CHAPTER V
5.1 INTRODUCTION

Injection laser has unique properties that make dynamic characteristics of the system. The spontaneous emission factor involved in the rate equation has been obtained without complex ideas. The injection lasers are emerging as the important lasers of the future because of their compactness, reliability, efficiency and low cost. They require very little auxiliary equipment. This type of laser systems involves high-density gain media. The use of injection laser includes high-speed computer networks, avionics system and high-definition television. The digitally modulated radiation of injection laser, scattered from the compact disc is collected by an optical sensor and processed through the audio amplifier system. This disc technique is also used for information storage and retrieval in computers and other systems. The use of injection laser includes high-speed printing, free space communication, pump source for solid state lasers, laser printers and various medical applications.

The objective of injection laser design is to minimise the electrical current required to produce sufficient gain to overcome the large inherent losses of the material. A theoretical formula for the spontaneous emission factor appearing in the rate equation of the injection laser is derived with the help of classical electromagnetic theory, such as,

\[
\frac{dN_i}{dt} = \frac{j}{(e^2d)} - \frac{N_i}{\tau_s} - \sum G_i
\]  

\[
\frac{dS_i}{dt} = (G_i - \Gamma_i) S_i + C_i \frac{N_i}{\tau_s} \quad (i = 2, 3, \ldots, M)
\]

where \(N_i\) is the injected carrier density, \(S_i\) is the photon density of the \(i^{th}\) mode, \(G_i\) is the gain of stimulated emission for the \(i^{th}\) mode, \(\Gamma_i\) is the cavity
loss of the photon for the $i^{th}$ mode, $C_i$ is the spontaneous emission factor for the $i^{th}$ mode, $j$ is the injection current density, $e^*$ is the electron charge, $d$ is the thickness of active region and $\tau_s$ is the spontaneous carrier lifetime [1]

Therefore, the closed-form of the spontaneous emission factor $C_i$ is defined as,

$$C_i = \frac{\text{(The rate of the increase of the photon density in the mode i due to the spontaneous emission)}}{\text{(The rate of the spontaneous photon emission)}}$$

The analysis is based on the following classical wave equation with respect to an electric field vector $E$ as:

$$\nabla^2 E - 2\kappa (\varepsilon_0 \mu_0 n^2) \frac{\partial E}{\partial t} - \varepsilon_0 \mu_0 n^2 \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2}$$

...\(5.3\)

where $\kappa$ is the decay constant of the mode energy by the absorption loss, $n$ is the refractive index, $\varepsilon_0$ and $\mu_0$ are the permittivity and permeability in vacuum respectively. $P$ is the polarisation vector characterised by an electric field vector $E$.

The spontaneous emission is treated as the radiation from dipoles located in the active region. The spontaneous emissions influence dynamic characteristics of the injection lasers mainly because the relaxation oscillation and the rise up property of the light output are affected remarkably by the spontaneous emission. The effect of the spontaneous emission is formulated in the rate equation where a certain part of photons emitted spontaneously contribute to excitation of lasing mode. As a result of the analysis, it was predicted that the relaxation oscillation would be suppressed when the spontaneous emission factor exceeds a certain value. Furthermore, the line-width of lasing mode is related to the spontaneous
emission factor. Therefore, the spontaneous emission factor is an important parameter to analyse the dynamic behaviour of injection lasers.

The spontaneous emission factor that appears in rate equation, indicates how much of the total radiated energy couples to a laser mode. It has been suggested that making the spontaneous emission factor close to unity can drastically reduce the threshold. To obtain a fast modulation response, it is effective to make the spontaneous lifetime shorter \([Z]\).

Keeping the energy confinement factor constant due to large refractive index difference, the spontaneous emission factor can be increased by the reducing volume of the active region. The value can be estimated from the refractive index, volume of the active region, the spectrum width of the spontaneous emission and wavelength of injection laser.

Then, the spontaneous emission factor is obtained from the shape of the spectrum of the spontaneous emission such that the injection ratio is

\[
C_i / C'_i = 1/3 \frac{1}{V} \int_{a}^{b} \int_{-\infty}^{\infty} f(\omega) \, d\omega
\]

The above equation is simplified as follows:

\[
C_i = \xi_i \lambda_c^4 / 4\pi^2 n_1^2 V \Delta \lambda
\]

where \(\lambda_c\) is the central lasing wavelength and \(\Delta \lambda\) is the spectrum width (HMFW) of the spontaneous emission, \(n_1\) and \(V\) are the refractive index and volume of the active region respectively. \(\xi_i\) is the energy confinement factor for the mode \(i\).
The photon density $S_i$ has been calculated relatively to the current density $j$ from equations (5.1) and (5.2), in which the $C_i'$ is taken to be a parameter. Then, $C_i$ of the injection laser is determined by comparing both the experimental and theoretical results [2]. The theoretical results are compared with the experimental measurements and they agree favourably.

### 5.2 SPONTANEOUS EMISSION FACTOR $C_i$

#### 5.2.1 Expression

In the theory of Suematsu and Furuya [3], the spontaneous emission factor of the $i^{th}$ mode is defined and formulated as

$$C_i = \frac{\text{radiation energy coupled to } i^{th} \text{ mode}}{\text{total radiation energy}}$$

$$= (\xi_i \lambda_i^4) \left(4\pi^2 n_{\text{eff}}^2 V \Delta \lambda_s\right)$$

where $\xi_i$ is the confinement factor of the mode $i$, $n_{\text{eff}}$ is the effective refractive index of mode, $\lambda_i$ and $\Delta \lambda_s$ are the peak wavelength and the width (FWHM) of the emission spectrum, respectively and $V$ is the volume of the active region. Equation (5.5) can be rewritten as

$$C_i = \frac{1}{2} [(1/2\pi)^3 \times (n_{\text{eff}}^3 V / \xi_i) \times (4\pi k_s^2 \cdot k^2 \Delta k)]^{-1}$$

$$= \frac{1}{2} [(\text{mode density in the free space})$$

$$\times (\text{effective mode volume})$$

$$\times (\text{wave number volume in the emission spectrum})]^{-1}$$

$$= \frac{1}{2} [(\text{total amount of modes in the emission spectrum})]^{-1}$$

Equation (5.7) implies that (5.6) is derived by assuming that modes distribute uniformly so that the same radiation energy couples to every mode. This assumption is valid when the laser cavity is much larger than the wavelength. However, it seems difficult to apply equation (5.6) to the estimation of the $C_i$.
Fig. 5.1 Structure of Laser cavity (assuming $n_2, n_3 < n_1$)
factor in the modeled cavity because the distribution of the mode density is altered too much from the uniform distribution in the free space. To obtain the $C_i$ factor, the radiation energy can be calculated that couples to the fundamental mode and compare it to the total radiation energy in the emission spectrum.

5.3 ANALYTICAL METHOD

For the analysis of spontaneous emission factor, the laser cavity is assumed to be rectangular dielectric wave-guide [4] of length $l$ as shown in figure 5.1. The coupling of a spontaneously radiating electron to the electromagnetic wave, which satisfies the resonance condition of the laser cavity, is treated classically using oscillating dipoles. Oscillating dipoles is assumed to appear successively in the active region and they disappear at the decay constant corresponding to the relaxation of the carrier in the crystal. It is assumed that there are no correlation's among phases of oscillating dipoles and vectors of the dipoles direct in $x$, $y$ and $z$ directions at equal probabilities.

Electromagnetic waves in the rectangular wave-guide can be separated into $x$ and $y$ polarised modes (constant in $z$-direction). A resonant mode of the laser cavity is represented as:

$$\begin{align*}
\left\{ E_x \right\} &= F_{sr}(x,y) \sin(q\pi z/l) e^{i\omega srq} \\
\left\{ E_y \right\}
\end{align*} \quad \text{(5.8)}$$

where $s$ and $r$ are transverse mode numbers and $q$ is the longitudinal mode number. $F_{sr}(x,y)$ represents the distribution of the electric field in a transverse cross section as:
\[ F_{sr}(x,y) = \begin{cases} \cos (\gamma_x x + \varphi_x) & \cos (\gamma_y y + \varphi_y) \text{ region I} \\ M_2 \cos (\gamma_x x + \varphi_x) e^{ky} & \text{ region II} \\ M_3 e^{kx} & \cos (\gamma_y y + \varphi_y) \text{ region III} \\ M_4 \cos (\gamma_x x + \varphi_x) e^{ky} & \text{ region IV} \\ M_5 e^{kx} & \cos (\gamma_y y + \varphi_y) \text{ region V} \end{cases} \] ...

where \( \varphi_x(\varphi_y) \) is zero when mode number \( s(r) \) is even, while \( \varphi_x(\varphi_y) \) is \( \pi/2 \) when \( s(r) \) is odd. \( M_2-M_3 \) are constants determined by boundary conditions.

From boundary conditions, the following equations are derived.

\[ \tan (\gamma_x a + \varphi_x) = \begin{cases} 1 & \{ [\omega^2 \varepsilon_0 (n_1^2 - n_2^2) \mu_0 - \gamma_x^2]^{1/2} \} / \gamma_x \ldots(5.10) \\ \left( \frac{n_1}{n_2} \right)^2 & \{ [\omega^2 \varepsilon_0 (n_1^2 - n_2^2) \mu_0 - \gamma_y^2]^{1/2} \} / \gamma_y \ldots(5.11) \end{cases} \]

and

\[ (q \pi / l)^2 = \omega^2 \varepsilon_0 n_1^2 \mu_0 - \gamma_x^2 - \gamma_y^2 \] ...

where \( a, b \) and \( l \) are dimensions of the laser cavity. For the specified mode number \( s \) and \( r \), the propagation constant \( (q \pi / l) \) is determined continuously as a function of the angular frequency \( \omega \). Then, we can determine the resonant angular frequency for given mode numbers \( s, r \), and the polarisation.
5.4 EXCITATION OF RESONANT MODE BY DIPOLE

The excitation of resonant mode by dipole corresponds to the coupling between the photon in the mode and the radiation electron, is analysed. A polarisation density is represented as follows.

\[ P(x,y,z,t) = P_p e^{(-\sigma + j\omega p)t} \delta(x-x_p, y-y_p, z-z_p) \]  \hspace{1cm} \text{(5.13)}

where \( P_p \) is the polarisation vector with the magnitude of the initial amplitude, and \( \omega_p \) and \( \sigma \) are the angular frequency and decay constant, respectively. \( 1/\sigma \) is the relaxation time and is of the order of \( 10^{-13} \) to \( 10^{-11} \) sec for GaAsAl injection lasers [5] and therefore \( \omega_p >> \sigma \). The delta function indicates that a dipole appears at a point \((x_p, y_p, z_p)\).

The electric field function \( E_x \) is expanded with resonant modes obtained in modes of dielectric wave-guide cavity as:

\[ E_x(x, y, z, t) = \sum_m D_m(t) F_m(x,y) \sin \left( q'n z/l \right) \]  \hspace{1cm} \text{(5.14)}

where \( D_m(t) \) is the mode amplitude of the mode \( m \) and depends on time \( t \). The expansion extends to the region of continuous spectrum of \( \omega_m \). The solution of the homogeneous equation is obtained by using the resonant modes from the above equations as:

\[
\begin{align*}
\sum_m (-\epsilon_0 n^2 \mu_0) [\ddot{D}_m + 2\kappa \dot{D}_m + \omega_m^2 D_m] F_m(x,y) \sin \left( q'n z/l \right) \\
= - \mu_\omega^2 P_p e^{(-\sigma + j\omega p)t} \delta(x-x_p, y-y_p, z-z_p) \hspace{1cm} \text{(5.15)}
\end{align*}
\]

where \( \dot{D} \) denotes the differentiation with respect to time \( t \), and \( \sigma << \omega_p \) is taken into consideration. Multiplying equation (5.15) by \( F_i(x,y) \sin(q'nz/l) \) and integrating it over the whole space, we obtain the following equation due to the orthogonality of the mode functions.
\[ \ddot{D}_i + 2\kappa_i \dot{D}_i + \omega_i^2 D_i = A e^{-(\sigma + j\omega_p) t} \] 

...(5.16)

Here

\[ A = (\omega_p^2 P_p / \epsilon_0 n_1^2) \{ [F_i(x_p, y_p) \sin (q\pi z_p / l)] \} \]

\[ I \{ \left[ \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{0}^{l} dz (n^2/n_1^2) F_i^2(x,y) \sin^2 (q\pi z / l) \right] \} \] 

...(5.17)

And \( 2\kappa_i \) is the coefficient of the net power loss of the mode \( i \) and is

\[ 2\kappa_i = \Gamma_i - G_i \] 

...(5.18)

with the initial conditions \( D(0) = \dot{D}(0) = 0 \), equation (5.16) is solved as

\[ D_i(t) = A \left\{ ((\sigma - \kappa_i)^2 + (\omega_i^2 - \omega_p^2) + 2j\omega_p(\kappa_i - \sigma))^2 \right\}^{-1} e^{-(\sigma + j\omega_p) t} \]

\[ + [2j\omega_i \{(\sigma - \kappa_i) + j(\omega_i - \omega_p)\}]^{-1} e^{(\kappa_i + j\omega_i t)} \]

\[ - [2j\omega_i \{((\sigma - \kappa_i) - j(\omega_i + \omega_p)\}]^{-1} e^{(\kappa_i - j\omega_i t)} \] 

...(5.19)

where the resonant angular frequency \( \omega_i \) deviates from \( \omega_i \) due to the loss and is given by

\[ \omega_i = (\omega_i^2 - \kappa_i^2)^{1/2} \] 

...(5.20)

By neglecting the non-resonant term and using \( \omega_i \approx \omega_p \) and \( \kappa_i \ll \omega_p \), the mode amplitude of the mode \( i \) excited by the polarisation is represented as follows:

\[ D_i = A l [2j\omega_i (\sigma - \kappa_i) + j(\omega_i - \omega_p)] \{ e^{(\kappa_i + j\omega_i) t} - e^{(\sigma + j\omega_p) t} \} \] 

...(5.21)
Then using the equation (5.21) and (5.17) the energy $W_i$ injected into mode $i$ by the polarisation equation (5.12) which is equal to the energy lost at the rate $2\kappa_i$ is calculated as follows:

$$W_i = \frac{1}{2} \int_{-\infty}^{\infty} 2\kappa_i \, dt \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\xi \, \varepsilon_0 n^2 E_{\xi} E_{\xi}^*$$

$$= \frac{1}{8} \left[ \frac{P_p^2}{(\varepsilon_0 n_1^2)} \right] \left\{ \left[ \omega_i^2 (\sigma + \kappa_i) \right] / \left[ (\sigma - \kappa_i)^2 \right] + (\omega_p - \omega_i)^2 \right\} \left\{ \int F_i^2(x_p, y_p) \sin^2(q\pi z_p / l) \, dx \, dy \int_0^l (n^2/n_1^2) \right\}$$

$$F_i^2(x_p, y_p) \sin^2(q\pi z_p / l) \, dx \, dy \, dz \}$$

...(5.22)

### 5.5 INJECTION RATIO

The total energy radiated by the dipole is as follows [6].

$$W_p = \int_0^\infty dt \left( \frac{\omega_p^4}{12\pi} \right) \mu_0 \varepsilon_0 \mu_0 \left[ \varepsilon_0 \mu_0 \right]^{1/2} |P_p|^2 e^{-2\sigma t}$$

$$= \omega_p^4 / (24\pi\sigma) \mu_0 \varepsilon_0 \mu_0 \left[ \varepsilon_0 \mu_0 \right]^{1/2} |P_p|^2$$

...(5.23)

Using equation (5.21) and (5.20), the ratio of the energy injected into mode $i$ to the total energy emitted by the dipole at point $(x_p, y_p, z_p)$ and oscillating at frequency of $\omega_p$.

$$C_i^+ = (\text{the energy injected into mode } i \text{ by the spontaneous emission})$$

$$/ (\text{the total energy of the spontaneous emission}) = W_i / W_p$$

$$= \left[ 3\pi\sigma C^3 (\sigma + \kappa_i) \right] / \left[ \left( \omega_i^2 n_1^3 \right) (\sigma + \kappa_i)^2 + (\omega_p - \omega_i)^2 \right] \left[ F_i^2(x_p, y_p) \sin^2(q\pi z_p / l) \right]$$

$$/ \left[ \int \int n^2/n_1^2 F_i^2(x, y) \sin^2(q\pi z / l) \, dx \, dy \right]$$

...(5.24)

where $C$ is the light velocity in the vacuum.

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5.6 AVERAGE WITH RESPECT TO LOCATION, FREQUENCY AND POLARISATION

It was reported that the relaxation oscillation at the high-speed direct modulation, does not occur in buried hetero lasers and narrow stripe lasers [7]. The probability that a dipole appears in the infinitesimal volume $dx_p \, dy_p \, dz_p$ around the point $(x_p, y_p, z_p)$ is $(dx_p \, dy_p \, dz_p / V)$, provided that the volume of the active region is equal to $V$. The oscillating frequency $\omega_p$ of a dipole is to fall into the range $\omega - \omega + d\omega$ at probability of $f(\omega) d\omega$, where $f(\omega)$ is the shape of the spectrum of the spontaneous emission normalised such that

$$\int_0^\infty f(\omega) \, d\omega = 1$$  \hspace{1cm} (5.25)

Furthermore, as for the polarisation, the probability that the vector of the dipole direct to the x-axis is $1/3$. Then, the spontaneous emission factor is obtained as follows:

$$C_i = \frac{1}{3} \frac{1}{V} \int_a^b dx_p \int_b^l dy_p \int_0^\infty dz_p \int_0^\infty d\omega_p f(\omega_p) C_i$$

$$\approx \left[ \xi_i \pi^2 C^3 f(\omega) \right] \left[ (\omega^2 n_1^3 V) \right]$$  \hspace{1cm} (5.26)

where $\xi_i$ is the energy confinement factor of the dielectric wave-guide and is given by

$$\xi_i = \frac{\int_a^b \int_{-b}^b \int_0^{l_1} n_{1}^2 F_1^2(x, y) \sin^2(q \pi z / l) \, dx \, dy \, dz}{\int_0^\infty \int_0^\infty \int_{-l}^l n^2 F_1^2(x, y) \sin^2(q \pi z / l) \, dx \, dy \, dz}$$  \hspace{1cm} (5.27)
In the derivation of equation (5.26), \( f(\omega) \) is taken out from the integral sign, since it varies slowly compared to the remaining factor. The spontaneous emission factor \( C_i \) depends neither \( \sigma \) nor \( \kappa_i \). In order to obtain the formula containing the spectrum width of the spontaneous emission, \( f(\omega) \) is approximated by the Lorentzian function. Replacing by the wavelength \( \lambda = 2\pi c/\omega \), the spontaneous emission factor \( C_i \) is given by

\[
C_i = \xi_i \lambda_c^{-4} (\Delta \lambda/2) / [8\pi^2 n_1^3 V (\Delta \lambda/2)^2 + (\lambda_i - \lambda_c)^2]
\] ...

(5.28)

where \( \lambda_c \) and \( \Delta \lambda \) are the centre wavelength and the spectrum width of the spontaneous emission, respectively. \( \lambda_i \) is the resonant wavelength of mode \( i \), \( n_1 \) and \( V \) are the refractive index and the volume of the active region, respectively, and \( \xi_i \) is the energy confinement factor of mode \( i \) given by equation (5.27). Especially, for the mode whose resonant wavelength is near to the centre wavelength, Equation (5.28) is simplified as follows.

\[
C_i = [\xi_i \lambda_c^{-4}] / [4\pi^2 n_1^3 V \Delta \lambda]
\] ...

(5.29)

Since the field distribution is separated into the x-and y-dependent functions as given by equation (5.8), the energy confinement factor \( \xi_i \) is represented as follows:

\[
\xi_i = \xi_{sx} = \xi_{sy} = \xi^y
\] ...

(5.30)

where \( \xi_{s}^{\alpha} \) (\( \alpha = x \) or \( y \)) \( j=0 \) for the transverse fundamental mode and \( j=1,2,3... \) for higher order modes is determined by the normalised waveguide thickness \( T \) [8]

\[
T^x = \sqrt{2\pi n_1 \Delta_{13}} (2a) / \lambda
\] ...

(5.31)

\[
T^y = \sqrt{2\pi n_1 \Delta_{12}} (2b) / \lambda
\] ...

(5.32)
5.2 Energy confinement factor for the transverse fundamental mode (assuming $\alpha = x_1 \Delta_3$ and $\alpha = y_1 \Delta_2$).

Fig. 5.2 Energy confinement factor for the transverse fundamental mode (assuming $\alpha = x_1 \Delta_3$ and $\alpha = y_1 \Delta_2$).
Fig. 5.3  Energy confinement factor for the transverse fundamental and higher order modes (assuming $\alpha = x.\Delta_{13}$ and $\alpha = y.\Delta_{12}$).
\[ \Delta_{1u} = \frac{(n_i^2 - n_u^2)}{(2n_i^2)} \cdot (u=2,3) \quad \ldots (5.33) \]

The measured and calculated values of $C_i$ is shown in Table 5.1

<table>
<thead>
<tr>
<th>Laser sample</th>
<th>$\lambda_c$(A)</th>
<th>$\Delta\lambda$(A)</th>
<th>Measured $C_i$</th>
<th>Volume of active region $V$(µm$^3$)</th>
<th>$n_1$</th>
<th>$\xi_i$</th>
<th>Calculated $C_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8818</td>
<td>5</td>
<td>1x$10^{-5}$</td>
<td>0.4x10x250</td>
<td>3.6</td>
<td>0.7</td>
<td>0.457x10$^{-5}$</td>
</tr>
<tr>
<td>B</td>
<td>8400</td>
<td>6.5</td>
<td>0.7x$10^{-5}$</td>
<td>0.5x10x200</td>
<td>3.6</td>
<td>0.8</td>
<td>0.333x10$^{-5}$</td>
</tr>
<tr>
<td>C</td>
<td>9000</td>
<td>12</td>
<td>(0.7-3)x$10^{-5}$</td>
<td>0.5x10x250</td>
<td>3.6</td>
<td>0.8</td>
<td>1.89x10$^{-5}$</td>
</tr>
<tr>
<td>D</td>
<td>8818</td>
<td>5</td>
<td>0.5x$10^{-5}$</td>
<td>0.4x10x250</td>
<td>3.6</td>
<td>0.9</td>
<td>0.591x10$^{-5}$</td>
</tr>
<tr>
<td>E</td>
<td>8500</td>
<td>12</td>
<td>0.8x$10^{-5}$</td>
<td>0.5x10x250</td>
<td>3.6</td>
<td>0.7</td>
<td>1.323x10$^{-5}$</td>
</tr>
<tr>
<td>F</td>
<td>9000</td>
<td>6.6</td>
<td>0.47x$10^{-5}$</td>
<td>0.5x10x200</td>
<td>3.6</td>
<td>0.8</td>
<td>0.351x10$^{-5}$</td>
</tr>
<tr>
<td>G</td>
<td>8500</td>
<td>10</td>
<td>(0.7±0.3)x$10^{-5}$</td>
<td>0.4x10x200</td>
<td>3.5</td>
<td>0.8</td>
<td>0.308x10$^{-5}$</td>
</tr>
</tbody>
</table>

If the active wave-guide consists with gain guiding or leaky wave guiding, the parameter $\xi_i$ could be counted [9] from the figure 5.2 for energy confinement factor (transverse fundamental mode) and figure 5.3 for normalised wave-guide thickness with higher order mode are shown.
5.7 CONCLUSION

The spontaneous emission factor $C_i$ of a micro cavity DBR surface-emitting laser with a three-dimensional model is analysed in order to investigate the possibility of spontaneous emission control and thresholdless lasing operation.

The radiation energy of spontaneous emission is formulated by introducing the distribution of mode density in $k$ space. Next, the distribution of mode density shows that strongly modified from that in free space by the micro cavity. It has found that the one-dimensional cavity of a pair of DBR's enhances emission at the Bragg wavelength and suppresses emission at wavelengths a little shorter than Bragg wavelength towards normal to the surface plane. This will be more remarkable by adopting two materials having a larger index difference. The fundamental mode in the three-dimensional cavity has defined and also that the resonant mode in the one-dimensional cavity is not sufficient to regard it as a lasing mode due to the lack of consideration for the transverse modes.

In order to perform the calculation of $C_i$, the expression based on the mode analysis is used. The upper limit of $C_i$ is determined by the polarisation probability of a linear polarized fundamental mode and it is 0.5 with the bulk active region and the square cross section of the cavity. The numerical analysis of $C_i$ provides the following results. 1) The characteristics of $C_i$ almost coincide with the experimental results by Suematsu and Furuya when $C_i$ is much smaller than the upper limit. 2) A large $C_i$ over 0.1 is realized with relatively realizable requirements for the transverse size of the active region ~ 0.5 μm, reflectivity of the DBR's > 90%, absorption loss < 2000 cm$^{-1}$, spectral width < 30 nm and tuning of the resonant wavelength to the peak emission wavelength within a 0.5% error. 3) $C_i$ ~0.5 cannot be obtained without a small width of the emission spectrum ($\Delta \lambda_e \ll 30$ nm). 4) If the cavity is buried by some cladding, $C_i$ decreases, but the change is no larger
than 32-41%. 5) The quantum-wire active region will raise the upper limit of $C_i$, close to unity.

By this enhancement of the C factor and decrease of $\tau_\text{s}$ a high-speed surface-emitting laser operating with extremely low threshold current $\sim \mu\text{A}$ and a quantum efficiency nearly equal to 1 will be possible.

According to the formula derived, the spontaneous emission factor is inversely proportional to the volume of the active region of the laser and the spectrum width of spontaneous emission and is proportional to the fourth power of the wavelength. The spontaneous emission factor is almost independent of the injection current. Ultimately, thresholdless lasers may be obtained.

The possibility of the suppression of the relaxation oscillation by reducing the volume of the active region was suggested theoretically in the present work.
REFERENCES


3. Y. Suematsu and K. Furuya, Trans. IEICE Japan, 60, 467 (1977)


