. Larger values denote higher degrees of the set membership. Such a function is called membership function, and the set defined by it is a fuzzy set. Therefore fuzzy set can be defined mathematically by assigning to each possible individual in the area of discourse a value representing its grade of membership in the fuzzy set, this grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set.

As already mentioned, these membership grades are very often represented by a real number values ranging in the closed interval \([0, 1]\). The most commonly used range of values of membership functions is the unit interval \([0, 1]\). In this case, each membership function maps elements of a given universal set \(X\), which is always a crisp set, into a real number in \([0, 1]\). The primary feature of fuzzy sets is that their boundaries are not precise.

As the complexity of a system increases, our ability to make precise and significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance become utmost mutually exclusive characteristics. Moreover, in constructing a model, we always attempt to maximize its usefulness.

This aim is closely related with the relationship among three key characteristics of every system model, namely complexity, credibility and uncertainty. Uncertainty has an essential role in any effort to maximize the usefulness of system models.
One of the meanings attributed to the term uncertainty is imprecision. That is, the difficulty of taking sharp or precise decision. This applies to many terms used in our day to day life such as the collection of all beautiful girls, costly shirt, close friend, etc. This imprecision or vagueness that is characteristic of natural language does not necessarily imply less accuracy or meaningless.

Zadeh’s ideas have been used in computer science, artificial intelligence, decision analysis, information science, system science, control engineering, expert systems, pattern recognition, management science, operations research, and robotics. The ideas of fuzzy set theory have been introduced in topology, abstract algebra, geometry, graph theory and analysis.

Fuzzy set theory provides us not only with a powerful representation of measurement of uncertainties, but also with a meaningful representation of measurement of blurred concepts expressed in natural languages.

Since every crisp set is fuzzy set but not conversely, the mathematical embedding of conventional set theory into fuzzy sets is as natural as the idea of embedding the real number, into complex plane. Hence, the idea of fuzziness is one of the enhancements, not of replacement.

**Fuzzy Set**

Let X be a nonempty set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function $\mu_A: X \to [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element $x$ in fuzzy set $A$ for each $x \in X$. Then

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}$$
**α-cut**

An α-cut of a fuzzy set $\tilde{A}$ is a crisp set that contains all the elements of the universal set $X$ that have a membership grade in $A$ greater than or equal to the specified value of $\alpha$. Thus,

$$A_\alpha = \{ X : \mu_A(x) \geq \alpha \}, 0 \leq \alpha \leq 1$$

![Figure 1.1 Triangular Fuzzy Parameters and its Alpha Cuts](image)

**Strong α-cut**

The Strong α-cut of a fuzzy set is a crisp set that contains all the elements of the universal set $X$ that have a membership grade in $A$ greater than the specified value of $\alpha$. Thus,

$$\mathcal{A}_\alpha = \{ X : \mu_A(x) > \alpha \}, 0 \leq \alpha \leq 1$$
1.3 FUZZY NUMBERS

Frequently fuzzy sets are represented by linguistic terms such as ‘low’, ‘medium’, ‘often’, ‘few’, ‘high’ and so on are employed to define states of a variable. The relevance of fuzzy variable is that they facilitate gradual transitions between states and consequently possesses a neutral capability to express and deal with observation and measurement of uncertainties. In the classical sense, computing involves manipulation of numbers and symbols.

But in contrast humans employ mostly words in computing, reasoning and arriving at natural language or having the form of mental perceptions. A key aspect of computing with words is that it involves a fusion of natural languages and computation with fuzzy variables. The notion of a granule plays a vital role in computing with words. According to Zadeh (1965), “graduation plays a key role in human cognition”. For humans it serves as way of achieving data comparison.

In most of the real world problems, the experts often, only imprecisely or ambiguously know the possible values of parameters of mathematical models. Hence, it would be certainly fitting to interpret the expert’s understanding of the parameters of fuzzy numerical data, which can be represented by means of fuzzy sets of the real line known as fuzzy numbers. The resulting mathematical programming problem involving fuzzy parameters would be viewed as a more realistic version than the conventional one. In uncertainty conditions, the fuzzy numbers play an important role in many applications including fuzzy queue, fuzzy control, decision-making, approximate reasoning and optimization (Zimmermann 2001).

Fuzzy sets, which are defined on the set R of real numbers, provide a special importance. Initiative conceptions of approximate numbers or intervals such as numbers that are close to 0’or numbers are around the
given real numbers’. Such notions are essential for characterizing states of fuzzy variables. A fuzzy number is the fuzzy subset of the real line where the highest membership values are clustered around a given real number. For a fuzzy number the membership function is monotonic on both sides of the central value. The following topic provides the definition of a fuzzy number, which is commonly accepted in literature.

**Fuzzy Number**

A fuzzy subset $A$ of the real line $R$ with membership function $\mu_A : R \rightarrow [0, 1]$ is called a fuzzy number if

(i) $A$ is normal, i.e., there exists an element $x_0 \in A$ such that $\mu_A(x_0) = 1$.

(ii) $A$ is fuzzy convex, i.e.

\[
\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \{\mu_A(x_1)^\mu_A(x_2)\}, \forall x_1, x_2 \in R \text{ and } \lambda \in [0, 1].
\]

(iii) $\mu_A$ is an upper semi continuous and

(iv) $\text{Supp } A$ is bounded where $\text{supp } A = \{x \in R : \mu_A(x) > 0\}$

**Triangular Fuzzy Number (TRFN)**

A triangular fuzzy number $\tilde{A}(x)$ is a fuzzy number represented by $(a, b, c)$ such that $a \leq b \leq c$, with membership function defined as

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x \leq b \\
1, & x = b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & \text{otherwise}
\end{cases}
\]
A trapezoidal fuzzy number $\tilde{A}(x)$ is a fuzzy number represented by $(a, b, c, d)$ such that $a \leq b \leq c \leq d$, with membership function defined as

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x \leq b \\
1, & b \leq x \leq c \\
\frac{d-x}{d-c}, & c \leq x \leq d \\
0, & \text{Otherwise.}
\end{cases}
$$

Zadeh’s Extension Principle

Let X and Y be crisp sets and let f be a mapping from X to Y,

\[ f: X \rightarrow Y \]  

such that for every \( x \in X \), \( f(x) = y \in Y \).

Let A be a fuzzy subset of X. Then using the extension principle, f(A) is defined as a fuzzy subset of Y such that

\[
f(A)(y) = \begin{cases} 
\sup_{x \in f^{-1}(y)} A(x) & \text{iff } f^{-1}(y) \neq \emptyset \\
0, & \text{Otherwise}
\end{cases}
\]

Where \( f^{-1}(y) = \{ x \in X / f(x) = y \} \)

If f is strictly increasing (or strictly decreasing) then

\[
f(A)(y) = \{ A(f^{-1}(y)) \text{ if } y \in \text{Range} f \}
\]

Fuzzy Structured Element

Even the fuzzy numbers are taken in different form, its Convex and normal membership function is the constant one. It is the structural characteristics of fuzzy numbers. Let f be a monotone function and A be an arbitrary convex normal fuzzy set.

Then from the extension Principle, f(A) is a convex normal fuzzy set. i.e. monotonic function of a convex normal fuzzy set is also convex and normal (Yang Yang et al 2010). Monotonic transformation of a fuzzy number is also a fuzzy number. This property is the base to construct the fuzzy structured element.
Consider the fuzzy set $E$ in a real number field $\mathbb{R}$ and its membership function denoted by $E(x)$.

If $E(x)$ satisfies the following properties:

1. $E(0) = 1$, $E(1+0) = E(-1-0) = 0$;
2. In interval $[-1, 0)$ $E(x)$ is a single growth by right-continuous function.
3. In interval $(0, 1]$, $E(x)$ is a single diminution by left-continuous function.
4. $E(x) = 0$ when $-\infty < x < -1$ and $1 < x < \infty$.

i.e. $E(x) = \begin{cases} 
1 + x, & x \in [-1,0) \\
1 - x, & x \in (0,1] \\
0, & \text{otherwise}
\end{cases}$

Then $E(x)$ is called the fuzzy structured element (Xiaohua Peng et al 2010).

![Figure 1.4 Membership of $E$](image)
Then E(x) is called the fuzzy structured element (Xiaohua Peng & Xinghan Tu 2010).

To each fuzzy structured element E and an arbitrary bounded closed fuzzy number A, there exists a monotone bounded function f in [-1, 1] such that \( \tilde{A} = f(E) \). Set E is the fuzzy set in a real number field R. \( \tilde{A} \) is a fuzzy number linearly spanned by fuzzy structured element E.

If \( f(x) = a + bx, b > 0 \), then we say that \( \tilde{A} = f(E) = a + bE \) is the fuzzy number formed with fuzzy structured element E, and it is easy to get the membership function \( \mu_{\tilde{A}}(x) = E(\frac{x-a}{b}) \). If the fuzzy number \( \tilde{A} = \bar{f}(E) \) =f (E), then the membership function \( \mu_{\tilde{A}}(x) = E(f^{-1}(x)) \), where \( f^{-1}(x) \) is a symmetric function of \( f(x) \) about the variable \( x \) and \( y \) and \( \bar{f}(E) \) is the prolongation of function f (Yang Yang et al 2010). Fuzzy structured element E(x) is symmetric if \( E(-x) = E(x) \).

### 1.4 FUZZY FUNCTION

Let X and Y be two non-empty crisp sets and f is a function from F(X) to F(Y). The function \( f: F(X) \rightarrow F(Y) \) is called a fuzzy function If for every fuzzy set

\[ A \in F(X) \exists \text{fuzzy set } B \in F(Y) \text{ such that } f(A) = B \]

Let X and Y be crisp sets. A fuzzy mapping \( f: F(X) \rightarrow F(Y) \) is said to be monotonically increasing if from \( A, A' \in F(X) \) and \( A \subseteq A' \) it follows that \( f(A) \subseteq f(A') \).

**Function Principle**

Though different methods are available for operations of fuzzy numbers, the function principle and the extension principle are used for
operations of fuzzy numbers in the present thesis. Function principle was introduced by Chen (1985) to treat the fuzzy diametrical operations with triangular, trapezoidal fuzzy number. Function principle has been used as the operations of addition, multiplication, subtraction, division of triangular and trapezoidal fuzzy numbers.

**Operations on Triangular Fuzzy Numbers**

Suppose $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ are two triangular fuzzy numbers. Then

1. Addition of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as
   $$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$
   
   Where, $a_1, b_1, c_1, a_2, b_2, c_2$ are any real numbers.

2. Multiplication of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as
   $$\tilde{A} \otimes \tilde{B} = (a_1a_2, b_1b_2, c_1c_2)$$
   
   Where $a_1, b_1, c_1, a_2, b_2, c_2$ are any positive real numbers

3. $-\tilde{B} = (-c_2, -b_2, -a_2)$
   
   Where, $a_2, b_2, c_2$ are any positive real numbers.

4. Subtraction of $\tilde{A}$ and $\tilde{B}$
   $$\tilde{A} - \tilde{B} = (a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$$
   
   Where $a_1, b_1, c_1, a_2, b_2, c_2$ are any positive real numbers.

5. Division of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as
   $$\tilde{A} \Theta \tilde{B} = \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right)$$
   
   Where $a_1, b_1, c_1, a_2, b_2, c_2$ are any positive real numbers.
6. For scalar multiplication, take $\alpha$ be any real number. Then for $\alpha \geq 0$, $\alpha \tilde{A} = (\alpha a_1, \alpha b_1, \alpha c_1) \alpha < 0$, $\alpha \tilde{A} = (\alpha c_1, \alpha b_1, \alpha a_1)$

7. The inverse of a fuzzy number $\tilde{A} = (a_1, b_1, c_1)$ is defined as

$$\tilde{A}^{-1} = \left( \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right)$$

where $a_1, b_1, c_1$ are any positive real numbers.

**Operations on Trapezoidal Fuzzy Numbers**

Suppose $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ are any two trapezoidal fuzzy numbers. Then

1. Addition of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as

$$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

where, $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$ are any real numbers.

2. Multiplication of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as

$$\tilde{A} \otimes \tilde{B} = (a_3, b_3, c_3, d_3), \text{ Where } X = (a_1a_2, a_1d_2, d_1a_2, d_1d_2)$$

$$Y = (b_1b_2, b_1c_2, c_1b_2, c_1c_2)$$

3. $a_3 = \min X, b_3 = \min Y, c_3 = \max Y, d_3 = \max X$

   Where $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$ are positive real numbers.

4. $-\tilde{B} = (-d_2, -c_2, -b_2, -a_2)$

   Where $a_2, b_2, c_2, d_2$ are any positive real numbers.

5. Subtraction of $\tilde{A}$ and $\tilde{B}$

$$\tilde{A} - \tilde{B} = (a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2)$$

$$= (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

Where $a_1, b_1, c_1, d_1, a_2, b_2, c_2$ and $d_2$ are positive real numbers.
6. Division of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is defined as

$$\tilde{A} \tilde{B} = \left( \frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2} \right)$$

where $a_1, b_1, c_1, d_1, a_2, b_2, c_2$ and $d_2$ are positive real numbers.

7. Scalar Multiplication

Let $\alpha$ be any real number. Then for $\alpha \geq 0$,

$$\alpha \tilde{A} = (\alpha a_1, \alpha b_1, \alpha c_1, \alpha d_1)$$

$$\alpha < 0, \alpha \tilde{A} = (\alpha d_1, \alpha c_1, \alpha b_1, \alpha a_1)$$

8. The inverse of a fuzzy number $\tilde{A} = (a_1, b_1, c_1, d_1)$ is defined as

$$\tilde{A}^{-1} = \left( \frac{1}{d_1}, \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right)$$

where $a_1, b_1, c_1, d_1$ are any positive real numbers (Abhinav Bansal 2011).

**Operations with Fuzzy Structured Element**

Let $M[-1,1]$ be the set of all monotone bounded functions with the sequence in $[-1,1]$.

Now ‘Same Order Monotone Function of $M$’ is defined

$$T_i : M[-1,1] \rightarrow M[-1,1], i = 0, 1, 2, 3 \forall f \in M[-1,1]$$

Such that $T_0(f) = f$, $T_1(f) = f^{T_1}$, $T_2(f) = f^{T_2}$, $T_3(f) = f^{T_3}$

$$f^{T_1}(x) = -f(-x), f^{T_2}(x) = \frac{1}{f(-x)}, (f(-x) \neq 0)$$

$$f^{T_3}(x) = -\frac{1}{f(x)}, (f(x) \neq 0), \forall x \in [-1,1]$$

$$T = \{T_0, T_1, T_2, T_3\}.$$
Multiplication operation of $T$ is defined as

$$T_i T_j(f) = f^{T_i T_j}(f^{T_i})^{T_j}$$

$T$ multiplication is closed and commutative (Wang Peizhuang 1983).

i. \( T_1 T_2 = T_2 T_1 = T_3, T_2 T_3 = T_3 T_2 = T_1, T_1 T_3 = T_3 T_1 = T_2 \)

ii. \( \forall k = 0, 1, 2, 3 \, T_0 T_k = T_k T_0 = T_k, \ i.e \ T_0 \text{ is unity} \)

iii. \( \forall k = 0, 1, 2, 3 \, T_k T_k = T_0 \ i.e \ the \ inverse \ exists. \)

Let $E$ be a symmetric fuzzy structured element and $f$ and $g$ be two monotonic functions with the sequence (assumed Monotone increasing function) in $[-1, 1]$.

Assume the fuzzy numbers $A = f(E)$, $B = g(E)$ and $f^{T_k}$ is the same sequence change of $f$ then

1. \( A + B = (f + g)(E) \)

2. \( A - B = (f + g^{T_1})(E) \)

3. If $A$ and $B$ are positive fuzzy numbers then

\[ AB = f(E) \cdot g(E) = [f \cdot g](E) \]

4. If $A$ and $B$ are negative fuzzy numbers then $AB = [f^{T_1} \cdot g^{T_1}](E)$

5. If $A$ is a negative fuzzy number and $B$ is a positive fuzzy number then

\[ AB = [(f^{T_1} \cdot g)^{T_1}](E) = [-f \cdot g^{T_1}](E) \]
6. If A and B are positive fuzzy numbers and commitment set of B does not contain 0 then
\[ A \div B = A \cdot \frac{1}{B} = f(E) \cdot g_{T_2}(E) = [f \cdot g_{T_2}](E) \]

7. If A and B are negative fuzzy numbers and commitment set of B does not contain 0 then
\[ A \div B = A \cdot \frac{1}{B} = f_{T_1}(E) \cdot g_{T_3}(E) = [f_{T_1} \cdot g_{T_3}](E) \]

8. If A is a negative fuzzy number and B is a positive fuzzy number and commitment set of B does not contain 0 then
\[ A \div B = (-f_{T_3})(E) \]

1.5 DEFUZZIFICATION

The aggregation defined by a triangular, trapezoidal fuzzy number has to be written by a single crisp value which provides the suitable corresponding average. This method is called de-fuzzification.

There are several existing methods for defuzzification taken into consideration that shape of the clipped fuzzy numbers, namely the length of supporting interval, the height of the clipped triangular, closeness to central triangular fuzzy numbers.

The most popular defuzzification methods are centre of area method, mean of maximum method, Graded mean integration method, height defuzzification method and robust ranking method. For simplest this work robust ranking method has been used to defuzzify the triangular and trapezoidal fuzzy numbers.
Robust Ranking Method

Ranking fuzzy numbers were first proposed by Jain (1976) for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Robust ranking method was proposed by Yager (1981) for ordering fuzzy sets based on the concept of area compensation. This method satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition.

For a convex fuzzy number $\tilde{A}$, the robust ranking index is defined by

$$R(\tilde{A}) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U) d\alpha$$

where $(a_\alpha^L, a_\alpha^U)$ is the $\alpha$-level cut of the fuzzy number $\tilde{A}$.

1.6 BASIC QUEUING THEORY

Classical Queues

Queuing theory constructs an efficient tool in modelling and system performance analysis of many complex systems, such as various computer networks, telecommunication systems, call centers, manufacturing systems and service systems. In recent years, queuing theory including queuing systems and networks motivates being the attention of mathematicians, computer engineers, and system designers.

Queuing models are constituted to assist the scientists and engineers to make the performance analysis of complex dynamic systems. Queuing models are dealing with the waiting and the consequence that occurs with their activities. The purpose of constructing a queuing model is to obtain reliable statistics that can be used to analyze, estimate and enhance the performance and the behaviour of the current or the new complex networks.
Queuing theory describes the most unpleasant experiences of our life, waiting. A queuing system can be defined as customers arriving for service, waiting for service if it is not immediate, and if having waited for service, leaving the system after being served.

The term customer is used in a general sense and does not imply a human customer. For example, a customer could be a ball bearing waiting to be polished, an airplane waiting in line to take off, a computer program waiting to be run on a time-shared basis. Wenstop (1976) indirectly analyzed fuzzy queuing systems.

Queuing theory problems was first raised by telephone calls and Danish Mathematician A.K. Erlang was the first who treated congestion problems in 1905. The field has developed to include the application of a variety of mathematical techniques to the study of waiting lines in different contexts. The mathematical techniques include Markov process, linear algebra, transform theory and asymptotic methods. The area of application includes computer, communication, production, manufacturing and health care systems.

**Characteristics of Queuing Processes**

Any queuing system can be completely described with the following six basic characteristics.

1. Arrival pattern of customers
2. Service pattern of customers
3. No of service channels
4. System capacity
5. Queue discipline
6. No of service stages
Arrival Pattern of Customers

Arrival pattern represents the way in which customers arrive and join the queuing system. It is usually measured in terms of the average number of customers per unit time called mean arrival rate. Equivalently, it can also be measured by inter-arrival time between successive customers, called mean inter-arrival time. If the arrival pattern does not change with time, it is called stationary arrival pattern and if it is time-dependent, it is referred as non-stationary.

Bulk Arrival

In this case, the input process is the possibility that arrivals come in batches instead of one at a time. In the event that more than one arrival can enter the system simultaneously, the input is said to occur in bulk or batches.

Queuing behaviour of customers plays a vital role in waiting-line analysis. This may be one of the following:

Jockeying  - When there are parallel queues, human customers may leave one queue and join another queue to reduce waiting time.

Balking    - Customers may not enter the queue at all because the queue is too long or they have no time to wait.

Reneging   - Owing to impatience in waiting, customers may leave the queue.
Service Pattern of Customers

The service pattern of the servers can be measured in terms of number of customers served per unit time, called the service rate or in terms of time required to serve a customer, called the inter-service time.

System Capacity

System capacity represents the number of customers in the system for getting service. It may be finite or infinite.

Queue Discipline

Queue discipline refers to the manner by which customers are selected for service when a queue has formed. It is also called service discipline. The most common disciplines that can be observed in everyday life are,

i) FCFS (First Come, First Served): who comes earlier leaves earlier

ii) LCFS (Last Come, First Served): who comes later leaves earlier

iii) SIRO (Service In Random Order): the customer is selected randomly

iv) Priority – Customer is served in preference over the other

There are two general situations in priority disciplines.

i) Pre-emptive Priority

ii) Non Pre-emptive Priority
Pre-emptive Priority

In Preemptive Priority case, the customer with the highest priority is allowed to enter service immediately even if a customer with lower priority is already in service when the higher priority customer enters the system; that is, the lower priority customer in service is preempted, his service stopped, to be resumed again after the higher priority customer is serviced.

Non- Pre-emptive Priority

In non pre-emptive priority case, the highest priority customer goes to the head of the queue but cannot get into service until the customer presently in service completed, even though this customer has a lower priority.

Symbolic Representation of Queuing Models

A queuing model can be represented in the form (a/b/c):(d/e/f)

Where,  
\(a\rightarrow\) distribution function of the interarrival times  
\(b\rightarrow\) distribution function of the service times  
\(c\rightarrow\) Number of servers  
\(d\rightarrow\) Capacity of the system (maximum number of customers allowed in the system)  
\(e\rightarrow\) Queue discipline  
\(f\rightarrow\) Population (size of input source, may be finite or infinite)

The first three elements were introduced by Kendall in 1953. The elements \(d\) and \(e\) were included by Lee in 1966 and Taha added the element \(f\) in 1968.
1.6.1 Performance Measures of Queuing Models

The main objective of all investigations in queuing theory is to obtain system performance measures which are the probabilistic properties (distribution function, density function, mean, variance) of the following random variables: number of customers in the system, number of customers in the queue, utilization of the server/s, response time of a customer, waiting time of a customer, idle time of the server, busy time of a server. These measures heavily depend on the assumptions concerning the distribution of inter arrival times, service times, number of servers, capacity and service discipline.

It is quite rare, except for elementary or Markovian systems, that the distributions can be calculated. In this work four performance measures are mainly focussed namely,

1. The average number of customers in the system \(L_s\)
2. The average waiting time in the system \(W_s\)
3. The average number of customers in the queue \(L_q\)
4. The average waiting time of a customer in the queue \(W_q\)

1.7 FUZZY QUEUES

In most of the practical situations, statistical information are expressed by linguistic terms such as ‘arrival is slow’, ‘service is slow’, ‘service is fast’, ‘service is not fast enough’ rather than probability distributions. Generally probability distributions require a prior predictable regularity or a posterior frequency distribution to construct. The inter arrival times and service times are more possibilitistic than probabilistic (Buckley 1990). To deal quantitatively with imprecise information in making decision, there is a need to introduce fuzzy logic in queuing models.
Fuzzy queues are used to represent the practical situations with the well-known established traditional queuing approaches which are rigorous but the assumptions are frequently too far from reality. Fuzzy queues would be potentially much more useful and realistic than the commonly used crisp queues. If the usual crisp queues were extended to fuzzy queues, queuing models would have even wider applications.

**Fuzzy Little’s Formula**

This formula states that (Nagoor Gani & Ashok Kumar 2009)

\[
\bar{W}_s = \frac{\bar{L}_s}{\bar{\lambda}}
\]

\[
\bar{W}_q = \frac{\bar{L}_q}{\bar{\lambda}}
\]

\[
\bar{W}_s = \bar{W}_q + \frac{1}{\bar{\mu}}
\]

\[
\bar{L}_s = \bar{L}_q + \frac{\bar{\lambda}}{\bar{\mu}}
\]

1.7.1 **FM/FM/1 Queue with Fuzzy Structured Element**

This model is a single server fuzzy queue with first in first out queue discipline and infinite calling source. Where FM denotes fuzzified time with arrival rate and service rates are fuzzy numbers.

Assume the fuzzy numbers \( \bar{\lambda} = a + bE = f(E), \bar{\mu} = c + dE = g(E) \) where E is the fuzzy structured element.

The performance measures of this queuing model are given by
The average number of customers in the system \( \bar{L}_s = \frac{\lambda}{\mu - \lambda} \)

The average waiting time in the system \( \bar{W}_s = \frac{1}{\mu - \lambda} \)

The average number of customers in the queue \( \bar{L}_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \)

The average waiting time of a customer in the queue \( \bar{W}_q = \frac{\lambda}{\mu(\mu - \lambda)} \)

### 1.7.2 Fuzzy non Pre-emptive Priority Queues

In non-preemptive priority discipline there is no interruption and the highest priority customer just goes to the head of the queue to wait his turn. In practical, the priority queuing model, the input data arrival rate, service rate are uncertainly known. Uncertainty is resolved by using fuzzy set theory. The various performance measures of non pre-emptive priority queues are as follows

Under the steady-state conditions \( \rho_k = \frac{\lambda_k}{\mu_k} < 1 \)

The expected priority queue size is

\[
L_q = \sum_{i=1}^{\infty} L_q^{(i)} - \sum_{i=1}^{\infty} \frac{\lambda_i \sum_{k=1}^{n} \rho_k}{(1-\sigma_{i-1})(1-\sigma_i)}
\]

and by using Little’s formula,

The expected waiting time in priority queue is

\[
W_q = \sum_{i=1}^{\infty} \frac{\lambda_i W_q^{(i)}}{\lambda}
\]

Where

\[
W_q^{(i)} = \frac{\sum_{k=1}^{n} \rho_k}{(1-\sigma_{i-1})(1-\sigma_i)}
\]
1.7.3 **Fuzzy Bulk Arrival Queue**

A queue in which customers arrive in batches with fuzzified Markovian arrival and service with single server and the service takes place with FCFS basis. The various performance measures are as follows.

\[
L_q = \frac{x[yE[K^2]] + 2x(E[K]^2 - yE[K])}{2y(y - xE[K])}
\]

\[
L_s = \frac{x(E[K] + (E[K]^2))}{2(y - xE[K])}
\]

\[
W_s = \frac{E[K] + E[K]^2}{2y(y - xE[K])}
\]

\[
W_q = \frac{yE[K^2] + 2x(E[K]^2) - yE[K]}{2y(y - xE[K])}
\]

1.8 **QUEUE NETWORKS**

Queue networks can be regarded as a group of ‘k’ inter-connected nodes, where each node represents a service facility of some kind with \( s_i \) servers at nodes \( i(S_i \geq 1) \).

**Series Queues with Blocking**

Series queue with blocking is a sequential queue model consisting of two service points \( S_1 \) and \( S_2 \), at each of which there is only one server and where no queue is allowed to form at either point. An entering customer will first go to \( S_1 \). After he gets the service completed at \( S_1 \), he will go to \( S_2 \) if it is empty or will wait in \( S_1 \) until \( S_2 \) becomes empty. This means that a potential customer will enter the system only when \( S_1 \) is empty, irrespective of whether \( S_2 \) is empty or not. Since the model is a sequential model, all the
customers require service at $S_1$ and then at $S_2$. The possible states of the system are (0,0), (1,0), (0,1), (1,1), (b,1) (blocking state).

Assume that potential customers arrive in accordance with a Poisson process with parameter $\lambda$ and the service time at $S_1$ and $S_2$ follow exponential distributions with parameters $\mu_1$ and $\mu_2$ respectively.

In this model Probability of blocking occurs if the first server is busy or the state is blocking (Agassi Melikov & AnarRustamov 2012).

Probability of blocking (PB) = $P(1,0)+ P(1,1)+ P(b,1)$

Case 1: $\mu_1 = \mu_2$

$$PB = \left(\frac{3\rho^2}{2} + \rho\right) P(0,0), \text{ Where } P(0,0) = \frac{1}{1 + 2\rho + \frac{3\rho^2}{2}}$$

Case 2: $\mu_1 \neq \mu_2$

$$PB = \left[\rho \rho_2 \left(1 + \frac{\mu_1}{\mu_2}\right) + \rho_1 (1 + \rho)\right]P(0,0),$$

Where $P(0,0) = \left[1 + (1 + \rho)(\rho_1 + \rho_2) + \frac{\mu_1}{\mu_2} \rho_2\right]^{-1}$

1.9 LITERATURE SURVEY

In recent years, there has been increasing interest in investigating fuzzy logic to get better decisions. A general approach for queuing systems in a fuzzy environment was proposed based on Zadeh’s extension principle and possibility concept. This section focuses on review of the techniques for fuzzy queuing system and its applications in computer networks. Further robust ranking technique for defuzzification of triangular and trapezoidal fuzzy numbers has also been reviewed.
1.9.1 Fuzzy Sets and Fuzzy Logic

Fuzzy set theory represents an attractive tool to aid research in queuing system when parameters are of imprecise in nature. As a methodology, fuzzy set theory incorporates imprecision and subjectively into the model formulation and solution process. In fuzzy logic, exact calculation is viewed as a limiting case of approximate calculation and all are matter of degrees. Using fuzzy logic, knowledge can be interpreted as a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.

Zimmermann (1976) presented a new tool for the formulation and solution of systems and decision problems which contain fuzzy components and fuzzy relationships. He described the basic theory of fuzzy sets, fuzzy relations, fuzzy graphs and the application of fuzzy set theory to the area of decision making, fuzzy control, pattern recognition and fuzzy set models in operations research.

Klir&Yuan (1995) recorded in the book not only the theoretical advances in some areas but also a broad variety of applications of fuzzy sets and fuzzy logic as well. It deals with probability theory and its intimate connection with fuzzy set theory. Possibility theory, various types of fuzzy propositions, fuzzy inference rules, fuzzy controllers, fuzzy automata and fuzzy neural networks are also covered. Basic ideas of fuzzy decision making, engineering applications, applications in medicine and economics are also overlooked in this book.

Ross Timothy (2000) proposed the basic concept of fuzzy logic and operations, a fundamental properties of fuzzy logic in engineering field such as classification, pattern recognition, optimization non linear simulation, knowledge-based systems, clustering, regression, decision making and possibility theory and defines them well.
He also discussed fuzzy relation, fuzzy arithmetic, fuzzy numbers and fuzzy vectors, extension to fuzziness to non fuzzy mathematical forms. He described the simple concepts in synthetic evaluation, ordering, preference and multi-objective decision making methods by fuzzifying this classic probabilistic approach. He classified and summarized them by using equivalence relations fuzzy optimization and fuzzy measures.

1.9.2 Fuzzy Queuing System

Gross & Harris (1998) described that queuing system has wider applications in service organizations as well as manufacturing firms, in that different types of customers are serviced by different types of servers according to specific queue discipline.

Li & Lee (1989) derived analytical results for two fuzzy queuing systems based on Zadeh’s extension principle. Negi and Lee (1992) proposed two methods, the α-cut and two-variable simulation to analyze fuzzy queuing systems. But their approach does not give the membership functions of performance measures completely.

Kao et al (1999) applied α-cut approach to decompose a fuzzy queue into a family of crisp queues. As the α-varies, the parametric programming technique was applied to describe the family of crisp queues and the concept was applied to four typical fuzzy queues, namely M/F/1, F/M/1, F/F/1 and FM/FM/1 where F denotes fuzzy time and FM denotes fuzzified exponential time.

Chen (2005) developed a nonlinear programming approach to derive the membership functions of the steady-state performance measures in bulk arrival queuing systems with varying batch sizes, in that the arrival rates and service rates are fuzzy numbers. Two pairs of Mixed Integer Non Linear
Programming (MINLP) with binary variables are formulated to calculate the upper and lower bounds of the system performance measure at possibility level $\alpha$.

Chen (2006) proposed a MINLP approach to find the system performances of multiple-channel queuing models with vagueness. The approach is based on the notion of classical queuing systems that are useful in capacity planning for designing production and service systems. Furthermore, the proposed approach can deal with the fuzzy input information.

Jau-Chuan et al (2006) proposed a pair of parametric nonlinear programs, using $\alpha$-cuts and Zadeh’s extension principle to derive the membership functions of various system performance measures of $FM^{[a]}/FM/C$ queuing models.

NagoorGani and Ashok Kumar (2009) developed a method to find the membership function of the system performance measures where the batch arrival size, arrival rates and service rates are fuzzy numbers. The idea is based on Zadeh’s extension principle to transform the bulk arrival fuzzy queue to a family of bulk arrival crisp queues that can be described by two pairs of MINLP models.

Jeeva&Rathnakumari (2012) worked on bulk arrival single server, Bernoulli feedback queue with fuzzy vacations and fuzzy parameters. They presented a mathematical non-linear programming method to construct the membership function of the system characteristics of $M/G/1$. A man power planning problem was solved to illustrate the validity of the proposed approach.

Ritha&Lilly Robert (2010) analyzed priority queuing models by using fuzzy set theory. They optimized fuzzy priority queuing models
(preemptive priority, non pre-emptive priority) in which arrival rates and service rates are fuzzy numbers. DSW (Dong, Shah and Wong) algorithm was used to define membership functions of the performance measures of priority queuing model. Algorithm is based on $\alpha$-cut representation of fuzzy sets in a standard interval analysis.

Srinivasan (2013) proposed a procedure to establish the membership functions of the system performance measures in queuing systems where the inter arrival time and service time are fuzzy numbers. DSW (Dong, Shah & Wong) algorithm was used to define membership functions of the performance measures for the queuing model FM/FM/1 in which the arrival rate and service rate are fuzzy numbers. DSW algorithm is based on the $\alpha$ – cut representation of fuzzy sets in a standard interval analysis.

Hsin-I Huang et al (2008) constructed the membership functions of the performance measures of non pre-emptive priority queuing system with J type jobs, in which the arrival rate and service rate for jobs are all fuzzy numbers. $\alpha$-cut approach has been used to transform a fuzzy queue into a family of conventional crisp (distinct) queues. A set of parametric nonlinear programs has been developed to describe the family of crisp queues with non pre-emptive priority.

Membership functions of performance measures for fuzzy non-preemptive priority queues was derived by Devaraj & Jayalakshmi (2012). They proposed a procedure to construct the membership functions of the system characteristics for 3-priority queues when the interarrival time and service time are fuzzy numbers. The basic idea is to reduce a fuzzy queue in to a family of crisp queues by applying $\alpha$-cut approach and Zadeh’s extension principle.
Maria Jose Pardo & David de la Fuente (2007) provided a more realistic description of priority queuing models by using fuzzy set theory. They developed two fuzzy queuing models with priority-discipline, a model with non pre-emptive priorities system and a model with pre-emptive priorities system. Zadeh’s extension principle is the basic approach to this into fuzzy stochastic processes.

Chen (2008) proposed a MINLP approach to obtain system performances of multiple-channel queuing models with fuzzy data. They suggested the idea to transform a multiple-server queue with imprecise data to a family of conventional crisp multiple-server queues by applying the $\alpha$-cut approach in fuzzy theory. On the basis of $\alpha$-cut representation and the extension principle, two pairs of parametric MINLP were constructed to describe the family of crisp multiple-channel queues, from which membership functions of the system performance measures were derived.

Lilly Robert & Ritha (2010) proposed a method for construction of system performance measures when breakdown rates and service rates are trapezoidal fuzzy numbers. Function principle was used as arithmetic operations for fuzzy numbers.

Abhinav Bansal (2011) worked on arithmetic behaviour of arbitrary trapezoidal fuzzy numbers. He reviewed the Basic mathematical operations and formulated trapezoidal fuzzy numbers which have direct applications in fuzzy linear and non linear equations. Advantage of this form of trapezoidal fuzzy numbers over others was also discussed and a category of fuzzy numbers that are neither positive nor negative was investigated in detail with a discussion of some of its interesting properties and applications.

Zhao Bao-fu & Zhang Yan-ju (2011) worked on fuzzy structured element for \((FM/FM/1):(\infty/FIFO/\infty)\) model with fuzzy arrival rate. They
evaluated membership functions for system characteristics such as expected queue length, expected system length, expected queue waiting time, and expected system waiting time based on fuzzy structured element.

There is an existence of a large number of queuing models in operations research using a great variety of methods for their solution. Developments on the representation of fuzzy numbers by random variables can be used to analyze the queuing system.

1.9.3 Robust Ranking Technique

Ranking fuzzy numbers are an important tool in decision making. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. It is true that fuzzy numbers are frequently partial order and cannot be compared like real numbers which can be linearly ordered.

In order to rank fuzzy quantities, each fuzzy quantity has to be converted into a real number and compared by defining a ranking function from the set of fuzzy numbers to a set of real numbers which assign a real number to each fuzzy number where a natural order exists. Usually by reducing the whole of any analysis to a single number, much of the information is lost and hence an attempt is to be made to minimize this loss.

Ranking procedures was developed since 1976 when the theory of fuzzy sets was first introduced by (Zadeh 1965). Ranking fuzzy numbers were first proposed by (Jain 1976) for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Cheng(1985) presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois
&Prade(1987) presented the mean value of a fuzzy number. Lee & Li(1988) presented a comparison of fuzzy numbers based on the probability measure of fuzzy events.

Liou&Wang(1992) presented ranking fuzzy numbers with integral value. Choobineh& Li (1993) presented an index for ordering fuzzy numbers. Chang & Lee 1994) presented ranking of fuzzy sets based on the concept of existence. Since then several methods have been proposed by various researchers which include ranking fuzzy numbers using area compensation (Fortemps&Roubens 1996), distance method, maximizing and minimizing set, decomposition principle, and signed distance. Cheng (1998) provided a new approach to defuzzify fuzzy number by distance method.

Wang & Kerre (2001) classified all the above ranking procedures into three classes. The first class consists of ranking procedures based on fuzzy mean and spread, and second class consists ranking procedures based on fuzzy scoring whereas the third class consists of methods based on preference relations and concluded that the ordering procedures associated with first class are relatively reasonable for the ordering of fuzzy numbers specially the ranking procedure presented by Adamo (1980) which satisfies all the reasonable properties for the ordering of fuzzy quantities. The methods presented in the second class are not doing well and the methods which belong to class three are reasonable.

Garcia & Lamata (2007) modified the index of Liou & Wang for ranking fuzzy numbers. Most of the methods presented above cannot discriminate fuzzy numbers, and some methods do not agree with human intuition, whereas some methods cannot rank crisp numbers which are a special case of fuzzy numbers.
Yager(1981) developed a procedure for ordering fuzzy subsets of the unit interval which is known as robust ranking method. It has compensation, linearity and additive property. This method was adopted to transform fuzzy numbers into crisp ones. Ruthisabels& Uthra (2012) used the robust ranking technique in the fuzzy assignment problem. Nagarajan & Solairaju (2010) used robust ranking technique on computing improved fuzzy optimal Hungarian assignment problems with fuzzy costs. In this research work, robust ranking technique has been used for defuzzification in order to reduce computation complexity.

1.9.4 Performance Evaluation in Computer Networks

Baruch Awerbuch et al (2005) analyzed performance evaluation of the Pulse protocol operating in a MANETs surroundings. The Pulse protocol uses a periodic flood initiated by the pulse source to offer both routing and synchronization to the MANETs. Pulse protocol technique provides increased scalability with respect to mobility, number of flows, node density and minimum power consumption. They used simulation technique to analyze performance of the protocol with respect to transmission ratio, packet delay, and power efficiency in MANETs.

Hekmat & An (2007) provided a mathematical structure for performance evaluation in ad hoc sensor networks. This structure can be used to compute the major factors such as interference, capacity, delay and energy consumption which affect performance metrics. They showed that low power consumption and high capacity are proportional indirectly and constructed the structure which can be used to design ad-hoc sensor networks with a balanced combination of major factors.

Rimantas Plestys & Rokas Zakarevicius (2010) constructed network testbeds which are used to evaluate the performance of MANETs. They
suggested Radio Frequency (RF) signals which travel through the cables among nodes in order to minimize the network. They designed and implemented the new desktop-size testbed. However large-scale testbed is hard to configure and control.

Nan Liu et al (2012) worked on hierarchical ad hoc network, which provides hierarchical on node management, network access authentication and authorization in emergency communications. They provided performance evaluation of hierarchical ad hoc network which was carried out from three aspects namely network packet overhead, path loss and conflict probability using simulation solutions.

Muhamad et al (2013) provided solutions for network bottleneck problems in low data rate ad hoc wireless sensor network IEEE 802.15.4. They conducted comparison study to analyze performance of the proposed ad hoc Wireless Sensor Network (WSN) between low data rate IEEE 802.15.4 and the high data rate IEEE 802.11.

Simulation technique was used for Performance evaluation which is based on throughput, packet delivery fraction, and end-to-end delay. They argued that performance of ad hoc WSN is better than of WSN in the case of throughput and packet delivery ratio. Moreover, it was concluded that performance of ad hoc WSN over IEEE 802.15.4 is similar to that over IEEE 802.11 on low data rate small scale networks.

Adrian Conway (1989) formulated a generic queuing network model which is used for the analytical performance evaluation networks according to Open System Interconnects (OSI) Reference Model. The queuing model, based on the generic specifications of the Reference Model, was used to evaluate mean performance measures, the underlying topology and the manner in which the processing capacity is distributed among the
various entities. He provided iterative decomposition algorithm for the solution of the network model.

Sidou et al (1991) presented architecture for creating an execution of protocols in OS1 reference model. They described the mechanism for intercommunication between layers. The implementation was presented in term of its support packages and tools. Performance measurements of the implementation were shown to highlight the features of the implementation, advantages and disadvantages of this architecture were discussed.

Yusheng Liu & Hoangt(1995) proposed a queuing network model for evaluation of performance measures of OSI Remote Procedure Call (RPC). They provided performance results which were obtained by applying different implementations of the model according to the OS1 RPC protocol specification. This model can be used to predict performance measures of a Communication Server Interface (CSI) RPC system and analyze performance effects of various implementation options.

Nuangjamnong et al (2008) analyzed OSI Network Management Model (OSI-NMM) which is the standard model for conceptual framework to organize diverse range of network resources. OSI-NMM has been used to evaluate large enterprise networks based on capacity and performance management.

Subayal Khan et al (2011) suggested complex distributed applications which provides scalability and performance metrics analysis at all the layers of OSI network model. They presented a technique to obtain performance of simulation based on the extensions of Abstract InstructionWorkLoad&executionPlatform based (ABSOLUT). They explained design accuracy of the modelled components and their application in the context of M3 (Multi-device, Multi-vendor, Multi-domain), which is a
tri-layered conceptual interoperability architecture for embedded devices. The simulated results analysis has been provided using network simulators such as ns-2, OMNeT++ and OPNET.

Xiaodongzhang & Yong yai (1995) proposed a technique which quantify the heterogeneity of networks and characterize the performance effects. They analyzed performance measures of heterogeneous Networks using different metrics such as speedup, efficiency and scalability. These techniques are used to cover performance evaluation of both homogeneous and heterogeneous computations.

António Serrador & Correia (2007) developed an approach to integrate a set of Key Performance Indicators (KPIs) into a single one by using a cost function. The proposed model enables the implementation of different Common Radio Resource Management (CRRM) policies, by manipulating KPIs according to user’s or controller’s perspectives to get better QoS. They have shown that various policies can be developed with a different impact on the network.

Samundiswary et al (2009) argued that simulation results in heterogeneous sensor networks outperformed homogeneous sensor networks with respect to different performance metrics such as energy consumption, end-to-end delay and delivery ratio with different node capability.

Qian Huang et al (2009) developed performance modelling which is important to create efficient tools for large complicated networks. Hierarchical overlay infrastructure and decomposition of the correlated traffic are the bases to establish the performance modelling for multiservice in hierarchical heterogeneous wireless networks with overflow traffic through simulation technique.

Dun Luo et al (2011) proposed Range Expansion (RE) to adopt the changes in the standardization process of heterogeneous networks. They
described heterogeneous deployment scenario and evaluate the merits and
demerits of introducing RE scheme. They designed Physical Downlink
Control Channel (PDCCH) and Physical Downlink Shared Channel (PDSCH)
with a novel PDCCH distribution policy.

Since past three decades many researchers have employed different
methods to establish membership functions of performance measures of
various fuzzy queuing systems. Mainly they have used α-cut approach to get
the membership function. Using α-cut approach, fuzzy queue has been
reduced to family of crisp queues with different α-cuts (Tsung-Yin Wang et al
2010). This approach is very hard to derive the membership functions of
performance measures. Various methods of performance evaluation of
computer networks were analyzed. All performance analysis provided the
results based on simulation techniques.

In addition to that, in usual practice the system designers are in
need of crisp data to take optimum decisions. In a review of lot of articles
published on this subject, no one has found the method for fuzzification as
well as defuzzification in a simple way. Also there was no evidence for
results without simulations. So, in this work, evaluation of performance
measures in terms of crisp values for various fuzzy queuing models is done in
a simple way and applications of fuzzy queuing models in various computer
networks to improve QoS has also been made with analytical(numerical)
results.

1.10 MOTIVATION AND SCOPE OF THE THESIS

In this thesis we expand the work initiated by Yager, in the
surcharges atmosphere of new arrival of research articles on fuzzy queuing
system by various researchers. Ronald. Yager motivated by his work of
ordering fuzzy subsets of the unit interval by robust ranking technique. Li
In real world applications, all the parameters of queuing models may not be known precisely due to uncontrollable factors. At the same time, in practical situations, the system designers wish to get precise data for decision making. To overcome this ambiguity, this thesis proposes performance measures of various fuzzy queuing models in terms of crisp values using robust ranking technique. Fuzzy structured element has been used to obtain system characteristics of (FM/FM/1): (∞/FIFO/∞) queues. In addition to that an effort has been made to apply series queues with blocking probability technique in MANETs and OSI network to improve Quality of Service (QoS) metrics in computer networks.

1.11 CONTRIBUTION OF THIS RESEARCH

In this research, an elaborate literature survey and background study was made on existing fuzzy queuing models, ranking techniques, queues with fuzzy structured element and series queues with probability of blocking in computer networks.

Membership function of performance measures for (FM/FM/1): (∞/FIFO/∞) model using fuzzy structured element has been made and crisp results have been presented using defuzzification.

An evaluation of performance measures in terms of crisp values for fuzzy non pre-emptive priority queues, fuzzy bulk arrival queues was made using robust ranking technique and results have been presented.

An evaluation of probability of packets blocking in MANETs was made using fuzzy queue and results have been presented.
Multi processing unit for reducing probability of packets blocking in Open System Interconnects (OSI) network was used and results of blocking probability have been presented using fuzzy queue.

An analysis of system characteristics of heterogeneous computing network using fuzzy queue with heterogeneous servers was made and results have been presented.

1.12 ORGANIZATION OF THE THESIS

The results embodied in this thesis are concerned with the study of fuzzy queuing models and applications of fuzzy queues in computer networks. The present thesis consists of seven chapters.

Chapter 1 is introductory in nature which unfolds the fundamental theoretical background of this thesis. In this chapter, a notion of triangular, trapezoidal fuzzy numbers which serve as a parameter for fuzzy queuing models is introduced (Dubois & Prade 1978). Fuzzy structured element and series queue with blocking probability in queue networks were also discussed.

This chapter gives exclusive literature survey on fuzzy queuing models and its applications in computer networks. The existing techniques are accounted vividly to construct the membership functions for fuzzy queuing models. In the previous literature, fuzzification of performance measures for traditional queuing theory was studied elaborately. Chapter 1 provides the summary and scope of the thesis as well as the applications used in the research.

Chapter 2 presents fuzzification of performance measures of (FM/FM/1):(∞/FIFO/∞) queue using fuzzy structured element and defuzzification using robust ranking technique. Construction of membership functions of system characteristics for queuing models by α- cut approach (T
sung Yin Wang et al 2010) provides heavy workload and it is difficult to directly reflect the characteristics of the structure.

In this work, membership function of performance measures for FM/FM/1 queue has been established based on fuzzy structured element theory. Fuzzy structured element theory has been applied and order-preserving transform of bounded monotone functions has been defined on the symmetric interval [-1, 1] (Xiaohua Peng & Xinghan 2010). Fuzzy arithmetic operations with fuzzy structured element were discussed. Arrival rates and service rates are fuzzy numbers represented by fuzzy structured element E.

Fuzzy structured element method provides analytic computation of the fuzzy number and fuzzy-valued functions. So far, many researchers have taken single fuzzy parameter to build the membership function using fuzzy structured element theory. However, in many real life situations the values of the parameters are vague. To overcome this ambiguity, in the present work, all the parameters have been taken as fuzzy numbers.

As arrival rates and service rates are considered fuzzy parameters, the results of system characteristics are more realistic in nature. Though fuzzy values are realistic, optimum decisions cannot be taken using fuzzy data. In order to obtain crisp data, defuzzification has been made for performance measures using robust ranking technique.

Chapter 3 has been divided into two parts. First part is concerned with fuzzy non pre-emptive priority queues (Ritha W & Robert L 2010) and second part is devoted to fuzzy bulk arrival queues (Chen 2006). So far many researchers have made fuzzification of performance measures of priority queues and have constructed membership functions in bulk arrival queuing system with varying fuzzy batch sizes.
All previous researches on fuzzy queuing models are focused on fuzzification of performance measures of ordinary queues. After that for practical use, defuzzification of fuzzy measures has been made.

In this work, arrival rates and service rates are taken as trapezoidal as well as triangular fuzzy numbers and fuzzy numbers have been defuzzified using robust ranking technique. Using the crisp data, various performance measures such as expected queue length, expected system length, expected waiting time in the queue and expected waiting time in the system of fuzzy non-pre-emptive priority queues and fuzzy bulk arrival queues with fuzzy varying batch sizes have been obtained in terms of crisp values. The present work describes input data as fuzzy numbers and output results as crisp values.

Chapter 4 describes Quality of Service (QoS) optimization in MANETs using fuzzy queue. MANET is an emerging technology because it holds dynamic infrastructure (ImrichChlamtac et al 2003). In MANETs data transmission need high bandwidth and reliable transfer because of large amount of data size and frequent path break due to mobility. When the Quality of Service metrics (QoS) in ad hoc networks is improved, longevity of the network increases.

Selecting suitable microcontroller and wireless transceiver chips play an important role in assembling ad hoc mobile devices, in which data control should be designed, so that packet loss is minimized. Available QoS metrics based on queuing or buffer management in wired and other wireless networks are not applicable because of its unique characteristics. In this work, the simplest QoS model buffer less system was proposed.

So far many researchers have taken crisp data for packets arrival rate. But in real time, the values of the parameters are vague. Literature search revealed that no researcher has taken packets arrival as trapezoidal fuzzy
number. Thus, there is a great need of fuzzy numbers, which will be helpful to explain uncertainty. Hence, in this work arrival rate of packets has been taken as trapezoidal fuzzy number.

In the present work, measurement of fuzzy probability of blocking of the arrival packets is suggested by using fuzzy queues. Series queue with blocking technique has been used between the network layers in the processor memory (Agassi Melikov & Anar Rustamov 2012). The processor memory is divided into two parts as separate service channels.

The Medium Access Control (MAC) sub layer has been determined as one server with service rate \( \mu_1 \) and rest of the network layers were taken as another server with service rate \( \mu_2 \). Arrival packets enter the MAC sub layer through the physical medium (physical layer) and waits in the queue if the system is busy. The blocking state happens when the first server completed its task, but the second server is still working. If the first server is busy, then arrival packet will be dropped.

Fuzzy probabilities of Blocking for different fuzzy arrival rates with three cases \( \mu_1 = \mu_2 = \mu, \mu_1 < \mu_2, \mu_1 \mu_2 \) have been found. Defuzzification has been made to obtain probability of blocking using robust ranking technique. According to the arrived results, when \( \mu_1 \mu_2 \), blocking probability is lesser than the other two cases. Thus service rate is increased in the MAC layer, QoS will be increased. Using these phenomena, system designers can design the microcontroller and wireless transceiver chips effectively.

Chapter 5 presents performance enhancement in Open System Interconnects (OSI) network layer using fuzzy queue. The OSI network layer is the backbone of data communication in network; in traditional networks size of packets is quite small to be delivered from the source node to the
destination node. In the case of multimedia data where size is measured in MegaByte (MB), it is a big challenge to transfer the data. In multimedia networks energy consumption still remains the main problem. As we increase hardware specifications, more energy will be consumed and vice versa. However, service time for each arrival packets increases as the size expands. Thus number of packets waiting in queue in the buffer is swelled. The loss of some video and voice packets might not seriously affect overall data in destination, so that remaining packet can convey the necessary message to the listener.

In the case of loss of image packet, it is difficult to rearrange whole frame in order to get original picture back. To reduce data loss, multiprocessing unit has been installed in OSI network layers (Kiln & Koh 1991).

In the processor memory, we took Physical layer and MAC sub layer together as one “service point” with service rate $\mu_1$ and remaining layers together as second “service point” with service rate $\mu_2$ to determine performance metric based on the data queue, stored in MAC layer. The temporary storage (buffers) device between two different layers (servers) consumes much electric power and more cost.

To avoid buffers between layers in OSI network, series queue with blocking technique has been used. According to arrived results in chapter 4, when $\mu_1 > \mu_2$ probability of blocking is decreased. In order to obtain the state $\mu_1 > \mu_2$ multiprocessing unit has been added in the MAC layer in OSI network model. The first service point performs with ‘S’ units. Then service at node 1 is $S\mu_1$.

Fuzzy blocking probability has been obtained for different fuzzy arrival rates with different values of S. Defuzzification has been done to get
blocking probability. When the value $S$ increases blocking probability decreases. Though installation cost of multiprocessing unit is higher than single processing unit, compared to data loss, installation of multiprocessing unit will be optimum for system designers.

Chapter 6 presents performance measures analysis of heterogeneous computing network using fuzzy queuing model. Heterogeneous computing network consists of more than one kind of servers with different service rates. It is a multi-core system which not only provides performance by just adding cores, but also gives specialized processing capabilities to handle particular tasks.

In the present work, heterogeneous server fuzzy queuing model has been used in heterogeneous computing network for analyzing performance measures of the network. Here data arrival rates and service rates are taken as trapezoidal fuzzy numbers. Thus the system is more realistic and general in nature. Crisp performance measures such as expected waiting time of a job in the system and expected system length have been evaluated for two servers with different arrival rates and difference service efficiencies.

According to the arrived results, conclusion has been made that when difference between service efficiencies is increased, waiting time and system length are reduced. Though installation of servers with high efficiency is costly, compared to system length and data waiting time; installation of high efficiency servers will give optimum solutions for system managers. As Performance measures are crisp values, the system managers can take optimum decisions.

Chapter 7 includes with conclusion and scope for future research. The work presented in chapters 1-6 of the thesis describes the development and validation of new analytical methodologies for determination of
membership functions, performance measures of various fuzzy queuing models, Quality of Service (QoS) in MANETs, performance enhancement in OSI network model and performance analysis of heterogeneous computing network. The methods are simple, rapid, reliable and validated. So, future works may address the way to find the performance measures of other fuzzy queuing models and apply fuzzy queues in various computer networks.