6.1 INTRODUCTION

Several unique concepts have been employed in the operation of the semiconductor lasers. It has grown and developed into a whole range of sophisticated optoelectronic devices. The semiconductor laser has provided spectroscopists with a very useful tunable light source and is making possible new advances in spectroscopy. It can be readily linked to optical fibres and can also be modulated. It is going to play a vital part in optical communication.

When semiconductor lasers are modulated with the change of injection current, the modulation frequency resembles approximately the resonance-like frequency. Recently a few other modulation schemes of semiconductor laser were suggested and it is hoped to switch them very fast with a high frequency beyond resonance like frequency [1]. The one is the switch of matrix element by applying electric field to shift the spatial distortion of electrons and holes in the quantum well and to prevent the recombination while the electric field is applied even when the constant current is injected into the active layer [2-4].

Another scheme proposed in the change of carrier flow by varying the collector voltage in the laser. Third possibility is the cavity Q switching. The maximum modulation frequencies for those schemes however have not well defined yet. The aim of this present work is therefore to obtain possible modulation frequencies associated with those modulation schemes and compared with the well-known criterion of the injection modulation.
6.1.1 Modulation Characteristics in some Parametric Modulation Schemes

The basic equations are described by the following rate equations, which include the effect of lateral carrier diffusion [5] as

\[ \frac{dS}{dt} = \int F^2 G (N - N_g) S dV - S/\tau_p \]  \hspace{1cm} \text{(6.1)}

\[ \frac{\partial N}{\partial t} = P - VF^2 G (N - N_g) S - N / \tau_s + D \cdot [\frac{\partial^2 N}{\partial x^2}] \]  \hspace{1cm} \text{(6.2)}

where \( S \) is the photon density, \( N \) is injected carrier density, \( \tau_p \) is photon lifetime, \( \tau_s \) is the carrier lifetime, \( V \) is the volume of the active region, \( P \) is pumping rate, \( G \) and \( N_g \) are the gain parameters, and \( D = L_D^2 / \tau_s \) is the diffusion constant with the diffusion length \( L_D \). The rectangular active waveguide is assumed which provides stable single transverse-mode operation and completes carrier confinement. The inhomogeneous carrier distribution is considered by taking the first two terms of its Fourier series as follows [6-8]:

\[ N = N_0 + N_1 \cos \left( \frac{2\pi x}{w} \right) \]  \hspace{1cm} \text{(6.3)}

where \( w \) is the width of active region.

Four types of parametric modulation are considered, with modulated parameters of pumping rate \( P \) (A: Injection Modulation), carrier lifetime \( \tau_s \) (B: Carrier Lifetime Modulation), gain coefficient \( G \) (C: Gain Modulation) and photon lifetime \( \tau_p \) (D: Cavity Q Modulation). The space-independent rate equation are obtained from equations (6.1) – (6.3). In order to obtain the maximum modulation speed in the modulation schemes of interest, the sinusoidal modulation of each parameter around the stationary value is considered as follows:
Fig. 6.1 Schematic diagram and the corresponding curve of modulation efficiency for lasing mode (a) injection modulation (b) carrier lifetime modulation (c) gain modulation (d) photon lifetime modulation

* Indicates for the particular modulation
Substituting the above equations to the rate equations under small signal assumption, the following equation can be obtained as

\[
\begin{align*}
S &= S_0 + s(t), \\
G &= G_0 + g(t) \\
N_o &= N_{oo} + n_o(t), \quad \frac{1}{\tau_p} = \frac{1}{\tau_{po}} + q(t) \\
N_1 &= N_{10} + n_1(t), \quad \frac{1}{\tau_s} = \frac{1}{\tau_{so}} + r(t) \\
P &= P_0 + p(t)
\end{align*}
\] ...

\[(6.4)\]

where \(\omega_{r0}\) is the resonance-like frequency without the lateral carrier diffusion and \(h = (2\pi L_D / w)^2\). Using these equations, the modulation efficiency of the photon density is calculated. Figure 6.1 shows the calculated modulation efficiency in each modulation scheme with the ratio of a stripe width \(w\) and diffusion length \(L_D\). The carrier diffusion effect can reduce the resonance-like peak that may provide somewhat flatter frequency response. When the stripe width is close to the diffusion length the resonance-like peak is minimised as shown by the solid line. This is as same as the direct injection modulation. The maximum modulation frequency for the gain and Q modulation can exceed more than ten times the resonance-like frequency \(f_r\).
6.1.2 Frequency Chirping in some types of External Modulator

The frequency chirping caused by direct laser modulator limits the transmission bandwidth of a single-mode fibre system. A directly modulated dynamic-single-mode laser is assumed to eliminate this problem even some types of external intensity modulator such as those employing electro-absorption [1,2]. The frequency chirping is caused by phase modulation due to a refractive-index change in a loss modulator [4].

Here the frequency chirping of some types of external intensity modulator such as the loss modulator, the directional-coupler type and the Mach-Zehnder interferometer type, is formulated and compared to the direct laser modulation. The relationship between the instantaneous intensity \( S \) and the phase \( \psi \) of three output light from a loss modulator is given by

\[
\frac{d\psi}{dt} = \left( \frac{\alpha}{2} \right) \left( \frac{1}{S} \right) \frac{ds}{dt} \quad \ldots (6.6)
\]

where \( \alpha \) is the relative change between the imaginary and real parts of the refractive index. This is the relation for the laser direct modulation.

Optical modulators built by electrically switching a directional coupler from the crossover state to the straight through state. The relation between the intensity \( S_i \) and the phase \( \psi \) of the input guide \((i=1)\) and coupled guide \((i=2)\) using the coupled-mode theory [5]. According to this theory, the frequency chirping can be completely eliminated in the output light of the coupled wave-guide. On the other hand, the instantaneous phase of the output light of the input wave-guide can be obtained under a small signal modulation as follows:

\[
\frac{d\psi}{dt} = \left( \frac{1}{2} \right) \left( \frac{1}{S_1} \right) \frac{dS_1}{dt} \quad \ldots (6.7)
\]
Thus the chirping in a directional coupler-type modulator can also be expressed by equation (6.6) with $\alpha = 0$ or $\alpha = 1$.

The frequency chirping of the Mach-Zehnder interferometer-type modulator of length $L$ is analysed where the intensity modulation results from changing the propagation constant of two wave-guides by the same amount ($\Delta \beta$) in different directions. The output electric field $E$ is given by

$$E \propto 2 \cos (\Delta \beta L) \exp (-j\beta L) \quad \ldots(6.8)$$

Thus pure intensity modulation without phase modulation can be obtained.

### 6.2 RATE EQUATIONS

In this analysis the rate equation is expressed by the following form

$$\frac{dS}{dt} = - \left( \frac{1}{\tau_p} \right) S + G SN + C N/\tau_s \quad \ldots(6.9)$$

$$\frac{dN}{dt} = - \left( \frac{1}{\tau_s} \right) N - G SN + P \quad \ldots(6.10)$$

The small variation of each parameter is appeared in the above equations around the stationary value, such as:

- Photon density, $S = S_0 + s(t)h \quad \ldots(6.11)$
- Carrier density, $N = N_0 + n(t)h \quad \ldots(6.12)$
- Photon lifetime ($\tau_p$), $1/\tau_p = 1/\tau_{po} + q(t)h \quad \ldots(6.13)$
- Carrier lifetime ($\tau_s$), $1/\tau_s = 1/\tau_{so} + r(t)h \quad \ldots(6.14)$
- Pumping rate, $P = P_0 + p(t)h \quad \ldots(6.15)$
Gain factor, \( G = G_0 + g(t)h \) \( \ldots (6.16) \)

First, the spontaneous emission term \( C \) is neglected existing in equation (6.9), while the spontaneous carrier loss has considered in equation (6.10), that might include non-radiative recombination. Here the parameter \( h \) is a dimensionless small number. The stationary values \( S_0, N_0, \tau_{po}, \tau_{so}, P_0 \) and \( G_0 \) satisfy the following equations:

\[
N_0 = \frac{1}{(G_0 \tau_{po})} \quad \ldots (6.17)
\]

\[
S_0 = \frac{1}{(G_0 \tau_{so})} \left( \frac{P_0}{P_{th}} - 1 \right) \quad \ldots (6.18)
\]

where \( P_{th} (= N_0/\tau_{so}) \) is the pumping rate necessary for reaching the laser oscillation.

The small-modulated terms are included and given by the following linearised rate equations:

\[
\frac{ds}{dt} = -(1/\tau_{po})s - qS_0N_0 + qS_0N_0 + G_0sN_0 + G_0S_0n \quad \ldots (6.19)
\]

\[
\frac{dn}{dt} = -(1/\tau_{so})n - rN_0 - (qS_0N_0 + G_0sN_0 + G_0S_0n) + p \quad \ldots (6.20)
\]

From these two linear equations we obtain a simple inhomogeneous oscillation equation, which express the change of photon density \( s(t) \) as follows:

\[
\frac{d^2 s}{dt^2} + \left( \frac{1}{T} \right) \frac{ds}{dt} + \omega_r^2 s = a[x + (b/\omega_r) \frac{dx}{dt}] \quad \ldots (6.21)
\]

where \( x \) denotes any one of \( g, q, r \) and \( p \) if we consider only one of them. Here \( a \& b \) are coefficients.
The dumping time $T$ and resonance-like angular frequency ($\omega_r$) are written by

$$T = \tau_{so} / (p_o/p_{th}) \quad \text{and} \quad \omega_r = (1/\tau_{so}) \left( (\tau_{so}/\tau_{po})(p_o/p_{th}-1) \right)^{1/2} \quad \ldots (6.22)$$

### 6.3 SMALL SIGNAL MODULATION

Here describing the variation $x$ in equation (6.21) treats the sinusoidal small signal modulation:

$$x(t) = (\tilde{x} / 2) \exp(j\omega t) + \text{complex conjugate} \quad \ldots (6.23)$$

$$s(t) = (\tilde{s} / 2) \exp(j\omega t) + c.c \quad \ldots (6.24)$$

where $\omega$ is the modulation angular frequency and $\tilde{x}$ and $\tilde{s}$ is slowly varying complex amplitudes that include the phase change from the origin of modulation. It can be solved from the inhomogeneous equation (6.21) by using equations (6.23) and (6.24).

By postulating that $|{(d\tilde{x} / dt) / \tilde{x}}|$ and $|{(d\tilde{s} / dt) / \tilde{s}}|$ are much smaller than $\omega$ the amplitude change of photon density $\tilde{s}$ is given by

$$\tilde{s} = a[1 + j(\omega / \omega_r) b] \tilde{x} / (\omega_r^2 - \omega^2 + j\omega / T) = M(\omega) \tilde{x} \quad \ldots (6.25)$$

where $M(\omega)$ is a kind of transfer function as a function of modulation frequency.

When normalise the above equation $M(\omega)$ for $\omega = 0$; e.g. $|M(0)| = a/\omega_r^2$, the amplitude of normalised modulation efficiency is obtained as:

$$|\tilde{M}(\Omega)| = \left\{ (1 + b^2 \Omega^2) / [(1 - \Omega^2)^2 + \Omega^2 (\omega_r T)^2] \right\}^{1/2} \quad \ldots (6.26)$$

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Here $\Omega = \omega/\omega_r$, the maximum modulation angular frequency $\Omega_m$ is obtained by putting $|\mathbf{M}(\Omega)| = 1$ for $\Omega > 1$, and written by

$$\Omega_m = \sqrt{2} \quad (\text{for } b = 0) \quad \ldots (6.27)$$

$$\Omega_m = b \quad (\text{for } b \neq 0) \quad \ldots (6.28)$$

where it is assumed that $1/ (\omega, T)^2 << 1$ and $\Omega >> 1$ for $b = 0$.

6.4 MODULATION EFFICIENCY OF LASING MODE

The injected modulation is expressed by the modulation of the pumping rate $p(t)$. If we put $x(t) = p(t)$, then we can obtain a well-known result given by Ikegami and Suematsu [1]. In this case, we have $b = 0$ and the modulation efficiency $|\mathbf{M}(\Omega)|$ and modulation curve are shown in figure 6.1 by assuming $\tau_{so}/\tau_{po} = 900$ and $P_o/P_{th} = 2$.

Another scheme proposed is the change of carrier flow by varying the collector voltage in the laser [3]. This situation may be described by the switching of non-radiative carrier lifetime with the injection of constant current into the active layer. We describe this situation as the change of carrier lifetime by putting $x(t) = r(t)$. From the solution of equation (6.21), we have $b = 0$. It is readily noted that the maximum modulation frequency cannot exceed the resonance-like frequency $f_r$ in this scheme although one may find some other merits in the laser device.

It is also proposed that applying an electric field to shift the spatial distribution of electrons and holes in a quantum well structure switch of matrix element of dipole transition. This may be attributed to the temporal change in the gain factor $G$ in the rate equations without any change of the pumping rate $P$ since $G$ is proportional to the squared absolute value of the matrix element associated with the dipole transition. In this case $x(t)$ may be
ideally represented by $x(t) = g(t)$. The solution of equation (6.21) is simply obtained and the coefficient $b$ is given by,

$$b = \tau_{so} \omega_r = [\tau_{so} / \tau_{po} (P_o / P_{th} - 1)]^{1/2}$$  \hspace{1cm} \ldots(6.29)$$

As seen from equation (6.26) the modulation efficiency $|\tilde{M}(\Omega)|$ is basically determined by the Lorentzian denominator in the equation (6.26) but it is drastically increased by the $dx / dt$ term in equation (6.21) which is the time derivative of modulation source $x(t)$.

The maximum modulation frequency $f_m$ is given by using equation (6.28),

$$f_m = b f_r = (1 / 2\pi \tau_{so}) (\tau_{so} / \tau_{po}) [(P_o / P_{th}) - 1]$$  \hspace{1cm} \ldots(6.30)$$

Next the modulation of cavity $Q$ is considered which may be represented by the variation of photon lifetime $\tau_p$. The solution of equation (6.21) with $x(t) = q(t)$ is given by

$$b = P_{th} / P_o [ \tau_{so} / \tau_{po} (P_o / P_{th} - 1)]^{1/2}$$  \hspace{1cm} \ldots(6.31)$$

The maximum frequency $f_m$ of this scheme is given by

$$f_m = (\omega_r / 2\pi) b = [1 / (2\pi \tau_{so})] (P_{th} / P_o) (\tau_{so} / \tau_{po}) (P_o / P_{th} - 1)$$  \hspace{1cm} \ldots(6.32)$$

It is noted that the modulation frequency is raised by the factor of $b$.

The carrier lifetime modulation has considered for LED of matrix element under spontaneous emission mode. Here the spontaneous emission rate $1/\tau_s$ is modulated. The parameter $b$, $\tilde{M}(1)$ and $f_m$ are shown in
Table 6.1. In this scheme the stimulated emission can be neglected in terms of equation (6.9) and (6.10) and the rate equations are given by

\[
\frac{ds}{dt} = -\left(\frac{1}{\tau_p}\right) S + C \left(\frac{1}{\tau_s}\right) N
\]

\[
\frac{dN}{dt} = -\left(\frac{1}{\tau_s}\right) N + P
\]

The stationary values are expressed as

\[
S_0 = \tau_{po} C P_0
\]

\[
N_0 = \tau_{so} P_0
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Injection modulation</th>
<th>Carrier life time modulation</th>
<th>Gain modulation</th>
<th>Cavity Q modulation</th>
</tr>
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<tbody>
<tr>
<td>Coefficient (b)</td>
<td>0</td>
<td>0</td>
<td>[\left(\frac{\tau_{so}}{\tau_{po}}\right)] [\left(\frac{p_o}{p_{th} - 1}\right)^{1/2}]</td>
<td>[\left(\frac{p_{th}}{p_0}\right)] [\left(\frac{\tau_{so}}{\tau_{po}}\right)] [\left(\frac{p_o}{p_{th} - 1}\right)^{1/2}]</td>
</tr>
<tr>
<td>Modulation frequency ((f_n))</td>
<td>[\left(\frac{p_{th}}{p_0}\right)] [\left(\frac{\tau_{so}}{\tau_{po}}\right)] [\left(\frac{p_o}{p_{th} - 1}\right)^{1/2}]</td>
<td>[\left(\frac{p_{th}}{p_0}\right)] [\left(\frac{\tau_{so}}{\tau_{po}}\right)] [\left(\frac{p_o}{p_{th} - 1}\right)^{1/2}]</td>
<td>[\left(\frac{p_{th}}{p_0}\right)^2] [\left(\frac{\tau_{so}}{\tau_{po}}\right)] [\left(\frac{p_o}{p_{th} - 1}\right)]</td>
<td>[\left(\frac{p_{th}}{p_0}\right)^2] [\left(\frac{\tau_{so}}{\tau_{po}}\right)] [\left(\frac{p_o}{p_{th} - 1}\right)]</td>
</tr>
<tr>
<td>Maximum frequency ((f_r))</td>
<td>[2^{1/2} f_r]</td>
<td>[2^{1/2} f_r]</td>
<td>[bf_r]</td>
<td>[bf_r]</td>
</tr>
</tbody>
</table>
In the similar way as described in the earlier equations the small-modulated terms are given by

\[ \frac{ds}{dt} = -(1/\tau_{po})s + (C/\tau_{so})n + C \tau N_o \quad \cdots (6.37) \]

\[ \frac{dn}{dt} = -(1/\tau_{so})n - \tau N_o \quad \cdots (6.38) \]

Here it is considered that only the modulation of spontaneous emission rate is expressed by equation (6.14). From these two equations, the change of photon density \( s(t) \) can be expressed as follows:

\[ \frac{d^2s}{dt^2} + (1/\tau_{po}) \frac{ds}{dt} + 1/(\tau_{so}\tau_{po}) s = C N_o \frac{dr}{dt} \quad \cdots (6.39) \]

Here we have assumed that \( \tau_{po} \ll \tau_{so} \). When we modulate \( r(t) \) by a small sinusoidal signal with an angular frequency \( \omega \), the amplitude of photon density \( s \) is given by

\[ \frac{\tilde{s}}{\tilde{r}} = C N_o / \omega \left[ 1/(\tau_{po}\tau_{so}) - \omega^2 + j\omega / \tau_{po} \right] \quad \cdots (6.40) \]

The modulated efficiency \( |M(\omega)| = |s/r| \) is then written as

\[ |M(\omega)| = |\tilde{s}/\tilde{r}| = \omega / \left[ (\omega^2 - 1/(\tau_{po}\tau_{so}))^2 + \omega^2 / \tau_{po}^2 \right]^{1/2} \quad \cdots (6.41) \]

The modulation efficiency has a peak at \( \omega = 1/(\tau_{so}\tau_{po})^{1/2} \). This is much larger than \( 1/\tau_{so} \) which is the case of direct modulation of LED. If we normalise \( M(\omega) \) by the peak value at \( \omega = 1/(\tau_{po}\tau_{so})^{1/2} \), i.e. \( |M| = C N_o \tau_{po} \), the normalised modulation efficiency \( |M(\omega)| \) is given by

\[ |\tilde{M}(\Omega)| = \Omega^{(\tau_{so}/\tau_{po})^{1/2}} / \left[ (\Omega^2 - 1)^2 + \Omega^2(\tau_{so}/\tau_{po}) \right]^{1/2} \quad \cdots (6.42) \]

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where it is defined as $\Omega = \omega/(1/\tau_{so}\tau_{po})^{1/2}$. The modulation efficiency $|M|$ peaks at the angular frequency $\omega = 1/(\tau_{so}\tau_{po})^{1/2}$ and extends approximately to $1/\tau_{po}$. The result shows the ultra-wide band modulation capability in LED scheme. The calculated value of the matrix element modulation is tabulated in Table 6.2. The modulation efficiency value may correspond to the Fourier transform of the modulated pulse shape obtained by Yamanishi et al. [9]

6.5 CONCLUSION

An attempt has been made in the present work to calculate the limit of modulation speed of semiconductor lasers with various modulation schemes such as (i) gain switching (ii) the modulation of non-radiative recombination life time of minority carriers and (iii) the cavity Q by small signal analysis and they are compared with the direct injection modulation. It has been made that the maximum modulation frequency for the gain and Q modulation can exceed the resonance-like frequency $f_m$ by the factor of $b$, which is the coefficient of the time derivative of modulation parameter. But the modulation of non-radiative lifetime of carriers is not different from the direct injection modulation. A solution for the carrier lifetime modulation of LED is obtained and it is found that there exists a possibility of wide band modulation in this scheme.

The other factors that limit the maximum modulation frequency, are (i) the coherent time of the matrix element associated with the dipole transition. At such a short period of time, the rate equation representation may not make sense and we have to discuss in terms of density matrix equations and (ii) CR time constant of the device.

The laser noise (frequency chirping caused by direct laser modulator) has been discussed only from the viewpoint of spontaneous emission. But it should be remembered that there might be other intensity and frequency fluctuations in the parameters with high frequency.
Therefore, there is a lot of scope to extend this present work on the study of modulation capability and noises of semiconductor lasers and LED's in Ultra-high frequency region.

Table 6.2 Calculated values of normalised modulation efficiency for LED mode associated with modulation angular frequency at τₚ₀/τₚ₀=900

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>τₚ₀ (assumed τₚ₀&gt;&gt;τₚ₀)</th>
<th>Ω (lower frequency)</th>
<th>Modulation frequency M(Ω)</th>
<th>Ω(higher frequency)</th>
<th>Modulation frequency M(Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>0.01</td>
<td>0.45</td>
<td>10</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0375</td>
<td>0.02</td>
<td>0.6</td>
<td>20</td>
<td>0.975</td>
</tr>
<tr>
<td>3</td>
<td>0.0429</td>
<td>0.03</td>
<td>0.7</td>
<td>30</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.04</td>
<td>0.75</td>
<td>40</td>
<td>0.925</td>
</tr>
<tr>
<td>5</td>
<td>0.06</td>
<td>0.05</td>
<td>0.85</td>
<td>50</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>0.075</td>
<td>0.06</td>
<td>0.9</td>
<td>60</td>
<td>0.85</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>0.07</td>
<td>0.925</td>
<td>70</td>
<td>0.75</td>
</tr>
<tr>
<td>8</td>
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<td>80</td>
<td>0.7</td>
</tr>
<tr>
<td>9</td>
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<td>0.09</td>
<td>0.975</td>
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<td>10</td>
<td>0.6</td>
<td>0.1</td>
<td>1.0</td>
<td>100</td>
<td>0.45</td>
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</tbody>
</table>
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