CHAPTER - V

MATHEMATICAL CONSIDERATIONS IN ŚULBASŪTRAS

5.1 PREAMBLE

Astronomy and mathematics had expanded greatly at the time of the vedic period. One can see many examples for this in *Saṁhitās, Brahmaṇās* and so on. *Śulbasūtras* explain many theorems and concepts of trigonometry, like Pythagoras theorem, and many properties about the sides of the triangle. Method of drawing a perpendicular bisector to a line, construction of a square and a rhombus, method of transforming a square into a circle or a circle into a square are described in *Sulbasutras.*

Mathematical and geometrical knowledge originated from the construction of altars. *Śulbasūtras* had summarized some of these informations. They are (1) various units for linear measurement, (2) knowledge of rational numbers like ½, 1/3 ... (3) Construction of a square, or rectangle using chord and peg (4) the ideas of irrational numbers (5) construction of squares and transformation of squares into rectangles (6) value of $\sqrt{2}$ as diagonal of a square of unit length and $\sqrt{3}$ as diagonal of a rectangle of sides $\sqrt{2}$ and 1.[66]

[66] *B.Sl.* 1.9 – 1.13 (Ref. A. K Bag , " Ritual geometry in India and its parallelism in other cultural areas" , Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian Journal of History of Science, 25(1-4) 1990))

Some rational triples are given in Āpa. Sl.[67] as (3,4,5), (12,5,13), (7,24,25), (12,35,37), (15,36,39)

5.2 ALTARS

Śulbasūtras describe the rules of construction of different types of altars, *pandals*, and places for sacred fire. The mathematical part of the construction especially the geometrical part is given only in *Śulbasūtras*. *Śulbasūtras* describe the shapes of *citis* or altars of sacred fires which may differ in shape but their areas will remain the same. The main *Śulbasūtras* dealing with mathematics are 1) *Baudhayana Śulbasūtra* 2) *Āpastamba Śulbasūtra* and 3) *Kātyayana Śulbasūtra*.

For some type of *yajnas* the shape of the altar was divided into one or more squares. For this the addition or subtraction of squares was necessary. This process was done based on the theorem of hypotenuse. The altar for the *Asvamedha* sacrifice must be twice or three times the area of the basic altar.[68]

The area of the altar to be constructed is either twice or thrice the area of the altar of the first basic altar which is of area 7.5 square *puruṣas*. The area of the *Sautramani vedi* is one-third of the area of the *Mahāvedi*.[69] The altar for the *Pitṛyajna* is to be formed with the one-third part of the side

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[67] *B. Sl* – 1-13.
[68] *K.Sl.* 5-2 (Ref. Kātyayana Śulbasūtra, Ed. Khadilkar S.D, p.56)
[69] *B.Sl.* p.56 (Ref. Kātyayana Śulbasūtra, Ed. Khadilkar S.D, p.56)

of the *Mahāvedi* and its area will be equal to the one ninth part of the *Mahāvedi.*[70]

The area of *the Mahāvedi* is 972 sqare *padas.*[71] The area of the *Sautramaṇi vedi* is 324 square *padas*. The area of the *vedi* for *Pitṛyajna* will be 1/9 x972 =108 square *padas*.

In earlier times Indians maintained three *Agnis* 1) *Gārhapathya* 2) *Āhavanīya* and 3) *Dakṣiṇāgni.* The third one is for sacred purposes. The shape of *Gārhapathya* is circular and that of *Āhavanīya* is square and that of *Dakṣiṇāgni* is semi circular. All the three are of equal areas.[72]

In the construction of *Paitṛki vedi,* first construct a square having an area of two square *puruṣas.*[73] It is clear that the constructions of all these *vedis* are not possible without the help of the knowledge of the proportion of the hypotenuse. The discovery of this proportion was before the period of *Baudhayana Śulbasūtra*. The shape of the altars may differ but their areas will remain the same. The period of development of Geometry in India is before the time of Pythagoras.

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[70] *B. Sl.* 1-82 (Ref. Katyayana Sulbasutra, Ed. Khadilkar S.D, p.56)
[71] *Ap. Sl.* p.83 (Ref. Katyayana Sulbasutra, Ed. Khadilkar S.D, p.57)
[72] Dr. M.M. Kane, *Dharmasastra*, Vol. 2 part II, p. 994. (Ref. *Katyayana Sulbasutra*, Ed. Khadilkar S.D, p.61)

[73] *K.Sl,* 2-6 (Ref. Katyayana Sulbasutra, Ed. Khadilkar S.D, Vaidika Samsodhana Mandala, TMV Nagar, Pune, p. 57)

The origin of Indian Geometry is connected with the perpetual daily sacred fires which are placed on altars of various shapes. Generally these have 5 layers and the number of bricks used are fixed. The construction of altars is based on different figures square, circle, semi-circle, triangles, rhombus, falcon, tortoise shape and so on. *Taittiriya Saṁhita*[74] says "He who desires heaven may construct falcon shaped altar because falcon is the best flier among the birds". The construction and calculation of area of altars involve many mathematical and geometrical problems. The ceremonies were performed on the altar in sacrificer's house or on a nearby plot. For keeping fire, it was made of bricks. There were two types of fire altars, (1) called *Nitya* (perpetual) which is performed daily and is of area one square *puruṣa*, (2) called *Kāmya* (optional) which is for wish fulfillment, and is of area 7½ sq. *puruṣa* or more and having a minimum of five layers of bricks. A class of structure known as *mahāvedi* and other related *vedis* were made by the side of the optional altar to reside the family of the organizer to reside in. We can see many descriptions of altars in *Śulbasūtras*. Some of them are tabulated below.[75]

[74] A.K Bag, *Ritual Geometry in India and its parallelism in other cultural areas*, Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian Journal of History of Science, 25 (1-4) 1990).

[75] Ibid.

5.2.1 Different Types of Altars and their Measurements

Altar	Shape/Horizontal Section	Area	Reference
I. Perpetual Fire Altar*:*			
(i*) Āhavanīya*	Square	One sq. *Vyayama*	B.Sl.3.1 – ; 7.4;7.5
(ii) *Gārhapatya*	Circles, square	----------	Ap.Sl. 4.4
(iii*) Dakṣiṇāgni*	Semi-circle	-----------	K.Sl. 1.11
II. ***Vedis***:			
(i) *Mahāvedi or Saumikyā vedi*	Isosceles trapezium a=face=24 *padas* b=base=*30 padas* c=height=36 *padas*	972 sq. *padas*	B.Sl. 4.3 B.Sl.3.11-13 Ap.Sl. 5.1-8 K.Sl.2.11;2.12
(ii) *Sautrāmaṇi vedi*	Isosceles trapezium (a=24/√3 b=30/√3 C=36/√3) or (a=8/√3 b=10/√3 C=12/√3)	324 sq. *padas*	
(iii*) Paitṛki vedi*	(i) Isosceles trapezium A=8, b=10, c=12 (ii) a square having four corners in four coordinate directions.	108 sq. *padas*	
(iv) *Prāgvaṃśa*	Rectangle 16 X 12 or 12 X 10	192 or 120 sq. *prakramas.*	

III. Optional Fire Altars:

(i) Caturasra-*śyenacit*	Hawk bird with sq. body, sq. wings and sq. tail	7½ sq. *puruṣa*	B.Sl. 8.1-1.19, 9.1-10, Ap.Sl.8.2- 15.7
(ii) *Vakrapakṣa-Vyastapuccha śyena*	Hawk bird with ben wings and out-spread tail.	7½ sq. *puruṣa*	B.Sl. 0.1- 20; 11.1 - 13 B.Sl. 11.1-13
(iii) *Kaṅkacit*	Hawk bird with curved wings and tail.	7½ sq. *puruṣa*	B.Sl. 12.1- 8; Ap.Sl. 21.1- 21
(iv) *Alajacit*	Alaja bird with curved wings and tail.	7½ sq. *puruṣa*	B.Sl 13.1-6; Ap.Sl. 21.1
(v) *Prauga*	Triangle	7½ sq. *puruṣa*	B.Sl 14.1-8; Ap.Sl.12.4-12
(vi)*Ubhayataprauga*	Rhombus	7½ sq. *puruṣa*	B.Sl 15.1– 6; Ap.Sl.12.7-12
(vii) *Rathacakracit*	Circle	7½ sq. *puruṣa*	B.Sl 15.1– 6; Ap.Sl.12.9-12
(viii) *Droṇacit*	Trough	7½ sq. *puruṣa*	B.Sl 17.1– 12; 18.1 – 15 Ap.Sl.13.4-13
(ix) *Smaśānacit*	Isosceless trap.	7½ sq. *puruṣa*	B.Sl. 9.1-11; Ap.Sl.14.7-14
(x) *Kūrmacit*	Tortoise	7½ sq. puruṣa	B.Sl.20.1– 21; 21.1-13

5.3 MEASUREMENTS OF BRICKS

Bricks of various sizes are used for optional fires to get the required shape. The number of bricks for the first construction is fixed as 200, covering an area of 7½ sq. *puruṣa*. Only sun dried bricks were used for this purpose. The square bricks *Caturthī* (one-fourth), *Puñcamī* (one-fifth), *Ṣaṣṭi* (one-sixth) and their subdivided bricks were manufactured and each was named separately.[76] Their dimensions and subdivisions are listed below.[77]

1. *Caturthī* (one-fourth, square brick, size 30 ang x ang).

   Five types were available

   (a) *Caturthī* (square quarter) = 30 x 30 (ang.)

   (b) *Ardhā* (triangular half) = 30 x 30 x 30√2 (ang.)

   (c) *Trasra pādyā* (triangular quarter) = 30 x 15√2 x 15√2 (ang.)

   (d) *Caturaśa pādyā* (four sided quarter) = 22½ x15 x15/2x15√2 (ang.)

   (e) *haṃsamūkhī* (pentagonal half brick)

   = 15√2 x 7½ x 15 x 7½ x 15√2 (ang.)

2. *Pañcamī* (one-fourth, square brick, 1/5 pu. x 1/5 pu;24 ang. x ang.)

   (a) *Pañcamī* (one-fourth, square brick) = 24 x 24 (ang.)

   (b) *adhyardhā-pañcamī* (rectangular brick, side longer by one-half) = 24 x 36 ang.

   (c) *pañcamī-sapādā* (rectangular brick, side longer by one-quarter) = 24 x 30 ang.

   (d) *pañcamī-ardhā* (triangular half brick) = 24 x 24 x 24√2 (ang.)

   (e) *pañcamī-pādyā* (triangular quarter bricks) = 24 x12√2x12√2 (ang.)

   (f) *adhyardhārdhā* (triangular half brick of *adhyardhā*) = 36 x 24 x 12√3 (ang.)

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[76] *B.Sl.* 10-2, 10-3, 11-5, 11-6

[77] A. K Bag, "*Ritual geometry in India and its parallelism in other cultural areas*", Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian journal of history of science, 25 (1-4) 1990)

(g) *dirghapādyā* (triangular quarter bricks of *adhyardhā* with larger base) = 36 x 6√13 x 6√13 (ang.)

(h) *śulapādyā* (triangular quarter brick of *adhyardha* with shorter base) = 24 x 6√13 x 6√13 (ang.)

(i) *ubhayī* (triangular brick when half brick of (g) and (h) are attached) = 30 x 12√2 x 6√13 (ang.)

(j) *pañcamī-aṣṭami* (one-eighth triangular brick of *pañcamī*) = 12 x 12 x 12√2 (ang.)

For falcon shaped fire altar, 187½ *Pañcamī* bricks were used to cover 7½ sq. *puruṣa* (187½ x 1/5 x 1/5 = 7½). Out of these 3½ *pañcamī* bricks were used for head, 52 for body, 117 for two wings, 15 for tail[78]. There are three types of *Nitya agni,* called *Gārhapathya, Āhavanīya* and *Dakṣināgni*.[79] *Gārhapathya citi* is constructed with 21 bricks[80]. The *saumiki vedi* has been constructed with face 24 *prakramas*, base 30 *prakramas* and height *36 prakramas*.[81] Some units[82] are given below:

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[78] *B.Sl.* 11-2, 11-3(Ref. A. K Bag, "Ritual geometry in India and its parallelism in other cultural areas", Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian journal of history of science, 25 (1-4) 1990))

[79] *Rgv* 1-15.12, 6.15.19, 5.11.2(Ref. A. K Bag, "Ritual geometry in India and its parallelism in other cultural areas", Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian journal of history of science, 25 (1-4) 1990))

[80] Taittiriya Samhita 5-2,3-5, *Maitrayani Samhita* 3.2.3, Katha Samhita- 20.1, Kapistala Samhita 32.3 (Ref. A. K Bag, "Ritual geometry in India and its parallelism in other cultural areas", Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian journal of history of science, 25 (1-4) 1990))

[81] *Taittiriya Samhita* 6.2.4.5 (Ref. A. K Bag, *Ritual geometry in India and its parallelism in other cultural areas*, Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian journal of history of science, 25(1-4) 1990))

[82] *B.Sl.* 1-3 (Ref. A. K Bag, "*Ritual geometry in India and its parallelism in other cultural areas*", Indian national science academy, Bahadur Shah Marg, New Delhi (Ref. Indian Journal of History of science, 25 (1-4) 1990))

1 *Prādeśa* = 12 *aṅgulas,*

1 pada = 15 *aṅgulas,*

1 iṣā = 188 *aṅgulas,*

1 *akṣa* = 104 *aṅgulas,*

1 *yuga* = 86 *aṅgulas,*

1 jā*nu* =32 *aṅgulas,*

1 *śamyā* = 36 *aṅgulas,*

1 *bāhu* = 36 *aṅgulas,*

1 *prakrama*= 2 *pada,*

1 *aratni* = 2 *prādeśa,*

1 puruṣa = 5 *aratni,*

1 vyā*yāma* = 4 *aratni,*

1 *angula* = *14 anus* =34 *tilas* = ¾ inch (approximately).

5.4 THE PROOF OF THE THEOREM OF HYPOTENUSE (PYTHAGORAS THEOREM)

The difference between Pythagoras theorem and theorem of hypotenuse stated by *Śulbakaras* is that they refer to a rectangle or to a square instead of the right-angled triangle. Even though the reference is made to a rectangle or to a square, their aim was to refer only to the two sides and the diagonal. If their aim was to refer to a rectangle or to a square, they will refer to all the four sides. The mathematical part of the *Śulbasūtras* contains theorems of squares and rectangles. Their aim was to find out the side of a square which is equal in area to the sum or difference of two squares, or to transform a circle into a square or a triangle into a square.

The Sulbakaras did not give any proof for the Pythagoras theorem (The theorem of hypotenuse). They gave importance to practical knowledge of making *vedis* instead of the mathematical aspect. Śulbasūtras give many sets of the measures of right angled triangles. One can find many sets in *Āpastambha Śulbasūtra.*[83] They are (1) If the sides of a rectangle are 3 and 4, then its diagonal is 5. (2) If the sides of a rectangle are 5 and 12, then its diagonal is 13. (3) If the sides of a rectangle are 8 and 15, then its diagonal is 17. (4) If the sides of a rectangle are 12 and 35, then its diagonal is 37. These measures are used in the construction of *vedis.* All these can be used for the construction of *Mahāvedi.*

5.4.1 Construction of a Square having an area of two times the area of a given square

In the case of *Paitṛki Vedi*[84] (to be constructed in the *Sakamedhaparvan* of *the caturmāsya* for the *Mahāpitṛyajna)* one should construct a square having an area of two square *puruṣas.*

CT =DT = *1 puruṣa.*

P,Q,R,S are the midpoints of AB, BC, CD, and DA. Square PQRS is a square having an area one square *puruṣa*. The proof is as follows.

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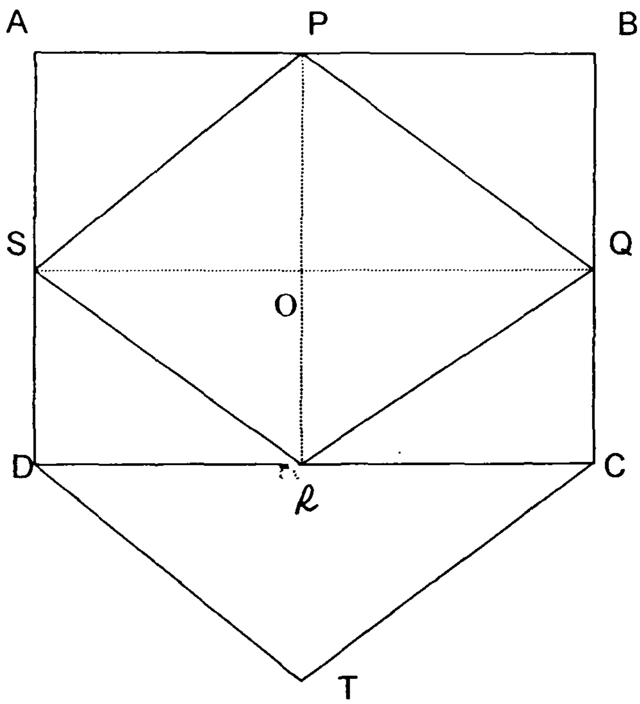
[83] *Ap. Sl.* 5-3, 5-4, 5-5, 5-6 (Ref. *K.Sl* p.53)

[84] *K.Sl.* 2-6 (Ref. Katyayana Sulbasutra, Ed. Khadilkar S.D, p.10)



From figure

Area of PBQO = 2x Area of triangle PBQ

=2 x Area of triangle POQ.

=2 x ¼ of the area of the square PQRS.

=1/2 of the area of square PQRS.

Area of square ABCD = 4 x Area of the square PBQO.

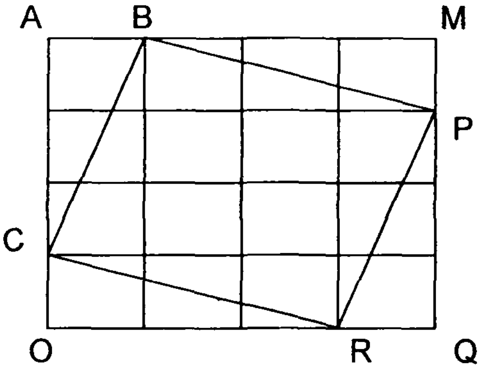
=4 x ½ Area of the square PQRS

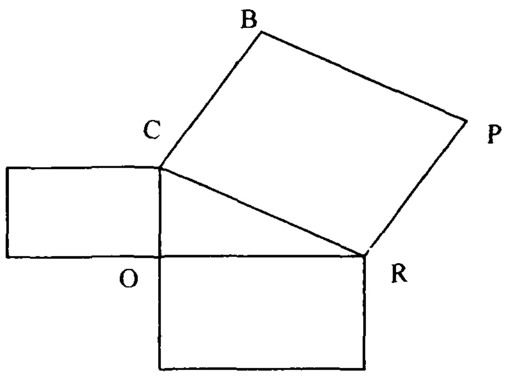
= 2 x Area of the square PQRS.

The figure is an indication that the square on the diagonal of a square is twice as large as that of the square.

5.4.2 Square produced on the Diagonal of a rectangle

Square produced on the diagonal is equal to the sum of the squares on the sides.





Another reference to the theorem of hypotenuse is in *Kātyayana Śulbasūtra*. This says that in a right angled triangle[85] whose side is one *pada* and the other side is 3 *padas*, its hypotenuse will produce an area of ten square *padas*. In the *Varunapraghasaparvam* of *Caturmāsya* sacrifice, the northern altar is a square having an area of ten square *padas*.

[85] *K.Sl.* 2-8 (Ref. Katyayana Sulbasutra, Ed. Khadilkar S.D, p.12)

Triangle ABC is a right angled triangle with one side one *pada* and the other side three *padas.* This is half of the rectangle with sides one *pada* and *three padas* respectively. The remaining three triangles (ΔBMP, ΔPQR, ΔCRO) are of the same size. Therefore the total area is 4 x 3/2 = 6 *padas.* So the rectangle BPRC is a square of area 16 - 6 = *10 padas.*

From this it is clear that the square produced on this diagonal is equal to the sum of the squares produced on the sides. Some other indications of the same theorem are as follows,

5.4.3 Some sets of the measures of the right angled triangles

In a right angled triangle whose one side is two *padas*, and the other side six *padas*, the hypotenuse will produce an area of forty square *padas.*[86] In a rectangle its diagonal produces by itself an area which its length and breadth produce seperately.[87]

In a square its diagonal produces an area which is double that of its side.[88]

In *Āpastambha Śulbasūtra*, there is a reference of right angled triangles which are used in the construction of *Mahāvedi.*[89] One of them says that the hypotenuse is 13 and the other sides are 5 and 12 units. The

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86 *K.Sl.* 2 - 9 (Ref. Kātyayana Śulbasūtra, Ed. Khadilkar S.D, p. 12)

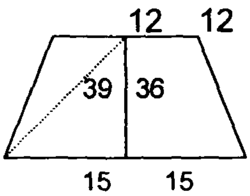
87 *K. Sl.* 2 - 11, *Baudh. Sl* 1-48 (Ref. Ibid., p.53)

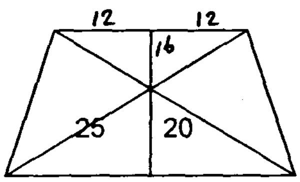
88 *K. Sl* 2-12 (Ref. Ibid., p. 15)

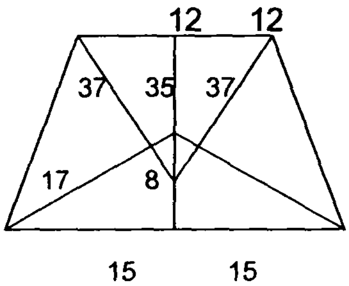
89 *Ap. Sl.* P. 81 (Ref. Ibid., p. 53)

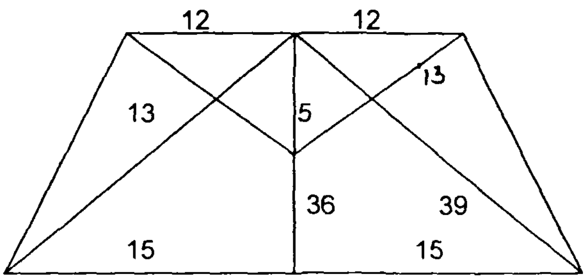
eastern corners are decided by making the sides three times the original length. Another one which is useful for fixing the western corners is such that the hypotenuse is 17, the length of whose sides is 8 and 15 units.

Another one for fixing the eastern corners is that the hypotenuse is 37, when the length of the sides is 12 and 35 units.[90] In *Āpastambha Śulbasūtra* there is a reference to the right angled triangle whose sides are 3, 4 and 5 which is used for constructing *Somavedi.*[91] There is a list of rightangled triangles in the *Baudhayana Sulbasutra*.The sides are listed as a) 3 and 4 b) 12 and 5 c) 15 and 8 d) 7 and 24 e) 12 and 35 f) 15 and 36. All these can be used for the construction of the *Saumiki vedi* or *Mahāvedi.*[92]









[90] *Ap. Sl.* p. 82 (Ref. Kātyayana Śulbasūtra, Ed Khadilkar S.D, p. 53)
[91] *Ap. Sl.* p. 76 (Ref. Ibid., p.53)
[92] *B.Sl.* 1-49 (Ref. *Ibid.*, p.54)

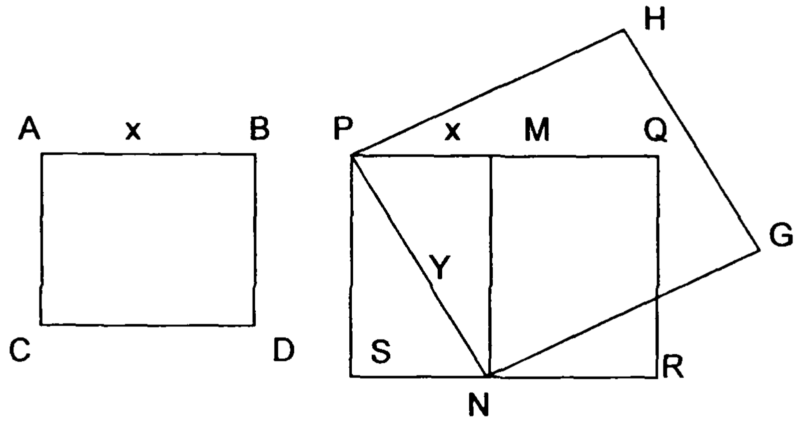
From these it is evident that ancient people in India were conscious about the construction of right angled triangles and the measurements of the hypotenuse and the other two sides.

5.5 SOME MATHEMATICAL AND GEOMETRICAL PROBLEMS IN ŚULBASŪTRAS

5.5.1. Construction of a square having the area as the sum of areas [93] of two squares.

Make a rectangle on one side of the large square having length of one side equal to the smaller square. Then the area of the square made by a diagonal of this rectangle is equal to the sum of the areas of the above two squares.

Let ABCD be the small square and let PQRS be the larger square such that AB = x and PQ = y



Let M and N are points on the sides PQ and RS such that PM = x and SN = x. Consider the rectangle PMNS having sides x and y respectively.

[93] *Nāna pramāṇayō schaturasrayossamāsa*
Hrasīyasa karaṇya varsīyaso vridhra
Mullikheth vṛidhṛasyā kṣṇayārajjurubhe
Samasyati taduktam (*Āpastambha Śulbasūtram* 2.4) [Ref : C. Krishnan Namboodiri, Bharatiya Sastracinta, 1998, Arshaprakasam Prasidhikarana Samiti, p.327]

Clearly PN be a diagonal of this rectangle. Construct a square having PN as a side, say PNGH.

$PN^2 = PM^2+MN^2 = x^2+y^2$

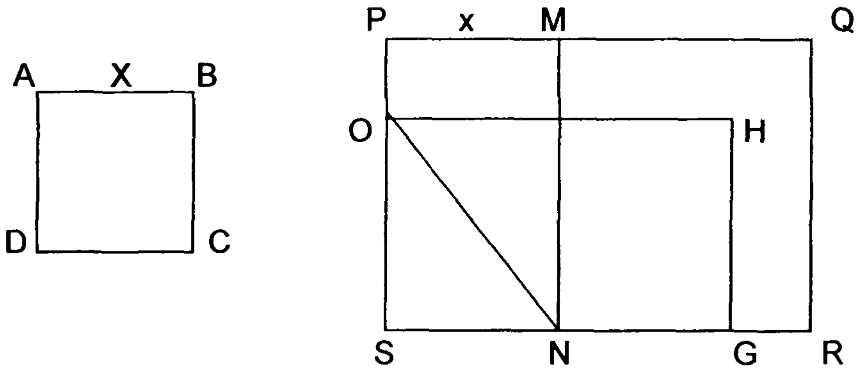
Area of the square PNGH $= PN^2=x^2+Y^2$

$= AB^2+PQ^2$

= Area of the square ABCD + Area of the square PQRS.

5.5.2 Construction of a square having area as the difference of areas[94] of two squares

Let ABCD be the small square and let PQRS be the larger square such that AB = x and PQ = y



Mark two points M and N on the sides PQ and RS respectively such that PM = SN = x. Let O be a point on the side PS such that NO = NM.

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94 *Caturaśra caturaśram*
Nirjihīrshan
Yavannirjihīrsheth
Tasyākaraṇyā
Varsīyaso vṛidhra
Mullikheth vridhrasya
Parśomānilmakṣṇa
Yetarath pārśwa
Mupasamharet sa yatra
Nipatet tadapachindyāt
Chinnaya nirustam (*Apastambha sullasutram* 2-5) (Ref: C. Krishnan Namboodiri, Bharatiya Sastracinta, 1998, Arshaprakasam Prasidheekarana samiti, Kozhikode, p.328.)

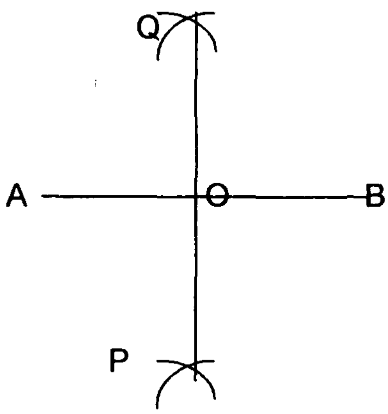
Construct a small square having OS as one side say OSGH. The area of the above two squares.

$$OS^2 = ON^2 - SN^2 = MN^2 - x^2 = y^2 - x^2$$

$$\therefore \text{Area of the square OSGH} = OS^2 = y^2 - x^2$$

5.5.3 Method of drawing perpendicular through the midpoint of a line

There is a method of drawing perpendicular through the midpoint of a line segment in *Baudhayana Śulbasūtra.*[95] First draw a line and fix two gnomons at the end points. By tying one end of a string on the gnomon draw arcs on the two sides using the other end. Similarly draw arcs by tying the string on the other gnomon. Join the points of intersection of the arcs. This line is perpendicular to the given line and passing through the midpoint.



We can prove this using the modern theorems. Triangles AQB and APB are equal since AP=AQ, AB=AB, BQ= BP(length of the string).

Therefore <PBA = < QBA.

Therefore, Triangles QOB and POB are equal. Since BP=BQ, BO = BO and the including angles PBO and QBO are also equal.

So <QOB = < POB = 90.

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[95] *B. Sl*, 1-4

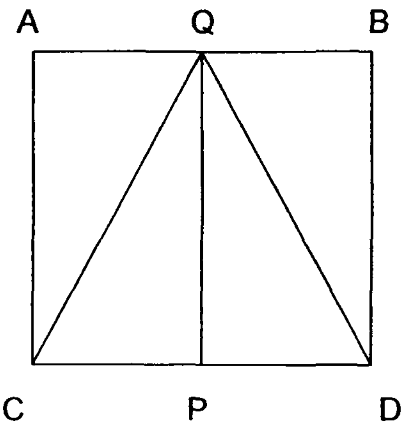
Therefore this line is perpendicular to the given line. Also AP=BP length of the string, OP=OP. One angle is right angle.

Therefore, < AOP and < BOP are equal.

Therefore, AO=BO. i.e., PQ is perpendicular to AB and passing through the midpoint.

5.5.4 Transformation of a Square into a Triangle of the same area

Transformation of a square into an isosceles triangle having the same area is described in *Āpastambha Śulbasūtra.*[96] Construct a square having area twice the area of *Agnivedi* constructed using the measurement scales *aratni and prādēsam*. Draw lines from the midpoint of the east side to the ends of west side. Then we obtain an isosceles triangle having an area of the *Agnivedi.*



From the figure,

Area of the *Agnivedi* = ½ Area of ABCD

= ½ CD × BD

=½ CD × PQ

[96] *Ap. Sl*, 12-5 (Ref: C.Krishnan Namboodiri, Bharatiya Sastracinta, 1998, Arshaprakasam prasidheskarana samiti, Kozhikode, p.344)

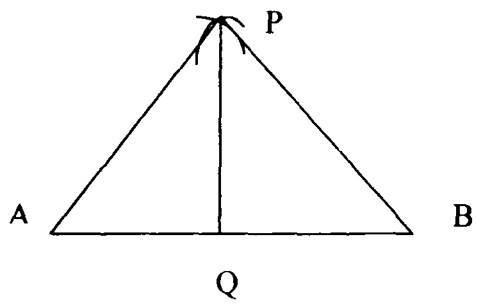
= ½ Base × Height of triangle CQD

= Area of triangle QCD

5.5.5 Construction of a Square having an area equal to the sum of the areas of the number of equal squares

Another method to construct a square having an area equal to the sum of areas of a number of equal squares is given in *Kātyayana Śulbasūtra*.[97]

If n is the number of equal squares having side length x, then draw a horizontal line of length (n-1)x. Draw two arcs at a distance (n+1)x/2 from the endpoints of the horizontal line. Draw a perpendicular from the point of intersection of these two arcs to the horizontal line. Then this perpendicular makes a square having an area as the sum of areas of n equal squares.



We can prove this using modern theorems. Let the horizontal line be AB.

AB= (n-1)x

Let P be the point of intersection of the arcs. Then AP = BP = (n+1)x/2.

Since PQ is the perpendicular from P on AB, we have <AQP = < BQP = 90

Also AP=BP, and PQ=PQ so the triangles APQ and BPQ are equal.

[97] *K. Sl.* 6-7.

Therefore, AQ= BQ =(n-1)x/2.

$PQ^2 = AP^2 - AQ^2 = ((n+1)x/2)^2 - ((n-1)x/2)^2$

$= nx^2$

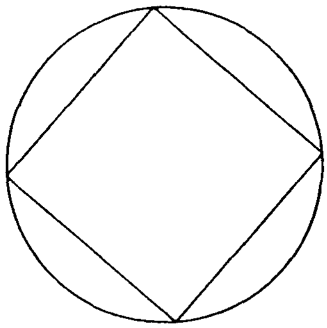
ie, PQ produces a square of area nx^2 which is the sum of areas of n equal squares of side length x.

5.5.6 Cyclic Quadrilaterals

A cyclic quadrilateral means a quadrilateral with its vertices on the circumference of a circle. The first idea of the construction of a square inscribed in a circle of a given diameter is in connection with the construction of *Gārhapathya* altar mentioned in *Āpastambha Śulbasūtra*[98]. Fixing a gnomon in the middle of the ground, a circle is drawn with half *vyāyāma* (approximately 3 feet as radius). The circumference is divided into 4 segments using the lines along the east-west and north-south directions. The points obtained on the circumference are the corners of the required square.

Kapardisvami (before 11th century AD), a commentator of *Āpastambha Śulbasūtra* calculated the length of the side of the square inscribe in the circle of diameter one *vyāyāma* as 68 *aṅgulas* less four *tila*. We can evaluate the value of $\sqrt{2}$ from this.

[98] A.Mukhopadhyay and M.R Adhikari, "*The concept of cyclic quadrilateral, its origin and development in India*", IJHS 32(1), 1997, Dec.



By Computation [99]

If x is the side length, then diagonal = √2x[1 *Vyāyāma=96 aṅgula,*

1 *aṅgulās=34 tila*]

But this diagonal is the diameter of the circle.

Therefore, Diameter = √2 X side length

=> One *Vyāyāma* = √2 X (68 X 34 - 4) *tila*

Therefore, √2= 96 X 34/((68 X 34) – 4)

= 1.414211438

Gārhapatya altar *and Rathacakra citi* mentioned in *Baudhayana Sulbasūtra* involves the construction of a square within the circle of a given diameter:

5.6 SOME TRIGONOMETRICAL CONSIDERATIONS IN *SULBASUTRAS*

We can see the references to trigonometry in many ancient texts. Especially in *Vāstuvidya* the roofing system is simply an application of trigonometry. Properties of triangles are used for getting the measurements of rafters.

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[99] *Ibid.*, p.54

5.6.1 Method of drawing perpendicular to a line segment through one end point

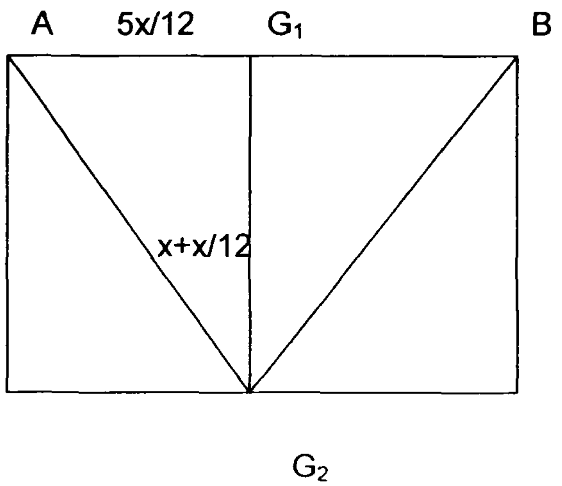
Method of drawing perpendicular to a line segment is given in *Āpastambha Śulbasūtra*. If x is the length of a line segment, then choose a string of length x+ x/2. Then put a mark on the string at the point x+ x/12 from one end. Fix two gnomons G_1 and G_2 at a distance x in the east – west direction. Tie the ends of the string on the gnomons. By stretching the string towards the south, fix a gnomon say A at the point of mark. In a similar way fix a gnomon say B on the north side also. Repeat the process and fix two gnomons on the other side[100], by putting a mark on the string at the point x+x/12 from the other end. Draw a rectangle by taking these gnomons as vertices.

$AG_1 = x/2 - x/12 = 5x/12$, $AG_2 = 13x/12$

$G_1 G_2 = x$

$(G_1 G_2)^2 + (AG_1)^2 = (AG_2)^2$

We have AG_1 is perpendicular to $G_1 G_2$.



[100] *Ap. Sl,* 1-2 (Ref. C.Krishanan Namboodiri, Bharathiya Sastracinta, 1998, Arshaprakasam Prasidheekarana Samithi, Kozhikode p. 271)

Method of construction of a square having an area twice the area of another square is given in *Āpastambha Śulbasūtra*. This says that the diagonal of a square makes a square having an area twice the area of the latter[101].

5.6.2 Construction of a Square

Construction of a square is described in *Āpastambha Śulbasūtra.*[102] Fix two gnomons in the east – west directions, say G1 and G2 at a distance x, Choose a string of length x. Tie the ends of the sting to the gnomons and put marks at the mid point and at the midpoints of the half portions and fix gnomons say G5,G3 and G4 at these points. Then tie the ends of the string on the last but one gnomons and stretch this towards the South. Put a mark at the midpoint say L_1. Tie the two ends on the middle gnomon and stretch the string towards the above mark, and fix a gnomon at the midpoint say L_2.

Tie one end of the string on this gnomon(gnomon at L_2) and tie the other end on the gnomon of the East (G1) and stretch this string towards south and fix a gnomon at the midpoint (say L_3). This point is the east – south corner. Similarly, we can find the west – south corner by tying one end at the gnomon of the west side. In a similar way we can find the other two corners. Connecting these corners we get a square.

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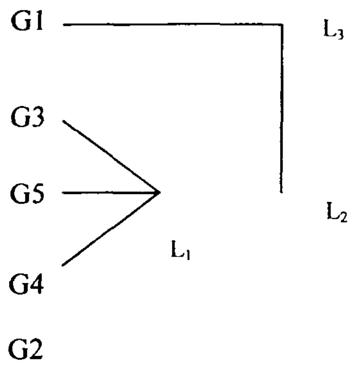
101 *Ibid.*, 1-5

102 *Ibid.*, 1-7.



$G_1G_2= x$, $G_1G_5 = x/2$,

$G_1G_3 = G_4G_5 = x/4$

$G_4L_1 = G_5G_1 = x/2$

And clealy G_5L_1 is perpendicular to G_3G_4. (Since $\Delta G_3L_1G_5$ and $\Delta G_4L_1G_5$ are similar, $<G_3G_5L_1 = <G_4G_5L_1 = 90^\circ$

Therefore G_5L_2 is equal to $x/2$ and perpendicular to G_1G_2.

$G_1L_3 = L_2L_3 = x/2$

Similarly finding other corners we get a square.

5.6.3 values of √2, √3 and √5

The three sacred fires are (1)*Gārhapathya* (2)*Āhavanīya and (3) Dakṣiṇāgni.*[103]

Baudhayana Śulbasūtra says that with the one third part of the length of the distance between *Āhavanīya* and *Gārhapathya*, make three squares touching each other. The place of the *Gārhapathya* is in the north-west

[103] *B. Sl* 1-67 (Ref: *Kātyayana Śulbasūtra*, Ed. Khadilkar S.D, p.53)

corner of the western square, *Dakṣiṇāgni* is in the south-west corner of the same square, the place of the *Āhavanīya* is in the north-east corner of the eastern square.[104]

The length of the diagonal AD will be equal to $\sqrt{5}a$, where a is the side length of the square. Similarly GD=$\sqrt{2}a$. There are some attempts to find the values of $\sqrt{2}$ and $\sqrt{5}$ in *Kātyāyana Śulbasūtra and Āpastamba śulbasūtra.*[105] By taking the distance between the centres of *Gārhapatya* and *Āhavanīya* as 8,11 and 12 units and divide the space between *Āhavanīya* and *Gārhapatya* into 6 or 7 parts and add one part to it. Take a chord of this length. Divide this into three equal parts, and a mark is made on the second part. Stretch the chord towards south and fix a pole on the point where the mark touches the ground. This is the place of *Dakṣiṇāgni.*

AG=8 AC=2/3 × 8 = 16/3

CD=8/3

(AD + DG) = 8 + 8/6 = 8 × 7/6

AD = 2/3 (8 × 7/6) = 56/9

DG=1/3 × (8×7)/6 = 28/9

$AC^2 + CD^2 = AD^2$

$(2\ CD)^2 + CD^2 = AD^2$

ie, AD/CD = $\sqrt{5}$

Also AD/CD = (56/9) × 3/8 = 2.333.....

So $\sqrt{5}$ = 2.333

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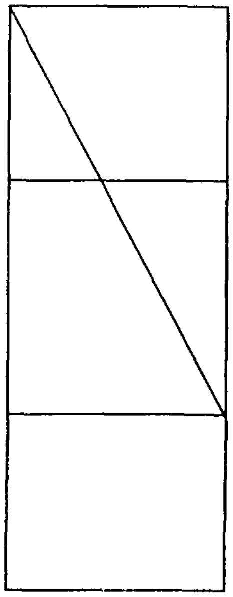
[104] *K. Sl*, 1-29.
[105] *Ibid.*, 1-27.

Also $DG^2 = 2\ CG^2$ ie, $DG/CG = \sqrt{2}$

Also $DG/CG = (28/9) \times 3/8 = 1.666......$

So $\sqrt{2} = 1.66.....$



When AG is 11 or 12, then also we will get the same value for $\sqrt{2}$ and $\sqrt{5}$. When the cord is increased by 1/7, in a similar way we will get the value of $\sqrt{5}$ as 2.2857 and $\sqrt{2}$=1.14285.

Another method is described in *Baudhayana Śulbasūtra.*[106] Increase the distance between *Āhavanīya* and *Gārhapathya* by 1/5th of it. The increased length is divided into 5.Put a mark on the third part from the eastern side and stretch the chord towards the south. Fix a point at the spot touched by the mark. This is the place of *Dakṣiṇāgni.*

If AG= 8, then AC= $8 \times 2/3$, CD = $8 \times 1/3$

AD+DG = 8 + 8/5 = 48/5

[106] *B. Sl* 1-67 (Ref: *Kātyayana Śulbasūtra*, Ed. Khadilkar S.D, p.81)

AD = 48/5 × 3/5 =144/25

DG= 48/5 × 2/5 = 96/25

From ΔACD,

$AD^2 = AC^2 + DC^2$

$AD^2 = 4\ DC^2+DC^2 = 5\ DC^2$

AD = $\sqrt{5}$DC or AD/DC =$\sqrt{5}$

But from the above AD/CD =(144/25)/(8/3) = 2.16

So $\sqrt{5}$ = 2.16

$GD^2 = CG^2 + CD^2 = 2\ CD^2$

GD/CD = $\sqrt{2}$

Also DG/CD =(96/25)/(8/3) = 1 44

So $\sqrt{2}$ = 1.44

If AG is taken as 11 or 12 then also we will obtain the same values for $\sqrt{5}$ and $\sqrt{2}$.But these values are much better approximations than that of above. (i.e., which is described in *Āpastambha Śulbasūtra* and *Kātyayana Śulbasūtra*). These procedures are based on the fact that in a square the hypotenuse produces an area which is twice the area of the square produced by its side. In a rectangle with sides in the ratio 1:2, its hypotenuse will produce an area which is 5 times the area produced by the shorter side. It is evident that ancient people in India had obtained a clear knowledge about the theorem of hypotenuse which is now known as Pythagoras theorem. There is given a concept of $\sqrt{3}$ in *Āpastambha*

Śulbasūtra[107] (2-2), which says that$\sqrt{3}$ is the length of the diagonal of a rectangle having sides 1 and $\sqrt{2}$.

According to Boudhayana Sulbasūtra

$\sqrt{2}=1+(1/3)+1/(3 \times 4)-1/(3 \times 4 \times 34)=577/408$

(*Pramāṇam tṛtīyēna vardhayēt tat ca caturthenatmacatustri n sonena.*)

Calculation of the value of $\sqrt{2} = 1 + 1/3 + 1/3.4 - 1/3.4.34$ (approximately).[108]

There were methods in *Sulbasūtras* for finding the approximate values of surds.[109] *Baudhayana Śulbasūtra* gives a method for finding the *dvikaraṇi* as "Increase the measure of which the *dvi-karaṇi* is to be found by its third part and again by the 4th part of this 3rd part less by the 34th part of the fourth part. The value thus obtained is approximate. *Dvi-Karaṇi* 'd' of 'a' is the length of the diagonal of a square of side 'a'.

i.e., $d= a + a/3 + a/3x4 - a/3x4 \times 34$ when $a = 1$, $d = \sqrt{2}$,

Therefore, $\sqrt{2} = 1 + 1/3 + 1/3 \times 4 - 1/3 \times 4 \times 34$

approximately = 1. 4142156.

According to modern calculation $\sqrt{2} = 1.414213$. It is clear that the calculated value is very near to the modern value.

Some approximate values occurring in the *Śulba* are

$\sqrt{2} = 7/5$, 1+11/5,

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[107] *Ap. Sl.* (2-2) (Ref: C. Krishnan Namboodiri, Bharatiya Sastracinta, 1998, Azshaprakasam Prasidheekarana samiti, Kozhikode, p.156)

[108] *B.Sl.* 2.1 – 2.2,

[109] Bibhutibhusan Datta and Awadesh Narayan Singh,"*Approximate values of Surds in Hindu Mathematics*", Indian Journal of history of Science, 28(3) 1993.

$\sqrt{29} = 5 + 7/18$,

$\sqrt{5} = 2 + 2/7$,

$\sqrt{61} = 7 + 5/6$.[110]

$\sqrt{3} = 1 + 2/3 + 1/3.5 - 1/3.5.52$[111]

$= 1.732053$

(In the works of Jainas between 500-300 BC there were applications of the formula $\sqrt{N} = \sqrt{a^2 + r} = a + r/2a$)[112]

Approximate value of a root is given in Bakhshāli treatise on arithmetic. "In the case of a non-square number, subtract the nearest square number, divide the remainder by twice the root of that number". Divide half the square of that by the sum of the root and fraction and subtract less the square of the last term.

$\sqrt{N} = \sqrt{a^2 + r} = a + r/(2a) - (r/2a)^2 / 2(a + r/2a)$ approximately.

5.7 COMPARISON WITH SOME OTHER BOOKS

5.7.1 Changing the square into an octagon having the same width

chatrasrēsya kamārdham
guhya kōnēṣu lānchayēt
astasro vyṣnavobhaga
sidhatyevanasamśaya

(Agnipurana : 53 –3 – 4)[113]

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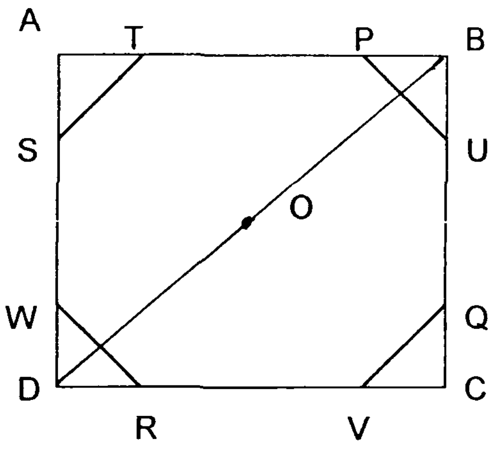
[110] Bibhutibhusan Datta and Awadesh Narayan Singh, " *Approximate values of Surds in Hindu Mathematics*", Indian Journal of history of Science 28(3) 1993 p.205

[111] *Ibid.*, p. 194

[112] *Ibid.*, p. 194

[113] C. Krishnan Namboodiri, *Bharathiya Sastracinta*, 1998, Arshaprakasam Prasidheckarana Samithi, Kozhikode.

Let ABCD be a square. Let BD be a diagonal and let O be the midpoint.



Choose points P, Q, R, S such that AP = BQ = CR = DS = BD/2. Similarly choose points T, U, V, W on the sides AB, BC, CD and DA respectively, such that BT = CU = DV = AW = BD/2. Join the point S and T, P and U, Q and V, and R and W. This is an octagon having the same width as the square.

By Computation

Let AB = x Then BD = $\sqrt{2}\,x$

$\therefore$ BD/2 = $\sqrt{2}\,x/2 = x/\sqrt{2}$

AP = $x/\sqrt{2}$, BT = $x/\sqrt{2}$

$\therefore$ AT = AB – BT = $x - x/\sqrt{2}$

$\therefore$ PT = AB – (AT + PB)

$= x - [\,(x - x/\sqrt{2}\,) + (x - x/\sqrt{2}\,)$

$= \sqrt{2}x - x$

Similarly we can prove that

$QV = RV = SW = \sqrt{2}\,x - x$

Clearly ΔPBU is a right-angled triangle.

$$PU^2 = PB^2 + BU^2 = (x - x/\sqrt{2})^2 + (x - x/\sqrt{2})^2$$

$$= 2\,[x - x/\sqrt{2}]^2$$

$$= \frac{2\,[\sqrt{2}\,x - x]^2}{\sqrt{2}^{\,2}} = \frac{2\,(\sqrt{2}x - x)^2}{2}$$

$$= (\sqrt{2}\,x - x)^2$$

$\therefore PU = \sqrt{2}\,x - x$

Similarly we can prove that QV = RW =ST = PU.

∴ All sides are equal; and the width of this octagon is the same as that of the square.

5.7.2 Changing of a square into an octagon having the same Area

kṣetrē tatra samantatō

dinakarāmśam nyasya

turyasritē konebhyo bhuja

sūthrakēṣu nihitaih swaih

karṇasūtrārdha kaih

Dvou dvou dikhṣu jhaṣha

prakalpya makareshvaspha

tairasutabhih sutrairiśwara

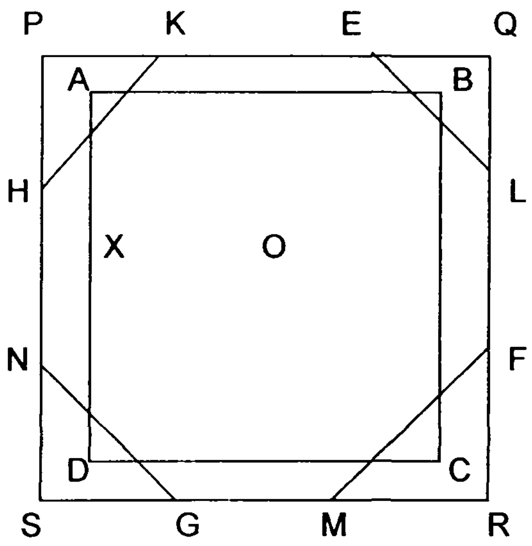
dingmukhē viracay

daṣhtāsra kuṇdam sudhih

(*Tantrasamuchayam* 12 – 32)[114]

[114] *Ibid.*,

Let ABCD be a square. Extend this square by extending each line on both sides by a length $1/12^{th}$ of it. That if AB = x, then extend AB on both sides by a length x/12 to form another square PQRS such that PQ = x +x/6.



By Computation

Choose points E, F, G, H on the side PQ, QR, RS and SP respectively such that

PE = QF = RG = SH = QS/2.

Similarly choose point K, L, M, N on these sides respectively such that QK = RL = SM = PN = QS/2. Join these points and we obtain an octagon KELFMGNHK having the same area as the square ABCD.

From the above problem we can prove that KELFMGNHK is an octagon.

Given that Area of the Octagon = Area of square ABCD

By computation

The area of this octagon = Area of the square PQRS – area of the four triangles.

The area of these four triangles are equal and is equal to ½ x PK x PH = ½ $(PK)^2$.

But PK = PH = EQ = QL = FR = RM = GS = SN.

$PK = PQ - QK = (x + x/6) - \frac{1}{2} \times \sqrt{2}\,(x + x/6)$

$= (x + x/6)\,[1 - 1/\sqrt{2}\,]$

$= (x + x/6)\,\frac{(\sqrt{2} - 1)}{\sqrt{2}}$

Sum of areas of four triangles

$= 4 \times \text{area of } \triangle\, PHK = 4 \times \frac{1}{2}(PK)^2 = 2\,(PK)^2$

$= 2 \times (x + x/6)^2 \frac{(\sqrt{2}-1)^2}{2} = (x + x/6)^2\,(\sqrt{2} - 1)^2$

Area of the octagon $= (x + x/6)^2 - (x + x/6)^2 (\sqrt{2} - 1)^2$

$= (x + x/6)^2\,[1 - (\sqrt{2} - 1)^2]$

$= (7x/6)^2 \times 2\,(\sqrt{2} - 1)$

Area of Octagon = Area of ABCD $\Rightarrow\ (7/6)^2\, 2(\sqrt{2}-1)\, x^2 =$

$x^2 \Rightarrow \sqrt{2} = 1.367346938$

5.7.3 Changing of a square into a hexagon having the same area

Ańkou prakalpya haripancama dikpadikstou

Saṣtāmśato bairatho nija madhyataścha

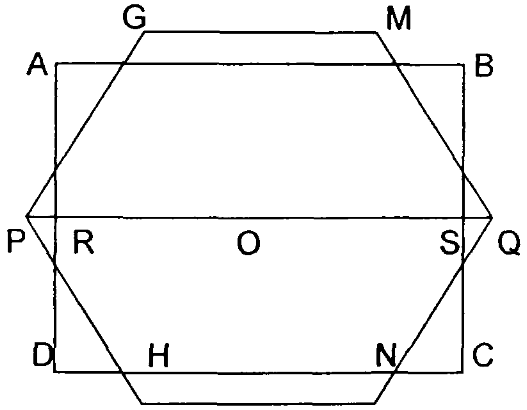
Dou dou chaṣou paradiśōrapi malsyachihna

Shad sūtra kairvirachayēd

Rasa kona kuṇḍam

(Tantrasamucayam 12 –30) [115]

[115] Ibid., p. 337.



Let ABCD be a square. Let R and S be the midpoints of AD and BC respectively. Extend RS on both sides by a length 1/6[th] of RS; to obtain the line PQ. M and N are two points such that they are at a distance OQ from both O and Q. Similarly G and H are at a distance OP (OQ) from both O and P. Then GMQNHPG form the required hexagon.

The triangles ΔPOG, ΔGOM, ΔMOQ, ΔQON, ΔNOH, ΔHOP are similar and having the same measure (equilateral).

OP = OG = $(x + x/3)/2$, clearly GM = ½ PQ = OP

$\therefore$ Area of the hexagon = 6x Area of ΔPOG.

Area of ΔPOG = ½ OP x OG Sin 60 = $\sqrt{3}/4\ [(x + x/3) / 2\ ]^2 = x^2/3\sqrt{3}$

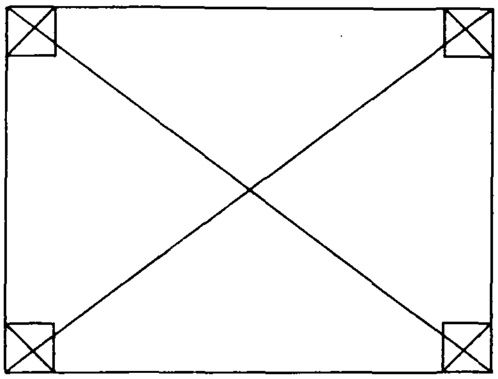
Area of hexagon = Area of Square ABCD $\Rightarrow 6\ x^2/3\sqrt{3} = x^2 \Rightarrow \sqrt{3}=2$

But the value of $\sqrt{3}$=1.732050807. The error is very large.

In *Tantrasamuccaya*[116] (2-125), there is described a method of construction of *Śivaliṅga*. Draw the diagonals of a square of equal sides

[116] *Ibid*

and angles. Make points on the diagonals at a distance equal to the side of the square from the corners. Draw lines parallel to the sides through these marks. Join the points of intersection of these lines with the sides and cut the corner portions outside the joining lines upto the 2/3rd length from the upper side, we get an octagon. Repeating this process to obtain *Shodaśāsram* and *Dwatriśāsram*. As the number of sides increases the upper side approaches the circular shape.



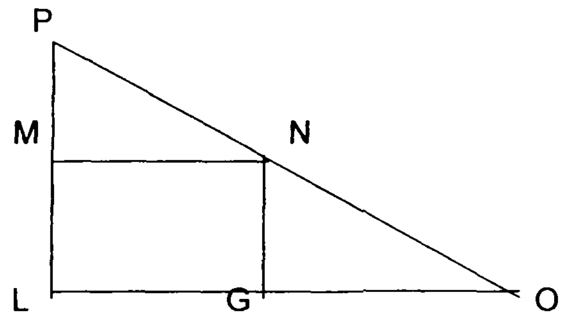
5.7.4 Some Trigonometrical Considerations and Various Properties of Similar Triangles.

There is a reference to the property of similar triangles in *Aryabhatīyam,* which is given below.

Consider the shadow of the gnomon caused by a lamp post. The length of the shadow is obtained by dividing the product of the height of the gnomon and the distance between the lamp post and the gnomon by the difference between the heights of gnomon and the lamp post.[117] Clearly

[117] Aryabhateeyam 2-15 (Ref. C.Krishanan Namboodiri, Bharathiya Sastracinta,1998, Arshaprakasam Prasidheekarana Samithi, Kozhikode. p.271)

this is just the property that the corresponding sides of two similar triangles are proportional.



By Computation

Let PL – Lamp Post

GN – Gnomon

GO – length of the shadow From figure ΔPMN and ΔNGO are similar

MN/GO =PM/GN

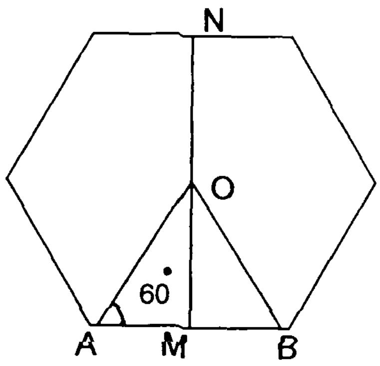
GO= MN x GN / PM

Let M be the foot of the perpendicular from N on PL. Triangles ΔPMN and ΔNGO are similar.

The value of √3

We can find or deduce the value of √3 from *Tantrasamuccaya*.[118] To construct a *Prāsāda* having 6 sides we have to divide the perimeter by 6 and construct the *Prāsāda* having this side length. Then its width is given by subtracting 2/15 of 2/6th of the perimeter from 2/6th of the perimeter.

[118] *Tantrasamuccayam*, 2-69 (Ref .Ibid, p.157)



By Computation

Let perimeter = x

Therefore, side length = x/6

Let O be the midpoint. Triangle AOB be an equilateral triangle.

Therefore, Tan60 = OM/AM

OM=AM tan60

=(x/12) X √3

=√3x/12

Therefore, MN=(√3x)/6

From the *sloka* MN=2 X x/6 - (2x/6) X 2 /15

=13x/45

From the above we have √3x/6 = 13x/45.

Therefore √3=78/45=1.7333

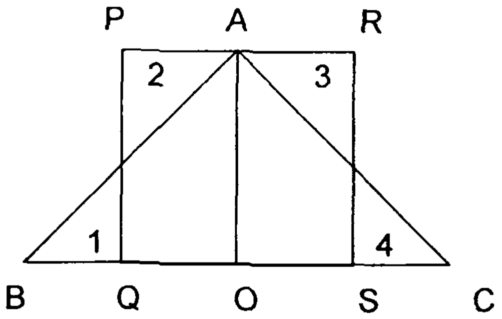
The accepted modern value for √3 is 1.732.

i.e., the computed value of √3 is correct upto 2 decimal places.

The area of a triangle is given in Aryabhatiya as ½ x base x height.[119] We can see a proof for this in *Yukti Deepika*.

Draw two vertical lines through the midpoints of the two sides of ΔABC, say PQ and RS.AO is ⊥ r to BC. Triangles (1) and (2) are equal, and (3) and (4) are equal.

Therefore, the area of the triangle ABC = area of the rectangle PQSR



BQ+SC=QO+OS=½ BC=QS (Since Q and S are midpoints of BO and OC)

Area= QS × AO = ½ BC × AO[120]

We can see the concept of right angle in *ṛgveda*. Consider a circle with 12 divisible places on the perimeter and having three *nabhis*. 360 pedals are fixed on this circle.[121] Another description is of a circle constructed by 4 *khaṇḍas* containing 90 *bhāgas*.[122]

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[119] *Ar.bh* 2-6 (Ref.C.Krishanan Namboodiri, Bharathiya Sastracinta, 1998, Arshaprakasam Prasidheekarana Samithi, Kozhikode, p.271)

[120] *Yuk.Dp* 2-531-534 (Ref. Ibid.)

[121] *Ṛgveda*. 1-164, 48(Ref. Ibid.)

[122] *Ṛgveda*, 1-155-6(Ref. Ibid.)