CHAPTER-4

COST BENEFIT ANALYSIS OF A TWO UNIT STANDBY MODEL WITH REPAIR, INSPECTION, POST-REPAIR AND RANDOM APPEARANCE AND DISAPPEARANCE OF REPAIRMAN IN THE SYSTEM
4.1 INTRODUCTION

Reliable functioning of embedded systems is of paramount concern to the billions of users that depend on these systems every day. Unfortunately most embedded systems still fall short of user’s expectation of reliability. The incorporation of redundancy is one of the important techniques to improve the reliability and hence the effectiveness of the system. The system models with standby redundancies have been frequently analyzed by various authors with different assumptions due to their vital existence in modern industries and business. Singh [160] considered profit evaluation of a two-unit cold standby system with random appearance and disappearance time of the service facility. Kour et.al. [100] provided the concept of reliability analysis of three non identical unit with random appearance and disappearance of repairman in the system. Chander et.al. [26] discussed the economic analysis of repairable systems with random appearance and disappearance of the service facility. Malik [117] investigated reliability modeling and profit analysis of a single-unit system with inspection by a server who appears and disappears randomly. While Goel et.al. [55] worked on Reliability analysis of a system with preventive maintenance, inspection and two types of repair. Gupta et.al. [78] discussed the concept of a two unit system with correlated failures and repairs and random appearance and disappearance of repairman. Agnihotri and Satsangi [3] considered two unit identical systems with priority based repair and inspection. Tuteja and Malik [172] discussed about reliability and profit analysis of two single unit models with three modes and different repair policies of repairman who appears and disappears randomly. Further, Goel et.al. [54] worked on cost analysis of a system with intermittent repair and inspection under abnormal weather. Joorel et.al. [88] put forth the concept of reliability analysis of a complex system composed of two parallel sub-systems without considering random appearance and disappearance of repairman in the system. Nakagawa and Osaki [136] investigated stochastic behavior of two unit priority standby redundant system with repair. Recently Gupta et.al. [73] provided the concept of analysis of a two unit standby system with correlated failure and repair and random appearance and disappearance of repairman. Kishan and Jain [97] analysed a two non-identical unit standby system model with repair, inspection and post-repair under classical and bayesian viewpoints. Kumar and Chowdhary [107] worked on stochastic analysis of a two unit standby system model with preventive maintenance and random
appearance and disappearance of repairman. The purpose of the present paper is to put forth the concept of random appearance and disappearance of repairman in the system. The system comprises of two non-identical units A and B, each unit having two modes Normal and Failure in which unit A is operative and unit B is kept in standby mode. Upon failure of operative unit, unit B becomes operative instantaneously. When unit A fails it goes for inspection to decide whether the repair is perfect or not. If repair is found perfect, the repaired unit becomes operative, otherwise it is sent for post repair and if the unit B fails it gets priority over unit A and after repairing it becomes operative. Single repair facility is available to repair the system which appears and disappears from the system randomly. Once the repairman starts the repair of the failed unit, he does not leave the system till all the units are repaired, inspected and post repaired during his stay in the system.

Using regenerative point technique the following important reliability characteristics of interest are obtained:

(i) Transition probabilities in transient and steady state
(ii) Mean sojourn time
(iii) Mean time to system failure (MTSF).
(iv) Point wise and steady-state availabilities of the system.
(v) Expected busy period of the repairman during \((0, t]\).
(vi) Expected number of visits for the repair facility.
(vii) Profit analysis of system.

4.2 ASSUMPTIONS AND SYSTEM DESCRIPTION

(i) The system comprises of two non-identical units A and B in which unit A is operative and unit B is kept in standby mode. Upon failure of operative unit, unit B becomes operative instantaneously.

(ii) Each unit has two modes Normal and Failure. When unit A fails it goes for inspection to decide whether the repair is perfect or not. If repair is found perfect, the repaired unit becomes operative, otherwise it is sent for post repair and if the unit B fails it gets priority over unit A and after repairing it becomes operative.
(iii) Single repair facility is available to repair the system which appears and disappears from the system randomly. Once the repairman starts the repair of the failed unit, he does not leave the system till all the units are repaired, inspected and post repaired during his stay in the system.

(iv) The failure time distributions of unit-A and B are taken exponential and inspection time of A is also taken as exponential.

(v) The distribution of time to repair of unit-A and B is assumed to be general.

4.3 NOTATION AND STATES OF THE SYSTEM

NOTATIONS

\[ \alpha_1 \] : Constant failure rate of unit-A.
\[ \alpha_2 \] : Constant failure rate of unit-B.
\[ \alpha_3 \] : Inspection rate of unit-A when unit is normal.
\[ \alpha_4 \] : Inspection rate of unit-A when unit goes for post repair.
\[ \beta(.) \] : Distribution of repair time and post repair time of unit-A
\[ \mu(.) \] : Distribution of repair time of unit-B
\[ \Psi_i \] : Mean sojourn time in state \( S_i \).
\[ \text{M}_i(t) \] : Probability that the system sojourns in state \( S_i \) upto time \( t \).
\[ \pi_i(t) \] : Cdf of time to system failure where starting from up state \( S_0 \)
\[ A_i(t) \] : \( P \) [system is up at epoch \( t \)]
\[ B_i(t) \] : \( P \) [repairman is busy in repair at an epoch \( t \)]

* : Symbol for Laplace transforms i.e. \( f^*(s) = \int_0^\infty e^{-st} f(t)dt \)
~ : Symbol for Laplace -Stieltjes transforms i.e.
\[ \tilde{F}(s) = \int_0^\infty e^{-st} dF(t) \]
© : Symbol for Laplace convolution
® : Symbol for Laplace-Stieltjes convolution
\[ p_j(t) \] : Probability that the system is in state \( S_j \) at time \( t \).
\[ Q_k(x, t)dx \] : Probability that the system is in state \( S_k \) at epoch \( t \) and has sojourned in this state for duration between \( x \) and \( x + dx \)
SYMBOLS FOR THE STATES OF THE SYSTEM

\( A_0 \) : Unit-A is in normal (N) mode and operative.
\( B_S \) : Unit-B is in standby mode.
\( A_f/A_{wr} \) : Unit-A is in failure (F) mode and under repair / waits for repair.
\( B_f/B_{wr} \) : Unit-B is in failure (F) mode and under repair / waits for repair.
\( A_I/A_{wi} \) : Unit-A is under inspection / waits for inspection.
\( A_{pr}/A_{wpri} \) : Unit-A is in failure (F) mode and under post repair / waits for post repair.

With the help of above symbols and keeping in view the assumptions, the possible states \( S_0 \) to \( S_9 \) of the system along with the transitions between the states and transition rates are shown in Fig. 1.

![Transition Diagram](image)

**Fig. 1**
STATES OF THE SYSTEM
The possible states of the system are:

\[ S_0 = \{A_0, B_0\} \quad S_1 = \{A_0, B_3\} \]
\[ S_2 = \{A_r, B_0\} \quad S_3 = \{A_{wr}, B_0\} \]
\[ S_4 = \{A_{wr}, B_{wr}\} \quad S_5 = \{A_{wr}, B_r\} \]
\[ S_6 = \{A_l, B_0\} \quad S_7 = \{A_{wl}, B_r\} \]
\[ S_8 = \{A_{pr}, B_0\} \quad S_9 = \{A_{wpr}, B_r\} \]

The states \( S_0, S_1 \) and \( S_2, S_3, S_6, S_8 \) are up states while \( S_4, S_5, S_7 \) and \( S_9 \) are down states. Further, all the states are regenerative states.

4.4 TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES
Let \( Q_{ij}(t) \) denotes the transition probability from state \( i \) to state \( j \) in transient state. To determine the various transition probabilities of different states, we proceed as follows.

Let \( T_0, T_1, T_2, \ldots \) denotes the regenerative epochs and \( X_n \) denotes the state visited at epoch \( T_{n+1} \) i.e just after the transition at \( T_n \). Then \( \{X_n, T_n\} \) constitute a Markov-Renewal process with state space \( E \), set of regenerative states and \( Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i] \) is the semi Markov kernel over \( E \).

The various transition probabilities, by simple calculations, can be obtained and written as follows:

\[ Q_{01}(t) = \alpha_1 \int_0^t e^{-(\alpha_1 + b)u} \, du \]
\[ Q_{02}(t) = b \int_0^t e^{-(\alpha_1 + b)u} \, du \]
\[ Q_{10}(t) = a \int_0^t e^{-(\alpha_1 + a)u} \, du \]
\[ Q_{13}(t) = \alpha_1 \int_0^t e^{-(\alpha_1 + a)u} \, du \]
\[ Q_{25}(t) = \alpha_2 \int_0^t e^{-\alpha_2 u} \beta(u) \, du \]
\[ Q_{26}(t) = \int_0^t e^{-\alpha_2 u} \, d\beta(u) \]
\[ Q_{32}(t) = a \int_0^t e^{-(\alpha_2 + a)u} \, du \]
\[ Q_{34}(t) = \alpha_2 \int_0^t e^{-(\alpha_2 + a)u} \, du \]
\[ Q_{45}(t) = a \int_0^t e^{-au} \, du \]
\[ Q_{60}(t) = \alpha_3 \int_0^t e^{-(\alpha_2 + \alpha_3 + \alpha_4)u} \, du \]
\[ Q_{67}(t) = \alpha_2 \int_0^t e^{-(\alpha_2 + \alpha_3 + \alpha_4)u} \, du \]

\[ Q_{68}(t) = \alpha_4 \int_0^t e^{-(\alpha_2 + \alpha_3 + \alpha_4)u} \, du \]

\[ Q_{80}(t) = \int_0^t e^{-\alpha_2 u} \, d\beta(u) \]

\[ Q_{89}(t) = \alpha_2 \int_0^t e^{-\alpha_2 u} \beta(u) \, du \]

\[ Q_{52}(t) = Q_{76}(t) = Q_{98}(t) = \int_0^t \mu(u) \, du \]  \hspace{1cm} (1)

STABLE STATE TRANSITION PROBABILITIES

By taking the limit as \( t \to \infty \), in transition probabilities (1), we get the following steady state probabilities:

\[ p_{01} = \frac{a_1}{a_1 + b} \hspace{1cm} p_{02} = \frac{b}{a_1 + b} \]

\[ p_{10} = \frac{a}{a_1 + a} \hspace{1cm} p_{13} = \frac{a_1}{a_1 + a} \]

\[ p_{25} = p_{99} = 1 - \beta(\alpha_2) \hspace{1cm} p_{26} = p_{80} = \beta(\alpha_2) \]

\[ p_{32} = \frac{a}{a_2 + a} \hspace{1cm} p_{34} = \frac{a_2}{a_2 + a} \]

\[ p_{60} = \frac{a_3}{a_2 + a_3 + a_4} \hspace{1cm} p_{67} = \frac{a_2}{a_2 + a_3 + a_4} \]

\[ p_{68} = \frac{a_4}{a_2 + a_3 + a_4} \hspace{1cm} p_{45} = p_{52} = p_{76} = p_{98} = 1 \]  \hspace{1cm} (2)

From these steady state probabilities the following relations can easily be verified:

\[ p_{01} + p_{02} = 1; \]

\[ p_{10} + p_{13} = 1; \]

\[ p_{25} + p_{26} = 1; \]

\[ p_{32} + p_{34} = 1; \]

\[ p_{60} + p_{67} + p_{68} = 1; \]

\[ p_{80} + p_{89} = 1; \]

\[ p_{52} = p_{45} = p_{76} = p_{98} = 1 \]  \hspace{1cm} (3)

MEAN SOJOURN TIME

The mean sojourn time in state \( S_i \), denoted by \( \Psi_i \), is defined as the expected time taken by the system in state \( S_i \) before transiting to any other state. To obtain mean sojourn time \( \Psi_i \) in state \( S_i \), we observe that as long as the system is in state \( S_i \), there is no
transition from $S_i$ to any other state. If $T_i$ denotes the sojourn time in state $S_i$ then mean sojourn time $\Psi_i$ in state $S_i$ is defined as

$$\Psi_i = E[T_i] = \int_0^\infty P(T_i > t)dt$$

By simple probabilistic arguments, the expressions for mean sojourn time in different states are given as:

$$\Psi_0 = \int_0^\infty e^{-(a_1 + b)t} dt = \frac{1}{(a_1 + b)}$$

$$\Psi_1 = \int_0^\infty e^{-(a_1 + a)t} dt = \frac{1}{(a_1 + a)}$$

$$\Psi_2 = \Psi_8 = \int_0^\infty e^{-a_2 t} \tilde{\beta}(t) dt = \frac{1}{a_2} [1 - \tilde{\beta}(a_2)]$$

$$\Psi_3 = \int_0^\infty e^{-(a_2 + a)t} dt = \frac{1}{(a_2 + a)}$$

$$\Psi_4 = \int_0^\infty e^{-at} dt = \frac{1}{a}$$

$$\Psi_5 = \Psi_7 = \Psi_9 = \int_0^\infty \tilde{\mu}(t) dt$$

$$\Psi_6 = \int_0^\infty e^{-(a_2 + a_3 + a_4)t} dt = \frac{1}{a_2 + a_3 + a_4}$$

(4)

**CONTRIBUTION TO MEAN SOJOURN TIME**

As we know that $m_{ij}$’s is the contribution to mean sojourn time in state $S_i$ when it transits directly to state $S_j$. Further, we also know that

$$m_{ij} = -\frac{d}{ds} q_{ij}(0) = -\tilde{Q}_{ij}(0)$$

we obtain the following expressions for different $m_{ij}$’s as given below:

$$m_{01} = a_1 \int_0^\infty t e^{-(a_1 + b)t} dt = \frac{a_1}{(a_1 + b)^2}$$

$$m_{02} = b \int_0^\infty t e^{-(a_1 + b)t} dt = \frac{b}{(a_1 + b)^2}$$

$$m_{10} = a \int_0^\infty e^{-(a_1 + a)t} dt = \frac{a}{(a_1 + a)^2}$$

$$m_{13} = a_1 \int_0^\infty e^{-(a_1 + a)t} dt = \frac{a_1}{(a_1 + a)^2}$$

$$m_{25} = m_{80} = \frac{1}{a_2} - a_2 \int_0^\infty t e^{-a_2 t} \beta(t) dt$$

$$m_{26} = m_{80} = a_2 \int_0^\infty t e^{-a_2 t} \beta(t) dt - \frac{1}{a_2} \tilde{\beta}(a_2)$$

$$m_{32} = a \int_0^\infty e^{-(a_2 + a)t} dt = \frac{a}{(a_2 + a)^2}$$

$$m_{34} = a_2 \int_0^\infty t e^{-(a_2 + a)t} dt = \frac{a_2}{(a_2 + a)^2}$$
From these expressions of $m_{ij}$'s, it can be easily verified that $\sum_j m_{ij} = \psi_i$. i.e.

\[
\begin{align*}
m_{01} + m_{02} &= \psi_0 \\
m_{10} + m_{13} &= \psi_1 \\
m_{25} + m_{26} &= \psi_2 \\
m_{32} + m_{34} &= \psi_3 \\
m_{45} &= \psi_4 \\
m_{52} &= \psi_5 \\
m_{60} + m_{67} + m_{68} &= \psi_6 \\
m_{76} &= \psi_7 \\
m_{80} + m_{89} &= \psi_8 \\
m_{98} &= \psi_9
\end{align*}
\]

### 4.5 MEAN TIME TO SYSTEM FAILURE

Let the random variable $T_i$ denotes the time to system failure when $E_0 = E_1 \in E$ and $\pi_i(t)$ is the c.d.f. of the time to system failure for the first time when the system starts operation from state $S_i$. To obtain the expressions of $\pi_i(t)$ for different values of $i$, the arguments of regenerative point processes has been used. First we will determine the expression for $\pi_0(t)$ and other may be obtained by similar approach.

In finding $\pi_0(t)$ we observe that the system transits from the state $S_0$ to state $S_j (j = 1,2)$.

(i) The system enters the state $S_1$ during $(u, u + du)$, and then starting from it fails before completing the time $(t-u)$ then the probability of this event is given by

\[
\int_0^t \pi_1(t-u) dQ_{01}(u) = Q_{01}(t) \otimes \pi_1(t)
\]
(ii) The system enters the state $S_2$ during $(u, u + du)$, and then starting from it fails before completing the time $(t-u)$ then, the probability of this event is given by

$$\int_0^t \pi_2(t-u) dQ_{02}(u) = Q_{02}(t) \otimes \pi_2(t)$$

Thus, $\pi_0(t)$ becomes

$$\pi_0(t) = Q_{01}(t) \otimes \pi_1(t) + Q_{02}(t) \otimes \pi_2(t)$$

By similar arguments, the following expression for $\pi_i(t)$ are obtained

$$\pi_1(t) = Q_{10}(t) \otimes \pi_0(t) + Q_{13}(t) \otimes \pi_3(t)$$
$$\pi_2(t) = Q_{25}(t) + Q_{26}(t) \otimes \pi_6(t)$$
$$\pi_3(t) = Q_{32}(t) + Q_{34}(t) \otimes \pi_4(t)$$
$$\pi_6(t) = Q_{60}(t) \otimes \pi_0(t) + Q_{67}(t) + Q_{68}(t) \otimes \pi_9(t)$$
$$\pi_8(t) = Q_{80}(t) \otimes \pi_0(t) + Q_{89}(t)$$

Taking Laplace-Stieltjes transform of above equations, we get a set of linear equations in $\pi_i(s)$ as

$$\tilde{\pi}_0(s) = \tilde{Q}_{01}(s)\tilde{\pi}_1(s) + \tilde{Q}_{02}(s)\tilde{\pi}_2(s)$$
$$\tilde{\pi}_1(s) = \tilde{Q}_{10}(s)\tilde{\pi}_0(s) + \tilde{Q}_{13}(s)\tilde{\pi}_3(s)$$
$$\tilde{\pi}_2(s) = \tilde{Q}_{25}(s) + \tilde{Q}_{26}(s)\tilde{\pi}_6(s)$$
$$\tilde{\pi}_3(s) = \tilde{Q}_{32}(s) + \tilde{Q}_{34}(s)\tilde{\pi}_4(s)$$
$$\tilde{\pi}_6(s) = \tilde{Q}_{60}(s)\tilde{\pi}_0(s) + \tilde{Q}_{67}(s) + \tilde{Q}_{68}(s)\tilde{\pi}_9(s)$$
$$\tilde{\pi}_8(s) = \tilde{Q}_{80}(s)\tilde{\pi}_0(s) + \tilde{Q}_{89}(s)$$

On solving the above equations for $\tilde{\pi}_0(s)$, we have

$$\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)}$$

where

$$N_1(s) = \tilde{Q}_{01}(s)\tilde{Q}_{13}(s)\tilde{Q}_{34}(s) + [\tilde{Q}_{25}(s) + \tilde{Q}_{26}(s)\tilde{Q}_{67}(s) + \tilde{Q}_{26}(s)\tilde{Q}_{68}(s)\tilde{Q}_{99}(s)]$$
$$[\tilde{Q}_{01}(s)\tilde{Q}_{13}(s)\tilde{Q}_{34}(s) + \tilde{Q}_{02}(s)]$$

$$D_1(s) = 1 - \tilde{Q}_{01}(s)\tilde{Q}_{10}(s) - [\tilde{Q}_{26}(s)\tilde{Q}_{67}(s) + \tilde{Q}_{26}(s)\tilde{Q}_{68}(s)\tilde{Q}_{80}(s)]$$
$$[\tilde{Q}_{01}(s)\tilde{Q}_{13}(s)\tilde{Q}_{34}(s) + \tilde{Q}_{02}(s)]$$

On taking $s \to 0$ in (6) and (7) and using the relation $\tilde{Q}_{ij}(s) \to p_{ij}$, as $s \to 0$ we have

$$N_1(0) = p_{01}p_{13}p_{34} + [p_{01}p_{13}p_{32} + p_{02}] [p_{25} + p_{26}p_{67} + p_{26}p_{68}p_{89}]$$
$$N_1(0) = 1 - p_{01}p_{10} - [p_{01}p_{13}p_{32} + p_{02}] [p_{26}p_{60} + p_{26}p_{68}p_{80}]$$
$$D_1(0) = 1 - p_{01}p_{10} - [p_{01}p_{13}p_{32} + p_{02}] [p_{26}p_{60} + p_{26}p_{68}p_{80}]$$
\[ \bar{\pi}_0(0) = \frac{N_1(0)}{D_1(0)} = 1 \]

Thus \( N_1(0) = D_1(0) \) showing that \( \bar{\pi}_0(0) = 1 \). Hence \( \pi_0(t) \) is a proper cdf.

Therefore, mean time to system failure when the initial state is \( S_0 \) is given by

\[ E(T) = -\frac{d\bar{\pi}_0(s)}{ds} \bigg|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \quad (8) \]

To obtain numerator of (8), we collect the coefficient of the relevant \( m_{ij} \)'s,

Where \( m_{ij} \)'s is the mean elapsed time of the system in the state \( S_i \) before transiting to other regenerative state \( S_j \) i.e

\[ m_{ij} = -Q_{ij}'(s) \bigg|_{s=0} = -\frac{d}{ds} \int e^{-st} dQ_{ij}(t) \bigg|_{s=0} \]

Also we know that \( \sum_j m_{ij} = \Psi_i \)

Thus the coefficient of various \( m_{ij} \)'s in \( D'_1(0) - N'_1(0) \) are:

Coefficient of \( m_{01} \)=1

Coefficient of \( m_{02} \)=1

Coefficient of \( m_{10} = p_{01} \)

Coefficient of \( m_{13} = p_{01} \)

Coefficient of \( m_{25} = (p_{01}p_{13}p_{32} + p_{02}) \)

Coefficient of \( m_{26} = (p_{01}p_{13}p_{32} + p_{02}) \)

Coefficient of \( m_{32} = p_{01}p_{13} \)

Coefficient of \( m_{34} = p_{01}p_{13} \)

Coefficient of \( m_{60} = p_{26}(p_{01}p_{13}p_{32} + p_{02}) \)

Coefficient of \( m_{67} = p_{26} (p_{01}p_{13}p_{32} + p_{02}) \)

Coefficient of \( m_{68} = p_{26} (p_{01}p_{13}p_{32} + p_{02}) \)

Coefficient of \( m_{80} = p_{68} p_{26} (p_{01}p_{13}p_{32} + p_{02}) \)

Coefficient of \( m_{89} = p_{68} p_{26} (p_{01}p_{13}p_{32} + p_{02}) \)

By collecting the above coefficients of \( m_{ij} \)'s, the expression for \( D'_1(0) - N'_1(0) \) can be written as:

\[ D'_1(0) - N'_1(0) = (m_{01} + m_{02}) + (m_{10} + m_{13})p_{01} + (m_{25} + m_{26})(p_{01}p_{13}p_{32} + p_{02}) + (m_{32} + m_{34})p_{01}p_{13} + (m_{60} + m_{67} + m_{68}) p_{26}(p_{01}p_{13} p_{32} + p_{02}) \]

Also we know that \( \sum m_{ij} = \Psi_i \)
Therefore, the mean time to system failure is found to be

\[ M.T.S.F = \frac{\Psi_0 + p_0\Psi_1 + (p_0p_1p_3p_2 + p_02)\Psi_2 + p_0p_1p_3\Psi_3 + (p_0p_1p_3}
\]
\[ p_2p_3 + p_02)p_26\Psi_6 + p_26 p_68(p_0p_1p_3p_2 + p_02)\Psi_8}{1 - p_0p_1p_0 - p_0p_1p_3p_2 + p_02} \]
\[ (9) \]

Now put value of \( \psi_i' \)'s and \( p_{ij}' \)'s in above equation, we get

\[ D'_1(0) - N'_1(0) = \alpha_2 [ (\alpha_1 + a)\alpha_1 ] (\alpha_1 + a) (\alpha_2 + \alpha_3 + \alpha_4) + \alpha_1\alpha_2\alpha_1(\alpha_2 + \alpha_3 + \alpha_4) +
\]
\[ \alpha_1\alpha_2 + b(\alpha_1 + a)(\alpha_2 + \alpha)][(\alpha_2 + \alpha_3 + \alpha_4)(1 - \beta(\alpha_2))] + (\alpha_2 + \alpha_4)
\]
\[ [1 - \beta(\alpha_2)]\beta(\alpha_2)]
\]
\[ D_1(0) = [(\alpha_1 + a)\alpha_1 + a\alpha_1\alpha_2(\alpha_2 + \alpha_3 + \alpha_4)(\alpha_2 + \alpha) - [\alpha_3 + \alpha_4\beta(\alpha_2)]\alpha_2\beta(\alpha_2)
\]
\[ [\alpha_1\alpha_1 + b(\alpha_1 + a)(\alpha_2 + \alpha)]
\]

### 4.6 AVAILABILITY ANALYSIS

We define \( A_1(t) \) as the probability that the system is up at epoch \( 't' \) when it initially started from regenerative state \( S_i \). It is also called pointwise availability of the system.

Now to obtain the recurrence relations among different pointwise availabilities we use the simple probabilistic arguments. As an illustration, we first find the expression for \( A_0(t) \) and then other expressions can be written accordingly.

As an illustration for \( A_0(t) \) can be expressed as the sum of the following probabilities:

(i) The system remains up in state \( S_0 \) without making any transition to any other regenerative state up to time \( 't' \), the probability of this event equals to

\[ M_0(t) = e^{-(\alpha_1 + b)t} \]

(ii) The system transits from state \( S_0 \) to state \( S_1 \) during \( (u, u + du) \), \( u < t \) and then starting from \( S_1 \) at epoch \( u \) it is available for remaining time \( (t - u) \), the probability of this event equals to

\[ \int_0^t q_{01}(u)duA_1(t - u) = q_{01}(t)A_1(t) \]

(iii) The system transits from state \( S_0 \) to state \( S_2 \) during \( (u, u + du) \), \( u < t \) and then starting from \( S_2 \) at epoch \( u \) it is available for remaining time \( (t - u) \), the probability of this event equals to

\[ \int_0^t q_{02}(u)duA_2(t - u) = q_{02}(t)A_2(t) \]

Therefore, \( A_0(t) \) become
\[ A_0(t) = M_0(t) + q_01(t) \circ A_1(t) + q_02(t) \circ A_2(t) \]

By similarly arguments, we have

\[ A_1(t) = M_1(t) + q_{10}(t) \circ A_0(t) + q_{13}(t) \circ A_3(t) \]
\[ A_2(t) = M_2(t) + q_{25}(t) \circ A_5(t) + q_{26}(t) \circ A_6(t) \]
\[ A_3(t) = M_3(t) + q_{32}(t) \circ A_2(t) + q_{34}(t) \circ A_4(t) \]
\[ A_4(t) = q_{45}(t) \circ A_5(t) \]
\[ A_5(t) = q_{52}(t) \circ A_2(t) \]
\[ A_6(t) = M_6(t) + q_{60}(t) \circ A_0(t) + q_{67}(t) \circ A_7(t) + q_{68}(t) \circ A_8(t) \]
\[ A_7(t) = q_{76}(t) \circ A_6(t) \]
\[ A_8(t) = M_8(t) + q_{80}(t) \circ A_0(t) + q_{89}(t) \circ A_9(t) \]
\[ A_9(t) = q_{98}(t) \circ A_8(t) \]

Taking Laplace Transformation of above equations, we get a set of linear equations in

\[ A_0^*(s) = M_0^*(s) + q_{01}^*(s)A_1^*(s) + q_{02}^*(s)A_2^*(s) \]
\[ A_1^*(s) = M_1^*(s) + q_{10}^*(s)A_0^*(s) + q_{13}^*(s)A_3^*(s) \]
\[ A_2^*(s) = M_2^*(s) + q_{25}^*(s)A_5^*(s) + q_{26}^*(s)A_6^*(s) \]
\[ A_3^*(s) = M_3^*(s) + q_{32}^*(s)A_2^*(s) + q_{34}^*(s)A_4^*(s) \]
\[ A_4^*(s) = q_{45}^*(s)A_5^*(s) \]
\[ A_5^*(s) = q_{52}^*(s)A_2^*(s) \]
\[ A_6^*(s) = M_6^*(s) + q_{60}^*(s)A_0^*(s) + q_{67}^*(s)A_7^*(s) + q_{68}^*(s)A_8^*(s) \]
\[ A_7^*(s) = q_{76}^*(s)A_6^*(s) \]
\[ A_8^*(s) = M_8^*(s) + q_{80}^*(s)A_0^*(s) + q_{89}^*(s)A_9^*(s) \]
\[ A_9^*(s) = q_{98}^*(s)A_8^*(s) \]

On solving the above equations, the Laplace-transformation of the point wise availability is

\[ A^*_0(s) = \frac{N_2(s)}{D_2(s)} \]  

where

\[ N_2(s) = (M_0^* + q_{01}^*M_1^* + q_{01}^*q_{13}^*M_3^*)(1 - q_{67}^*q_{76}^*)(1 - q_{89}^*q_{98}^*)(1 - q_{25}^*q_{52}^*) + (q_{01}^*q_{13}^*q_{32}^* + q_{01}^*q_{13}^*q_{34}^*q_{45}^*q_{52}^* + q_{02}^*)(1 - q_{67}^*q_{76}^*)(1 - q_{89}^*q_{98}^*)M_2^* + (1 - q_{89}^*q_{98}^*)q_{26}^*M_6^* + q_{26}^*q_{68}^*M_8^*) \] 

\[ D_2(s) = (1 - q_{03}^*q_{30}^*)(1 - q_{67}^*q_{76}^*)(1 - q_{89}^*q_{98}^*)(1 - q_{25}^*q_{52}^*) - [(1 - q_{89}^*q_{98}^*) \text{terms}] \]
The steady state availability of the system will be given by
\[ A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} s A_0^*(s) = N_2(0)/D_2(0) \]

As we know that, \( q_{ij}(t) \) is the pdf of the time of transition from state \( S_i \) to \( S_j \) and \( q_{ij}(t)dt \) is the probability of transition from state \( S_i \) to \( S_j \) during the interval \((t, t + dt)\), thus
\[ q_{ij}(s)_{s=0} = q_{ij}^*(0) = p_{ij} \text{ and } -q_{ij}(s)_{s=0} = m_{ij} \]

Also, \( \lim_{s \to 0} M_i^*(0) = \int_0^\infty M_i(t)dt = \Psi_i \), therefore we have
\[
\begin{align*}
M_0^*(0) &= \int_0^\infty e^{-(\alpha_1 + b)t} dt = \Psi_0 \\
M_1^*(0) &= \int_0^\infty e^{-(\alpha_1 + a_1)u} \bar{\beta}(t) dt = \Psi_1 \\
M_2^*(0) &= \int_0^\infty e^{-(\alpha_2 + a_2)u} \bar{\beta}(t) dt = \Psi_2 \\
M_3^*(0) &= \int_0^\infty e^{-(\alpha_1 + \alpha_2 + a_3)u} \bar{\beta}(t) dt = \Psi_3 \\
M_4^*(0) &= \int_0^\infty e^{-(\alpha_1 + \alpha_2 + a_3)u} \bar{\beta}(t) dt = \Psi_4 \\
M_8^*(0) &= \int_0^\infty e^{-(\alpha_2 + a_3)u} \bar{\beta}(t) dt = \Psi_8
\end{align*}
\]

On using the above relations and simplifications and we get \( N_2(0) \) and \( D_2(0) \) in the following form.
\[
\begin{align*}
N_2(0) &= (\Psi_0 + p_{01}\Psi_1 + p_{01}p_{13}\Psi_3)(1 - p_{25})(1 - p_{67})(1 - p_{99}) + (1 - p_{01})p_{10} \\
&\quad \left[(1 - p_{67})(1 - p_{99})\Psi_2 + (1 - p_{25})(1 - p_{99})\Psi_6 + (1 - p_{25})p_{68}\Psi_8\right] \quad (14) \\
D_2(0) &= (1 - p_{10}p_{01})(1 - p_{25})(1 - p_{67})(1 - p_{99}) - p_{26}\left[p_{60}(1 - p_{89}) + p_{68}p_{80}\right] \\
&\quad [p_{01}p_{13}p_{34} + p_{01}p_{13}p_{32} + p_{02}] \\
D_2(0) &= (1 - p_{10}p_{01})(1 - p_{25})(1 - p_{67})(1 - p_{99}) - (1 - p_{10}p_{01})(1 - p_{25})(1 - p_{67}) \\
&\quad (1 - p_{89}) \\
&= 0 \quad (15)
\end{align*}
\]

The steady state probability that the system will be up in the long run is given by
\[ A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} s A_0^*(s) \]
\[ A_0 = \lim_{s \to 0} sN_2(s)/D_2(s) \]

As \( s \to 0, D_2(s) \) becomes zero. Thus above equation becomes indeterminate form.

Hence on using L’Hospital’s rule, \( A_0 \) becomes
\[ A_0 = N_2(0)/D_2(0) \quad (16) \]

Where, \( N_2(0) \) has already been obtained and is given by equation (14).
Now to obtain $D_2'(0)$, we first collect the coefficient of relevant $m_{ij}$'s, where $m_{ij}$ is the mean elapsed time of the system in state $S_i$ before transiting to state $S_j$.

Thus, the coefficient of various $m_{ij}$'s in $D_2'(0)$ can be obtained and written as

Coefficient of $m_{01}$ in $D_2(0)$ = $(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{02}$ in $D_2(0)$ = $(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{10}$ in $D_2(0)$ = $p_{01}(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{13}$ in $D_2(0)$ = $p_{01}(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{25}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{67})(1 - p_{89})$

Coefficient of $m_{26}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{67})(1 - p_{89})$

Coefficient of $m_{32}$ in $D_2(0)$ = $p_{01}p_{13}(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{34}$ in $D_2(0)$ = $p_{01}p_{13}(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{45}$ in $D_2(0)$ = $p_{01}p_{13}p_{34}(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{52}$ in $D_2(0)$ = $p_{01}p_{13}p_{34}(1 - p_{67})(1 - p_{89})(1 - p_{25})$

Coefficient of $m_{66}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{25})(1 - p_{89})$

Coefficient of $m_{67}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{25})(1 - p_{89})$

Coefficient of $m_{68}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{25})(1 - p_{89})$

Coefficient of $m_{76}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{25})(1 - p_{89})p_{67}$

Coefficient of $m_{80}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{25})p_{68}$

Coefficient of $m_{89}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{25})p_{68}$

Coefficient of $m_{98}$ in $D_2(0)$ = $(1 - p_{01}p_{10})(1 - p_{25})(1 - p_{89})p_{68}$

By collecting the above coefficients we get

$D_2'(0) = (m_{01} + m_{02})(1 - p_{25})(1 - p_{67})(1 - p_{89}) + (m_{10} + m_{13})p_{01}(1 - p_{25})$

$(1 - p_{67})(1 - p_{89}) + (m_{25} + m_{26})(1 - p_{01}p_{10})(1 - p_{67})(1 - p_{89}) + (m_{32} + m_{34})p_{01}p_{13}(1 - p_{25})(1 - p_{67})(1 - p_{89}) + (m_{45} + m_{52})p_{01}p_{13}$

$(1 - p_{25})(1 - p_{67})(1 - p_{89})p_{34} + (m_{60} + m_{67} + m_{68})(1 - p_{01}p_{10})$

$(1 - p_{25})(1 - p_{89}) + m_{76}p_{67}(1 - p_{01}p_{10})(1 - p_{25})(1 - p_{89}) + p_{68}$

$(m_{80} + m_{89})(1 - p_{01}p_{10})(1 - p_{25}) + m_{98}(1 - p_{01}p_{10})(1 - p_{25})p_{89}p_{68}$

$D_2'(0) = (1 - p_{25})(1 - p_{67})(1 - p_{89})\Psi_0 + p_{01}(1 - p_{25})(1 - p_{67})(1 - p_{89})\Psi_1$

$(1 - p_{67})(1 - p_{01}p_{10})(1 - p_{89})\Psi_2 + p_{01}p_{13}(1 - p_{25})(1 - p_{67})(1 - p_{89})$

$\Psi_3 + p_{01}(1 - p_{25})p_{13}(1 - p_{67})(1 - p_{89})\Psi_4 + p_{01}p_{13}p_{34}(1 - p_{25})$

$(1 - p_{67})(1 - p_{89})(\Psi_4 + \Psi_5) + (1 - p_{01}p_{10})(1 - p_{25})(1 - p_{89})\Psi_6 +$
\[ p_{67}(1 - p_{01}p_{10})(1 - p_{25})(1 - p_{99}) \Psi_7 + (1 - p_{01}p_{10})(1 - p_{25})p_{68} \Psi_8 + \\
(1 - p_{01}p_{10})(1 - p_{25})p_{89}p_{68} \Psi_9 \]

\[ D'_2(0) = [\Psi_0 + p_{01} \Psi_1 + p_{01}p_{13} \Psi_3 + p_{01}p_{13} p_{34}(\Psi_4 + \Psi_5)](1 - p_{25})(1 - p_{67}) \]

\[ (1 - p_{89}) + (1 - p_{01} p_{10})(1 - p_{67})(1 - p_{99}) \Psi_2 + (1 - p_{25})(1 - p_{89}) \]

\[ (\Psi_6 + p_{67} \Psi_7) + (1 - p_{25}) p_{68}[\Psi_8 + p_{89} \Psi_9] \] (17)

Therefore on using the expressions (14) and (17) in (16), point wise availability can be written as

\[ A_0 = \frac{N_2(0)}{D'_2(0)} \]

where,

\[ N_2(0) = (\Psi_0 + p_{01} \Psi_1 + p_{01}p_{13} \Psi_3)(1 - p_{25})(1 - p_{67})(1 - p_{99}) + (1 - p_{01} p_{10}) \]

\[ [(1 - p_{67})(1 - p_{89}) \Psi_2 + (1 - p_{25})(1 - p_{99}) \Psi_6 + (1 - p_{25}) p_{68} \Psi_9] \]

\[ D'_2(0) = [\Psi_0 + p_{01} \Psi_1 + p_{01}p_{13} \Psi_5 + p_{01}p_{13} p_{34}(\Psi_4 + \Psi_5)](1 - p_{25})(1 - p_{67}) \]

\[ (1 - p_{89}) + (1 - p_{01} p_{10})(1 - p_{67})(1 - p_{99}) \Psi_2 + (1 - p_{25})(1 - p_{89}) \]

\[ (\Psi_6 + p_{67} \Psi_7) + (1 - p_{25}) p_{68}[\Psi_8 + p_{89} \Psi_9] \]

Again substituting values of \( p_{ij} \)'s and \( \Psi_i \)'s in above equation, we get

\[ N_2(0) = [(\alpha_1 + a)(\alpha_2 + a) + \alpha_1(\alpha_2 + a) + \alpha_1 \alpha_1 \alpha_2 \beta(\alpha_2) \beta(\alpha_2)(\alpha_3 + \alpha_4) + \\
(\alpha_2 + a)\{(\alpha_1 + a)(\alpha_1 + b) - a \alpha_1\}([1 - \beta(\alpha_2)] \beta(\alpha_2)(\alpha_3 + \alpha_4) + \\
\alpha_2 \beta(\alpha_2) \beta(\alpha_2) + \alpha_4 [1 - \beta(\alpha_2)] \beta(\alpha_2)) \]

\[ D'_2(0) = \left[(\alpha_1 + a)(\alpha_2 + a) + \alpha_1(\alpha_2 + a) + \alpha_1 \alpha_1 + \alpha_1 \alpha_2 \alpha_1 \left(\frac{1}{a} + \int_0^\infty \mu(u) \, du \right) \right] \]

\[ \beta(\alpha_2) \beta(\alpha_2) \alpha_2(\alpha_3 + \alpha_4) + \{(\alpha_1 + a)(\alpha_1 + b) - a \alpha_1\} \{\beta(\alpha_2) \}
\]

\[ (\alpha_3 + \alpha_4)[1 - \beta(\alpha_2)] + \alpha_2 \beta(\alpha_2) \beta(\alpha_2) \left(1 + \alpha_2 \int_0^\infty \mu(u) \, du \right) + \alpha_4 \]

\[ (1 + \alpha_2)[1 - \beta(\alpha_2)] \beta(\alpha_2) \int_0^\infty \mu(u) \, du \}
\]

Now the mean up time during \((0, t]\) is

\[ \mu_{up} = \int_0^t A_0(u) \, du \]

so that, \( \mu_{up}(s) = A_0^*(s)/s \)

and the down time during \((0,t]\) is

\[ \mu_{dn}(t) = t - \mu_{up}(t) \]

so that, \( \mu_{dn}(s) = t/s^2 - \mu_{up}^*(s) \)
4.7 BUSY PERIOD ANALYSIS FOR REPAIRMAN

Define $B_i(t)$ as the probability that the repairman is busy in the repair of the failed unit when the system initially starts from state $S_i \in E$. To illustrate the calculations we consider $B_0(t)$ as the sum of following probabilities:

(i) Probability that the system starting from state $S_0$ transits to regenerative state $S_1$ during $(u, u + du)$ and the system is busy at epoch $t$, which is given by

$$\int_0^t q_{01}(u)B_1(t - u)\,du = q_{01}(t)\,B_1(t)$$

(ii) Probability that the system starting from state $S_0$ transits to state $S_2$ during $(u, u + du)$ and the system is busy at epoch $t$, which is given by

$$\int_0^t q_{02}(u)B_2(t - u)\,du = q_{02}(t)\,B_2(t)$$

Thus, we get

$$B_0(t) = q_{01}(t)\,B_1(t) + q_{02}(t)\,B_2(t)$$

By similar argument, we have

$$B_1(t) = q_{10}(t)\,B_0(t) + q_{13}(t)\,B_3(t)$$

$$B_2(t) = M_2(t) + q_{25}(t)\,B_5(t) + q_{26}(t)\,B_6(t)$$

$$B_3(t) = q_{32}(t)\,B_2(t) + q_{34}(t)\,B_4(t)$$

$$B_4(t) = q_{45}(t)\,B_5(t)$$

$$B_5(t) = M_5(t) + q_{52}(t)\,B_2(t)$$

$$B_6(t) = M_6(t) + q_{60}(t)\,B_0(t) + q_{67}(t)\,B_7(t) + q_{68}(t)\,B_8(t)$$

$$B_7(t) = M_7(t) + q_{76}(t)\,B_6(t)$$

$$B_8(t) = M_8(t) + q_{80}(t)\,B_0(t) + q_{89}(t)\,B_9(t)$$

$$B_9(t) = M_9(t) + q_{98}(t)\,B_8(t)$$

(18)

where

$$M_2(t) = e^{-\alpha_2 u} \bar{\beta}(u);$$

$$M_6(t) = e^{-(\alpha_2 + \alpha_3 + \alpha_4) u};$$

$$M_8(t) = e^{-\alpha_2 u} \bar{\beta}(u);$$

$$M_5(t) = M_7(t) = M_9(t) = \int_0^\infty \mu(u)\,du;$$

Taking Laplace Transformation of above equations, we get a set of linear equations in $B_i^*(s)$ as:

$$B_0^*(s) = q_{01}^*(s)B_1^*(s) + q_{02}^*(s)B_2^*(s)$$

$$B_1^*(s) = q_{10}^*(s)B_0^*(s) + q_{13}^*(s)B_3^*(s)$$

$$B_2^*(s) = M_2^*(s) + q_{25}^*(s)B_5^*(s) + q_{26}^*(s)B_6^*(s)$$

$$B_3^*(s) = q_{32}^*(s)B_2^*(s) + q_{34}^*(s)B_4^*(s)$$

$$B_4^*(s) = q_{45}^*(s)B_5^*(s)$$

$$B_5^*(s) = M_5^*(s) + q_{52}^*(s)B_2^*(s)$$

$$B_6^*(s) = M_6^*(s) + q_{60}^*(s)B_0^*(s) + q_{67}^*(s)B_7^*(s) + q_{68}^*(s)B_8^*(s)$$

$$B_7^*(s) = M_7^*(s) + q_{76}^*(s)B_6^*(s)$$

$$B_8^*(s) = M_8^*(s) + q_{80}^*(s)B_0^*(s) + q_{89}^*(s)B_9^*(s)$$

$$B_9^*(s) = M_9^*(s) + q_{98}^*(s)B_8^*(s)$$
On solving the above equations for $B_0^*(s)$ we have

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)} \tag{19}$$

where

$$N_3(s) = q_{01}^* q_{13}^* q_{34}^* q_{45}^* (1 - q_{67}^* q_{68}^*) (1 - q_{89}^* q_{98}^*) (1 - q_{25}^* q_{52}^*) M_5^* + \{(M_2^* + q_{25}^* M_5^*) (1 - q_{67}^* q_{76}^*) (1 - q_{89}^* q_{98}^*) (1 - q_{25}^* q_{52}^*) \}
+ \{(M_6^* + q_{67}^* M_7^*) q_{26}^* (1 - q_{89}^* q_{98}^*) (1 - q_{52}^* q_{25}^*) \}
+ \{(M_8^* + q_{89}^* M_9^*) q_{26}^* q_{68}^* (q_{01}^* q_{13}^* q_{34}^* q_{45}^* q_{52}^* + q_{01}^* q_{13}^* q_{32}^* + q_{02}^*) \}
$$

and $D_2(s)$ is same as in availability analysis which is given by (12)

Using the result, $\lim_{s \to 0} M_i^*(s) = \int_0^\infty M_i(u) \, du = \Psi_i^*$, and $q_{ij}^*(0) = p_{ij}$, we get

$$N_3(s) = q_{01}^* q_{13}^* q_{34}^* q_{45}^* (1 - q_{67}^* q_{68}^*) (1 - q_{89}^* q_{98}^*) (1 - q_{25}^* q_{52}^*) \Psi_5^* + \{(\Psi_2^* + q_{25}^* \Psi_5^*) (1 - q_{67}^* q_{76}^*) (1 - q_{89}^* q_{98}^*) + (\Psi_6^* + q_{67}^* \Psi_7^*) q_{26}^* (1 - q_{89}^* q_{98}^*) \}
+ \{(\Psi_8^* + q_{89}^* \Psi_9^*) q_{26}^* q_{68}^* (q_{01}^* q_{13}^* q_{34}^* q_{45}^* q_{52}^* + q_{01}^* q_{13}^* q_{32}^* + q_{02}^*) \}
$$

Thus in the long run, the fraction of time for which system is under repair is given by

$$B_0 = \lim_{t \to \infty} B_0(t) = \lim_{s \to 0} sB_0^*(s) = \frac{N_3(0)}{D_2(0)} \tag{20}$$

where

$$N_3(0) = p_{01} p_{13} p_{34} (1 - p_{25})(1 - p_{67})(1 - p_{89}) (\Psi_5 + (1 - p_{01} p_{10}) (\Psi_2 + p_{25} \Psi_5)
+ (1 - p_{67})(1 - p_{89}) + (\Psi_6 + p_{67} \Psi_7)(1 - p_{25})(1 - p_{89}) + (\Psi_8 + p_{89} \Psi_9) p_{68}(1 - p_{25}) \}
$$

$$D_2^*(0) = \{(\Psi_0 + p_{01} \Psi_1 + p_{01} p_{13} \Psi_3 + p_{01} p_{13} p_{34} (\Psi_4 + \Psi_5))(1 - p_{25})(1 - p_{67})
(1 - p_{89}) + (1 - p_{01} p_{10}) (1 - p_{67})(1 - p_{89}) (\Psi_2 + (1 - p_{25})(1 - p_{89}) (\Psi_6 + p_{67} \Psi_7) + (1 - p_{25}) p_{68}[\Psi_8 + p_{89} \Psi_0]) \}
$$

$D_2^*(0)$ is same as in the case of availability which is given by (17)

Again substituting values of $p_{ij}'s$ and $\Psi_i's$ in above equation, we get
Now the expected duration of the busy time of the repairman in (0,t]

\[ N_3(0) = \{ \alpha_1 \alpha_2 \alpha_3 \beta_2 (\alpha_3 + \alpha_4) \beta (\alpha_2) + [(\alpha_1 + a)(\alpha_1 + b) - a\alpha_1] \beta(\alpha_2)[1 - \beta(\alpha_2)](\alpha_3 + \alpha_4)(\alpha_2 + a)\alpha_2 \int_0^\infty \bar{\mu}(u) du + [(\alpha_1 + a)(\alpha_1 + b) - a\alpha_1] (\alpha_2 + a)\beta(\alpha_2)(1 + \alpha_2 \int_0^\infty \bar{\mu}(u) du ) + (\alpha_3 + \alpha_4) \beta(\alpha_2)[1 - \beta(\alpha_2)] + \alpha_4 \beta(\alpha_2)(1 + \alpha_2)[1 - \beta(\alpha_2)] \int_0^\infty \bar{\mu}(u) du \} \]

\[ D'_2(0) = \left[ (\alpha_1 + a)(\alpha_2 + a) + \alpha_1 (\alpha_2 + a) + \alpha_1 \alpha_1 + \alpha_1 \alpha_2 \alpha_1 \left( \frac{1}{\alpha} + \int_0^\infty \bar{\mu}(u) du \right) \right] \]

\[ \bar{\beta}(\alpha_2) \beta(\alpha_2)(\alpha_3 + \alpha_4) + [(\alpha_1 + a)(\alpha_1 + b) - a\alpha_1] (\alpha_2 + a)\beta(\alpha_2) (\alpha_3 + \alpha_4)[1 - \beta(\alpha_2)] + \alpha_2 \bar{\beta}(\alpha_2) \beta(\alpha_2) \left( 1 + \alpha_2 \int_0^\infty \bar{\mu}(u) du \right) + \alpha_4 (1 + \alpha_2)[1 - \beta(\alpha_2)] \beta(\alpha_2) \int_0^\infty \bar{\mu}(u) du \} \]

Now the expected duration of the busy time of the repairman in (0,t]

\[ \mu_b(t) = \int_0^t B_0(u) du \]

so that, \[ \mu_b^* = B_0^*/s \]

### 4.8 Expected Number of Visits by Repairman

We define \( V_i(t) \) as the expected number of visits by the repairman during time (0,t], given that the system initially starts from regenerative state \( S_i \). To obtain for \( V_i(t) \),\n
\( i=0,1,2,3...9 \), we first find the value for \( V_0(t) \) and then other can be obtain on similar pattern.

To illustrate the calculation of these expressions, we consider \( V_0(t) \), noting that a fresh visit by the regular repairman occurs whenever the state \( S_0 \) is left.

(i) The system transits from regenerative state \( S_0 \) to \( S_1 \) but there is no visit of the repairman during the time interval \((0,t]\) when the system initially starts from regenerative state \( S_0 \) which is given by

\[ \int_0^t q_{01}(u)V_1(t-u)du = q_{01}(t)\otimes V_1(t) \]

(ii) Similarly, the system transits from regenerative state \( S_0 \) to \( S_2 \) during time interval \((u, u + du]\) and let \( u< t \) and during this time interval repairman completes one visit. Further starting from state \( S_2 \), we may count the expected no. of visits (t-u) which is given by

\[ \int_0^t q_{02}(u)[1 + V_2(t-u)]du = q_{02}(t)\otimes [1 + V_2(t)] \]

Therefore, we can write

\[ V_0(t) = q_{01}(t)\otimes V_1(t) + q_{02}(t)\otimes [1 + V_2(t)] \]
By similar probabilistic argument, we have

\[ V_1(t) = q_{10}(t) \odot V_0(t) + q_{13}(t) \odot V_3(t) \]
\[ V_2(t) = q_{25}(t) \odot V_5(t) + q_{26}(t) \odot V_6(t) \]
\[ V_3(t) = q_{32}(t) \odot [1 + V_2(t)] + q_{34}(t) \odot V_4(t) \]
\[ V_4(t) = q_{45}(t) \odot [1 + V_5(t)] \]
\[ V_5(t) = q_{52}(t) \odot V_2(t) \]
\[ V_6(t) = q_{60}(t) \odot V_0(t) + q_{67}(t) \odot V_7(t) + q_{68}(t) \odot V_8(t) \]
\[ V_7(t) = q_{76}(t) \odot V_6(t) \]
\[ V_8(t) = q_{80}(t) \odot V_0(t) + q_{89}(t) \odot V_9(t) \]
\[ V_9(t) = q_{98}(t) \odot V_8(t) \]

Taking Laplace Transformation of above equations, we get a set of linear equations in \( V^*_s \) (s) as:

\[ V^*_1(s) = q_{01}^*(s)V^*_1(s) + q_{02}^*(s)[1 + V^*_2(s)] \]
\[ V^*_2(s) = q_{10}^*(s)V^*_0(s) + q_{13}^*(s)V^*_3(s) \]
\[ V^*_3(s) = q_{25}^*(s)V^*_5(s) + q_{26}^*(s)V^*_6(s) \]
\[ V^*_4(s) = q_{32}^*(s)[1 + V^*_2(s)] + q_{34}^*(s)V^*_4(s) \]
\[ V^*_5(s) = q_{45}^*(s)[1 + V^*_5(s)] \]
\[ V^*_6(s) = q_{52}^*(s)V^*_2(s) \]
\[ V^*_7(s) = q_{60}^*(s)V^*_0(s) + q_{67}^*(s)V^*_7(s) + q_{68}^*(s)V^*_8(s) \]
\[ V^*_8(s) = q_{76}^*(s)V^*_6(s) \]
\[ V^*_9(s) = q_{80}^*(s)V^*_0(s) + q_{89}^*(s)V^*_9(s) \]
\[ V^*_9(s) = q_{98}^*(s)V^*_8(s) \]

By solving the above equation for \( V^*_0(s) \) the Laplace Transformation of the expected number of visits by the service facility is given by

\[ V^*_0(s) = \frac{N_4(s)}{D_2(s)} \]  

where

\[ N_4(s) = (q_{02}^* + q_{01}^* q_{13}^* q_{32}^* + q_{01}^* q_{13}^* q_{34}^* q_{45}^*)(1 - q_{25}^* q_{52}^*)(1 - q_{67}^* q_{76}^*)(1 - q_{89}^* q_{98}^*) \]

\[ D_2(s) \] is same as in availability analysis given by (12)

Also as, \( s \to 0, q_{ij}^*(s) \to p_{ij} \) we have

\[ N_4(0) = (p_{20} + p_{01} p_{13} p_{32} + p_{01} p_{13} p_{34} p_{45} ) (1 - p_{25} p_{52}) (1 - p_{67} p_{76}) (1 - p_{89} p_{98}) \]
\[ N_4(0) = (1 - p_{01} p_{10} ) (1 - p_{25}) (1 - p_{67}) (1 - p_{89}) \]
\( D'_2(0) \) is same as availability analysis which is given by (17).

In steady state, number of visits per unit time is given by
\[
V_0(0) = \lim_{t \to -\infty} \frac{V_0(t)}{t} = \frac{N_4(0)}{D'_2(0)}
\]  
(23)

\[
N_4(0) = (1 - p_{01}p_{10})(1 - p_{25})(1 - p_{67})(1 - p_{89})
\]

\[
D'_2(0) = [\psi_0 + p_{01}\psi_1 + p_{01}p_{13}\psi_3 + p_{01}p_{13}p_{34}(\psi_4 + \psi_5)](1 - p_{25})(1 - p_{67})
\]
\[
(1 - p_{89}) + (1 - p_{01}p_{10})((1 - p_{67})(1 - p_{89})\psi_2 + (1 - p_{25})(1 - p_{89})
\]
\[
(\psi_6 + p_{67}\psi_7) + (1 - p_{25})p_{68}[\psi_8 + p_{89}\psi_0]
\]

\( D'_2(0) \) is same as in the case of availability which is given by (17).

Again substituting values of \( p_{ij}'s \) and \( \psi'_i's \) in above equation, we get

\[
N_4(0) = [(\alpha_1 + a)(\alpha_1 + b) - a\alpha_1]\alpha_2(\alpha_2 + a)(\alpha_3 + \alpha_4)\tilde{\beta}(\alpha_2)\tilde{\beta}(\alpha_2)
\]

\[
D'_2(0) = \left[ (\alpha_1 + a)(\alpha_2 + a) + \alpha_1(\alpha_2 + a) + \alpha_1\alpha_1 + \alpha_1\alpha_2\alpha_1 \left( \frac{1}{a} + \int_0^\infty \bar{\mu}(u) \, du \right) \right]
\]
\[
\tilde{\beta}(\alpha_2)\tilde{\beta}(\alpha_2)\alpha_2(\alpha_3 + \alpha_4) + [(\alpha_1 + a)(\alpha_2 + b) - a\alpha_1]((\alpha_2 + a)\tilde{\beta}(\alpha_2)
\]
\[
(\alpha_3 + \alpha_4)[1 - \tilde{\beta}(\alpha_2)] + \alpha_2\tilde{\beta}(\alpha_2)\tilde{\beta}(\alpha_2) \left( 1 + \alpha_2\int_0^\infty \bar{\mu}(u) \, du \right) + \alpha_4
\]
\[
(1 + \alpha_2)[1 - \tilde{\beta}(\alpha_2)] \tilde{\beta}(\alpha_2)\int_0^\infty \bar{\mu}(u) \, du \}
\]

Now the expected number of visits by the repairman in (0,t]
\[
\mu_v(t) = \int_0^t V_0(u) \, du
\]
so that, \( \mu^*_v = V^*_0/s \)

### 4.9 PROFIT ANALYSIS

The expected uptime, down time of the system and busy period of the repairman in (0, t] is given as:

\[
\mu_{up} = \int_0^t A_0(u) \, du
\]
\[
\mu_{dn}(t) = t - \mu_{up}(t)
\]
\[
\mu_b(t) = \int_0^t B_0(u) \, du
\]
\[
\mu_v(t) = \int_0^t V_0(u) \, du
\]
so that,
\[
\mu^*_{up}(s) = A^*_0(s)/s
\]
\[
\mu^*_{dn}(s) = t/s^2 - \mu^*_{up}(s)
\]
\[
\mu^*_b = B^*_0/s
\]
\[ \mu_v^* = V_0^* / s \]

The expected profits incurred in \((0, t] = \) expected total revenue in \((0, t] - \) expected total repair in \((0, t] - \) expected cost of visits by repairman in \((0, t] \)

Therefore, profit analysis of the system can be written as:

\[ P_1 = K_0A_0 - K_1B_0 - K_2V_0 \]

where

\( K_0 = \) Revenue per unit up time of the system,
\( K_1 = \) Cost per unit time for which the repair is busy,
\( K_2 = \) Cost per unit visits by the repairman.

4.10 GRAPHICAL STUDY OF SYSTEM BEHAVIOUR

The behavior of MTSF, Availability and Profit analysis of the system is studied graphically in this section and to plot their graphs, the failure time distributions are assumed to be distributed exponentially. The graphs of MTSF, availability and that of profit are depicted with respect to the different parameters. It is observed that the MTSF decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF increases with increasing repair rates. Thus, we can conclude that the expected life of the system can be increased by increasing repair rate of the unit. Further, it is observed that the availability of the system gradually decreases with increasing failure rates irrespective of type of failure and increases with increasing repair rate of the unit. Also, it is seen that profit analysis of the system decreases as failure rate increases irrespective of the other parameters and increases with increasing repair rate of the unit. Hence, it can be concluded that the expected life of the system can be increased by decreasing failure rate and increasing repair rate of the unit which in turn will improve the reliability and hence the effectiveness of the system.
1) For fixed values of the parameters $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a, b$ and changing $\alpha_1$, table-1 is obtained.

TABLE-1: Effect of $\alpha_1$ and fixed parameters $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a$ and $b$ on MTSF.

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>MTSF</th>
<th>$\alpha_2 = 0.02, \alpha_3 = 0.04, \alpha_4 = 0.05, \gamma_1 = 0.40, \gamma_2 = 0.56, a = 0.05, b = 0.07$</th>
<th>$\alpha_2 = 0.08, \alpha_3 = 0.06, \alpha_4 = 0.04, \gamma_1 = 0.77, \gamma_2 = 0.65, a = 0.03, b = 0.07$</th>
<th>$\alpha_2 = 0.07, \alpha_3 = 0.05, \alpha_4 = 0.03, \gamma_1 = 0.77, \gamma_2 = 0.65, a = 0.03, b = 0.09$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>377.711</td>
<td>63.841</td>
<td>64.2412</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>245.846</td>
<td>40.4821</td>
<td>46.7111</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>202.392</td>
<td>30.1488</td>
<td>36.2712</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>185.858</td>
<td>25.9495</td>
<td>31.7165</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>177.309</td>
<td>23.6979</td>
<td>29.2103</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>172.112</td>
<td>22.2978</td>
<td>27.6314</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>168.626</td>
<td>21.3439</td>
<td>26.5472</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>166.127</td>
<td>20.6525</td>
<td>25.7753</td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>164.249</td>
<td>20.1285</td>
<td>25.1564</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>162.786</td>
<td>19.7177</td>
<td>24.6841</td>
<td></td>
</tr>
</tbody>
</table>
In Fig 2, we plot MTSF w.r.t. $\alpha_1$ and fixed values of parameter $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a, b$.

It is observed that MTSF of the system decreases w.r.t. $\alpha_1$ irrespective of the other parameters so that we conclude that expected life of the system increases with decreasing failure rate of unit in minor failure mode ($\alpha_1$).
2) For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, a, b$ and changing $\gamma_1$, table-2 is obtained.

**TABLE-2:** Effect of $\gamma_1$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, a, b$ on MTSF

<table>
<thead>
<tr>
<th>$\gamma_1$</th>
<th>MTSF</th>
<th>$\gamma_1$</th>
<th>MTSF</th>
<th>$\gamma_1$</th>
<th>MTSF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1 = 0.77, \alpha_2 = 0.68,$</td>
<td>$\alpha_1 = 0.82, \alpha_2 = 0.55,$</td>
<td>$\alpha_1 = 0.96, \alpha_2 = 0.62,$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_3 = 0.52, \alpha_4 = 0.66,$</td>
<td>$\alpha_3 = 0.72, \alpha_4 = 0.46,$</td>
<td>$\alpha_3 = 0.83, \alpha_4 = 0.55,$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma_2 = 0.02, a = 0.48,$</td>
<td>$\gamma_2 = 0.07, a = 0.68,$</td>
<td>$\gamma_2 = 0.05, a = 0.78,$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$b = 0.53,$</td>
<td>$b = 0.83,$</td>
<td>$b = 0.43,$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>2.01619</td>
<td>1.49508</td>
<td>2.01119</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>2.2772</td>
<td>1.93001</td>
<td>2.35121</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>2.5146</td>
<td>2.31658</td>
<td>2.65651</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>2.73078</td>
<td>2.66319</td>
<td>2.93198</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>2.92814</td>
<td>2.9761</td>
<td>3.18171</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>3.10884</td>
<td>3.26018</td>
<td>3.40908</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>3.27482</td>
<td>3.51934</td>
<td>3.61695</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>3.42775</td>
<td>3.75679</td>
<td>3.8077</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>3.56906</td>
<td>3.97517</td>
<td>3.98337</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>3.7</td>
<td>4.17672</td>
<td>4.14565</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In Fig 3, we plot MTSF $\gamma_1$ w.r.t. and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, n_1, n_2$. It is quite clear that MTSF of the system increases w.r.t. $\gamma_1$ irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate of unit in repair mode ($\gamma_1$).
3) For fixed values of the parameters $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a, b$ and changing $\alpha_1$, table-3 is obtained.

**TABLE-3: Effect of $\alpha_1$ and fixed parameters $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a, b$ on Availability.**

| $\alpha_1$ | Availability | $\alpha_2 = 0.57, \alpha_3 = 0.63, \alpha_4 = 0.72, \gamma_1 = 0.09, \gamma_2 = 0.02, a = 0.28, b = 0.73, $ | $\alpha_2 = 0.52, \alpha_3 = 0.64, \alpha_4 = 0.75, \gamma_1 = 0.08, \gamma_2 = 0.03, a = 0.37, b = 0.86, $ | $\alpha_2 = 0.67, \alpha_3 = 0.73, \alpha_4 = 0.82, \gamma_1 = 0.07, \gamma_2 = 0.03, a = 0.38, b = 0.63, $ |
|---|---|---|---|
| 0.1 | 0.833508 | 0.836443 | 0.835206 |
| 0.2 | 0.82431 | 0.835033 | 0.83144 |
| 0.3 | 0.786785 | 0.813845 | 0.80204 |
| 0.4 | 0.747233 | 0.78784 | 0.767618 |
| 0.5 | 0.711883 | 0.762333 | 0.734875 |
| 0.6 | 0.681616 | 0.739028 | 0.70564 |
| 0.7 | 0.655942 | 0.718286 | 0.680091 |
| 0.8 | 0.634118 | 0.699993 | 0.657888 |
| 0.9 | 0.61545 | 0.683882 | 0.638571 |
| 1.0 | 0.599356 | 0.669663 | 0.621695 |
BEHAVIOUR OF AVAILABILITY W.R.T $\alpha_1$ FOR DIFFERENT VALUES OF $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a, b$

In Fig 4, we plot Availability w.r.t. $\alpha_1$ and fixed values of parameter $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, n_1, n_2$. It is observed that Availability of the system decreases w.r.t. $\alpha_1$ irrespective of the other parameters. Therefore, we conclude that expected life of the system increases with decreasing failure rate of unit in failure mode ($\alpha_1$).
4) For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, a, b$ and changing $\gamma_1$, table-4 is obtained.

**TABLE-4: Effect of $\gamma_1$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, a, b$ on Availability.**

<table>
<thead>
<tr>
<th>$\gamma_1$</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 0.77, \alpha_2 = 0.68, \alpha_3 = 0.52, \alpha_4 = 0.66, \gamma_2 = 0.02, a = 0.48, b = 0.53,$</td>
<td>0.666965</td>
</tr>
<tr>
<td>$\alpha_1 = 0.082, \alpha_2 = 0.55, \alpha_3 = 0.72, \alpha_4 = 0.46, \gamma_2 = 0.07, a = 0.68, b = 0.83,$</td>
<td>0.679154</td>
</tr>
<tr>
<td>$\alpha_1 = 0.96, \alpha_2 = 0.62, \alpha_3 = 0.83, \alpha_4 = 0.55, \gamma_2 = 0.05, a = 0.78, b = 0.43,$</td>
<td>0.68935</td>
</tr>
<tr>
<td>0.1</td>
<td>0.666965</td>
</tr>
<tr>
<td>0.2</td>
<td>0.679154</td>
</tr>
<tr>
<td>0.3</td>
<td>0.68935</td>
</tr>
<tr>
<td>0.4</td>
<td>0.698006</td>
</tr>
<tr>
<td>0.5</td>
<td>0.705446</td>
</tr>
<tr>
<td>0.6</td>
<td>0.711909</td>
</tr>
<tr>
<td>0.7</td>
<td>0.717576</td>
</tr>
<tr>
<td>0.8</td>
<td>0.722585</td>
</tr>
<tr>
<td>0.9</td>
<td>0.727045</td>
</tr>
<tr>
<td>1.0</td>
<td>0.731042</td>
</tr>
</tbody>
</table>
In Fig 5, we plot Availability $\gamma_1$ w.r.t. and fixed values of parameter $\alpha_1, \alpha_2, \lambda, n_1, n_2$. It is quiet clear that Availability of the system increases w.r.t. $\gamma_1$ irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate of unit in repair mode ($\gamma_1$).
5) For fixed values of the parameters $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a, b, k_0, k_1, k_2$ and changing $\alpha_1$, table-5 is obtained.

**TABLE-5:** Effect of $\alpha_1$ and fixed parameters $\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, a, b, k_0, k_1, k_2$ on Profit.

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1 = 0.02, \alpha_3 = 0.04, \alpha_4 = 0.05, \gamma_1 = 0.40, \gamma_2 = 0.56, \ a = 0.05, \ b = 0.07, k_0 = 1000, k_1 = 100, k_2 = 50,$</td>
<td>$a = 0.08, \alpha_3 = 0.06, \alpha_4 = 0.04, \gamma_1 = 0.77, \gamma_2 = 0.65, \ a = 0.03, \ b = 0.07, k_0 = 900, k_1 = 500, k_2 = 100,$</td>
</tr>
<tr>
<td>0.1</td>
<td>892.492</td>
</tr>
<tr>
<td>0.2</td>
<td>891.179</td>
</tr>
<tr>
<td>0.3</td>
<td>890.385</td>
</tr>
<tr>
<td>0.4</td>
<td>890.023</td>
</tr>
<tr>
<td>0.5</td>
<td>889.82</td>
</tr>
<tr>
<td>0.6</td>
<td>889.691</td>
</tr>
<tr>
<td>0.7</td>
<td>889.603</td>
</tr>
<tr>
<td>0.8</td>
<td>889.538</td>
</tr>
<tr>
<td>0.9</td>
<td>889.488</td>
</tr>
<tr>
<td>1.0</td>
<td>889.449</td>
</tr>
</tbody>
</table>
In Fig 6, we plot profit w.r.t. \(\alpha_1\) and fixed values of parameter \(\alpha_2, \alpha_3, \alpha_4, \gamma_1, \gamma_2, n_1, n_2, k_0, k_1, k_2\). It is observed that profit of the system decreases w.r.t. \(\alpha_1\) irrespective of the other parameters so that we conclude that expected life of the system increases with decreasing failure rate of unit in minor failure mode (\(\alpha_1\)).
6) For fixed values of the parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, n_1, n_2, k_0, k_3, k_2$ and changing $\gamma_1$, table-6 is obtained.

**TABLE-6:** Effect of $\gamma_1$ and fixed parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, n_1, n_2, k_0, k_1, k_2$ on Profit.

<table>
<thead>
<tr>
<th>$\gamma_1$</th>
<th>Profit</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1 = 0.72, \alpha_2 = 0.64,$</td>
<td>$\alpha_1 = 0.82, \alpha_2 = 0.55,$</td>
<td>$\alpha_1 = 0.96, \alpha_2 = 0.62,$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_3 = 0.95, \alpha_4 = 0.40,$</td>
<td>$\alpha_3 = 0.72, \alpha_4 = 0.46,$</td>
<td>$\alpha_3 = 0.83, \alpha_4 = 0.55,$</td>
</tr>
<tr>
<td></td>
<td>$\gamma_2 = 0.06, a = 0.69,$</td>
<td>$\gamma_2 = 0.07, a = 0.68,$</td>
<td>$\gamma_2 = 0.05, a = 0.78,$</td>
</tr>
<tr>
<td></td>
<td>$b = 0.75, k_0 = 1000$</td>
<td>$b = 0.83, k_0 = 900$</td>
<td>$b = 0.43, k_0 = 1000$</td>
</tr>
<tr>
<td></td>
<td>$k_1 = 100, k_2 = 50,$</td>
<td>$k_1 = 150, k_2 = 80,$</td>
<td>$k_1 = 120, k_2 = 90,$</td>
</tr>
<tr>
<td>0.1</td>
<td>672.309</td>
<td>565.613</td>
<td>661.422</td>
</tr>
<tr>
<td>0.2</td>
<td>678.111</td>
<td>567.417</td>
<td>663.804</td>
</tr>
<tr>
<td>0.3</td>
<td>682.841</td>
<td>568.772</td>
<td>665.743</td>
</tr>
<tr>
<td>0.4</td>
<td>686.771</td>
<td>569.826</td>
<td>667.352</td>
</tr>
<tr>
<td>0.5</td>
<td>690.088</td>
<td>570.67</td>
<td>668.709</td>
</tr>
<tr>
<td>0.6</td>
<td>692.925</td>
<td>571.361</td>
<td>669.868</td>
</tr>
<tr>
<td>0.7</td>
<td>695.38</td>
<td>571.937</td>
<td>670.869</td>
</tr>
<tr>
<td>0.8</td>
<td>697.524</td>
<td>572.424</td>
<td>671.744</td>
</tr>
<tr>
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<td>572.842</td>
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<td>1.0</td>
<td>701.09</td>
<td>573.205</td>
<td>673.197</td>
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In fig 7, we plot profit w.r.t. $\gamma_1$ and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \gamma_2, n_1, n_2$. It is quite clear that profit of the system increases w.r.t. $\gamma_1$ irrespective of the other parameters so that we conclude that expected life of the system increases with increasing repair rate of unit in repair mode ($\gamma_1$).