Chapter 4

Section A: Microscopic Nucleon Nucleus
Optical Potential

Section B: Microscopic description of proton scattering at 295 MeV from Pb isotopes
This chapter is divided into two major parts. **Section A** describes the formal expressions used to calculate the microscopic optical potential for finite nuclei in the folding model approach. We also report our results for the calculated nucleon optical potential in BHF using two and two plus three-body force. We also present results of our analysis of the p-\(^{40}\)Ca elastic scattering data at 65 and 200MeV using BHF potentials obtained with two and three body forces. **Section B** describes the application of the calculated potential for analyzing the proton nucleus optical potential for analyzing proton nucleus scattering data from the isotopes of Pb at 295MeV. Our main concern in Section B is to extract neutron skin for Pb-isotopes.

**(4.1) Introduction:**

Nucleon-Nuclear matter optical potential can be calculated (as described in detail in Chapter 2) as a function of matter density and local momentum starting from a realistic inter-nucleon interaction within the framework of first order Brueckner theory. In this chapter, we describe the procedure for obtaining the radial dependence of effective interaction (g-matrices) and the local density approximation to obtain different components of the optical potential for the scattering of nucleons from finite nuclei. The calculation of optical potential for a finite nucleus essentially involves various forms of local density approximation [1-6]. We have used the approach suggested by Brieva and Rook [2-4]. It should be noted that we do not use the generalized reference spectrum method [2-4] but solve the integral equation using matrix inversion technique described in chapter 2. The basic idea behind this approach is that radial dependence of the effective interaction, the g-matrix, is obtained by imposing the condition that the matrix element of g reproduces the nucleon-nuclear matter optical potential. The g-matrix is then folded over the point nucleon density in the target to obtain the nucleon-nucleus optical potential. The microscopic optical potential thus obtained is used to calculate differential cross-section, analyzing power and reaction cross section for elastic scattering of nucleons.

In section 4.2 and 4.3 we describe briefly the procedure for obtaining radial dependence of two nucleon g-matrices and the Local Density Approximation (LDA) for calculating different components of theucleon-nucleus optical potential.
In sections 4.4 and 4.5 we describe the relevant equations for obtaining direct and exchange parts of Real and Spin orbit potential.

In section 4.6, we describe our results for the calculated proton–nucleus optical potential in the energy region 21.0 MeV-225.0 MeV for the scattering of protons from $^{40}$Ca in first order Brueckner theory, using Argonne v18[7], Reid93 and Nijm II[8] inter-nucleon interactions.

In section 4.7, we describe our results for the calculated nucleon optical potential using two and two plus three-body force for p-$^{40}$Ca at 65 and 200MeV. To study the effect of three body forces on the scattering observables, we make an analysis of p-$^{40}$Ca data at 65 and 200MeV using two body and two plus three body forces.

**4.2 Radial Dependence of g– Matrices:**

In order to obtain the radial dependence of the nucleon-nucleon effective interaction, g-matrix, we follow the approach proposed by Siemens [9] and used by Brieva and Rook[2-4]. Instead of imposing the requirement that the approximate g should reproduce the binding energy of nuclear matter when it is used in lowest order Born approximation we define g so as to reproduce the nuclear matter complex optical potential. This condition allows us to obtain radial dependence of g-matrices [2-4] as described below.

We consider a nucleon with energy E and momentum k moving in an infinite symmetric nuclear matter of matter density $\rho_{NM}$ and Fermi momentum $k_F$ related by

$$\rho_{NM} = \frac{2}{3\pi^2} k_F^3$$

(4.1)

The energy E and momentum k are related (on-shell choice):

$$E = \frac{k^2}{2m} + \text{Re}[U(k_F; k, E)],$$

(4.2)

where $m$ is the nucleon mass and $U(k_F; k, E)$ is the single particle complex optical potential. The incident nucleon interacts with a bound nucleon with momentum $p$ with $|p| \leq k_F$. We introduce the total and relative momentum for the nucleon pair,

$$K_o = k + p,$$

(4.3a)

$$k_o = (k - p) / 2.$$  

(4.3b)
The radial part of the correlated wave function of the two nucleons \( U^J_{L,S,J,\alpha} (r) \) is calculated as discussed in Chapter 2. L, S and J refer to the orbital angular momentum, total spin and total angular momentum respectively of the nucleon pair. Angular momentum \( L' \) allows for the tensor coupling in the inter-nucleon interaction and \( \alpha \) represents the dependence of the wave function on \( E, k, p \) and \( \rho_{NM} \).

A diagonal representation of \( g \) in coordinate space is easily obtained from the equation:

\[
< \Phi | g | \Phi > = < \Phi | v | \Psi > , \tag{4.4}
\]

where \( | \Phi \rangle \) is a plane wave state characterized by the relative momentum of the pair, \( v \) is the realistic inter-nucleon potential and \( | \Psi \rangle \) is the correlated two-nucleon wave-function.

We obtain using Eq.(4.4), in states of L, S, J quantum state of the two interacting nucleons [2-4].

\[
g^{J,S}_L (r; \rho_{NM} , E ) = \frac{\sum_{p<k_F} \sum_{J=|J-S|}^{J+S} \frac{1}{k_o^2} I_L (r) \psi^{J,S}_{LL'} (r) U^{J,S}_{LL',\alpha} (r)}{\sum_{p<k_F} \frac{1}{k_o^2} I_L^2 (r)} \tag{4.5}
\]

where \( I_L (r) = k_o r J_L (k_o r) \), with \( J_L (x) \) the spherical Bessel function of order L, and \( \psi^{J,S}_{LL'} (r) \) are the matrix elements of realistic inter-nucleon potential and \( U^{J,S}_{LL',\alpha} (r) \) is the radial part of the correlated two nucleon wave function. For singlet states, the sum over \( L' \) in Eq. (4.5) does not apply. In triplet states it is convenient to have a J-independent interaction, namely

\[
g^{S=1}_L (r; \rho_{NM} , E ) = \frac{\sum_{J=|L-1|}^{L+1} \frac{[2J + 1]}{3[2L + 1]} g^{J,S=1}_L (r; \rho_{NM} , E )}{3[2L + 1]} \tag{4.6}
\]

For practical purposes and computational simplicity an L-independent effective interaction can be defined in states of spin S and isospin T.
\[
g^{ST}(r; \rho_{NM}, E) = \frac{\sum_L [2L + 1] g^S_L(r; \rho_{NM}, E) \sum_{p<k_F} \frac{I}{k_F^2} I^2_L(r)}{\sum_L [2L + 1] \sum_{p<k_F} \frac{I}{k_F^2} I^2_L(r)}, \quad (4.7)
\]

where the sum over \( L \) is over even or odd values so as to have negative total parity. The \( g^{ST} \) effective interactions are complex and function of density and energy.

For incident protons we define the quantities, \( g_C^{D,PP} \) and \( g_C^{D,PN} \), the direct part of the central pp and pn effective interactions respectively [2-4]:

\[
g_C^{D,PP} = \frac{1}{4} (g^{01} + 3 g^{11}) \quad (4.8)
\]

and

\[
g_C^{D,PN} = \frac{1}{8} (3g^{10} + g^{01} + g^{00} + 3g^{11}), \quad (4.9)
\]

For the exchange part of the central pp and pn effective interactions we can define the quantities \( g_C^{EX,PP} \) and \( g_C^{EX,PN} \) as:

\[
g_C^{EX,PP} = \frac{1}{4} (g^{01} - 3 g^{11}), \quad (4.10)
\]

\[
g_C^{EX,PN} = \frac{1}{8} (3g^{10} + g^{01} - g^{00} - 3g^{11}) \quad (4.11)
\]

Similarly, \( g_S^{D,PP} \) and \( g_S^{D,PN} \) are the direct parts of the spin-orbit pp and pn effective interaction respectively and are defined by [6]:

\[
g_S^{D,PP} = g_S^{11} \quad (4.12)
\]

and

\[
g_S^{D,PN} = \frac{1}{4} (3 g_S^{11} + g_S^{10}) \quad (4.13)
\]

and \( g_S^{EX,PP} \) and \( g_S^{EX,PN} \), the exchange part of the spin-orbit pp and pn effective interactions respectively are defined by:

\[
g_S^{EX,PP} = - g_S^{11} \quad (4.14)
\]
and 
\[ g^{\text{EX, } P} = \frac{1}{4} \left( g^{10}_{\text{SO}} - 3 g^{11}_{\text{SO}} \right). \] (4.15)

Thus using Eq. (4.7) we can easily calculate different components of effective interaction. Eqs. (4.8-4.15) are used in the next section to calculate the proton-nucleus optical potential.

(4.3) Folding Procedure:

The nucleon-nucleus optical potential, \( M \) is written as the sum of a local direct term and a non-local exchange term [2-4], namely
\[
M \psi (r_1) = \sum_n \int \phi_n^* (r_2) g^D \left( |r_1 - r_2|, \rho (R), E \right) \phi_n (r_2) d^3 r_2 \psi (r_1)
\]
\[ + \sum_n \int \phi_n^* (r_2) g^\text{EX} \left( |r_1 - r_2|, \rho (R), E \right) \psi (r_2) d^3 r_2 \phi_n (r_1) \] (4.16)

where \( r_1 \) and \( r_2 \) refer to the radial coordinates of the incident and the bound nucleons respectively, \( \phi_n (r_2) \) is the bound-state single-particle wave function with \( n \) representing the appropriate quantum numbers and \( g^D \) and \( g^\text{EX} \) are the direct and exchange effective nucleon-nucleon interactions. Both \( g^D \) and \( g^\text{EX} \) have essentially the following structure:
\[
g( r_1, r_2 :E) = g^c ( r_1, r_2: E ) + g^\text{so} ( r_1, r_2: E ) L . S + \text{other terms} \] (4.17)

From Eq. (4.16), it is convenient to define a local equivalent optical potential \( U \) by
\[
U ( r_1, E ) \psi (r_1) = \int M ( r_1, r_1 ', E ) \psi (r_1 ') r_1 ' \] (4.18)

where \( \Psi (r_1) \) is the scattering wave function of the incident nucleon. The nucleon-nucleus optical potential can now be written in the standard form (neglecting the tensor part):
\[
U( r_1, E) = U_c( r_1, E) + U_{so}( r_1, E) \] (4.19)

where
\[
U_c( r_1, E) = - V( r_1, E) - iW( r_1, E) \] (4.20)

and
\[
U_{so}( r_1, E) = \left[ V_{so}( r_1, E) + iW_{so}( r_1, E) \right] l_1. s_1 \] (4.21)

refer to the central and the spin-orbit component of the nucleon-nucleus optical potential. In Eq. (4.21), \( l_1 \) and \( S_1= \frac{\hbar}{2} \sigma \) are the incident nucleon orbital angular momentums and spin.
respectively. The evaluation of the effective NN interaction, \( g^D \) and \( g^{EX} \), in finite nuclei is quite difficult. However, a hypothesis [2-4] is made that these effective interactions in finite nuclei can be approximated by the local, density and energy dependant effective interactions calculated in infinite nuclear matter, i.e.

\[
g_{C,SO}^{D,EX} (r_1, r_2; E) \approx g_{C,SO}^{D,EX} (|r_1 - r_2|; \rho (R), E) \tag{4.22}
\]

where \( \rho(R) \) is the nuclear matter density at

\[
R = \frac{(r_1 + r_2)}{2} \tag{4.23}
\]

Using Eqs.(4.16), (4.18) and (4.22) we can obtain the local equivalent nucleon-nucleus optical potential \( U(r_1, E) \).

**4.4 Central Optical Potential:**

Here we give the basic equations used for the calculating the direct and exchange components of the central optical potential [2,4].

The expression for the direct central optical potential is given by:

\[
U^D_C (r_1, E) = \sum_n \int \phi^* (r_2) g_D^C (|r_1 - r_2|; \rho (R), E) \phi_n (r_2) d r_2 , \tag{4.24}
\]

where \( \phi_n (r_2) \) is the wave function of the bound nucleon in the target.

Eq. (4.24) can be written, in term of the single-particle density distribution in the target nucleus as,

\[
U^D_C (r_1, E) = \int \rho (r_2) g_D^C (|r_1 - r_2|; \rho (R), E) d r_2 , \tag{4.25}
\]

where

\[
\rho (z) = \sum_n \phi_n^* (z) \phi_n (z) , \tag{4.26}
\]

is the single-particle density distribution. For incident protons we incorporate in Eq. (4.25) the differences between neutron and proton matter densities and pp and pn effective interactions.

The expression for the direct component of the central optical potential for incident proton can be written as;
\[ U_{C}^{D,P}(r_1, E) = \int \rho_p(r_2) g_{C}^{D,PP}( \left| r_1 - r_2 \right| ; \rho(R), E) dr_2 \]
\[ + \int \rho_n(r_2) g_{C}^{D,PN}( \left| r_1 - r_2 \right| ; \rho(R), E) dr_2. \] 

(4.27)

Eq. (4.27) coupled with Eqs. (4.8) and (4.9) is used to calculate the direct part of central component of the optical potential.

The exchange part of the central optical potential can be written as:

\[ U_{C}^{EX}(r_1, E)\psi(r_1) = \sum_n \int \phi_n^*(r_2) g_{C}^{EX}( \left| r_1 - r_2 \right| ; \rho(R), E) \phi_n(r_1) \psi(r_2) dr_2 \]

(4.28)

We use the equivalent local approximation [2-4] to factorize out \( \psi(r_1) \) from Eq. (4.28) using the expansion for \( \psi(r_2) \). In order to include the difference between proton and neutron matter densities and the difference between pp and pn central effective interactions, we write the above Eq. (4.29) in the following form [2,4]

\[ U_{C}^{EX,P}(r_1, E) = \int \rho_p(r_1, r_2) g_{C}^{EX,PP}( \left| r_1 - r_2 \right| ; \rho(R), E) j_0(k \left| r_1 - r_2 \right|) dr_2 \]
\[ + \int \rho_n(r_1, r_2) g_{C}^{EX,PN}( \left| r_1 - r_2 \right| ; \rho(R), E) j_0(k \left| r_1 - r_2 \right|) dr_2, \] 

(4.29)

where

\[ \rho(x, y) = \sum_n \phi_n^*(x) \phi_n(y) \]

(4.30)

is the single particle mixed density. The proton and neutron single-particle mixed densities in Eq. (4.29), in the first approximation, are given by the first term of an expansion proposed by Negele and Vautherin [10] i.e

\[ \rho_{i=(P,N)}(r_1, r_2) \approx \rho_{i=(P,N)} \left( \frac{r_1 + r_2}{2} \right) \frac{3}{(sk_F)^3} \left[ \sin(sk_F) - sk_F \cos(sk_F) \right] \]

(4.31)

with \( s = |r_1 - r_2| \), and \( k_F \) is the magnitude of Fermi momentum.
(4.5) Spin-Orbit Optical Potential:

We present the commonly used prescription [2-4] for obtaining the direct and exchange part of the spin-orbit potential.

The direct part of the spin-orbit optical potential [2-4] is given by:

\[
U_{SO}^D (r_1, E) = \sum_n \int \phi_n^* (r_2) g_{SO}^D \left( r_1 - r_2 \right| \rho, E ) l.s \phi_n (r_2) dr_2
\]  

(4.32)

The product l.s in Eq. (4.32) can be written as:

\[
l.s = r \times p.s = \frac{1}{2} (r_1 - r_2) \times (p_1 - p_2). (s_1 + s_2)
\]  

(4.33)

where \( p_1 (p_2) \) and \( s_1 (s_2) \) refer to the momentum and spin vectors of the incident (bound) nucleons. Changing the integration variables in Eq. (4.32) to \( x = r_2 - r_1 \) we obtain:

\[
U_{SO}^D (r_1, E) = -\frac{1}{2} \int \rho \left( |r_1 + x| \right) g_{SO}^D (x; \rho, E) x dx \times (p_1 - p_2). (s_1 + s_2)
\]  

(4.34)

where

\[
\rho ( |r_1 + x| ) = \sum_n \phi_n^* (r_1 + x) \phi_n (r_1 + x)
\]  

(4.35)

is the density distribution in the target nucleus. We consider only spin zero nuclei, and hence the sum over \( s_2 \) is zero. Further, since no direction of nucleons in the target is specified the integration over \( p_2 \) vanishes. This gives us:

\[
U_{SO}^D (r_1, E) = -\frac{1}{2} \int \rho \left( |r_1 + x| \right) g_{SO}^D (x; \rho, E) x dx \times p_1.s_1
\]  

(4.36)

The integration in Eq.(4.36) must be in the direction of \( r_1 \) to contribute to the spin-orbit potential. We finally obtain.

\[
U_{SO}^D (r_1, E) = -\frac{1}{2} A(r_1) \frac{l_1.s_1}{r_1}
\]  

(4.37)

where \( l_1 = r_1 \times p_1 \) is the orbital angular momentum of the incident nucleon, and

\[
A(r_1) = \int \rho (|r_1 + x|) g_{SO}^D (x; \rho, E) x dx
\]  

(4.38)

Equation (4.37) is our basic equation to calculate the spin-orbit part of the nucleon-nucleus optical potential. Once \( g_{SO}^D \) is calculated using first order Brueckner theory, Eq.(4.37) along with Eq.(4.38) can be used to calculate the direct part of spin orbit potential without any
further approximation. This equation has also been discussed by Brieva and Rook [2-4]. We have used Brieva and Rook [2-4] approximation to calculate the exchange part of the spin-orbit potential. When the difference between the proton and neutron densities and the difference between exchange part of pp and pn spin-orbit effective interactions are included, the expression for the exchange spin-orbit optical potential for the incident proton can be written as:

\[
U_{SO}^{EX,P}(r_i, E) = -\frac{2}{3} \pi \left[ \frac{3}{k} \int \frac{\partial}{\partial r_i} \rho_p(r_i) \right] \]

\[
+ \frac{3}{k} \int g_{SO}^{EX,PN}(x; \rho, E) j_1(kx) x^3 dx \frac{1}{r_i} \frac{\partial}{\partial r_i} \rho_n(r_i) \]

\[
1_s \]  

(4.39)

(4.6) Results for Optical Potential from Two Body Forces:

In this section we describe our results for the calculated proton–nucleus optical potential in the energy region 21.0 MeV-225.0 MeV for the scattering of protons from \( ^{40}\text{Ca} \), using three inter-nucleon local potentials namely Argonne v18[7], Reid93 and Nijm II[8]. To study the energy dependence we present, in the following, our results for the p-\(^{40}\text{Ca} \) optical potential in the energy region 21-225MeV.

Central Optical Potential:

The radial shape of the calculated real part of the central optical potential is shown in Figs. 1(a), 2(a) and 3(a) for p-\(^{40}\text{Ca} \) in the energy range 21-225MeV, calculated using Argonne v18, Reid93 and Nijm II inter-nucleon interactions respectively. The nucleon density for \(^{40}\text{Ca} \) were obtained from relativistic mean field theory RMF [11,12]. We observe that the potential and its shape in the nuclear interior changes rapidly with energy and the strength of these attractive potentials decrease with increasing energy. The shape of the potential cannot be described by a simple Saxon-Woods form even at low energies. The real potential is assuming the well known wine bottle bottom shape in the interior region of the nucleus as the incident energy increases. The potential remains attractive at all incident energies ranges from 21 MeV to 225.0 MeV. The value of the real central optical potential is about
-2.24MeV from Av-18 and about – 8.9MeV from Reid 93 and NijmII at 225MeV. Thus the calculated real central potentials using Reid93 and NijmII interactions are more attractive than Av18 in the interior of the nucleus. At low energies, the central optical potential decreases smoothly with distance and the difference between the potentials calculated using different NN interactions is not significant. The oscillatory behavior of the potentials at larger energies is attributed to the oscillatory features of the RMF density [11,12] used in our calculation.

We also show our results for imaginary part of our calculated optical potential for p-\( ^{40}\text{Ca} \) in the energy range 21-225MeV in Figs. 1(b), 2(b) and 3(b) using Argonne v18, Reid93 and Nijm II respectively. Our results show that the imaginary central optical potential also exhibits strong energy and radial dependence. The imaginary central potential is always attractive and its strength in the interior of nucleus increases with increasing incident energy. The imaginary central potential shows mild surface enhancement at low energies. As the incident energy increases the position of the peak slowly shifts towards nuclear interior and decreases in magnitude. For \( E > 85.0 \text{ MeV} \) the imaginary central potential shows a slightly smooth radial dependence and the surface enhancement disappears. Comparison of Fig. 2(a), 2(b) and 2(c) indicates that most of the features of imaginary central potentials obtained from all the three inter-nucleon interactions are similar.

**Spin-Orbit Optical Potential:**

The real part of our calculated spin-orbit potential for p-\( ^{40}\text{Ca} \) at incident energies (\( E_p = 21\text{-}225\text{MeV} \)) is shown in Fig. 1(c), 2(c) and 3(c) respectively for Argonne v18, Reid93 and Nijm II respectively. We observe that the radial shape of the calculated real spin-orbit potential is nearly Thomas form at all energies for the three interactions considered here. The strength of the real spin-orbit potential decreases very slowly with increasing incident energy. Our results show that this energy dependence is mainly due to the energy dependence of the exchange part of the spin orbit potential. The direct part of the real spin-orbit potential is only mildly energy dependent. The radial behaviour of the imaginary spin orbit potential for \( ^{40}\text{Ca} \) in the above mentioned energy range is shown in Figs. 1(d),2(d) and
3(d) using Argonne v18, Reid93 and Nijm II inter-nucleon interactions respectively. The strength of the imaginary spin-orbit potential decreases with increasing incident energy. This behavior is same for all the three Hamiltonians used in our calculations.

(4.7) Optical potential from two plus three body forces:

In this section we describe our results for the calculated microscopic optical potential at 65MeV and 200 MeV for the scattering of protons from $^{40}$Ca using Argonne v18 NN interaction plus a three-body force (TBF) namely; the Urbana VII model [13] (Av18 plus UVII) (described in chapter 3.) The calculated real and imaginary central and spin-orbit part of optical potential at 65 MeV are shown in Fig. 4(a) and at 200 MeV in Fig. 4(b). The calculated potentials using Av18 plus UVII are shown by a red line. For comparison we also show our results obtained with the use of only two-body force by solid back line. We observe that the strength of real and imaginary central optical potential decreases as a result of three body forces at short and middle distances for both energies (65 and 200 MeV). We also observe that the real and imaginary part of spin-orbit optical potential at both energies exhibits only marginal changes due to inclusion of three-body forces in the Hamiltonians.

We have also done an analysis of the differential cross section, polarization and spin rotation data of p-$^{40}$Ca at 65MeV (Fig. 5(a)) and at 200MeV (Fig. 5(b)) using the potentials calculated from Av18 plus three body force. For comparison we have also done this analysis using only two body force Av18. We observe that there is no major effect on proton nucleus scattering observables due to inclusion of three body forces in the NN interaction at both energies. Both potentials give satisfactory agreement with experiments as shown in Figs. 5(a) and 5(b). This implies that the data analyzed is not very sensitive to the interior of the target nuclei.
Fig.1. Energy dependence of calculated Central (a) Real (b) Imaginary and Spin orbit (c) Real and (d) Imaginary optical potential for $^p{}^{40}\text{Ca}$ in the energy range 21-225MeV using Argonne v18 inter nucleon interaction.
Fig. 2. Same as Fig. 1 but using Reid 93 inter nucleon interaction.
Fig. 3. Same as Fig. 1 but using NijmII inter nucleon interaction.
Fig. 4(a). Calculated central real and imaginary and spin-orbit parts of optical potential for p-\(^{208}\)Pb at 65 MeV using three body force: Av18 plus UVII (red line). Black line is result using Av18 (only two-body force).
Fig. 4(b). Same as that of Fig. 4(a) but at 200MeV.
Fig 5(a). Calculated (a) Differential cross section (b) Polarization and (c) Spin rotation for scattering of protons from $^{40}$Ca isotopes using Av 18 with and without three body force.
Fig. 5(b). Same as that of Fig. 5(a) but at 200MeV.
Section B: Microscopic description of proton scattering at 295 MeV from Pb isotopes

(4.8) Introduction:
Nucleon density in nuclei is a property of fundamental importance in nuclear physics. Experimentally, the charge (and hence proton) density can be measured to a high degree of accuracy through electron scattering [14,15]. A number of such measurements have been carried out in the past, and the corresponding model independent charge densities have been reported. On the other hand, for the neutron density distributions, one needs to rely on indirect methods, and hence the extracted neutron densities are model dependent. Hadron scattering in the intermediate energy region has proved to be an important tool in studying the neutron density distributions in nuclei [16-18]. However, the extracted neutron densities are not as accurate as the corresponding proton density distributions.

Heavy nuclei are expected to have neutron skin which is strongly correlated with the nuclear symmetry energy of the equation of state. Further, a precise knowledge of neutron skin in $^{208}$Pb would be helpful in understanding the cooling mechanism and properties of neutron stars [19]. In view of the above Terashima et al. [20] and Zenihiro et al. [21] have measured and analyzed the 295-MeV-proton scattering data to extract neutron densities in Sn and Pb isotopes, respectively. The method of analysis used in Refs. [20,21] is based on the relativistic impulse approximation (RIA) using the Love-Franey interaction as proposed by Murdock and Horowitz [22]. A medium modification of the (NN) interaction has been included in a phenomenological manner by introducing density dependence in the coupling constants and masses of $\sigma$ and $\omega$ mesons and calibrated to fit the proton scattering data from $^{58}$Ni. The proton and neutron density distributions were assumed to have the same radial shape in $^{58}$Ni. Using this medium modified interaction Zenihiro et al. [21] have parametrized the neutron densities as a sum of twelve Gaussians with eleven independent parameters for each isotope of Pb ($^{204,206,208}$Pb). The parameters of the Gaussians for each isotope have been determined by minimizing $\chi^2$ per degree of freedom to reproduce the experimental data.
Using these as free parameters for each isotope, they have been able to obtain neutron densities and a reasonably good agreement with both the differential cross-section and analyzing-power data for the scattering of 295-MeV protons from $^{204,206,208}$Pb. We have analysed the same data using BHF with RMF densities. Microscopic analysis of the recently reported 295-MeV-proton scattering data from Pb isotopes and $^{58}$Ni is presented within the framework of the Brueckner-Hartree-Fock theory. The effective interaction (g matrix) has been calculated using three Hamiltonians with Urbana v-14, Argonne v-18, and Ried93 internucleon potentials. The microscopic optical potential is calculated by folding the effective interactions over nucleon density distributions obtained in the relativistic mean field framework. The Argonne v-18 and Ried93 interactions have been used for the first time to calculate the nucleon-nucleus optical potential. The calculations reproduce the experiment well thus revalidating the use of microscopic optical potential in such analyses.

(4.9) Method of calculation:

The method of calculating the microscopic optical potential follows the procedure described in detail in Section A of this chapter. We calculate the effective interaction in the BHF approach using the Urbana v-14 [23], Argonne v-18 [7], and Reid93 [8] internucleon potentials. Besides NN interaction the other important inputs required to calculate the nucleon-nucleus optical potential are the proton and neutron density distributions in the target nuclei. The densities employed here have been obtained using the relativistic mean field (RMF) theory [11,12] along with the NL3 Lagrangian parameter set [24]. The calculated central [real $V(E, r)$ and imaginary $W(E, r)$] and spin-orbit parts [real $V_{so}(E, r)$ and imaginary $W_{so}(E, r)$] of the optical potential are multiplied by normalization constants ($\lambda$’s), which are adjusted to minimize $\chi^2$ per degree of freedom to reproduce the scattering data. Thus the optical potential used to calculate the desired observables is

$$U(E, r) = \lambda_r V(E, r) + i\lambda_s W(E, r) + \lambda_{so}^s V_{so}(E, r) + i\lambda_{so}^l W_{so}(E, r)$$

Equation (1) shows that there are four free parameters ($\lambda$’s) to obtain best fit to the data. However, we have kept $\lambda_{so}^l =1$ in all the cases due to its insensitivity to data at 295 MeV.
and hence there are only three parameters to minimize $\chi^2/DF$ (DF=degrees of freedom).

**(4.10) Results and Discussion:**

The results for the differential cross sections, analyzing power, and spin-rotation parameter (only for $^{58}$Ni) for the scattering of protons from $^{204,206,208}$Pb and $^{58}$Ni are presented in Figs. 1–3, respectively. The figures show satisfactory agreement with the scattering data for all nuclei considered here with only three parameters for each Hamiltonian. The values of the scaling parameters ($\lambda$’s) are given in Table I. It is observed that there are only minor differences between the scaling parameters ($\lambda$’s) for the three Hamiltonians considered here. Further, the scaling parameters are less than unity for all three inter-nucleon potentials used. This in turn implies that the calculated potential is larger than that required by the data. This is consistent with the findings of the recent calculation [25], indicating that the inclusion of three-body forces in BHF leads to a reduction in the strength of the central potential at high densities. Hence the inclusion of three-body forces is expected to bring the normalization parameters close to unity.

The RMF densities calculated with the NL3 parameter set used here yield slightly larger neutron skin ($\Delta r_{np}$) for Pb isotopes as compared to those obtained from other sources employing semiempirical and theoretical methods (e.g., proton elastic scattering [26], antiprotonic atoms [27], DD-ME2 [28], FSUGold [29], SkM* [30], SkP [31], Sly4 [32], and Ref. [21]) as shown in Fig.4(a). The same figure reveals that there are still substantial uncertainties in the neutron skin. Further, the error bars corresponding to the semiempirical values of the neutron skin [21] show that our results are close to the upper limit of $\Delta r_{np}$ [21]. It is important to experimentally measure the large angle cross-section, analyzing-power and spin-rotation data as it may help in reducing the uncertainties in the neutron skin. The parity violating elastic electron scattering experiment recently performed at the Jefferson Laboratory has not been able to give a precise value of the neutron skin in Pb [33,34].

The calculated (RMF) proton and neutron density distributions for $^{208}$Pb along with the corresponding SOG distributions [21] are presented in Fig. 4(b). As there are considerable differences in the neutron density distributions, we have calculated, using Reid93 inter nucleon potential, the differential cross sections, and analyzing power for $^{208}$Pb using the
same (RMF) proton density and both the RMF and SOG neutron density distributions. The results are presented in Fig. 5. Clearly, both neutron density distributions yield qualitatively similar results. However, there are minor differences at larger angles (center of mass angles $>20^\circ$), and the results with the RMF densities are in better agreement with the experiment. In order to test the predictions of the present microscopic optical potential further, the reaction cross section for $p-^{208}\text{Pb}$ scattering has been calculated, using $\nu$-18 Hamiltonian in the energy region $20 < E < 500$ MeV and compared in Fig. 6 with the corresponding experimental data [35–37]. The results for other Hamiltonians used in the present work are similar. We note that although the reaction cross-section data has not been included in $\chi^2$ minimization, the agreement with data over the entire energy region is quite satisfactory. In summary we have been able to obtain a satisfactory agreement with the proton scattering from Pb isotopes at 295 MeV in the BHF approach using RMF densities with three (Reid93, Argonne $\nu$-18, and Urbana $\nu$-14) inter nucleon potentials.
TABLE 1. Normalization constants that give best fit to the 295 MeV Proton scattering data from $^{204,206,208}$Pb and $^{58}$Ni for three Hamiltonians in BHF.

<table>
<thead>
<tr>
<th>Target</th>
<th>Reid93</th>
<th>Argonne v18</th>
<th>Urbana v14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{204,206,208}$Pb</td>
<td>$\lambda_v$</td>
<td>$\lambda_w$</td>
<td>$\lambda_{SO}$</td>
</tr>
<tr>
<td></td>
<td>0.85</td>
<td>0.76</td>
<td>0.72</td>
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<td>$^{58}$Ni</td>
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<td>0.88</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>0.70</td>
<td>0.83</td>
<td>0.70</td>
</tr>
<tr>
<td>$^{204,206,208}$Pb</td>
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<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td>$^{58}$Ni</td>
<td>0.70</td>
<td>0.97</td>
<td>0.70</td>
</tr>
</tbody>
</table>
References:


Fig. 1. Differential cross sections for scattering of protons from $^{58}$Ni and Pb isotopes using Reid93, Argonne v-18, and Urbana v-14 internucleon potentials in the BHF framework.
Fig. 2. Same as for Fig. 1 but for the proton analyzing-power data at 295 MeV.
Fig. 3. Same as for Fig. 1 but for proton $p-^{58}\text{Ni}$ spin-rotation parameter at 295MeV.
Fig. 4(a). Neutron skin thicknesses for $^{204, 206, 208}$Pb.

Fig. 4(b). Proton (p) and neutron (n) densities in $^{208}$Pb from RMF calculations and SOG[25].
Fig. 5. Differential cross section and analyzing power for $p$-$^{208}$Pb obtained using our RMF densities and the sum of Gaussian (SOG) neutron density [25] for the Reid93 potential in BHF.
Fig. 6. Calculated reaction cross section for $p^{-208}$Pb using the Argonne v-18 interaction in BHF. Solid line shows our result whereas solid circles are experimental data taken from Refs. [33-35].