Systematic procedure of physical oceanographic research involves collection of data on the independent parameters of the physical properties of sea water and the computation of the dependent parameters, leading ultimately to information on the dynamics of the oceans. The data collected at sea include, among others, values of temperature and salinity at surface and subsurface levels. Physical oceanographic measurements made at sea are subjected to careful examinations, corrections and conversions which require considerable amount of practical experience to judge the reliability of the records from instruments which operate blindly below the sea surface. Corrections are made for instrumental errors and for errors inherent in the methods of obtaining the data. The oceanographer is thus equipped with the basic data of temperature and salinity for different depths at various stations in the sea. The processing procedures do not ultimately provide perfectly accurate information on these parameters including temperature, salinity, depth, station position, etc. These procedures provide the data within certain error limits.

The basic data are then converted to standard depths to facilitate comparison with other oceanographic
data. This is followed by a series of calculations required to derive the dependent quantities like specific volume, density, and currents. These calculations have been highly systematised through practice by the physical oceanographers. This branch of physical oceanography is known as dynamical oceanography. Dynamical oceanography discusses the water properties and water movements and their temporal and spatial variations in the world oceans.

Dynamical oceanography has almost always neglected to consider as to what happens to the random errors, inherent in the basic data in the process of the series of calculations involved in the computational practices. It is conveniently assumed that the final results are not much affected by these errors. The fact that even small errors can, at times, lead to highly erroneous results through the propagation of errors in the computational procedures is often overlooked by oceanographers. Very few authors have made any serious mention on the error component of the derived results in dynamical oceanography. Jakhelln (1936), Thompson (1939) and Dobrovolskii (1949) have mentioned this possibility, while Roman (1964) has given serious consideration to this problem. The present studies aim to examine the limits of errors contained in
the basic data of physical oceanography and the magnitude of the error component in the results derived in dynamical oceanography. The study also suggests a graphical method of smoothing of the derived parameters within the limits of errors to increase the reliability of the derived results.

1.1. Physical Oceanographic Practices

Customarily and scientifically, the practical procedures in Physical Oceanography include collection of the various data during oceanographic cruises. Of these, those that are relevant in the context of errors and their propagation will be discussed in this section.

1.1.1. Station position

The place where an oceanographic vessel is stopped to carry out observations and collection of data and samples is known as an oceanographic station. The geographic location of the station is a primary requirement in oceanography. When land is visible and if recognisable features are accurately located on land, the position of the vessel is determined by means of horizontal angles and bearings on shore features. Out of sight of land,
the ship's position is determined by astronomic sights or by radio direction-finder bearings. Between positions established in these ways, the location at any time is obtained by dead-reckoning, from the course steered and distance run. More recently, satellite navigation systems are available which provide more accurate information on the location of oceanographic stations. The values of latitudes are of particular interest as it appears in the dynamic computation as a term in the Coriolis force component. The station positions also determine the values of distance between stations which again is a term in the dynamic computation for estimating relative currents.

1.1.2. Depth

Depth of the bottom at the station is obtained in shallow regions by the use of the classical method of lead sounding. More accurate and easy method is provided by echosounding in which the time interval between the generation of a sound impulse and the reception of its echo is used as a direct measure of the depth, using a constant sounding velocity. Depth of the oceanographic station does not directly appear in the dynamic computations and therefore does not require detailed considerations in the present study.
1.1.3. Temperature

Measurement of surface temperature is carried out using accurate mercury-in-glass thermometers. Thermometers used for measuring temperatures at subsurface levels are of the reversing type and are generally mounted upon water sampling bottles so that temperature and the water for salinity and other physical and chemical tests are obtained at the same level. Serial data on temperature and salinity are thus obtained from a hydrographic cast of water samplers arranged in series on a wire rope, to each of which are attached the protected and unprotected reversing thermometers.

The protected reversing thermometer is essentially a double ended thermometer. It is sent down to the required depth in the set position and consists of a large reservoir of mercury connected by means of a fine capillary to a small bulb at the upper end. Just above the large reservoir, the capillary is constricted and branched with a small arm, and above this, the thermometer tube is bent into a loop, from which it continues straight and terminates in the smaller bulb. In the set position, mercury fills the reservoir, the capillary and part of the bulb. The amount of mercury above the constriction
depends upon the temperature of the surrounding water. When the thermometer is reversed at the required depth along with the water bottle by sending a messenger weight down the wire rope, the mercury column breaks at the point of constriction and runs down, filling the bulb and part of the graduated capillary, thus indicating the temperature at reversal. The loop in the capillary which is generally of enlarged diameter, is designed to trap any mercury that is forced past the constriction if the temperature is raised after the thermometer has been reversed. In order to correct the reading for the changes resulting from difference between the temperature at reversal and surrounding temperature at the time of reading, a small standard thermometer, known as the auxiliary thermometer, is mounted alongside the reversing thermometer. The reversing thermometer and the auxiliary thermometer are enclosed in a heavy glass tube that is partially evacuated except for the portion surrounding the reservoir of the thermometer, and this part is filled with mercury to serve as a thermal conductor between the surroundings and the reservoir. The thermometer tube eliminates the effect of hydrostatic pressure. Seawater temperature in situ is obtained from the reading of a protected reversing
thermometer by applying corrections for instrumental error and for thermal expansion subsequent to reversal.

Reversing thermometers were first introduced by Negretti and Zambra in 1874 and have since been improved so that well made instruments are now accurate to within ±0.01°C. Recently electronic instruments are also being used to obtain records of subsurface temperature.

1.1.4. **Depth of Sampling**

By depth of sampling we mean the subsurface depth at which the reversing bottle along with the reversing thermometer is made to reverse collecting subsurface water sample and recording the temperature. The wire rope carrying the equipment is payed out through a meter wheel which measures the length of wire rope that has run out. The depth of reversal, as obtained from the meter wheel readings, may be erroneous due to non-vertical running out of the wire rope in the presence of ship drift or ocean currents. A correction for this can be obtained in the surface layers by measuring the wire angle. Depth of reversal is more accurately found by comparing the corrected reading of protected thermometer with the corrected reading of unprotected thermometer which is paired with a protected thermometer.
Unprotected reversing thermometers are identical with the protected reversing thermometers but have open protective tubes. Because of the difference in the compressibility of glass and mercury, thermometers subjected to pressure give a fictitious temperature reading that is dependent upon the temperature and pressure. The unprotected reversing thermometers are so designed that the apparent temperature increase due to hydrostatic pressure is about 0.01°C/m. The readings obtained from the unprotected thermometers also have to be corrected for thermal expansion and instrumental errors.

1.1.5. Salinity

Water samples for estimation of salinity are obtained in oceanographic vessels using subsurface cast of Nansen bottles in series. The Nansen bottle is a reversing bottle which can be reversed at the desired depth by sending a messenger weight down the wire rope. On reversal the bottle closes entrapping the water at that depth.

Salinity is directly proportional to chlorinity, which is determined by the titration of the water sample with silver nitrate in the presence of a suitable indicator.
Recently electrical conductivity is used as a measure of salinity and STD or CTD recorders are available for speedy collection of hydrographic data.

1.2. Oceanographic computations

There are a series of computations which the oceanographers do to derive the values of the various dependent quantities from the hydrographic data. These operations begin with the interpolation of the values of temperature and salinity at standard depths and conclude with the computation of currents from the distribution of density as obtained from the temperature and salinity data.

1.2.1. Conversion to standard depths

The first step in the processing of serial data on temperature and salinity is to prepare plots for the vertical distribution of the variables. Such plots of temperature and salinity as a function of depth are useful to detect incorrect values resulting from faulty operation of thermometers and water bottles. Another use of these plots is to scale off depths of decided values of the variables which are necessary for drawing diagrams of horizontal distributions. The main purpose of plotting
variables against depth is to obtain interpolated values of temperature and salinity at 'Standard depths'. The International Association of Physical Oceanography has defined standard depths as: Surface, 10, 20, 30, 50, 75, 100, 150, 200, (250), 300, 400, 500, 600, (700), 800, 1000, 1200, 1500, 2000, 2500, 3000, and 4000 meters and intervals of 1000 meters thereafter to the greatest depth of sampling. The National Oceanographic Data Centre has accepted the following standard depths viz., Surface, 10, 20, 30, 50, 75, 100, 125, 150, 200, 250, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200, 1300, 1400, 1500, 1750, 2000, 2500, 3000, 4000 meters and intervals of 1000 m to the greatest depth of sampling, which are currently being used.

Temperature and salinity data are also plotted on a T-S diagram. Introduced by Helland-Hansen (1916), the temperature-salinity diagrams are plotted with salinity on the X-axis and temperature on the Y-axis. When salinities are plotted against temperatures, the points generally lie on a well-defined curve. The T-S curve gives the temperature-salinity relationship of the sub-surface water in the area under study. Surface data have to be omitted because annual variations and local
modifications lead to discrepancies. The f-S diagram helps to detect errors and to bring out watermass characteristics of the data. After the T-S curve has been drawn for the observed data, corresponding interpolated values of temperatures and salinity read from the vertical curves are also plotted. If these data do not fall on the T-S curve, certain adjustments must be made in the construction of the vertical distribution curves.

1.2.2. Computation of specific volume and specific volume anomaly

After obtaining the values of depth, temperature and salinity as detailed above, certain calculations are necessary to derive the values of the various dependent variables commonly used to describe the field of mass in the sea. These variables include: specific volume, anomaly of specific volume from a standard value, density in situ and \( \sigma_t \), which represents density at surface pressure.

Specific volume is volume per unit mass. Specific volume in situ in the sea is expressed by the symbol \( \alpha_{s,t,p} \), where the subscripts indicate salinity, temperature and pressure of the sample. Specific volume is computed by expressing it as a known specific volume under given conditions plus a series of correction terms for the
dependent variables of temperature, salinity and pressure. These terms may be grouped, computed, and added as follows to give specific volume in situ.

\[ \alpha_{s,t,p} = (\alpha_{35,0,0} + \delta_p) + (\delta_s + \delta_{c,t} + \delta_{s,t}) + \delta_{s,p} + \delta_{t,p} + \delta_{s,t,p} \]

In the first term, \( \alpha_{35,0,0} \) is a constant (0.97264) and \( \delta_p \) represents the effect of pressure at standard salinity (35%) and at \( 0^\circ C \).

The next term depends only on salinity and temperature and may be summed to give \( \Delta_{s,t} \) or \( \Delta_T \), known as the thermosteric anomaly. This term is found from values of temperature and salinity by means of tables or graphs (Sverdrup, 1933). The salinity-pressure term, \( \delta_{s,p} \) and the temperature-pressure term \( \delta_{t,p} \) are also found from tables or graphs. The last term, \( \delta_{s,t,p} \), is so small that it is always neglected.

The sum of the terms \( \Delta_{s,t}, \delta_{s,p} \) and \( \delta_{t,p} \) constitutes the anomaly of specific volume from the standard \( \alpha_{35,0,p} \) and is designated by the symbol \( \delta \). For computing currents, the variations in specific volume along isobaric surfaces are required. Since pressure is constant along any given isobaric surface, the term \( \delta_p \) is a constant. It is sufficient therefore to calculate the specific
volume anomaly, $\delta$, since the standard term does not contribute to variation in specific volume along an isobaric surface.

Graphs or tables are available for the calculation of specific volume anomaly (Sund, 1926; LaFond, 1940; Callaway, 1950). LaFond (1951) presents the oceanographic tables for finding out the values of the various terms, the sum of which gives the specific volume anomaly.

1.2.3. Computation of density and sigma-$t$

Density in situ is the reciprocal of specific volume in situ. Another way to express the density is by the symbol $\sigma_{s,t,p}$. By definition $\sigma_{s,t,p}$ is equal to $10^3 (\rho_{s,t,p}-1)$. This expression has the advantage that the numerical value contains fewer digits and is easier to handle.

$\sigma_t$ represents the density of water of given salinity and temperature at surface pressure. In oceanography, $\sigma_t$ assumes significance because the motion along $\sigma_t$ surfaces involves little change in energy and therefore mixing of water masses tends to take place along these surfaces. After calculating $\Delta_{s,t}, \sigma_t$ is obtained
directly from the table for $\sigma_t$ for values of $10^5 \Delta s, t$ (Sverdrup, 1933). Based on Knudsen's equations (Knudsen, 1901), several authors have compiled tables for $\sigma_t$ for values of temperature and salinity (Mc Ewen, 1929; Mathews, 1932; Fleming, 1939; Ennis, 1944; Bumpus and Martineau, 1948).

1.2.4. Computation of currents

Dynamic computations provide information on relative currents pertaining to the distribution of mass in the sea. Such relative currents are deduced from a consideration of the balance of forces in the sea. The forces considered are those which act along an isobaric surface. When the isobaric surface is not level, a component of gravity acts downward along it. This is balanced by the Coriolis force so that the slope of the surface is maintained.

The thickness of an isobaric layer, which is the layer between two isobaric surfaces, depends upon the average specific volume of the layer. Therefore, the slope of an isobaric surface relative to another, which is assumed to be level, may be found. Since the dynamic height is a measure of the work performed against gravity in moving unit mass from one level to another, the component
of the force of gravity acting down the slopping isobaric surface between two stations is the difference in dynamic height of the surface at the two stations divided by the distance between the stations. Equating this expression to the expression for Coriolis force, we can obtain the component of current normal to the line joining the two stations (Sandstrom and Helland-Hansen, 1903). This current is at the upper isobaric surface and is relative to any current which is present at the lower reference surface. In dynamic computation, the isobaric surface is assumed level at some depth where the motion is negligible and the dynamic slope of an upper isobaric surface is found from the variation of specific volume along the isobaric layer. Thus, the current at the upper surface relative to any possible current at the lower surface is determined.

For each oceanographic station, the dynamic thickness of the isobaric layer is calculated by means of the equation,

\[ D_2 - D_1 = \int_{P_1}^{P_2} \alpha \, dp \]

where \( D_2 - D_1 \) is the dynamic thickness of the isobaric layer, \( \alpha \) is the specific volume and \( dp \) is the pressure.
interval. Since \( a = a_{35,0,p} + \delta \), the total dynamic thickness of the layer may be considered as the sum of the dynamic thickness of the layer of standard specific volume and the increment in dynamic thickness due to the anomaly of specific volume from the standard. Since the dynamic thickness of the standard layer is the same at every station, the differences in dynamic height between stations are given by the differences in the increments which can be obtained as \( \int_{P_1}^{P_2} \delta dp \). In this, metres of depth are substituted for decibars of pressure.

The dynamic computation involves the following procedures. The specific volume anomaly, \( \delta \), at each depth is calculated. The mean specific volume anomaly, \( \bar{\delta} \), for each depth interval is determined by averaging the two bounding values. This is multiplied by the depth interval to get the anomaly of the dynamic height, \( \Delta D \), for each small depth layer. Total \( \Delta D \) for each station is obtained by adding the anomalies of dynamic height from the selected reference level to the level at which relative currents are to be computed. The relative current velocity normal to a line joining two stations is obtained in metres/second as
\[ v = \frac{10(\Delta D_A - \Delta D_B)}{L \omega \sin \phi} \]

where \( \Delta D_A - \Delta D_B \) is equal to the difference in the anomalies in the dynamic height at stations A and B in dynamic metres, \( L \) is the distance between the stations in metres, \( \omega \) is the angular velocity of the earth \( (0.789 \times 10^{-4} \text{ radians/sec.}) \) and \( \phi \) is the mean latitude between the stations.

1.3. **Scheme of the present work.**

The results of an investigation on the limits of the random errors contained in the basic data of Physical Oceanography and their propagation through the computational procedures are presented in this thesis. It also suggests a method which increases the reliability of the derived results. The thesis is presented in eight chapters including the introductory chapter. Chapter 2 discusses the general theory of errors that are relevant in the context of the propagation of errors in Physical Oceanographic computations. The error components contained in the independent oceanographic variables namely, temperature, salinity and depth are delineated and quantified in chapter 3. Chapter 4 discusses and derives
the magnitude of errors in the computation of the dependent oceanographic variables, density in situ, \( \sigma_t \), specific volume and specific volume anomaly, due to the propagation of errors contained in the independent oceanographic variables. The errors propagated into the computed values of the derived quantities namely, dynamic depth and relative currents, have been estimated and presented chapter 5. Chapter 6 reviews the existing methods for the identification of level of no motion and suggests a method for the identification of a reliable zero reference level. Chapter 7 discusses the available methods for the extension of the zero reference level into shallow regions of the oceans and suggests a new method which is more reliable. A procedure of graphical smoothening of dynamic topographies between the error limits to provide more reliable results is also suggested in this chapter. Chapter 8 deals with the computation of the geostrophic current from these smoothened values of dynamic heights, with reference to the selected zero-reference level. The summary and conclusion are also presented in this chapter.