5.1 INTRODUCTION

A comprehensive account of the Rayleigh-Taylor instability under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar (1961).

The influence of viscosity on the stability of the plane interface separating two incompressible superposed conducting fluids of uniform densities when the whole system is immersed in a uniform magnetic field has been studied by Bhatia (1974). He has carried out the stability analysis for two highly viscous fluids of equal kinematic viscosities and different uniform densities.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stokes (1966) has formulated the theory of couple-stress fluid. One of the applications of a couple-stress fluid is its use in the study of mechanisms of lubrication of synovial joints, which has become the objective of scientific research. A human joint is a dynamically loaded bearing which has the articular cartilage as the bearing and the synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, hip, knee and ankle joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear.

A normal synovial fluid is a viscous, non-Newtonian fluid and is generally clear or yellowish. According to the theory of Stokes (1966), couple-stresses are found to appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in the synovial fluid. Walicki and Walicka (1999) modelled synovial fluid as a couple-stress fluid in human joints. The use of a magnetic field is being made of clinical purposes in detection and cure of certain diseases with the help of magnetic field devices/instruments. Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent (1966) has studied the effect of a horizontal magnetic field,
which varies in the vertical direction, on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of a magnetic field the system is known to be stable. In all the above studies, the medium has been considered to be non-porous.

In recent years, the investigations of flow of fluids through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. McDonnell (1978) pointed out the physical properties of comets; meteorites and interplanetary dust strongly suggest the significance of the effects of porosity in the astrophysical context. A great number of applications in geophysics may be found in the books by Phillips (1991), Ingham and Pop (1998) and Nield and Bejan (1999). Several researchers [Vaghela and Chhajlani (1988), Samaria et al. (1990), Sharma and Kumar (1997), and Khan and Bhatia (2003)] have studied the effects of the permeability of a porous medium on different problems in hydrodynamic and hydromagnetic stability in view of the importance of such studies in rocks heavy oil recovery. When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous and couple-stress viscous terms in the equations of couple-stress fluid motion are replaced by the resistance term \(-\frac{1}{k_1}(\mu - \mu' \nabla^2)q\), where \(\mu\) and \(\mu'\) are the viscosity and couple-stress viscosity, \(k_1\) is the medium permeability and \(q\) is the Darcian (filter) velocity of the couple-stress fluid. Recently, Khan et. al (2010) have studied the instability of superposed streaming fluids through a porous medium and have shown that the streaming motion tends to further destabilize the unstable arrangement of the superposed fluids.

Keeping in mind the importance and applications of couple-stress fluids, porous medium and magnetic field mentioned above, a study has been made to investigate instability of a plane interface between viscous (Newtonian) and viscoelastic (couple-stress) fluid numerically in the presence of uniform two-dimensional horizontal magnetic field saturating porous medium in the present chapter.
5.2 FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

The initial stationary state whose stability we wish to examine is that of an incompressible couple-stress fluid layer consisting of an infinitely conducting (hydromagnetics) fluid of density \( \rho \), arranged in horizontal strata and acted on by a uniform viscosity \( \mu \), through a porous medium in the presence of uniform two dimensional magnetic field \( \mathbf{H}(H_x, H_y, 0) \) and acted on by gravity field \( \mathbf{g}(0, 0, -g) \). This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity \( \varepsilon \) and medium permeability \( k_1 \). Then the equations of motion, continuity, incompressibility of the fluid and the Maxwell's equations through the porous medium are

\[
\rho \frac{\partial q}{\partial t} = -\nabla p - \rho \mathbf{g} + (\nabla \times \mathbf{H}) \times \mathbf{H} - \frac{1}{k_1} (\mu - \mu' \nabla^2) q, 
\]

(5.1)

\[
\nabla \cdot q = 0, 
\]

(5.2)

\[
\varepsilon \frac{\partial \rho}{\partial t} + (q \cdot \nabla) \rho = 0, 
\]

(5.3)

\[
\varepsilon \frac{\partial \mathbf{h}}{\partial t} = (\mathbf{H} \cdot \nabla) q, 
\]

(5.4)

\[
\nabla \cdot \mathbf{h} = 0. 
\]

(5.5)

The initial stationary state solution is given by

\[
q = (0, 0, 0), \quad \rho = \rho(z), \quad p = p(z). 
\]

(5.6)

The character of the equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then by following its further evolution.

This initial state is given infinitesimally small disturbances. Let \( \mathbf{h}(h_x, h_y, h_z) \), \( q(u, v, w) \), \( \delta \rho \) and \( \rho' \) are the perturbations, respectively, in magnetic field \( \mathbf{H} \), Darcian velocity \( q \), density \( \rho(z) \) and pressure \( p(z) \). Then the linearized hydromagnetic perturbation equations governing the motion of the couple-stress fluid through a porous medium are

\[
\rho \frac{\partial \delta q}{\partial t} = -\nabla \delta \rho + \mathbf{g} \delta \rho + (\nabla \times \mathbf{h}) \times \mathbf{H} - \frac{1}{k_1} (\mu - \mu' \nabla^2) q, 
\]

(5.7)
\[ \nabla \cdot q = 0, \quad \text{(5.8)} \]
\[ \varepsilon \frac{\partial}{\partial t} (\delta \rho) + (q \cdot \nabla) \rho = 0, \quad \text{(5.9)} \]
\[ \varepsilon \frac{\partial h}{\partial t} = (H \cdot \nabla) q, \quad \text{(5.10)} \]
\[ \nabla \cdot h = 0. \quad \text{(5.11)} \]

Analyzing in terms of normal modes, we assume that the perturbed quantities have the space \( x, y, z \) and time \( t \) dependence of the form
\[ f(z) \exp (i k_x x + i k_y y + n t), \quad \text{(5.12)} \]
where \( f(z) \) is some function of \( z \), \( k_x \) and \( k_y \) are the horizontal wavenumber \( k^2 = k_x^2 + k_y^2 \), and \( n \) is the rate at which the system departs away from equilibrium.

Eliminating \( u, v, h_x, h_y, h_z, \delta \rho \) and \( \delta p \) from the set of equations (5.7) - (5.11) using expression (5.12), and after a little algebra, we obtain
\[ \frac{n}{\varepsilon} [\rho k^2 w - D(\rho Dw)] - \frac{g \kappa^2}{\varepsilon n} (D \rho) w - \frac{1}{\varepsilon n} \left[ (k_x H_x + k_y H_y)^2 (D^2 - k^2) w \right] - \frac{1}{k_1} [D(\mu - \mu') (D^2 - k^2) D w - k^2 (\mu - \mu' (D^2 - k^2)) w] = 0. \quad \text{(5.13)} \]

### 5.3 Two Uniform Viscous and Viscoelastic Fluids Separated by a Horizontal Boundary

Here we consider the case in which two superposed fluids of densities \( \rho_1 \) and \( \rho_2 \), viscosities \( \mu_1 \) and \( \mu_2 \) and two-dimensional uniform magnetic fields \( H_x \) and \( H_y \) occupying the regions, \( z < 0 \) and \( z > 0 \), are separated by a horizontal boundary at \( z = 0 \).

In the two regions of constant \( \rho, \mu, \mu' \) and \( H \), equation (5.13) reduces to
\[ (D^2 - k^2) (D^2 - M'^2) w = 0, \quad \text{(5.14)} \]
where
\[ M'^2 = k^2 + \frac{k_1 n}{\varepsilon v'} + \frac{k_1 n (k \cdot H)^2}{\varepsilon v' \rho n} + \frac{v}{v'}, \]
and \( v = \frac{\mu}{\rho}, \quad v' = \frac{\mu'}{\rho} \) are the coefficients of viscosity and couple-stress viscoelasticity.
The general solution of equation (5.14) is a linear combination of integrals $e^{\pm k}z$ and $e^{\pm M'z}$.

Since $w$ must be bounded both when $z \to +\infty$ in the upper fluid and $z \to -\infty$ in the lower fluid, the solution of equation (5.14), which remains bounded in the two regions, are

$$w_1 = A_1 e^{kz} + B_1 e^{M_1z}, \quad z < 0,$$

$$w_2 = A_2 e^{-kz} + B_2 e^{-M_2z}, \quad z > 0,$$

where $A_1, B_1, A_2$ and $B_2$ are constants and $M_1$ and $M_2$ are respectively, the square roots of $M'^2$ for the two regions.

The expressions determining $M_1$ and $M_2$ are

$$M_1^2 = k^2 + \frac{k_1 n}{\nu_1} + \frac{k_1 (k_x h_x + k_y h_y)^2}{\nu_1 \rho n},$$

$$M_2^2 = k^2 + \frac{k_1 n}{\nu_2} + \frac{k_1 (k_x h_x + k_y h_y)^2}{\nu_2 \rho n}.$$  

Since $\mu_2 = 0 \Rightarrow M_2^2 = k^2.$

In writing the solutions (5.15) and (5.16), it was assumed that $M_1$ and $M_2$ were so defined that their real parts were positive.

The solutions (5.15) and (5.16) must satisfy certain boundary conditions for the present problem which require that

(i) $w$ must be bounded both when $z \to +\infty$ (in the upper fluid) and $z \to -\infty$ (in the lower fluid).

(ii) $w, Dw, \{\mu - \mu'(D^2 - k^2)\}(D^2 + k^2)w$ must be continuous at the interface $z = 0.$

Since the constitutive equations for the couple stress fluid are

$$\tau_{ij} = (2\mu - 2\mu' \nabla^2) e_{ij}, \quad e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),$$

where $\tau_{ij}$ and $e_{ij}$ denote, respectively, the shear stress tensor and rate-of-strain tensor.

The conditions on a free surface are

$$\tau_{xx} = (\mu - \mu' \nabla^2) \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial x} \right) = (\mu - \mu'(D^2 - k^2))(Du + i k_x w),$$

$$\tau_{yy} = (\mu - \mu' \nabla^2) \left( \frac{\partial v}{\partial y} + \frac{\partial w}{\partial y} \right) = (\mu - \mu'(D^2 - k^2))(Dv + i k_y w),$$

must be continuous.
(iii) Integrating equation (5.13) between $z^\prime - \eta_1^\prime$ and $z + \eta_1^\prime$ and passing the limit $\eta_1^\prime = 0$, across the interface $z = 0$, we obtain another jump condition

$$\Delta_s \left[ \frac{n}{e} \rho + \frac{\rho}{k_1} (v - v'(D^2 - k^2)) Dw \right] - \frac{1}{en} \left[ (k_x H_x + k_y H_y)^2 \Delta_s (Dw) \right] = -\frac{gk^2}{en} (\Delta_s \rho) \text{ (for } z = 0).$$

which implies that using the solutions (5.15) and (5.16) that

$$\left\{ \begin{array}{l} 
\mu_1 - \mu_1^\prime (D^2 - k^2) Dw_1 = 0, \\
(\rho_2 Dw_2 - \rho_1 Dw_1) + \frac{gk^2}{n^2} (\rho_2 - \rho_1) w_0 - \frac{(k_x H_x + k_y H_y)^2}{n^2} (Dw_2 - Dw_1) + \frac{\epsilon}{k_1 n} [\mu_2 Dw_2 - \\
\frac{\mu_1 (D^2 - k^2) Dw_1)] = 0, 
\end{array} \right.$$

while the equation valid everywhere else $z \neq 0$ is

$$D \left[ \frac{n}{e} \rho + \frac{\rho}{k_1} (v - v'(D^2 - k^2)) Dw \right] - k^2 \left[ \frac{n}{e} \rho + \frac{\rho}{k_1} (v - v'(D^2 - k^2)) \right] w - (k_x H_x + k_y H_y)^2 (D^2 - k^2) w = -\frac{gk^2}{en} (D\rho) w,$$

remembering the configuration that the upper fluid is Newtonian and the lower fluid is couple stress fluid.

Applying the boundary conditions (5.21) and (5.25) to the solutions given in equations (5.15) and (5.16), we obtain

$$A_1 + B_1 = A_2 + B_2,$$

$$kA_1 + M_1 B_1 = -kA_2 - M_2 B_2,$$

$$[\mu_1 (2k^2 A_1 + (M_1^2 + k^2) B_1) - \mu_1^\prime (M_1^2 - k^2)(M_1^2 + k^2) B_1] = [\mu_2 (2k^2 A_2 + (M_2^2 + k^2) B_2)],$$

$$[\rho_2 (-kA_2 - M_2 B_2) - \rho_1 (kA_1 - M_1 B_1)] + \frac{gk^2}{2n^2} (\rho_2 - \rho_1) (A_1 + B_1 + A_2 + B_2) +$$

$$\frac{(k_x H_x + k_y H_y)^2}{n^2} (kA_1 + M_1 B_1 + kA_2 + M_2 B_2) + \frac{\epsilon}{k_1 n} [\mu_2 (-kA_2 - M_2 B_2) - \mu_1 (kA_1 + M_1 B_1) +$$

$$\mu_1^\prime (q_1^2 - k^2) kA_1 + M_1 B_1] = 0.$$

On eliminating the constant $A_1, B_1, A_2, B_2$ and evaluating the determinant of the given matrix of the coefficients in equations (5.27) - (5.29), we obtain the following characteristic equation

$$2k^2 (\alpha_1 v_1 + \alpha_2 v_2) \left\{ -\frac{k}{k_1} \left( 1 + \frac{ev_2}{nk_1} \right) + \frac{(k v_2 A)^2}{n^2 k} \right\} - \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\} \left[ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right]$$

$$= \frac{\epsilon}{k_1 n} \alpha_1 v_1 (M_1^2 - k^2) \left\{ -\alpha_2 v_2 (M_2 + k) \right\} - 2k \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)]^2 \right\}$$

$$= \frac{\epsilon}{k_1 n} \alpha_1 v_1 (M_1^2 - k^2) \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$

$$- \frac{k^2}{n^2} \left\{ -\alpha_2 v_2 (M_1 + k) \right\} + \left\{ [\alpha_1 v_1 (M_1 + k) - \alpha_1 v_1^\prime (M_1 + k) (M_1^2 + k^2)] \right\} \left\{ R - 1 + \frac{2(k v_2 A)^2}{n^2} \right\}$$
\[ \frac{\varepsilon}{k_1 n} (\alpha_1 v_1 + \alpha_2 v_2) + \frac{\varepsilon}{k_1 n} \alpha_1 v_1 (M_1^2 - k^2) - \left\{ - \frac{\alpha_1}{k_1} \left( 1 + \frac{\varepsilon v_1}{k_1 n} \right) + \frac{(k V_n)^2}{n^2 k} + \frac{\varepsilon \alpha_1 v_1'(M_1^2 - k^2)}{n k_1} \right\} [2k^2(\alpha_1 v_1 + \alpha_2 v_2)] = 0. \]  
(5.30)

where \( R = \frac{g k^2}{n} (\alpha_2 - \alpha_1), \quad \alpha_2 = \frac{\rho_2}{\rho_1 + \rho_2}, \quad \alpha_1 = \frac{\rho_1}{\rho_1 + \rho_2}, \quad (k V_n)^2 = \frac{(k_x h_x + k_y h_y)^2}{\rho_1 + \rho_2}. \)

Substituting the values of \( M_1 \) and \( M_2 \) from the expressions (5.12), (5.13) and making the assumptions for mathematical simplicity that the kinematic viscosities of two fluids are the same i.e. \( v_1 = v_2 = v \) and that the fluids are of high viscosity and high viscoelasticity. Under these assumption we have,

\[ M_1 = k \left[ 1 + \frac{k_x n}{2e v_1 k^2} + \frac{k_x (k V_n)^2}{2e v_1 \alpha_1 n k^2} + \frac{v_1}{2v_1 k^2} \right], \]

\[ M_2 = k. \]  
(5.32)

Substituting the values of \( M_1 \) and \( M_2 \), given by equations (5.31) and (5.32), in equation (5.30), we obtain

\[ A_6 n^6 + A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0, \]

(5.33)

where

\[ A_6 = k \alpha_1^2 k_1^3 \left\{ 2\alpha_2 v_1^2 + v_1^2 (\alpha_2 - 1) \right\}, \]

\[ A_5 = \alpha_1 \varepsilon k_1^2 \left[ \alpha_1 v_1^2 \left[ k (v_1 + 6k^2 v_1')(2\alpha_2 - 1) + 2k^2 v_1 + \alpha_2 v_1 + 2k^2 \alpha_2 v_2 + 6k^3 \alpha_1 v_1 + k \alpha_1 v_1' \right] - \alpha_1^2 v_2 \right] = \alpha_1 v_1^2 \left( k (v_1 + 6k^2 v_1')(2\alpha_2 - 1) + 2k^2 v_1 + \alpha_2 v_1 + 2k^2 \alpha_2 v_2 + 6k^3 \alpha_1 v_1 + k \alpha_1 v_1' \right) - \alpha_1^2 v_2 \right], \]

\[ A_4 = \alpha_1 k_1 \left[ 2k v_1^2 k_1^2 \left( k_x h_x + k_y h_y \right)^2 - 4k^3 \varepsilon^2 \alpha_2 v_1^2 v_2 v_1' \left( \alpha_2 - 1 \right) + \left( 2k^3 \varepsilon^2 \alpha_1 v_1^2 v_1' + 2k^5 \varepsilon^2 \alpha_1 v_1^2 v_1' \right) - k_1^2 \right] \left\{ 2\alpha_2 - 1 \right\} + 6k^3 \varepsilon^2 \alpha_1 \alpha_2 v_1^2 v_2 v_1' + 4k^4 \varepsilon^2 \alpha_1 v_1^2 v_1' \left( 3\alpha_2 v_2 + 2 \right) + 2k^2 \varepsilon^2 \alpha_1 v_1^2 + \alpha_2 v_1^2 k_1^2 \left( k_x h_x + k_y h_y \right)^2 + 4k^2 \varepsilon^2 \alpha_1 \alpha_2 v_1^2 v_1' + \varepsilon^2 \alpha_1 \alpha_2 v_1^4 + 16k^6 \varepsilon^2 \alpha_1 \alpha_2 v_1^2 v_2 v_1' + 2k^2 \varepsilon^2 \alpha_1 \alpha_2 v_3 v_1' - k_1^2 \alpha_1 v_1^2 k_1 \left( k_x h_x + k_y h_y \right)^2 + k_1^2 \alpha_1 v_1^2 k_1^2 g (\alpha_2 - \alpha_1) + 3k \alpha_1 v_1^2 k_1^2 \left( k_x h_x + k_y h_y \right)^2 + 10k^3 \varepsilon^2 v_1^2 v_1' + 8k^5 \varepsilon^2 \alpha_1^2 v_1^2 v_2^2 + 2k \alpha_1 v_1^2 k_1^2 \left( k_x h_x + k_y h_y \right)^2 \right\}, \]
\[ A_3 = \alpha_1 \varepsilon \left[ k_1 v_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} (2\alpha_2 - 1) + v_1^3 k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} (2\alpha_2 - \alpha_1) + 2k^3 \varepsilon^2 \alpha_2 v_1^3 v_2 v'_1 (3\alpha_1 + 2) + 8k^3 \alpha_2 v_1^3 v_2 v'_1 k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} - 4k^3 \varepsilon^2 \alpha_2^2 v_2^2 v'_1 v_1' (k - 1) + k\alpha_2 v_1^3 v_2 k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} + 2k^2 v_1^3 k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} \varepsilon^2 \alpha_2 v_1 v_2 k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} + k\varepsilon^2 \alpha_1 \alpha_2 v_1^4 v'_1 + 12k^3 \alpha_1 v_1^2 v_1' k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} + 6k^4 \alpha_1 v_1^3 v_1' k_2^2 g(\alpha_2 - \alpha_1) + 6k^3 v_1^2 v_1' k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} - 8k^5 \varepsilon^2 \alpha_1 \alpha_2 v_1^2 v_2 v_1' + 8k^3 \varepsilon^2 \alpha_1^2 v_1^4 v_1' \right] ,

\[ A_2 = v_1^2 k_1^3 \left( \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} \right) v_1^2 v_2 v_1' k_1 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} (3k - 2) + 3k^3 \varepsilon^2 \alpha_1 \alpha_2 v_1^2 v_2 v_1' k_1 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} (2k + 3) - 4k^3 \alpha_1 \alpha_2 v_1^2 v_2 v_1' k_1 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} (\alpha_2 - \alpha_1) + k\varepsilon^2 \alpha_2 v_1^2 v_2 v_1' k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} + 8k^5 \varepsilon^2 \alpha_1^2 v_1^2 v_2 v_1' k_1 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} + 5k\alpha_1 v_1^2 k_1^3 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} - \varepsilon^2 \alpha_1^2 v_1^3 v_1' k_1 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} + 2k^4 \varepsilon^2 \alpha_1^2 v_1^2 v_2 v_1' k_1 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} g(\alpha_2 - \alpha_1) + 8k^6 \varepsilon^2 \alpha_1^2 v_1^2 v_2 v_1' k_1 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} g(\alpha_2 - \alpha_1) + 2k^2 \alpha_1 v_1^2 k_1^3 g(\alpha_2 - \alpha_1) \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} - k\varepsilon^2 \alpha_1^2 v_1^3 k_1^3 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} ,

\[ A_1 = \varepsilon \alpha_1 k_1^2 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} \left( -4k^3 \varepsilon v_1^2 v_1' \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} - 4k^2 v_1^2 v_1' \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} - 2v_1^3 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} \right) + 6k^4 v_1^2 v_1' g(\alpha_2 - \alpha_1) + 3k v_1^3 \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} + k\varepsilon v_1^2 k_1 \left( \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} \right)^2 (18k^2 \alpha_1 v_1' - \alpha_2 v_2 k_1) ,

\[ A_0 = v_1^2 k_1^3 \left( \frac{(k_x H_z + k_y H_y)^2}{\rho_1 + \rho_2} \right)^3 (k^2 + 6k + 1) .

Equation (5.33) is the required dispersion relation which involves sixth order in \( n \) and the coefficients are very lengthy and complicated to find critical wavenumber. It is thus not feasible to analyze the dispersion relation analytically. We therefore solve it numerically, for different values of the parameters, for an unstable arrangement of superposed fluids, i.e. a top-heavy configuration.
5.6 NUMERICAL RESULTS AND DISCUSSION

We are interested in the qualitative behavior of the various parameters on the instability of the configuration. Therefore, the dispersion relation, given by equation (5.33), is numerically solved to ascertain the values of the growth rate versus the wavenumbers for various values of one parameter, taking other parameters fixed, using the software Mathematica-5.2. The dispersion relation is first non-dimensionalized by measuring $n$ and the parameters in terms of $\sqrt{g}$.

For the potentially unstable arrangement, figure 1 shows the variation of growth rate $n$ with respect to wave number $k$ satisfying equation (5.33) for fixed permissible values of $v_1 = 1, v_2 = 1, v_i = 5, H_x = 10, H_y = 10, g = 980 \text{cm/}sec^2$,

$k_1 = 5, \alpha_1 = 0.3, \alpha_2 = 0.7, k_x = k/\sqrt{2}, k_y = k/\sqrt{2}, \rho_1 = 1, \rho_2 = 1.2$ for three different values of the medium porosity $\varepsilon = 0.1, 0.5, 0.9$, respectively. The graph shows that the growth rate increases with the increase in medium porosity for a fixed wavenumber implying thereby the destabilizing effect on the system for the most unstable mode.

![Figure 1: Variation of growth rate $n_r$ (real part of $n$) with respect to wavenumber $k$ for three values of medium porosity $\varepsilon = 0.1, 0.5, 0.9$.](image)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Variation of growth rate $n_r$ (real part of $n$) with respect to wavenumber $k$ for three values of medium porosity $\varepsilon = 0.1, 0.5, 0.9$.}
\end{figure}
Figure 2: Variation of the growth rate $n_r$ (real part of $n$) with respect to wavenumber $k$ for three values of for kinematic viscosity $v_1 = 2, 3, 4$.

Figure 2 and 3 depict the variation of the growth rate $n$ with respect to wavenumber $k$ for three different values of the kinematic viscosity $v_1 = 2$, 3, 4 for a fixed $v_2 = 1$ and $v_2 = 2$, 3, 4 for a fixed of the lower fluid and upper fluid for respectively. For fixed permissible values of $v_1 = 1, \varepsilon = 0.1, v_2 = 1, v_1' = 10, H_x = 10, H_y = 10, g = 980 \text{ cm/s}^2, k_1 = 5, \alpha_1 = 0.3, \alpha_2 = 0.7, k_x = k/\sqrt{2}, k_y = k/\sqrt{2}, \rho_1 = 1, \rho_2 = 1.2$. It is clear from the graphs that the growth rate increases with increase in kinematic viscosities of both the fluids implying thereby the destabilizing effect of the kinematic viscosities of the fluids on system.
Figure 3: Variation of the growth rate $r_r$ (real part of $n$) with respect to wave number $k$ for three values of for kinematic viscosity $v_2 = 2, 3, 4$.

In figure 4 variation of growth rate with respect to wavenumber is plotted for different three values of the kinematic viscosity of couple-stress $v_1 = 3, 4, 5$ and fixed permissible values of $v_1 = 1$, $v_2 = 1$, $H_x = 10$, $H_y = 10$, $g = 980 \text{ cm/s}^2$, $k_1 = 5$, $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, $k_x = k/\sqrt{2}$, $k_y = k/\sqrt{2}$, $\rho_1 = 1$, $\rho_2 = 1.2$, $\varepsilon = 0.1$, respectively. It is depicted from the graphs that the growth rates decreases with the increase in viscosity of couple-stress fluid for a fixed wavenumber showing thereby the stabilizing effect (slight) of the system.
Figure 4: Variation of growth rate $n_r$ (real part of $n$) with respect to wavenumber is plotted for different three values of the kinematic viscosity of couple-stress $\nu_i = 3, 4, 5$.

5.7 CONCLUSIONS

The numerical calculations, although carried out for a few representative values of the physical parameters involved, reveal the tendencies of the nature of the physical effect of the instability of the superposed viscous and viscoelastic fluids saturating porous media in the presence of two dimensional uniform magnetic field. The principal conclusions drawn are as under:

(i) The medium porosity, kinematic viscosities of lower fluid and upper fluid have a destabilizing effect (of approximate similar order of magnitude) on the system for a fixed wavenumber.
(ii) The viscoelasticity of the couple-stress fluid has a stabilizing effect (to a small extent) on the system for a fixed wavenumber. It is also observed that none of the parameters signify the complete stability of the system.