Chapter 2

Average Modeling and Small-signal Analysis of Non Minimum Phase DC-DC Converters Considering Inductor’s ESR with Constant Power Load

2.1 Introduction

Except buck converter, the boost, buck-boost, boost-buck, Z-source dc-dc converters exhibit non minimum second order phase system. Due to non minimum phase behavior of converter, linear techniques yield right half plane zeros in the converter transfer functions, which tend to cause instability. Therefore, Control issues for non minimum phase converters are more challenging than buck converter. Elimination of non minimum instability is the most desirable for the operation. So it is required to study all the converters possessing non minimum second order phase system. The first dc-dc converter of such type is boost converter described in section 2.2. The basic topology with inductor’s ESR is considered for average state-space analysis followed by small-signal analysis for linearization and control with constant power load. In further sections 2.3 and 2.4 the same analysis is done for Z-source dc-dc converter with inductor’s and capacitor’s ESR and buck-boost dc-dc converter with constant power load. The steady state response is analyzed on the basis of small-signal model.

2.2 Boost DC-DC Converter

The dc-dc converter used for step-up the input voltage is known as boost dc-dc converter. The basic circuit configuration of ideal boost dc-dc converter with constant power load is shown in figure 2.1.
In the basic circuit diagram of boost dc-dc converter, if the ESR of inductor is taken into account then the diagram can be represented as in figure 2.2.

Now considering the switch on condition and by applying KVL (Kirchhoff’s voltage law) and KCL (Kirchhoff’s current law) in above circuit, the expression in the terms of the inductor current and capacitor voltage is as follows,

\[ V_{in} - L \frac{di}{dt} - i_L(R_L + R_S) - V_S = 0 \]  \hspace{1cm} (2.1)

\[ i_C + i_{LOAD} = 0 \]  \hspace{1cm} (2.2)

\[ C \frac{dv_c}{dt} + \frac{v_c}{R_{LOAD}} = 0 \]  \hspace{1cm} (2.3)
Where \( L, i_l, V_{in}, R_L, R_S, V_S, V_D, i_C, i_{LOAD}, R_{LOAD}, v_c, C \) are inductance in henry, inductor current in ampere, input voltage in volt, inductor resistance in ohm, switch resistance in ohm, on-state voltage drop in switch in volt, on-state voltage drop in diode in volt, capacitor current in ampere, load current in ampere, load resistance in ohm, capacitor voltage in volt and capacitance in farad respectively. When the switch is off,

\[
V_{in} - L \frac{di_l}{dt} - i_l(R_L + R_D) - V_D - v_c = 0 \tag{2.4}
\]

\[
i_l = i_C + i_{LOAD} \tag{2.5}
\]

\[
i_l = C \frac{dv_c}{dt} + \frac{v_c}{R_{LOAD}} \tag{2.6}
\]

From (2.1), (2.3), (2.4), (2.6) state-space form can be derived for both on and off period of switch as follows,

\[
\begin{bmatrix}
\frac{di_l}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = \begin{bmatrix}
-\frac{(R_L + R_S)}{L} & 0 \\
0 & -\frac{1}{R_{LOAD} C}
\end{bmatrix} \begin{bmatrix}
i_l \\
v_c
\end{bmatrix} + \begin{bmatrix}
\frac{(V_{in} - V_S)}{L} \\
0
\end{bmatrix} \tag{2.7}
\]

Where \( A_1 = \begin{bmatrix}
-\frac{(R_L + R_S)}{L} & 0 \\
0 & -\frac{1}{R_{LOAD} C}
\end{bmatrix} \) and \( B_1 U_1 = \begin{bmatrix}
\frac{(V_{in} - V_S)}{L} \\
0
\end{bmatrix} \)

Similarly

\[
\begin{bmatrix}
\frac{di_l}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = \begin{bmatrix}
-\frac{(R_L + R_D)}{L} & -\frac{1}{L} \\
\frac{1}{C} & -\frac{1}{R_{LOAD} C}
\end{bmatrix} \begin{bmatrix}
i_l \\
v_c
\end{bmatrix} + \begin{bmatrix}
\frac{(V_{in} - V_D)}{L} \\
0
\end{bmatrix} \tag{2.8}
\]

Where \( A_2 = \begin{bmatrix}
-\frac{(R_L + R_D)}{L} & -\frac{1}{L} \\
\frac{1}{C} & -\frac{1}{R_{LOAD} C}
\end{bmatrix} \) and \( B_1 U_1 = \begin{bmatrix}
\frac{(V_{in} - V_D)}{L} \\
0
\end{bmatrix} \)

Assuming the continuous conduction mode, the averaged model is obtained by substituting \( d \) for \( q_1 \) and \( (1-d) \) for \( q_2 \). The state-space average model is represented as follows,
\[
\dot{X} = AX + BU \\
Y = CX
\]

Where,
\[
A = dA_1 + (1 - d)A_2 \quad \text{(2.9)}
\]
\[
BU = dB_1 U_1 + (1 - d)B_2 U_2
\]

Let \( R_S = R_D \) and \( V_S = V_D \)

\[
\begin{bmatrix}
\frac{di_l}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{(R_L + R_S)}{L} & \frac{(1-d)}{C} & \frac{L}{R_{LOAD}} & 1 \\
\frac{L}{(1-d)} & \frac{1}{C} & 0 & 0
\end{bmatrix} \begin{bmatrix}
i_l \\
v_c
\end{bmatrix} + \begin{bmatrix}
\frac{(V_{in} - V_S)}{L} \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
i_l \\
v_c
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
i_l \\
v_c
\end{bmatrix}
\]

Where
\[
C = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

The small-signal model of above state-space average model is expressed as follows,

\[
\begin{bmatrix}
\frac{d(i_l + \hat{i}_l)}{dt} \\
\frac{d(v_c + \hat{v}_c)}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{(R_L + R_S)}{L} & \frac{-1 - \hat{d}}{C} & \frac{L}{R_{LOAD}} & 1 \\
\frac{L}{(1-D)} & \frac{1}{C} & 0 & 0
\end{bmatrix} \begin{bmatrix}
i_l + \hat{i}_l \\
v_c + \hat{v}_c
\end{bmatrix} + \begin{bmatrix}
\frac{(V_{in} - V_S)(1-D)}{L} \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{d\hat{i}_l}{dt} \\
\frac{d\hat{v}_c}{dt}
\end{bmatrix} = \begin{bmatrix}
0 & \frac{(1-D)}{L} & 1 \\
\frac{(1-D)}{C} & 0 & -1
\end{bmatrix} \begin{bmatrix}
\hat{i}_l \\
\hat{v}_c
\end{bmatrix} + \begin{bmatrix}
\frac{L}{C} \\
-\hat{d}
\end{bmatrix}
\]

Where \( i_l, \hat{v}_c \) and \( \hat{d} \) are small signal parameters of inductor current, capacitor voltage and duty ratio respectively.

**Steady-State Analysis**

The steady-state equations can be derived from average state-space model (2.10) and represented as follows,

\[
V_c = V_o = \frac{(V_{in} - V_S)(R_{LOAD} (1-D))}{(R_L + R_S) + (R_{LOAD} (1-D))^2}
\]

Where, \( V_o \) is the output voltage,
\[ I_I = \frac{V_c}{R_{LOAD}(1-D)} \] (2.13)

When the inductor’s ESR and switch resistance values \((R_L + R_S)\) become zero, then the system is considered as ideal. For such ideal system, the steady state curve leads to infinity, with \(V_s = 0.707V\). The graph shown in figure 2.3 represents various behaviors of non-zero values of inductor’s ESR and switch resistance, considered as real or practical converter. The transfer functions in component symbol and parameter values are given in (a) of appendix-A. This is the steady state graph of output voltage \((V_c)\) versus duty cycle \((D)\) for both ideal and real boost dc-dc converter. From above analysis it is concluded that the input current does not depend on the parasitic elements but the output voltage is dependent on the parasitic elements.

Figure. 2.3. Steady-state graph of output voltage \((V_c)\) versus duty cycle \((D)\) for both ideal and real boost converter.
2.3 Z-source DC-DC Converter

The similar analysis is performed with another type of dc-dc converter i.e. Z-source dc-dc converter [66]. A simplified equivalent model of Z-source dc-dc converter with inductor’s and capacitor’s ESR with R-L load is presented in figure 2.4.

![Circuit diagram of Z-source converter with inductor’s and capacitor’s ESR.](image)

Figure. 2.4. Circuit diagram of Z-source converter with inductor’s and capacitor’s ESR.

The operation can be explained in two modes of Z-source dc-dc converter with R-L (resistive-inductive) load. Now assuming continuous conduction mode, there are two modes one is called shoot through mode, where load is shorted by turning on the switch-1 and other is input connected mode where switch-2 is on to connect the input to load. This converter has three energy storage elements so three state variables are needed to represent state-space model. The ESR (equivalent series resistance) of inductor and capacitor are taken into consideration for designing state-space average model of Z-source dc-dc converter [82]. The KVL, KCL expressions in terms of the inductor current, load current and capacitor voltage when switch-1 is on (switch-2 is off) are,

\[
L \frac{di_l}{dt} + R_l i_l = v_c - R_c i_l 
\]  \hspace{1cm} (2.14)

\[
L_o \frac{di_o}{dt} + R_o i_o = 0 
\]  \hspace{1cm} (2.15)
\[ C \frac{dv_c}{dt} = i_c = -i_l \]  \hspace{1cm} (2.16)

Where \( L, i_l, V_{in}, i_C, i_o, v_c \), \( C \) are inductance in henry, inductor current in ampere, input voltage in volt, capacitor current in ampere, load current in ampere, capacitor voltage in volt and capacitance in farad respectively. \( R_o, L_o \) are the resistive-inductive load of Z-source dc-dc converter. The \( R_i \) and \( R_c \) represent inductor’s ESR and capacitor’s ESR of converter circuit respectively.

The (2.14), (2.15) and (2.16) can be represented in state-space form as,

\[ \dot{X} = AX + BU \quad \text{and} \quad Y = CX \]

Where, \( A = dA_1 + (1 - d)A_2 \) and \( BU = dB_1U_1 + (1 - d)B_2U_2 \), then

\[
\begin{bmatrix}
\frac{di_l}{dt} \\
\frac{di_o}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = \begin{bmatrix}
-\frac{(R_c+R_l)}{L} & 0 & \frac{1}{L} \\
0 & -\frac{R_o}{L_o} & 0 \\
-\frac{1}{C} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
i_l \\
i_o \\
v_c
\end{bmatrix}
\]  \hspace{1cm} (2.17)

Where \( A_1 = \begin{bmatrix}
-\frac{(R_c+R_l)}{L} & 0 & \frac{1}{L} \\
0 & -\frac{R_o}{L_o} & 0 \\
-\frac{1}{C} & 0 & 0
\end{bmatrix} \) and \( B_1U_1 = \begin{bmatrix}0 \\0 \end{bmatrix} \)

Similarly when switch-1 is off mode and the switch-2 is conducting mode, the equations can be written based on KCL and KVL as,

\[ C \frac{dv_c}{dt} = i_c = i_l - i_o \]  \hspace{1cm} (2.18)

\[ L_o \frac{di_o}{dt} + R_o i_o + L \frac{di_l}{dt} + R_i i_l = v_c + (i_l - i_o)R_c \]  \hspace{1cm} (2.19)

\[ L \frac{di_l}{dt} = V_{in} - v_c - i_l(R_l + R_c) + i_o R_c \]  \hspace{1cm} (2.20)

Solving (2.19),
\[ \frac{di_o}{dt} = \frac{2v_c - v_{in} + 2i_lR_c - 2i_oR_c - i_oR_o}{L_o} \]  

(2.21)

Now the state-space form is,

\[
\begin{bmatrix}
\frac{dt}{dt} \\
\frac{di_o}{dt} \\
\frac{di_c}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{(R_c + R_l)}{L} & -\frac{R_c}{L} & -\frac{1}{L} & \frac{V_{in}}{L} \\
\frac{2R_c}{L_o} & \frac{-2(2R_c + R_o)}{L_o} & \frac{2}{L_o} & \frac{V_{in}}{L_o} \\
\frac{1}{C} & \frac{-1}{C} & 0 & \frac{0}{C} \\
\frac{1}{C} & \frac{-1}{C} & 0 & \frac{0}{C}
\end{bmatrix} \begin{bmatrix}
i \\\nI_o \\\nv_c \\\nv_{in}
\end{bmatrix} + \begin{bmatrix}
i_c \\\nl_o \\
v_c \\\nv_{in}
\end{bmatrix}
\]  

(2.22)

Where \( A_2 = \begin{bmatrix}
\frac{-1}{L} & \frac{-1}{L} & \frac{1}{L} & \frac{V_{in}}{L} \\
\frac{2R_c}{L_o} & \frac{-2(2R_c + R_o)}{L_o} & \frac{2}{L_o} & \frac{V_{in}}{L_o} \\
\frac{1}{C} & \frac{-1}{C} & 0 & \frac{0}{C} \\
\frac{1}{C} & \frac{-1}{C} & 0 & \frac{0}{C}
\end{bmatrix} \) and \( B_2U_2 = \begin{bmatrix}
V_{in} \\
V_{in} \\
0 \\
0
\end{bmatrix} \)

Let \( q_1 \) and \( q_2 \) be the switching functions of switch-1 and switch-2 respectively. Let’s assume it to be continuous conduction mode and the averaged modeling is done by substituting \( d \) for \( q_1 \) and \((1-d)\) for \( q_2 \). Now the state-space average model is,

\[ \dot{X} = AX + BU \]  

(2.23)

\[ Y = CX \]

Where, \( A = dA_1 + (1 - d)A_2 \) and \( BU = dB_1U_1 + (1 - d)B_2U_2 \), then rearranging the equations,

\[
A = \begin{bmatrix}
\frac{-1}{L} & \frac{-1}{L} & \frac{1}{L} & \frac{V_{in}(1-d)}{L} \\
\frac{2R_c}{L_o} & \frac{-2(2R_c + R_o)}{L_o} & \frac{2}{L_o} & \frac{-V_{in}(1-d)}{L_o} \\
\frac{1}{C} & \frac{-1}{C} & 0 & 0 \\
\frac{1}{C} & \frac{-1}{C} & 0 & 0
\end{bmatrix} \) and \( BU = \begin{bmatrix}
V_{in}(1-d) \\
V_{in}(1-d) \\
0 \\
0
\end{bmatrix} \)

(2.24)

Hence the average form is,
Again the small-signal model of above state-space average model is expressed as follows,

\[
\frac{d(i_l + \hat{i}_l)}{dt} = \begin{bmatrix}
\frac{-(R_c+R_l)}{L} & \frac{R_c(1-d)}{L} & \frac{(1-d)-d}{C} \\
\frac{2R_c(1-d)}{L} & \frac{-R_c(1-d)+R_o}{L} & \frac{-(1-d)-d}{C} \\
\frac{-(D-(1-D)+2\hat{d})}{C} & \frac{(1-d)-d}{C} & 0
\end{bmatrix} \begin{bmatrix}
\hat{i}_l \\
\hat{i}_o \\
\hat{V}_c
\end{bmatrix} + \begin{bmatrix}
\frac{V_{in}(1-d)}{L} \\
\frac{V_{in}(1-d)}{L} \\
0
\end{bmatrix}
\] (2.25)

\[
\begin{bmatrix}
\hat{i}_l \\
\hat{i}_o \\
\hat{V}_c
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\hat{i}_l \\
\hat{i}_o \\
\hat{V}_c
\end{bmatrix}
\] (2.26)

Now neglecting higher order perturbation terms, the small-signal model is,

\[
\begin{bmatrix}
\frac{d\hat{i}_l}{dt} \\
\frac{d\hat{i}_o}{dt} \\
\frac{d\hat{V}_c}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{-(R_c+R_l)}{L} & \frac{R_c(1-D)}{L} & \frac{(1-D)-2}{C} \\
\frac{2R_c(1-D)}{L} & \frac{-R_c(1-D)+R_o}{L} & \frac{-(1-D)-2}{C} \\
\frac{-(D-(1-D)+2\hat{d})}{C} & \frac{(1-D)-2}{C} & 0
\end{bmatrix} \begin{bmatrix}
\hat{i}_l \\
\hat{i}_o \\
\hat{V}_c
\end{bmatrix} + \begin{bmatrix}
\frac{2V_{in}-2V_c+R_c\hat{V}_o}{L} \\
\frac{V_{in}-(D-(1-D)+2\hat{d})}{L} \\
\frac{-(1-D)-2}{C}
\end{bmatrix} \hat{d} \] (2.29)
\[
\begin{bmatrix}
\dot{i}_l \\
\dot{I}_o \\
\dot{v}_c
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}\begin{bmatrix}
\dot{i}_l \\
\dot{I}_o \\
\dot{v}_c
\end{bmatrix}
\]

(2.30)

**Steady-State Analysis**

The steady-state equations can be derived from averaged model. Simplifying this,

\[
I_l = \frac{(1-D)I_o}{(1-2D)}
\]

(2.31)

Where \(I_o\) is the steady state load current of Z-source dc-dc converter with R-L load.

\[
V_c = \frac{(A+B)(1-D)(1-2D)\Delta V_{in}}{2A(1-D)(1-2D)+B(1-2D)^2}
\]

(2.32)

Where \(A = 2D^2R_c - 2DR_c - R_l + DR_l\)

\(B = 6D^2R_c - 10DR_c - R_o + 2DR_o\)

Under ideal condition (assuming ESR of inductor and capacitor is zero) then \(A=0; B=R_o (2D-1)\), which gives:

\[
V_c = \frac{(1-D)\Delta V_{in}}{(1-2D)} = I_o R_o
\]

(2.33)

Selections of passive parameters of X-shaped network are based on minimization of current ripples and voltage ripples. It is also shown that the change of passive parameters affect the transient responses. It is important to carefully select the \(L\) and \(C\) values of Z-source converter to achieve a good compromise between oscillatory response, ripple factor and non minimum phase effect. Initially selection is based on ripple factor. A battery of 12V input is used as source to control voltage across load resistor. Let us assume that the voltage across the load of \(R_o\) (10Ω) is increased to 5 times (60V). The load current is \(I_l=6A\). At rated voltage operation the voltage gain of Z-source dc-dc converter is found to be \(\pm 5\) (60/12).

Under ideal condition and found to be \(D=0.445\) for 60V and 0.545 for 60V with \(I_l = \pm 6A\). Let the current ripple of \(I_l\) and the capacitor voltage ripples (\(v_c\)) are assumed to be maximum limit of \(\pm 1\%\), then \(\Delta I_l = .06A\) and \(\Delta v_c = 1.2V\). The operations of switch occur with switching frequency of 10 kHz under full rated voltage condition. The ESR of capacitor
depends on switching frequency and capacitance value of capacitor. The mathematical expression of ESR of capacitor is,

$$R_c = \frac{1}{\omega^2 R_{\text{leak}} C^2}$$

(2.34)

The ESR effect of capacitor is negligible if the switching frequency is very high. The ESR of inductor depends on number of turns, diameter of wire and also switching frequency. The ESR of inductor increases with increase in frequency. The ESR sensitivity graph is shown in figure 2.5 (a, b). Where the x-axis presents the duty ratio and y-axis presents the voltage gain for both the cases. The boost capability decreases with increase the ESR of capacitor and also for ESR of inductor. The ESR of inductor is more dominant on decrease of the boost capability of such converter compared to ESR of capacitor as the Z-source converter posses boost capability with same polarity as input in the duty ratio range between 0<D<0.5 and both boost and buck capability with reverse polarity in the duty ratio range between 0.5<D<1.
Figure 2.5. (a, b). ESR sensitivity graph of Z-source converter inductor’s ESR and capacitor’s ESR respectively.

At D=0.4 using the parameters as given in table 2.1 which is justified from the Simulation model.

Table 2.1: Z-source converter parameters for R-L load

<table>
<thead>
<tr>
<th>V_{in}</th>
<th>R_{o}</th>
<th>RL(ESR)</th>
<th>L</th>
<th>L_{o}</th>
<th>C</th>
<th>RC(ESR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12V</td>
<td>10Ω</td>
<td>0.02 Ω</td>
<td>4mH</td>
<td>2mH</td>
<td>1.375mF</td>
<td>0.12 Ω</td>
</tr>
</tbody>
</table>

The transfer functions are derived from the small signal model and presented in their component symbol in (b) of appendix-A. The transfer functions in their parameter values are presented as follows,

\[ G_{ii} \frac{d(s)}{d(s)} = \frac{1.26 \times 10^4 s^2 + 6.42 \times 10^7 s + 5.52 \times 10^9}{s^3 + 5.1 \times 10^3 s^2 + 4.45 \times 10^5 s + 4.29 \times 10^7} \]  \hspace{1cm} (2.35)
\[ G_{li} \frac{d(s)}{d(s)} = \frac{i_c(s)}{d(s)} = \frac{-2.62 \times 10^4 s^2 - 6.88 \times 10^6 s + 7.14 \times 10^8}{s^3 + 5.1 \times 10^5 s^2 + 4.45 \times 10^5 s + 4.29 \times 10^7} \]  

(2.36)

\[ G_{vc} \frac{d(s)}{d(s)} = \frac{v_c(s)}{d(s)} = \frac{-1.14 \times 10^4 s^2 - 4.52 \times 10^7 s + 7.25 \times 10^9}{s^3 + 5.1 \times 10^5 s^2 + 4.45 \times 10^5 s + 4.29 \times 10^7} \]  

(2.37)

Where \( G_{li} \frac{d(s)}{d(s)} \), \( G_{io} \frac{d(s)}{d(s)} \) and \( G_{vc} \frac{d(s)}{d(s)} \) is denoted as control to inductor current, control to load current, control to capacitor voltage transfer functions respectively.

### 2.4 Buck-boost DC-DC Converter

In previous dc-dc converters, the effect of ESR of inductor in boost converter and ESR of capacitor in Z-source converter with R-L load is clearly visible. Here one buck-boost dc-dc converter is modeled. The ESR of inductor and capacitor of the converter are kept very low as negligible to observe the behavior of the converter. The basic circuit diagram of buck-boost converter with R-load is presented in figure 2.6.

![Circuit diagram of buck-boost converter with R-load](image)

Figure. 2.6. Circuit diagram of buck-boost converter with R-load.

The operation of dc-dc buck-boost converter with R-load can be explained as follows for continuous conduction mode. The switch and the diode are turned on and off in a cyclic and complementary manner. When the switch is closed (the diode is open), the inductor current ramps up and stores the electrical energy. When the switch is open (the diode closed), the inductor current ramps down by transferring the electrical energy to capacitor. The duty ratio defined as the fraction of a switching period determines the stepping up or stepping down the voltage and power transfer. When the switch (S) is on, the equations are,
\[ L \frac{di}{dt} = V_{in} \]  

(2.38)

\[ C \frac{dv_c}{dt} = i_c = -\frac{v_c}{R_{LOAD}} \]  

(2.39)

Where \( L, i_l, V_{in}, i_C, R_{LOAD}, v_c, C \) are inductance in henry, inductor current in ampere, input voltage in volt, capacitor current in ampere, load resistance in ohm, capacitor voltage in volt and capacitance in farad respectively. The (2.38) and (2.39) can be written in state space form as,

\[
\begin{bmatrix}
\frac{di_l}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
0 & -\frac{1}{R_{LOAD}C}
\end{bmatrix}
\begin{bmatrix}
i_l \\
v_c
\end{bmatrix} +
\begin{bmatrix}
V_{in} \\
L
\end{bmatrix}
\]  

(2.40)

Where \( A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_{LOAD}C} \end{bmatrix} \) and \( B_1 U_1 = \begin{bmatrix} V_{in} \\ L \end{bmatrix} \)

The similar procedure is followed for switch-1 off and diode on condition and the derivation is as follows,

\[ L \frac{di}{dt} = v_c \]  

(2.41)

\[ -C \frac{dv_c}{dt} = i_l + \frac{v_c}{R_{LOAD}} \]  

(2.42)

Representing (2.41) and (2.42) in state space form is as follows,

\[
\begin{bmatrix}
\frac{di_l}{dt} \\
\frac{dv_c}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{1}{L} \\
-1 & \frac{1}{R_{LOAD}C}
\end{bmatrix}
\begin{bmatrix}
i_l \\
v_c
\end{bmatrix}
\]  

(2.43)

Where \( A_2 = \begin{bmatrix} 0 & \frac{1}{L} \\ -1 & \frac{1}{R_{LOAD}C} \end{bmatrix} \) and \( B_2 U_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)

Average model can be derived by the equations represented as,

\[ A = dA_1 + (1 - d)A_2 \text{ And } BU = dB_1 U_1 + (1 - d)B_2 U_2 \]
‘d’ is the turn-on period of switch. Hence the average model of buck-boost converter is,

\[
\begin{bmatrix}
\frac{di}{dt} \\
\frac{d\bar{v}_c}{dt}
\end{bmatrix} = \begin{bmatrix}
0 & \frac{1-d}{L} \\
-\frac{1}{C} & \frac{-1}{R_{LOAD} C}
\end{bmatrix} \begin{bmatrix}
\bar{i} \\
\bar{v}_c
\end{bmatrix} + \begin{bmatrix}
\frac{V_{in}}{L} \\
0
\end{bmatrix} d
\]

(2.44)

The state-space average model of buck-boost converter with R-load shown in (2.44) has non-linear in nature as the control parameter (d) is in matrix A. Therefore small-signal analysis is required to make the state-space to be linear. Due to small variation of \(\hat{d}\) in steady state D, the state variables are changed to \(i_l + \hat{i}_l\) and \(v_c + \hat{v}_c\). Now the state-space average model with small perturbation is modified as follows:

\[
\begin{bmatrix}
\frac{d(i_l+\hat{i}_l)}{dt} \\
\frac{d(v_c+\hat{v}_c)}{dt}
\end{bmatrix} = \begin{bmatrix}
0 & \frac{1-(D+\hat{d})}{L} \\
-\frac{1}{C} & \frac{1}{R_L C}
\end{bmatrix} \begin{bmatrix}
\hat{i}_l + \hat{v}_c \\
\hat{v}_c + \hat{v}_c
\end{bmatrix} + \begin{bmatrix}
\frac{V_{in}}{L} (D + \hat{d}) \\
0
\end{bmatrix} 
\]

(2.45)

Subtracting (2.44) from (2.45) and neglecting higher order perturbation terms the small-signal model of buck-boost converter can be written as follows:

\[
\begin{bmatrix}
\frac{d\hat{i}_l}{dt} \\
\frac{d\hat{v}_c}{dt}
\end{bmatrix} = \begin{bmatrix}
0 & \frac{1-D}{L} \\
-\frac{1}{C} & \frac{-1}{R_{LOAD} C}
\end{bmatrix} \begin{bmatrix}
\hat{i}_l \\
\hat{v}_c
\end{bmatrix} + \begin{bmatrix}
\frac{V_{in} - v_c}{L} \\
\frac{i_l}{C}
\end{bmatrix} \hat{d}
\]

(2.46)

**Steady-State Analysis**

The steady-state behavior is presented as follows,

\[
I_l = \frac{V_c}{R_L (1-D)}
\]

(2.47)

\[
\frac{V_c}{V_{in}} = -\frac{D}{1-D}
\]

(2.48)

The (2.47) is also known as the load line. It is the locus of inductor current and capacitor voltage for different duty cycle. The transfer functions are derived from the small signal model and presented in their component symbol in (c) of appendix-A. The transfer functions in their parameter values are presented as follows,
\[
\frac{i_d(S)}{d(S)} = \frac{-3.478e004 S + 1.763 \times 10^5}{S^2 + 41.68 S + 1.043 \times 10^5}
\] (2.49)

\[
\frac{v_c(S)}{d(S)} = \frac{18700 S + 4.174 \times 10^7}{S^2 + 41.68 S + 1.043 \times 10^5}
\] (2.50)

2.5 Conclusion

Due to non minimum phase behavior of converter, linear techniques yield right half plane zeros in the converter transfer functions, which tend to cause instability. Therefore, Control issues for non minimum phase dc-dc converters are more challenging than buck converter. Elimination of non minimum instability is the most desirable for the operation. So it is required to study all the converters possessing non minimum second order phase system. The first dc-dc converter of such type is boost converter described in section 2.2. The basic topology with ESR is considered for average state-space analysis followed by small-signal analysis for linearization and control with constant power load. In coming sections 2.3 and 2.4 the same is done for Z-source and buck-boost dc-dc converter with ESR and constant power load. The steady state analysis is presented for all the proposed dc-dc converters.