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FUZZY EOQ MODEL WITH THE IMPACT OF STOCHASTIC LEAD TIME REDUCTION ON INVENTORY COST UNDER ORDER CROSSOVER

For the fixed order size inventory models, the economic order quantity (EOQ) model is most well known. The Yager's ranking method (1981) for fuzzy numbers is utilized to find the optimal inventory policies. We assume that the lead time is made of one or several components and is the time between when the need of a replenishment order is determined to the time of receipt. A set of numerical data is employed to analyse the characteristics of proposal models.

5.1. EOQ MODEL WITH THE IMPACT OF STOCHASTIC LEAD TIME REDUCTION

The order cross over transforms the original lead times into 'effective lead times' whose mean is the same as that of the parent lead time but whose variance is lower discussed by Hayya et al., [55]. We use the model in Silver et al [110] and Ishii, H., and Konno, T., [65], Jack, C., Hayya Terry, P., Harrison, X., James He., [66] where the shortage penalty is applied per unit short and where the demand rate is constant so far, research on lead time reduction has dealt only with deterministic lead times by Lan, S.P., Chu et al [77] where these authors portray cost as a piecewise linear function of lead time $L$.

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5.2. FUZZY TOTAL INVENTORY COST WITH THE LEAD TIME REDUCTION UNDER ORDER CROSSOVER

If the lead time were exponentially distributed, the total cost in the first stage including shortage and lead time reduction cost would have been

\[ C(Q, Z_0) = \frac{AD}{Q} + h \left[ \frac{Q}{2} + Z_0 (a'D + b'Q) \right] + \frac{DB_z}{Q} (a'D + b'Q) G(Z_0) + \frac{D}{Q} R(L) \]

The objective is to find the optimal order quantity which minimize the total cost by Petrovic, D., Petrovic, R., and Vujosevic, M., [99] and Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E., [111], Wee, H.M., Chung, S.L., Yang, P.C., [126-128]

The necessary condition for minimum \( \frac{\partial C(Q, Z_0)}{\partial Q} = 0. \)

Therefore the optimal order quantity is

\[ Q^* = \sqrt{\frac{2\left[ AD + a'B_2 + D^2 G(Z_0) + DR(L) \right]}{h(1 + 2Z_0b')}} \]

5.2.1. The EOQ model with the stochastic lead time reduction and fuzzy demands

Let \( \tilde{D} \) be a normal fuzzy number with parameters \( \tilde{D} = (l, m, n, u) \), then the membership function of \( \tilde{D} \) can be defined by a left shape function \( L(x) \) and a right shape function \( R(x) \) as:
\[ \mu_b(x) = \begin{cases} L(x), & 1 \leq x \leq m \\ 1, & m \leq x \leq n \\ R(x), & n \leq x \leq u \end{cases} \]

The above equation can also be described by the terms of \( \alpha \)-level cut of \( \tilde{\lambda} \) as:

\[ D(\alpha) = \left[ \min \cdot \mu_0^{-1}(\alpha), \max \cdot \mu_0^{-1}(\alpha) \right] \]

\[ = \left[ L^{-1}(\alpha), R^{-1}(\alpha) \right], \quad 0 \leq \alpha \leq 1 \]

Firstly, we discussed the EOQ model with fuzzy demand, according to the extension principle, the model can be described by Woo, Y.Y., Hsu, S.L., Wu, S.S., [129] terms of \( \alpha \) as

\[ \tilde{T}(\alpha) = \left\{ C\left[ Q \mid D = L^{-1}(\alpha) \right], C\left[ Q \mid D = R^{-1}(\alpha) \right] \right\}, \quad 0 \leq \alpha \leq 1 \]

where \( C(Q) = \frac{AD}{Q} + \frac{hQ}{2} \)

Since the annual cost function \( \tilde{T}(\alpha_0) \) is a fuzzy number, we can compare \( \tilde{T}(\alpha_0) \) of different \( Q \) by using some ranking methods to find the optimal solution \( Q^* \) with a minimal total cost, of which, not every method is applicable to rank \( \tilde{T}(\alpha_0) \) of all possible \( Q \). The method proposed by Yager, R.R., [130] does not need to know the explicit form of the membership functions, and can thus be applied here.

The Yager's ranking index ranks the fuzzy numbers by an area measurement defined as
\[ I(\tilde{T}) = \frac{l_L(\tilde{T})}{2} + \frac{l_R(\tilde{T})}{2} \]

where \( I_L(\tilde{T}) \) represents the area bounded by the left shape function of \( \tilde{T}(\alpha_0) \), the x axis, the y axis and the horizontal line \( \mu_l = 1 \) and \( I_R(\tilde{T}) \) represents the area bounded by the right shape function of \( \tilde{T}(\alpha_0) \), the x axis, the y axis and the horizontal line \( \mu_l = 1 \).

The Yager’s ranking index of \( \tilde{T}(\alpha_0) \) thus can be calculated as

\[
I(\tilde{T}) = \frac{1}{2} \int_0^1 [C[Q \mid D = L_0^{-1}(\alpha_0)] \, d\alpha_0 + \frac{1}{2} \int_0^1 [C[Q \mid D = R_0^{-1}(\alpha_0)] \, d\alpha_0
\]

Let \( K(\Box_0) = \frac{1}{2} \int_0^1 [L_0^{-1}(\alpha_0) + R_0^{-1}(\alpha_0)] \, d\alpha_0 \)

Taking the partial derivative of \( \tilde{T}(\alpha_0) \) with respect to \( Q \) and setting to zero, the necessary condition of optimal solution of \( \tilde{T}(\alpha_0) \) can be found as

\[
Q^* = \sqrt{\frac{2A}{h}} \cdot K(\alpha_0)
\]

\[
\tilde{T}^*(\alpha) = \{ C[Q^* \mid D = L^{-1}(\alpha)], C[Q^* \mid D = R^{-1}(\alpha)] \}, 0 \leq \alpha \leq 1
\]

Now if, shortage cost is permitted, the model can be described by terms of \( \alpha \) as

\[
\tilde{T}_{b_2}(\alpha) = \{ C_{b_2}[Q, Z_0 \mid D = L^{-1}(\alpha)], C_{b_2}[Q, Z_0 \mid D = R^{-1}(\alpha)] \}, 0 \leq \alpha \leq 1
\]
The Yager's index of $\tilde{T}_{b_1}(\alpha)$ then can be derived as

$$l(\tilde{T}_{b_1}) = \frac{1}{2} \int_{0}^{1} C_{b_2} \left[ Q, Z_0 | D = L'(\alpha) \right] d\alpha + \frac{1}{2} \int_{0}^{1} C_{b_2} \left[ Q, Z_0 | D = R'(\alpha) \right] d\alpha$$

Let $K_2(\alpha) = \frac{1}{2} \int \left[ \left( L'(\alpha) \right)^2 + \left( R'(\alpha) \right)^2 \right] d\alpha$

The necessary conditions for $l(\tilde{T}_{b_1})$ equal to attain the minimum are

$$l'_Q(\tilde{T}_{b_1}) = 0$$

which can be calculated as follows.

$$Q^2 = \frac{1}{h} \left[ 2A \cdot K_1(\alpha) + (h + B_2)Z_0^2 \cdot K_2(\alpha) \right]$$

$$Q = \sqrt{\frac{1}{h} \left[ 2A \cdot K_1(\alpha) + (h + B_2)Z_0^2 \cdot K_2(\alpha) \right]}$$

The sufficient conditions for the $l(\tilde{T}_{b_1})$ to attain the minimum are $l''_Q(\tilde{T}_{b_1}) > 0$.

Because of $K_1(\alpha) > 0$ and $K_2(\alpha) > 0$, the sufficient conditions are clearly held from the above equations. The optimal solutions $(Q^*, t)$ that can be found and the optimal annual cost can be calculated as,

$$\tilde{T}_{b_2}(\alpha) = \left\{ C_{b_2} \left[ Q^*, Z_0 | D = L'(\alpha) \right], C_{b_2} \left[ Q^*, Z_0 | D = R'(\alpha) \right] \right\}, 0 \leq \alpha \leq 1$$

To show the characteristics of proposed models a trapezoidal fuzzy demand is employed. Let $\tilde{\lambda}$ be the trapezoidal fuzzy demands with parameters:

$$\tilde{D} = [l, m, n, u]$$

It is easy to find that $K_1(\alpha) = \frac{1 + m + n + u}{4}$
For the EOQ model with $\tilde{D}$, we have

$$Q^* = \sqrt{\frac{2A}{h} \left[ \frac{1}{4}(1 + m + n + u) \right]}$$

If $\tilde{D}$ is a symmetrical fuzzy number then

$$u - n = m - l$$

ie., $u + l = m + n$

Let $D_0 = \frac{m + n}{2}$, the mean of $\tilde{D}$, then

$$Q^* = \sqrt{\frac{2A}{h} \left[ \frac{1}{4}(m + u) \right]} = \sqrt{\frac{2A}{h} D_0}$$

$Q^*$ is the conventional EOQ with crisp demands, $\frac{m + n}{2}$.

This result implies that no matter what the spreads of fuzzy demands, as long as the fuzzy demands are symmetric with the same mean, the $Q^*$ will be the same and equal to the conventional EOQ with the mean of fuzzy demands. The fuzzy number of annual cost can be calculated as

$$\tilde{T}^*(\alpha) = \{C[Q^* | D = l + \alpha(m - l)], C[Q^* | D = u - \alpha(u - n)]\}, \; 0 \leq \alpha \leq 1$$

The above equation shows that the annual cost will also be a trapezoidal fuzzy number and with parameter as

$$\tilde{C}(Q^*) = \left[ \frac{A_l}{Q^*} + \frac{hQ^*}{2}, \frac{A_m}{Q^*} + \frac{hQ^*}{2}, \frac{A_n}{Q^*} + \frac{hQ^*}{2}, \frac{A_u}{Q^*} + \frac{hQ^*}{2} \right]$$
This implies that the shape of membership function of \( \tilde{C}(Q') \) is the same as the \( \tilde{D} \), but with a different scale. Accordingly, the spread of \( \tilde{C}(Q') \) will vary according to the spread of \( \tilde{D} \).

\[
C_{\theta_z}(Q, Z_o) = \frac{AD}{Q} + h \left( \frac{Q}{2} + Z_o \left( a'D + b'Q \right) \right) + \frac{DB}{Q} \left( a'D + b'Q \right) \cdot G(Z_o) + \frac{D}{Q} \cdot R(L)
\]

Furthermore, besides demands, the other coefficients may also be fuzzy. Let \( \hat{h} \) and \( \hat{B}_2 \) be the fuzzy number of holding cost and shortage cost of models and be defined as

\[
\hat{B}_2 \left( \alpha_{\hat{b}_2} \right) = \left[ \min \cdot \mu_{\hat{b}_2}^{-1}(\alpha_{\hat{b}_2}), \max \cdot \mu_{\hat{b}_2}^{-1}(\alpha_{\hat{b}_2}) \right]
\]

\[
\left[ L^{-1}_{\hat{b}_2}(\alpha_{\hat{b}_2}), R^{-1}_{\hat{b}_2}(\alpha_{\hat{b}_2}) \right], \ 0 \leq \alpha_{\hat{b}_2} \leq 1
\]

and

\[
h(\alpha_n) = \left[ \min \cdot \mu_n^{-1}(\alpha_n), \max \cdot \mu_n^{-1}(\alpha_n) \right]
\]

\[
\left[ L^{-1}_n(\alpha_n), R^{-1}_n(\alpha_n) \right], \ 0 \leq \alpha_n \leq 1
\]

Let

\[
K_2(\alpha_0) = \frac{1}{2} \int_0^1 \left\{ \left[ L^{-1}_n(\alpha_0) \right]^2 + \left[ R^{-1}_n(\alpha_0) \right]^2 \right\} \, d\alpha_0
\]

\[
K_3(\alpha_0, \alpha_{\hat{b}_2}) = \frac{1}{4} \int_0^1 \left\{ \left[ L^{-1}_{\hat{b}_2}(\alpha_{\hat{b}_2}) \right] \, d\alpha_{\hat{b}_2} \cdot \left[ L^{-1}_n(\alpha_0) \right]^2 \, d\alpha_0 + \left[ R^{-1}_{\hat{b}_2}(\alpha_{\hat{b}_2}) \right] \, d\alpha_{\hat{b}_2} \cdot \left[ R^{-1}_n(\alpha_0) \right]^2 \, d\alpha_0 \right\}
\]

\[
K_4(\alpha_n) = \frac{1}{2} \int_0^1 \left[ L^{-1}_n(\alpha_n) + R^{-1}_n(\alpha_n) \right] \, d\alpha_n
\]
Then the Yager’s ranking index can be derived as

\[
I(\tilde{t}) = \frac{AD}{Q}K_1(a_d) + \frac{Q}{2}K_4(a_n) + Z_0 \cdot K_s(a_n, a_d) + \frac{D^2B_2}{Q}G(Z_0)K_6(a_n, a_d) + DB_2G(Z_0)K_3(a_d, a_{b_2})
\]

and the optimal solution can be found as:

\[
Q^* = \sqrt{\frac{1}{h} \left[ \frac{2AK_1(a_d)}{K_4(a_n)} + \frac{(h + B_2)K^2_3(a_n, a_d)}{K_4(a_n)K_6(a_n, a_d) + K_3(a_d, a_{b_2})} \right]}
\]

5.2.2. Numerical Example

\[D = 600, \ h = 20, \ A = 200, \ B_2 = 1000, \ b' = 0.396, \ a' = 0.360, \ R(L) = 5.6, \]
\[Z_0 = 2.3, \ G(2.3) = 0.0036, \ L = 6, \ \sigma = 5.999 \]
\[Q^* = 144.5, \]
\[C(Q, Z_0) = 18951.06, \ \tilde{D} = (560, 580, 620, 640), \]
\[\tilde{C}(Q^*, Z_0) = (17758.57, 18351.23, 19553.07, 20172.25) \]

The purpose of this model is to study the EOQ model with impact of stochastic lead time reduction on inventory cost under order crossover and fuzzy demands. Because demands are fuzzy, the quantities of all usually treated as a decision variable will be also fuzzy. This study focuses on
possibilistic situations where the demand is described by the membership function and uncertain demand causes and uncertain total cost function. By using the Yager's Ranking Method for the stochastic Lead Time Inventory Model under fuzzy demand, we can analyse the effects of demand fuzziness on optimum order quantity and optimum total cost.