Chapter – 1

INTRODUCTION
1.1 INTRODUCTION

Controlling and maintaining inventory of physical goods, is a problem common to different organisations like manufacturing enterprises, retail establishments and other business environments. Inventory is part and parcel of every facet of business life. Inventory is also inevitable as far as various sectors of the economy such as industry, agriculture and defence are concerned. The term inventory, in a broader sense, may be considered as a physical stock of assets which is kept for the smooth and efficient running of future affairs of an organisation. This could be in the form of physical resources such as raw materials, semifinished goods used in the production processes, finished products ready for delivery to consumers, human resources such as unutilised labour or financial resources such as working capital etc. To be precise, a stock of physical assets having some economic value and awaiting use can be called inventory.

The reasons, why organisations should maintain inventory of goods, are varied. One of the main reasons is that, it is either physically impossible or economically unsound to have goods arrive in a given system precisely when demands for them occur. Without inventories, customers would have to wait until their orders were filled from a source or manufactured. Hence inventories of goods help to maintain customer goodwill, which in turn increases the sales and profits. In production systems, inventory helps to facilitate smooth economic production runs. Inventories are sometimes carried because of anticipated changes in the cost of commodities. For instance, a manufacturer who believes that his supplier of an important raw material will raise the price may stock unusually large quantities to take advantage of the lower price. Anticipation stock can also occur because of seasonality on the supply side.
An inventory system may be regarded as a complex system consisting of inflow, accumulation, and outflow of a certain commodity. The inflow of the commodity is by some type of replenishment. This may be by self production or purchase from some external source. The outflow is induced by demands associated with customer orders or production orders. Inventory systems, being a complex system in the real world, its representation in the form of a mathematical model depends on various assumptions and restrictions. In many cases, the model is not a complete representation of the true situation in all its complexity, but a simplified version amenable to analysis. However, they are interesting and relevant because they exhibit certain theoretical properties, which are helpful in understanding the nature of inventory systems.

Analysis of inventory systems usually involves a study of the system to determine the basic structure, the objective thought important in the system operation and the variables that affect the degree of attainment of these objectives. Some of the variables are under firms control, others are not. It is assumed that the demand is independent of the firms control. Even with this restriction, there may have the choice of assuming that the demand can be perfectly predicted or that it is not known in advance. In the latter case, it is convenient to assume the demand as a random variable with a known probability distribution.

Because of the stochastic nature of the demand pattern, there are times when demands occur and the system is out of stock. Demands occurring during this period are either completely backordered or lost. In the former case, the customers are allowed to wait until the inventory system attains sufficient stock to meet the demands. This is the case of complete backlogging. Demands are said to be lost when customers try to satisfy
there needs from some other source and the firm loses all the demands during the period. There is yet another situation where a few customers are impatient and others may be willing to wait till the next arrival of stock. This situation is referred to as a mixture of backorders and lost sales.

In inventory systems, the inventory is created at the storage point, as it is generally impossible to balance exactly the inflow with the outflow. The procurement of items in inventory may be instantaneous or it may take some time. The time that elapses from the moment an order is placed until it is physically on the shelf for satisfying customer demands is called the lead time. Lead times play a crucial role in inventory management. They affect work-in-process as well as finished goods inventories, quality management practices and customer service. Silver and Peterson (1985) indicate that in general, it is convenient to think of the lead time as being made up of five distinct components: administrative time at the stocking point, transit time to the supplier, time at the supplier, transit time back to the stocking point and time from order receipt until it is available on the shelf.

In most inventory models, very often the solution is a rule indicating when to replenish and how much to be replenished. The decisions regarding these factors are usually made on the basis of the costs and revenues associated with the system. The revenues are by way of the sales of the items stocked. Major cost components associated with an inventory system are the procurement cost, the holding cost and the shortage cost. Holding cost usually increases or decreases in proportion to the amount of inventory that is carried. Shortage cost is assumed to be proportional to the length of time of backorders. Various assumptions regarding the cost components appear.
reasonable in different contexts. In a specific problem, a realistic cost function may be considered as made up of these cost components, each of which is either concave, convex or linear.

The earliest known analysis of an inventory system was given by Harris, by deriving the classical lot size formula. The formula is often called the Wilson's lot size formula as Wilson also derived the same formula and presented its application in inventory control problems. There were certain shortcomings of this formula as it can be used only under certain restrictive assumptions. Interest in the study of inventory systems has increased and it was during World War II that the researchers concentrated on the stochastic nature of inventory problems. The publication of the paper, optimal inventory policy by Arrow, Harris and Marschak (1951) marks the beginning of the modern analysis of inventory systems.

When randomness is introduced into the demand pattern, i.e., in stochastic inventory models, it is no longer possible to know the state of the system at each point in time unless each transaction is recorded and reported as it occurs. Inventory policies are decision rules that come handy in this direction. Stochastic inventory policies that are commonly in use are classified into two as periodic review policy and continuous review policy. When a periodic review policy is used, the state of the system is examined only at discrete equally spaced intervals of time. Decisions concerning the operation of the system such as whether or not to place an order are made only at these review times. When a continuous review inventory policy is used, the inventory level is updated continuously until a certain lower limit is reached, at which point a new order is placed. The lot size reorder point policy or the (Q, r) policy and the reorder point order
level policy or the (s,S) policy are the main type of policies commonly used. Several variations of the above two policies are also used in practice. With the present day technology of high speed computers, continuous review policies are easy to implement and hence are preferred over periodic review policies.

The use of (Q, r) policies has been propagated since the seminal paper of Galliher, Morse and Simond (1959) and the classical textbook by Hadley and Whitin (1963). In a continuous review (Q, r) policy, the demand is assumed to be continuously monitored and an order of fixed size Q is placed when the inventory position reaches the reorder point. The amount Q is received after a lead time L, which is either a constant or a variable. The fixed order quantity has advantages in terms of less likelihood of error and also predictability of production requirement on the part of the supplier [Silver and Peterson (1985)]. (Q, r) policies are widely used in industry and extensively studied in the literature. An efficient algorithm for computing the optimal (Q, r) policy in a continuous review stochastic inventory system by Federgruen and Zheng (1992), Modelling of (Q, r) inventory systems by Strybosch and Heuts (1992), a partial backorder control model for continuous review (Q, r) inventory system by Rabinowitz et al (1995), the problem of order cross over in (Q, r) systems advocated by Hayya et al (1995), the work by Bookbinder and Cakanyildirim (1999) on random lead times and expedited orders in (Q, r) inventory systems, a study of (Q, r) inventory models with lost sales by Melchiors et al (2000), to name a few, are some of the latest developments.

For the traditional inventory models, lead time is an insignificant factor in inventory modeling and implementation. The early models developed were based on the simple assumption that the lead time was constant. But such simplifying assumptions
are scarcely true only of inventory systems. In practice, lead times in many situations are found to exhibit significant variability. But in the beginning, researchers and practitioners in inventory theory emphasized only the variability of demand, usually neglecting that of lead time. However, later on, there has been an awakening of interest in the role of lead time variability. Bramson (1962) gives an expository survey of the literature on the variable lead time problem in inventory control. In a simulation model of a periodic review inventory system, Gross and Soriano (1969) note that an inventory system is more sensitive to lead time variability than to demand variability. Vinson (1972) warns of great financial damage to a company that ignores lead time variability. Vinson also finds the lead time variability to be more important in influencing inventory costs than mean lead time or demand variability. The static analysis of the effect of lead time by Das (1975), the stochastic lead time problem discussed by Cawdery (1976), stock control procedures for variable lead times developed by Magson (1979), the solution of inventory models for variable lead time described by Sphicas (1982), the effect of lead time uncertainty in stochastic inventory models investigated by Song (1994), the probabilistic models for random lead times developed by Bookbinder and Cakanyildirim (1999) and several other studies highlight the impact of lead time variability in the analysis of inventory systems.

When lead times are stochastic and independent, the orders may not be received in the same sequence in which they are placed, a phenomenon known as order cross over. Various modeling approaches have been used to reduce or eliminate this complication. Hadley and Whitin (1963) note that when the interval between successive orders is large enough, the probability of crossover is negligible and can be omitted. Order crossover makes the analysis of i.i.d. lead time models still more complex. The
prevailing approach is a single cycle approximation that ignores order crossover. While doing so, it overestimates the true cost and the order quantity when lead time variability is high. But the calculation of probabilities of order crossover can shed light on the magnitude of the departure from the true cost. Hayya et al. (1995) present closed form expressions for the probabilities of order crossover in continuous review inventory models. He et al. (1998) developed a multicycle analysis to tackle the problem of order crossover in a continuous review (Q, r) system with constant demand and i.i.d. exponential/uniform lead times.

In the next section, we present a brief review of the literature work related to the inventory models presented in the thesis. Most of the models described are of continuous review (Q, r) type with variations over lead time and demand structures.

1.2 Review of Literature

As mentioned earlier, the modern analysis of stochastic inventory models originated with the work of Arrow, Harris and Marshack (1951), a detailed description of which is available in Arrow et al. (1958). In the studies on stochastic inventory models reported in the literature, much attention has been given to the form of the probability distribution of the demand during lead time (LTD). Often the LTD is assumed to follow a certain convenient distribution, and computational procedures are developed based on the assumed distribution. The LTD distribution takes into account the distribution of both the demand and the lead time. A detailed description of stochastic inventory models under Poisson and normal lead time demand structure is available in Hadley and Whitin (1963). Since then, a variety of distributions to represent the LTD have been considered by different authors. Elaborating the inappropriateness
of normal and negative exponential distributions in modeling LTD, Burgin (1975) justified the applicability of gamma distribution to inventory control problems Snyder (1984) also describes the use of gamma distribution in the analysis of inventory systems Tadikamalla (1978) elaborated the applications of the Weibull distribution while Fortuin (1980) gave a comparison of the use of five probability distributions viz gamma, Weibull, logistic, Gaussian and lognormal

Several authors modeled LTD as mixture/compound distribution with different distributions representing the demand and the lead times. When demand is Poisson and the lead time is gamma, the distribution of demand during lead time is negative binomial [Hadley and Whittin(1963)] Burgin (1972) obtained exact expressions for protection and potential lost sales for a reorder level system for normal demand and gamma lead time. For Poisson demand and normal lead time, the distribution of LTD is Hermite [Bagchi et al (1983)], under certain conditions. Using Laplace transforms, Murphy (1975) presents an approximate method to calculate the expected lost sales for a reorder level system when both demand and the lead time are gamma distributed. By extending Murphy's work, Kottas and Lau (1979) present practical approaches for determining the expected lost sales for a reorder level system of inventory control. Bagchi and Hayya (1984) addressed the problem of the distribution of demand during lead time when unit demand is normal and the lead time is gamma. Bagchi et al (1984) describe the LTD as a compound distribution involving the order intensity, order size and the lead time. An advantage of the compound distribution approach is that the components can be modeled individually and their parameters estimated.
Each of the distributions used to model lead time demand has its own advantages and disadvantages. Quite often the choice is determined by empirical meaningfulness and computational ease. In cases where the precise knowledge of the specific distribution of the lead time demand is difficult to get, the distribution free method is used to analyse some of the inventory models. Scarf (1958) addressed the newsboy problem where only the mean and the variance of the demand are known without any further assumption about the form of the distribution of the demand. Taking a conservative approach, the problem was modeled as that of finding the order quantity that maximises the expected profit against the worst possible distribution of the demand with the mean and the variance. The procedure acquired the name 'minmax distribution free approach'. Gallego (1992) applied the minmax distribution free procedure to a continuous review (Q,r) model with backorder costs. Scarf's ordering rule was not so popular until Gallego and Moon (1993) provided a compact proof and extended the analysis in several directions, which helped to disseminate Scarf's idea. Later on, Moon and Gallego (1994) used the distribution free procedure to solve various other inventory models. Moon and Choi (1995) described the distribution free newsboy problem with balking. Later they [Moon and Choi (1998)] developed a minmax distribution free procedure for a continuous review inventory model where both lead time and order quantity are considered as decision variables. Bookbinder and Lordhal (1989) illustrated the estimation of inventory reorder levels using the bootstrap statistical procedure, where also the approach is distribution free. Recently, Fricker and Goodhart (2000) applied the bootstrap approach for setting inventory reorder points in military supply systems.
Suggesting that the use of traditional theoretical distributions of one and two parameters are overly restrictive and unrealistic, Kottas and Lau (1980) proposed the use of four parameter family of distributions to model LTD. They mention that by using four parameter family of distributions to model lead time demand, there is no need to sacrifice modeling versatility and realism in the name of mathematical and computational convenience. It is well known that to fit the four basic characteristics - the location, dispersion, skewness and kurtosis - a theoretical distribution must have at least four free parameters. Based on this idea, many families of distributions with four or more parameters have been developed in the literature [Ramberg and Schmeiser (1974), Johnson and Kotz (1970), Burr (1970)]. Kottas and Lau (1980) recommended the use of four parameter Schmeiser-Deutsch (S-D) distributions to represent stochastic lead time demands. Kumaran and Achary (1996a, 1996b) demonstrated the analysis of standard inventory models using the generalised lambda distribution (GLD). GLD is a generalisation of Tukey's lambda distribution [Freimer et al. (1988)] and it was developed by Ramberg and Schmeiser (1974). The reason that a wide variety of curve shapes are possible with the distribution makes it useful for the representation of the data when the underlying model is unknown. GLD provides a good approximation to many of the commonly used symmetric and asymmetric distributions for modeling LTD. It is also useful in Monte-Carlo studies of the robustness of statistical procedures and for sensitivity analysis in simulation studies. Recently, Shore (1999) developed a new four parameter family of distributions and elaborated its usefulness in inventory decision making where he assumes only that the first two moments, partial and complete, are known.
The inventory models so far considered in this section are confined to replenishment from only one source/supplier. However, there are occasions where the use of more than one source or supplier may be necessary and also economical. Associated with the idea of using several suppliers, many advantages have been cited in the literature. These include supplier reliability, price and quality competitiveness and even political stability in case of items like crude oil. When lead times are stochastic, the simultaneous use of two sources tends to reduce the uncertainty in the lead times.

Sculli and Wu (1981) studied the problem of lead time reduction in a two vendor system where the lead times of the suppliers are normally distributed. Their study investigates the mean and standard deviation of the effective lead time, which will be the minimum of the set of random variables representing the lead time of each supplier. This seems to be the earliest paper which specifically considered the analytical problem of order splitting. Later Fong (1992) showed that the mean and variance of the effective lead times considered in Sculli and Wu (1981) can be computed in closed form. Sculli and Shum (1990) presented numerical results on the effect of using multiple suppliers on the lead time demand. They developed approximate expressions to calculate the means and variances of the effective lead time, in case of several suppliers with non-identical normal lead times. By introducing split orders among two or more vendors, Kelle and Silver (1990) examined the reductions in safety stock for the particular case of Weibull distributed lead times. The simulation study by Hayya et al. (1987) demonstrates that the use of two vendors can reduce inventory investment. Based on order statistics, Pan et al. (1991) presented a technique for the estimation of parameters of the lead time distribution in a multiple sourcing environment. Fong and Ord (1993) proposed a Bayesian approach to estimate the
moments of the effective lead time for a stock control model with independent normal lead times. Guo and Ganeshan (1995) gave decision rules to find the optimal number of suppliers by keeping the lead time mean and variance at specific levels, when lead times are uniformly or exponentially distributed. Using Gaussian quadrature rules, Fong and Gempesaw (1996) provided an efficient algorithm to evaluate the effective lead time mean and variance, reorder level and the probability of stockout during effective lead time. With Poisson and deterministic demand processes, Hill (1996) considered an order splitting situation where both lost sales and backorder situation are discussed. Instead of placing the split orders at the same time to identical suppliers, he explored the possibility of placing orders at different times to a single supplier. Extending the work of Hayya et al (1987), Fong et al (2000) developed a dual sourcing inventory model for normal unit demand and Erlang mixture lead times. Quite recently, Janssen et al (2000) considered an approximate analysis of the delivery splitting model where they investigate the order splitting concept from the suppliers point of view.

Ramasesh et al (1991) analysed the dual sourcing models with constant demand and uniform and exponential distributions to represent the lead times. Their study indicates that dual sourcing offers savings in inventory holding and backordering costs. Further, savings from dual sourcing are higher for the case of exponentially distributed lead times as compared with that of uniformly distributed lead times, which throws light on the fact that dual sourcing is more attractive when the lead time distributions are skewed and long tailed. Ramasesh, Ord and Hayya (1993) presented a dual sourcing inventory model with nonidentical suppliers. Chiang and Benton (1994) investigated the relative performance of the sole-sourcing versus dual sourcing inventory control policies for normally distributed demand and shifted exponential lead times. Lau and
Zhao (1993) discussed the problem of determining the optimal ordering policies with two suppliers which handles any stochastic form of demand and lead times. Most of the works presented in these discussions are focused on the reduction of safety stock in the multiple sourcing set up. Chiang and Chiang (1996) instead, considered the reduction of cycle stock by order splitting, in the sole sourcing environment.

The models described above are all such that the replenishment lead time is usually prescribed, whether deterministic or probabilistic. i.e. it is not subject to control. In fact, the lead time usually consists of the following components order preparation, order transit, supplier lead time, delivery time and set up time. In many practical situations, the lead time can be reduced at an added crashing cost i.e. the lead time is controllable. The crashing time idea originated in project management where the duration of some activities can be reduced by allocating more resource at an extra direct cost for these activities. Recently, several models considering lead time as a decision variable, have been developed in the literature. Liao and Shyu (1991) note that there is a lack of an appropriate inventory model that treats lead time as a decision variable and propose a model that can be used to determine the length of lead time which minimizes the expected total cost in a continuous review policy. The lead time is the only decision variable in their model. They consider the lead time as decomposed into 'n' components each having a different crashing cost for reduced lead time. Ben Daya and Raouf (1994) developed a model that considers both lead time and order quantity as decision variables. Considering shortages, Ouyang et al (1996) extended Ben Daya and Raouf's model where the total amount of stockout is considered as a mixture of backorders and lost sales. They assume a given service level and hence the reorder point is fixed. Ouyang and Wu (1997) presented a mixture inventory model for variable lead time with
a service level constraint. Later on, they [Ouyang and Wu (1998)] developed a minmax
distribution free procedure for a mixed inventory model with variable lead time. In a
recent article, Hanga and Ben Daya (1999) considered the analysis of classical
continuous and periodic review models with variable lead time. Recently, Lan et al (1999)
developed a simple method to locate the optimal order quantity and optimal lead times
of a mixture inventory model, which very much simplifies the solution procedure of
ordering cost reductions in a continuous review inventory system with partial
backorders.

In the review presented so far, we see that some of the models discussed are
with complete backordering or lost sales while others form a mixture of backorders and
lost sales, where both the provisions exist. In case of backorders, even though the
customers are ready to wait, the loss incurred due to the non-availability of items may
increase with time, as in the stocking of raw materials or spare parts of machines etc. In
such cases, apart from the number of backorders, the duration of time for which a
backorder exists must also be considered. Such models are known as time weighted
backorder models. The model has been originally introduced in Hadley and Whittin (1963)
which provide a time weighted \((Q, r)\) policy with constant lead time, when the LTD
follow a Poisson or a normal distribution. Kim and Park (1985) described the \((Q, r)\) policy
with a mixture of lost sales and time weighted backorder models. In the above two
models, the solution is based on iterative procedures. Das (1983) developed a non-
iterative procedure, known as the quadratic method, to solve the time weighted
backorder models. Observing that Das's procedure cannot be regarded as general,
Shore (1986a) provided non-iterative solution procedures, applicable to any lead time.
demand distribution provided its first three moments are known. However, Shore concludes his observation with the remark that, "if so desired, the three moment approximation must be replaced by four moment ones, with ensuing increase in accuracy and diminished algebraic simplicity."

The review presented in this section has covered a brief survey of the related literature. In the next section, we give a summary of the thesis.

1.3 SUMMARY OF THE THESIS

We note from the literature that several inventory models have been analysed using the generalised lambda distribution. Due to the analytical and computational tractability of the distribution, it would be interesting to see its further application in several other generalised inventory models such as multiple supplier models, models with variable lead time etc. It would also be interesting to examine the efficiency of GLD to represent different other distributions not considered in earlier studies, in the context of inventory analysis. The study presented in the thesis has been motivated by these aspects.

The results contained in the thesis are organised along the following lines:

1. Analysis of continuous review (Q, r) policy for Weibull lead time demand using the GLD

2. Analysis of base stock systems using GLD

3. Comparison of GLD and Murphy's approximation for solving a (Q, r) system when both demand and lead time follow gamma distributions
4 Derivation of moments of order statistics from the GLD family and its usefulness in representing the lead times in the multiple supplier inventory models.

5 Analysis of a two supplier inventory model when the orders to the suppliers are placed at different time points

6 Analysis of dual sourcing inventory models with variable lead time, when the lead time demand follows a normal distribution

7 Continuous review (Q, r, L) policy with variable lead time analysed using GLD

8 Analysis of periodic review (T,S,L) policy with variable lead time, using GLD.

9 The usefulness of GLD in approximating certain complicated integrals arising in the analysis of time weighted backorder models, it’s verification using simulation techniques

10 Analysis of time weighted backorder models by Kim and Park (1985) and the multi-item HMMS (Holt, Modigliani, Muth and Simon) model using GLD approximations

The thesis is divided into five chapters where each chapter is further divided into several sections and subsections

In chapter 2, we consider the analysis of some continuous review inventory models using GLD approximations. A description on the use of four parameter family of distributions and the use of distribution free procedures in inventory decision making is given in the introductory section. In section 2.2, we describe the GLD family and its properties in brief. In section 2.3, we consider the analysis of a continuous review (Q, r) inventory model with mixture of backorders and lost sales where we assume the LTD as
Weibull. The optimal values of the policy parameters are obtained by representing the Weibull distribution as a member of the GLD family. We also obtain the optimal solution under distribution free procedures and under the exact case. Further, the expected value of additional information (EVAI), which serves as a measure of efficiency of the procedure is provided for both the GLD and the distribution free procedures. In section 2.4, we describe the base stock system, which is a special case of the (Q, r) model with Q = 1. Assuming Weibull LTD, we describe the analysis of the system using GLD. The optimal base stock level and the system cost are obtained. Here also we provide optimal solutions under distribution free approach and the exact case. The computations performed also include the EVAI. In section 2.5, a study to examine the closeness of the approximation given by Murphy (1975), for the loss function in the (Q, r) model when both the demand and the lead time following a gamma distribution, and that given by the GLD approximation is embarked on. We show that the two approximations are in close agreement while the latter has computational advantages and can be used for any combination of the demand and the lead time distributions. The results of all the numerical studies carried out are also included.

In chapter 3, we describe the analysis of multiple supplier inventory models. The importance of using multiple suppliers in inventory management problems and a review of the related works are presented in the introductory section. The review reveals that order statistics play an important role in the analysis of multiple supplier inventory models. In section 3.2, we derive the moments of order statistics from the GLD family and give expressions for the calculation of the first four moments of the jth order statistics. In section 3.3, we describe the lead times in the multiple sourcing models and the computation of moments of lead times using the proposed procedure.
The computational advantage of the procedure is illustrated through numerical examples. We compute the first four moments of the supplier lead times, which turn out to be the moments of the order statistics. Computational studies were performed in the case of gamma and Weibull distributions by assuming them as parent distributions of lead times and using the corresponding GLD approximation. The accuracy of the procedure is tested by comparing with the exact values available. The remaining discussion in this section is based on the work of Guo and Ganeshan (1995). They give expressions for the percentage decrease in the variance of the first lead time, when an additional supplier is included each time, for uniform and exponential lead times. Using the results on order statistics from GLD, we present the generalisation of these expressions to any arbitrary distribution of lead time. For comparison purpose, in the illustrative numerical examples given, we consider uniform, exponential and Weibull lead time distributions. Results of the numerical studies presented show that our approximation procedure agrees very much with the exact values of Guo and Ganeshan (1995). In the last section of this chapter, we develop a two supplier inventory model where the orders to the suppliers are placed at two different time points. Assuming the lead time demand distribution to be gamma and allowing complete backordering of shortages, we obtain the order quantity, reorder point, the optimal proportion of split given to each supplier and the system cost through numerical studies. Here we also compare the model with the single reorder point model developed by Lau and Zhao (1993), where we consider only the case of deterministic lead times.

In chapter 4, we describe inventory models where the lead time is treated as a decision variable. In section 4.1, we describe the importance of treating lead time as a decision variable and the available literature on such models. In section 4.2, we consider
the analysis of continuous review (Q, r, L) inventory model with variable lead time using GLD. Assuming the demand distribution to be a member of the GLD family, we describe a unique procedure of analysing the model. The procedure is an alternative approach to Hariga and Ben Daya (1999)'s method. Even though Hariga and Ben Daya's procedure is general, its implementation in case of distributions other than normal looks complicated. We show here that the proposed procedure is easy to implement and makes the computational aspects very much simpler. In section 4.3, we describe the periodic review (T, S, L) policy for variable lead time and its analysis using GLD approximations. Illustrative numerical examples are provided to test the performance of the proposed procedure in case of normal, gamma and Weibull distributions. In section 4.4, we present a continuous review two supplier inventory model with shortages, where the order quantity and the lead times are treated as decision variables. Following Ben Daya and Raouf (1994), a procedure is described, to find the optimal order quantity and the lead times. A numerical example is provided to illustrate the working of the procedure.

In the final chapter, we present the analysis of time weighted backorder models using GLD. In section 5.1, we give the general introduction on inventory models with mixture of lost sales and backorders and also the importance of time weighted backorder models. In section 5.2, we describe the (Q, r) model with a mixture of lost sales and time weighted backorders as given in Kim and Park (1985) and its analysis using GLD approximations. We note from Kim and Park's model that the optimal determination of policy parameters requires the evaluation of certain complicated integrals and an iterative procedure is used in the solution procedures. Using GLD, we describe here an approximate procedure which makes the computation of the above
integrals much simpler and which results in a non-iterative solution procedure. We investigated the validity of the procedure in computing the above integrals, comparing it with Monte-Carlo simulation procedures. Numerical examples to this effect are provided for Gamma, Normal, and Weibull distributions. With the proposed approximations, a straightforward method of determining the optimal order quantity and the reorder point is discussed. Illustrative numerical examples are provided and a sensitivity analysis is also included. In section 5.3, we present the multi-item HMMS model, as given in Das (1983), and its analysis using GLD approximations. Results of the numerical study are presented in all the cases.

All computations mentioned in this study are performed using computer programs developed in C and MATLAB package. The programs developed are given in the APPENDIX.

Some parts of the work presented in this thesis have been published/accepted for publication in the following journals:


2. On Two supplier inventory models with different reorder levels – Published in International Journal of Information and Management Sciences (1999), Vol 10, No 3, 53-63

3. A note on the analysis of (Q,r) policy GLD approximation Vs distribution free approach – Accepted for publication in OPSEARCH