APPENDIX
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Optimum Interpolation method

Most of the analysis schemes perform well, when good quality of data well distributed around the grid point are available. However, when the data density and quality becomes poor, the performances of these schemes differ widely. The main advantages of the OI scheme over others are: firstly, the weighting factors are obtained not empirically, but on the basis of climatological characteristics (of the given parameters) or by the error characteristics of the model. Secondly, there is an inherent way in OI scheme to take care of the errors in the observational systems by reducing the weights when mean random errors of the observational system increases in magnitude and thirdly, the analyses of wind field and mass field can be done simultaneously to obtain balanced fields. Elliasen (1954) and Gandin (1963) have independently suggested this OI scheme in which the weights are optimally determined in order to minimize in a statistical sense the resulting interpolation error at each grid point by taking into account the spatial structure of the meteorological field. The analysis of various meteorological parameters could be carried out separately so that the observations of one parameter do not influence the observations of the other parameter. This is known as univariate optimum interpolation scheme.

The analysis scheme used in this study is based on Gandin’s (1963) Optimum Interpolation method. He derived an expression for the weighting function for the observing stations with respect to the grid points incorporating the physical characteristics of the parameter over the region. This is done by computing the structure functions and autocorrelation functions. Here the covariances over the analysis domain are assumed to be both homogeneous and isotropic and the variances to be homogeneous.

Let the scalar field \( f \) denotes correlated variables such as height, wind (u and v components). The OI scheme utilises an analysis equation of the form
\[ f_x^a = f_x^p + \sum_{i=1}^{n} W_i \left( f_i^o - f_i^p \right) \]  

where \( n \) is the number of observations affecting a particular grid point. \( f_i^o - f_i^p \) denotes the difference between observed and the initial guess at the \( i^{th} \) location, \( f_i^p \) is the initial guess value at the grid point and \( f_x^a \) is the resulting grid point analysis. The \( W_i \)'s are the so called "weights" which are chosen in such a manner that the mean square of the analysis error is minimised. The weights satisfy the \( n \) equations

\[ \sum_{j=1}^{n} (\mu_{ij} + \lambda^2\delta_{ij}) W_j = \mu_{xi} \quad (i = 1, 2, \ldots n) \]  

subscripts \((i,j)\) refer to the observation points and \( x \) refers to the grid point. \( \mu_{ij} = \frac{(f_i^o - f_j^o)}{\sigma^2} \) is the spatial correlation coefficient. \( \mu_{xi} \) is the vector correlation between grid and observation point. \( \lambda^2 \left( -\frac{\sigma^2}{\sigma^2} \right) \) is the normalised observational error variance, \( \sigma^2 \) is the observational error variance. \( \sigma^2_{\delta} = \sigma^2_i - \sigma^2_{\delta} \) is the corrected variance and \( \delta_{ij} \) is the kroneker delta. The covariances \( \mu_{ij} \) denote the value of \( \frac{(f_i^o - f_i^p)(f_j^o - f_j^p)}{\sigma^2} \), where bar represents average over number of cases. The observational error variance \( \sigma^2_{\delta} \) required for calculation of \( \lambda^2 \) is obtained from structure function. The true structure function

\[ Q(\rho) = \left( f_i^o - f_j^o \right)^2 \]

and the estimated structure function

\[ \hat{Q}(\rho) = \left( f_i^o - f_j^o \right)^2 \]

are related through the following relation.

\[ \hat{Q}(\rho) = \hat{Q}(\rho) + 2\sigma^2_{\delta} \]

where \( f_i^o \) and \( f_j^o \) are the anomalies of the true values \( f_i \) and \( f_j \) at \( i^{th} \) and \( j^{th} \) locations. \( Q(\rho) \) becomes zero when \( \rho \) becomes zero but \( \hat{Q}(\rho) \) need not become zero at the
same location. So, when $\rho$ becomes zero, $\hat{Q}(0) = 2\sigma_z^2$. In other words $2\sigma_z^2$ is estimated by fitting a curve to the computed structure function plotted against distance $\rho$ and extrapolating the curve until it intersects the axis of $\hat{Q}(\rho)$ at $\rho = 0$. The covariances $\hat{f}_i\hat{f}_j$ for u and v components of the wind are computed for every station with respect to every other station over the domain of the study. They are normalised by dividing by $\sigma_0^2$ before plotting them against distance. These are scattered points. Hence points within 4° segment are averaged with middle points located at a distance $d=2, 3, 4, \ldots 30^\circ$ and determine values of $\mu(d)$ in each interval by means of a function from Petersen & Truske (1969).