CHAPTER 2

PSO BASED ECONOMIC LOAD DISPATCH PROBLEMS

2.1. INTRODUCTION

The main aim of electric power utilities is to provide high-quality, reliable power supply to the consumers at the lowest possible cost while operating to meet the limits and constraints imposed on the generating units. This formulates the economic load dispatch (ELD) problem for finding the optimal combination of the output power of all the online generating units that minimizes the total fuel cost, while satisfying an equality constraint and a set of inequality constraints. As the cost of power generation is exorbitant, an optimum dispatch results in economy.

In recent years, with an increasing awareness of the environmental pollution caused by thermal power plants, limiting the emission of pollutants is becoming a crucial issue in economic power dispatch. The conventional economic power dispatch cannot meet the environmental protection requirements, since it only considers minimizing the total fuel cost. The multiobjective generation dispatch in electric power systems treats economic and emission impact as competing objectives, which requires some reasonable tradeoff among objectives to reach an optimal solution. This formulates the combined economic emission dispatch (CEED) problem with an objective to dispatch the electric power considering both economic and environmental concerns.

Practically, the real world input–output characteristics of the generating units are highly nonlinear, nonsmooth and discrete in nature owing to prohibited operating zones, ramp rate limits and multifuel effects. Thus the resultant ELD is a challenging nonconvex optimization problem, which is difficult to solve using the traditional methods.

In this work, particle swarm optimization (PSO) algorithm is proposed to solve the various types of economic load dispatch problems in power systems such as
economic load dispatch (ELD) for combined cycle cogeneration plant (CCCP), combined economic emission dispatch (CEED) and the economic load dispatch (ELD) with prohibited operating zones considering ramp rate limits. The feasibility of the proposed method is demonstrated on six different systems and the numerical results were compared with classical and other evolutionary computing techniques.

2.2. ECONOMIC LOAD DISPATCH PROBLEM

2.2.1. Problem Description

Economic load dispatch problem is the sub problem of optimal power flow (OPF). The main objective of ELD is to minimize the fuel cost while satisfying the load demand with transmission constraints.

2.2.2. Objective Function

The classical ELD with power balance and generation limit constraints has been formulated [82] as follows.

Minimize $F = \sum_{i=1}^{d} F_i(P_i)$

where $F_i$ is the total fuel cost of generation, $F_i(P_i)$ is the fuel cost function of $i^{th}$ generator, $a_i, b_i, c_i$ are the cost coefficients of $i^{th}$ generator, $P_i$ is the real power generation of $i^{th}$ generator, $d$ represents the number of generators connected in the network.

The minimum value of the above objective function has to be found by satisfying the following constraints.

The power balance constraint [82]

$$\sum_{i=1}^{d} P_i = P_D + P_L$$  \hspace{1cm} (2.3)
where $P_D$ is the total load of the system and $P_L$ is the transmission losses of the system.

The total transmission loss [83]

$$P_L = \sum_m \sum_n P_{i,m} B_{mn} P_{i,n}$$  \hspace{1cm} (2.4)

where $P_{i,m}$ and $P_{i,n}$ are the real power injections at $m^{th}$ and $n^{th}$ buses and $B_{mn}$ are the B-coefficients of transmission loss formula.

The inequality constraint on real power generation $P_i$ for each generator [82] is

$$P_i^{mn} \leq P_i \leq P_i^{max}$$  \hspace{1cm} (2.5)

where $P_i^{mn}$ and $P_i^{max}$ are, respectively, minimum and maximum values of real power allowed at generator $i$.

A. Economic Load Dispatch Problem with CCCP

Cogeneration units play an increasingly important role in the utility industry. The mutual dependencies of the multiple demand and heat-power capacity of the cogeneration units introduce a complication of integrating the system for economic power dispatch. The cost characteristics of CCCP system (two 75 MW gas turbines and one 50 MW steam turbine) [82] is obtained and hence can be found that they are not differentiable. So the lambda-iterative method will fail in obtaining solution for the ELD of the above problem. The solution for this problem is obtained by formulating the cost equations by curve fitting technique and implementing the proposed PSO algorithm for the optimal scheduling of generators.
B. Constraint Satisfaction Technique

To satisfy the equality constraint of equation (2.3), loading of any one of the units is selected as the dependent loading $P_{du}$, and its present value is replaced by the value calculated according to the following equation [83]:

$$\sum_{i=1, i \neq du}^{d} P_i$$

(2.6)

where, $P_{du}$ can be calculated directly from the equation (2.6) with the known power demand $P_D$ and the known values of remaining loading of the generators. Therefore, the dispatch solution always satisfies the power balance constraint provided that $P_{du}$ also satisfies the operation limit constraint as given in equation (2.5). An infeasible solution is omitted and above procedure is repeated until $P_{du}$ lies within its operational limit. As $P_{L}$ also depends on $P_{du}$, an expression for $P_{L}$ can be substituted in terms of $P_1, P_2, ..., P_{du}, ..., P_d$ and $B_{mn}$ coefficients. After substituting $P_{L}$ in the equation (2.6), the independent and dependent generator terms are separated to obtain a quadratic equation for $P_{du}$. The power balance equality condition is exactly met by solving the quadratic equation for $P_{du}$.

2.2.3. Features of Particle Swarm Optimization (PSO)

Particle swarm optimization was first introduced by Kennedy and Eberhart in the year 1995 [5]. It is an exciting new methodology in evolutionary computation and a population-based optimization tool like GA. PSO is motivated from the simulation of the behaviour of social systems such as fish schooling and birds flocking [84]. The PSO algorithm requires less computation time and less memory because of its inherent simplicity. The basic assumption behind the PSO algorithm is that birds find food by flocking and not individually. This leads to the assumption that information is owned jointly in the flocking. The swarm initially has a population of random solutions. Each potential solution, called a particle (agent), is given a random velocity and is flown through the problem space. All the particles have memory and each particle keeps track of its previous best position (pbest) and the corresponding fitness value. The swarm has another value called gbest, which is the best value of all the
Particle swarm optimization has been found to be extremely effective in solving a wide range of engineering problems and solves them very quickly.

In a PSO system, population of particles exists in the n-dimensional search space. Each particle has certain amount of knowledge and will move about the search space on the basis of this knowledge. The particle has some inertia attributed to it and hence will continue to have a component of motion in the direction it is moving. The particle knows its location in the search space and will encounter with the best solution. The particle will then modify its direction such that it has additional components towards its own best position, pbest and towards the overall best position, gbest. The particle updates its velocity [64] and position [64] using the following equations (2.7) and (2.8)

\[
V_{i}^{(k+1)} = WV_i^k + c_1 \text{Rand}_1() \times (\text{pbest}_i - S_i^k) + c_2 \text{Rand}_2() \times (\text{gbest}_i - S_i^k) \quad (2.7)
\]

\[
S_i^{(k+1)} = S_i^k + V_i^{k+1} \quad (2.8)
\]

where \(V_i^k\) is the velocity of individual \(i\) at iteration \(k\)

- \(k\) is pointer of iterations,
- \(W\) is the weighing factor,
- \(c_1, c_2\) are the acceleration coefficients,
- \(\text{Rand}_1()\), \(\text{Rand}_2()\) are the random numbers between 0 and 1,
- \(S_i^k\) is the current position of individual \(i\) at iteration \(k\),
- \(\text{pbest}_i\) is the best position of individual \(i\)
- \(\text{gbest}\) is the best position of the group

In equation (2.7), \(c_1\) has a range (1.5, 2), which is called self-confidence range; \(c_2\) has a range (2, 2.5), which is called swarm range. The coefficients \(c_1\) and \(c_2\) pull each particle towards \(\text{pbest}\) and \(\text{gbest}\) positions. Low values of acceleration coefficients allow particles to roam far from the target regions, before being tugged back. On the other hand, high values result in abrupt movement towards or past the target regions. Hence, the acceleration coefficients \(c_1\) and \(c_2\) are often set to be 2 according to past experiences. The term \(c_1 \text{Rand}_1() \times (\text{pbest}_i - S_i^k)\) is called particle memory influence or cognition part which represents the private thinking of the
particle itself and the term $c_2 \text{Rand}_1() \times (\text{gbest} - S_i)$ is called swarm influence or the social part which represents the collaboration among the particles.

In the procedure of the particle swarm paradigm, the value of maximum allowed particle velocity $V_{\text{max}}$ determines the resolution, or fitness, with which regions are to be searched between the present position and the target position. If $V_{\text{max}}$ is too high, particles may fly past good solutions. If $V_{\text{max}}$ is too small, particles may not explore sufficiently beyond local solutions. Thus, the system parameter $V_{\text{max}}$ has the beneficial effect of preventing explosion and scales the exploration of the particle search. The choice of a value for $V_{\text{max}}$ is often set at 10–20% of the dynamic range of the variable for each problem.

Suitable selection of inertia weight $W$ provides a balance between global and local explorations, thus requiring less iteration on an average to find a sufficiently optimal solution. Since $W$ decreases linearly from about 0.9 to 0.4 quite often during a run, the following weighing function [64] is used in (2.7):

$$W = W_{\text{max}} - \frac{W_{\text{max}} - W_{\text{min}}}{\text{iter}_{\text{max}}} \times \text{iter}$$  \hspace{1cm} (2.9)

where $W_{\text{max}}$ is the initial weight,
$W_{\text{min}}$ is the final weight,
$\text{iter}_{\text{max}}$ is the maximum iteration number,
$\text{iter}$ is the current iteration number.

Fig. 2.1 shows a concept of modification of a searching point by PSO [64] and Fig. 2.2 shows a searching concept with agents in a solution space. Each particle changes its current position using the integration of vectors [64] as shown in Fig. 2.2.
The equation (2.7) is used to calculate the particle's new velocity according to its previous velocity and the distances of its current position from its own best experience (position) and the group's best experience. Then the particle flies towards a new position according to (2.8). The performance of each particle is measured according to a predefined fitness function, which is related to the problem to be solved.

The step by step procedure of PSO algorithm is given as follows:

1. Initialize a population of particles with random values and velocities within the d-dimensional search space. Initialize the maximum allowable velocity magnitude of any particle $V^{\text{max}}$. Evaluate the fitness of each particle and assign
the particle's position to pbest position and fitness to pbest fitness. Identify the best among the pbest as gbest.

2. Change the velocity and position of the particle according to equations (2.7) and (2.8), respectively.

3. For each particle, evaluate the fitness, if all decisions variables are within the search ranges.

4. Compare the particle's fitness evaluation with its previous pbest. If the current value is better than the previous pbest, then set the pbest value equal to the current value and the pbest location equal to the current location in the d-dimensional search space.

5. Compare the best current fitness evaluation with the population gbest. If the current value is better than the population gbest, then reset the gbest to the current best position and the fitness value to current fitness value.

6. Repeat steps 2–5 until a stopping criterion, such as sufficiently good gbest fitness or a maximum number of iterations/function evaluations is met.
The general flowchart of PSO is illustrated as follows:

Fig. 2.3. Flowchart of PSO method

Advantages of PSO:

PSO is a population-based evolutionary technique that has many advantages over other optimization techniques. PSO is a derivative-free algorithm unlike many conventional techniques and is less sensitive to the nature of the objective function, viz., convexity or continuity. These swarm intelligence based methods have few parameters to adjust and escape local minima. The proposed method is easy to implement and program with basic mathematical and logical operations. It can also handle objective functions with stochastic nature and does not require a good initial solution to start its iteration process.

2.2.4. Implementation of PSO for ELD solution

The main objective of ELD is to obtain the amount of real power to be generated by each committed generator, while achieving a minimum generation cost
within the constraints. The details of the implementation of PSO components are summarized in the following subsections.

2.2.4.1. Representation of an Individual String

For an efficient evolutionary method, the representation of chromosome strings of the problem parameter set is important [42]. Since the decision variables of the ELD problems are real power generations, the generation power output of each unit is represented as a gene, and many genes comprise an individual in the swarm. Each individual within the population represents a candidate solution for an ELD problem. For example, if there are \( d \) units that must be operated to provide power to loads, then the \( i^{th} \) individual \( P_{gi} \) can be defined [42] as follows:

\[
P_{gi} = [P_{1i}, P_{2i}, \ldots, P_{di}] \quad i=1, 2, \ldots, n
\]

(2.10)

where \( n \) means population size, \( d \) is the number of generator, \( P_{di} \) is the generation power output of \( d^{th} \) unit at individual \( i \). The dimension of a population is \((n \times d)\). These genes in each individual are represented as real values. The matrix representation of a population is as follows:

<table>
<thead>
<tr>
<th>Individual number</th>
<th>( P_{11} )</th>
<th>( P_{12} )</th>
<th>\ldots</th>
<th>( P_{1(d-1)} )</th>
<th>( P_{1d} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>420.03</td>
<td>150.32</td>
<td>\ldots</td>
<td>75.12</td>
<td>45.55</td>
</tr>
<tr>
<td>2</td>
<td>390.28</td>
<td>165.35</td>
<td>\ldots</td>
<td>80.23</td>
<td>41.93</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>n</td>
<td>412.88</td>
<td>156.84</td>
<td>\ldots</td>
<td>78.11</td>
<td>42.78</td>
</tr>
</tbody>
</table>

2.2.4.2. Evaluation Function

The evaluation function for evaluating the minimum generation cost of each individual in the population is adopted [42] as follows:

\[
\text{Minimize } F_i = \sum_{i=1}^{d} F_i(P_i)
\]

(2.11)
2.2.5. Algorithm of the Proposed Method

The search procedure for calculating the optimal generation quantity of each unit is summarized as follows:

1. In the ELD problems the number of online generating units is the 'dimension' of this problem. The particles are randomly generated between the maximum and the minimum operating limits of the generators and represented using equation (2.10).

2. To each individual of the population calculate the dependent unit output $P_{du}$ from the power balance equation and employ the $B$-coefficient loss formula to calculate the transmission loss $P_L$ using constraint satisfaction technique.

3. Calculate the evaluation value of each individual $P_g$ in the population using the evaluation function $f$, given by equation (2.11).

4. Compare each individual's evaluation value with its $pbest$. The best evaluation value among the $pbest$ is identified as $gbest$.

5. Modify the member velocity $V$ of each individual $P_g$ according to the following equation:

$$V_{id}^{(t+1)}=W V_{id}^{(t)}+c_1 \text{Rand}(\cdot)(pbest_{id}-P_{g_{id}}^{(0)})+c_2 \text{Rand}(\cdot)(gbest_d-P_{g_{id}}^{(0)})$$

$$i=1,2,...,n, \quad d=1,2,...,m \quad (2.12)$$

where $n$ is the population size, $m$ is the generator units.

6. Check the velocity constraints of the members of each individual from the following conditions [42]:

If $V_{id}^{(t+1)} > V_d^{\text{max}}$, then $V_{id}^{(t+1)} = V_d^{\text{max}}$,

If $V_{id}^{(t+1)} < V_d^{\text{min}}$, then $V_{id}^{(t+1)} = V_d^{\text{min}}$,  \hspace{1cm} (2.13)

where $V_d^{\text{min}} = -0.5 P_d^{\text{min}}$

$V_d^{\text{max}} = +0.5 P_d^{\text{max}}$
7. Modify the member position of each individual \( P_p \) [42] according to the equation
\[
P_{s_{a}}^{(t+1)} = P_{s_{a}}^{(t)} + V_{s_{a}}^{(t+1)}
\] (2.14)

\( P_{s_{a}}^{(t+1)} \) must satisfy the constraints, namely the generating limits, described by equation (2.5). If \( P_{s_{a}}^{(t+1)} \) violates the constraints, then \( P_{s_{a}}^{(t+1)} \) must be modified towards the nearest margin of the feasible solution.

8. If the evaluation value of each individual is better than previous pbest, the current value is set to be pbest. If the best pbest is better than gbest, the best pbest is set to be gbest.

9. If the number of iterations reaches the maximum, then go to step 10. Otherwise, go to step 2.

10. The individual that generates the latest gbest is the optimal generation power of each unit with the minimum total generation cost.

2.2.6. Numerical Examples, Simulation Results and Analysis

The study has been conducted on test cases with 3-unit thermal, 6-unit thermal and two thermal units with 1-unit as combined cycle cogeneration plant system. The description of the test systems are described in the following sections.

Test Case 1: Three-Unit Thermal System

The cost coefficients of 3-unit thermal system are taken from [82]. The cost equations are given below in Rs/h:

\[
F_1 = 0.00156 P_1^2 + 7.92 P_1 + 561 \text{ Rs/h}
\]
\[
F_2 = 0.00194 P_2^2 + 7.85 P_2 + 310 \text{ Rs/h}
\]
\[
F_3 = 0.00482 P_3^2 + 7.97 P_3 + 78 \text{ Rs/h}
\]
\[ \mathbf{B}_{nn} \text{ coefficient matrix:} \]
\[
\begin{bmatrix}
0.000075 & 0.000005 & 0.0000075 \\
0.000005 & 0.000015 & 0.0000100 \\
0.000075 & 0.000010 & 0.0000450
\end{bmatrix}
\]

The unit operating ranges are
\[
100 \text{ MW} \leq P_1 \leq 600 \text{ MW} ;
\]
\[
100 \text{ MW} \leq P_2 \leq 400 \text{ MW} ;
\]
\[
50 \text{ MW} \leq P_3 \leq 200 \text{ MW} ;
\]

Test Case 2: Two Thermal Units and One CCCP System

In this case, the first two units are the same as 3-unit system and the third unit is replaced with a combined cycle cogeneration plant (CCCP). In CCCP, gas and steam turbines are working in combination to generate electric power. CCCP has two 75 MW gas turbine units and one 50 MW steam turbine unit [82]. The fuel cost characteristics of this plant is shown in Fig. 2.4

![Fig. 2.4. Fuel cost characteristics of CCCP system](image-url)
By the method of curve fitting, the cost equation for third plant is formed as follows.

\[ F_3 = 8.517P_3 + 62.75 \text{ Rs/h} \]
\[ = 605.67 \text{ Rs/h} \]
\[ = 24.08 P_3 - 1390.04 \text{ Rs/h} \]
\[ = 9.18 P_3 + 6.829 \text{ Rs/h} \]
\[ = 1452.84 \text{ Rs/h} \]
\[ = 17.62 P_3 - 1660 \text{ Rs/h} \]

50 MW ≤ \( P_3 \) ≤ 63.75 MW;
63.75 MW ≤ \( P_3 \) ≤ 82.875 MW;
82.875 MW ≤ \( P_3 \) ≤ 93.75 MW;
93.75 MW ≤ \( P_3 \) ≤ 157.5 MW;
157.5 MW ≤ \( P_3 \) ≤ 176.625 MW;
176.625 MW ≤ \( P_3 \) ≤ 200 MW;

Test Case 3: Six-Unit Thermal System

The system tested consists of six-thermal units [83]. The cost coefficients of the system are given below in Rs/h:

\[ F_1 = 0.15247P_1^2 + 38.53973 P_1 + 756.79886 \text{ Rs/h} \]
\[ F_2 = 0.10587P_2^2 + 46.15916 P_2 + 451.32513 \text{ Rs/h} \]
\[ F_3 = 0.02803P_3^2 + 40.39655 P_3 + 1049.9977 \text{ Rs/h} \]
\[ F_4 = 0.03546P_4^2 + 38.30553 P_4 + 1243.5311 \text{ Rs/h} \]
\[ F_5 = 0.02111P_5^2 + 36.32782 P_5 + 1658.5596 \text{ Rs/h} \]
\[ F_6 = 0.01799P_6^2 + 38.27041 P_6 + 1356.6592 \text{ Rs/h} \]

The unit operating ranges are

10 MW ≤ \( P_1 \) ≤ 125 MW;
10 MW ≤ \( P_2 \) ≤ 150 MW;
35 MW ≤ \( P_3 \) ≤ 225 MW;
35 MW ≤ \( P_4 \) ≤ 210 MW;
130 MW ≤ \( P_5 \) ≤ 325 MW;
125 MW ≤ \( P_6 \) ≤ 315 MW;
B_{mn} Coefficient matrix:

\[
B_{mn} = \begin{bmatrix}
0.000140 & 0.000017 & 0.000015 & 0.000019 & 0.000026 & 0.000022 \\
0.000017 & 0.000060 & 0.000013 & 0.000016 & 0.000015 & 0.000022 \\
0.000015 & 0.000013 & 0.000065 & 0.000017 & 0.000024 & 0.000019 \\
0.000019 & 0.000016 & 0.000017 & 0.000071 & 0.000030 & 0.000025 \\
0.000026 & 0.000015 & 0.000024 & 0.000030 & 0.000069 & 0.000032 \\
0.000022 & 0.000020 & 0.000019 & 0.000025 & 0.000032 & 0.000085 \\
\end{bmatrix}
\]

To verify the feasibility of the proposed PSO method, three different power systems were tested, under the same evaluation function and individual definition. 50 trials were performed to observe the evolutionary process and to compare their solution quality, convergence characteristic, and computation efficiency. From the experiences of many experiments the following parameters are selected for the particle swarm optimization algorithm to solve the above test cases and are given in Table 2.1. For implementing the above algorithm, the simulation studies were carried out on P-IV, 2.4 GHz, 512 MBDDR RAM system in MATLAB environment.

| Table 2.1. Parameters used in PSO method - 3, 6 and CCCP unit systems |
|------------------|------------------|
| Parameter        | Value            |
| Population size  | 10               |
| W_{max}          | 0.9              |
| W_{min}          | 0.4              |
| Acceleration Coefficients c_1, c_2 | 2.0, 2.0 |

2.2.6.1. Test Case 1: Three-Unit Thermal System

The economic load dispatch for the first test case with the corresponding loads is given as 585 MW, 700 MW and 800 MW, respectively. The proposed PSO method is applied to obtain the minimum generation cost. Table 2.2. provides the results of optimal scheduling of generators obtained by proposed PSO method for three thermal units system with losses neglected. Table 2.3. provides a comparison of economic load dispatch results obtained by various optimization methods for a three unit thermal system with losses neglected.
The above system is solved for a load demand of 585.33 MW using the proposed PSO method with the inclusion of transmission loss. The optimal scheduling of generators obtained by the PSO algorithm for a three-unit thermal system was shown in Table 2.4. By following the above procedure, the solution obtained by the proposed method for a three unit thermal unit system with losses included is given in Table 2.5.
Table 2.4. Optimal scheduling of generators including losses – 3-unit system

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load Demand $P_D$ (MW)</th>
<th>$P_1$ (MW)</th>
<th>$P_2$ (MW)</th>
<th>$P_3$ (MW)</th>
<th>$F_t$ (Rs/h)</th>
<th>Loss, $P_L$ (MW)</th>
<th>Execution Time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>585.33</td>
<td>233.2</td>
<td>267.8</td>
<td>90.84</td>
<td>5886</td>
<td>6.95</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 2.5. Solution of different methods including losses – 3-unit system

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load Demand $P_D$ (MW)</th>
<th>Conventional Method $[82]$ (Rs/h)</th>
<th>GA Method $[82]$ (Rs/h)</th>
<th>Proposed PSO Method (Rs/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>585.33</td>
<td>5890.06</td>
<td>5890.09</td>
<td>5886.9</td>
</tr>
</tbody>
</table>

Fig. 2.5. shows the convergence characteristics of the proposed algorithm for a load demand ($P_D$) of 585 MW with losses neglected. Fig. 2.6. shows the reliability of the proposed algorithm for different runs of the program. The figure shows that the algorithm is capable of obtaining a faster convergence for the three unit thermal system in a very few generations and the solution is consistent.

![Convergence Property](image)

Fig. 2.5. PSO based ELD convergence characteristics – 3-unit system
2.2.6.2. Test Case 2: Three-Unit System with CCCP

The economic load dispatch is solved using a proposed PSO algorithm for a three unit system with CCCP having system load as 680 MW, 750 MW and 869 MW, respectively. Table 2.6. summarizes the optimal dispatch of load among the available generating units. The simulation results were studied and the obtained values of cost of generation of different methods are given in Table 2.7. The cost was found to be minimum in the PSO based method.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load Demand (MW)</th>
<th>P₁ (MW)</th>
<th>P₂ (MW)</th>
<th>P₃ (MW)</th>
<th>Loss Pₖ (MW)</th>
<th>F₁ (Rs/h)</th>
<th>Execution Time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>680</td>
<td>283.6643</td>
<td>323.1737</td>
<td>82.8777</td>
<td>9.7157</td>
<td>6588.3</td>
<td>0.1560</td>
</tr>
<tr>
<td>2.</td>
<td>750</td>
<td>273.0729</td>
<td>311.4274</td>
<td>176.6250</td>
<td>11.1253</td>
<td>7235.1</td>
<td>0.1410</td>
</tr>
<tr>
<td>3.</td>
<td>869</td>
<td>328.6344</td>
<td>378.8490</td>
<td>176.6274</td>
<td>15.1108</td>
<td>8346.8</td>
<td>0.1570</td>
</tr>
</tbody>
</table>
Table 2.7. Solution of different methods including CCCP – 3-unit system

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load Demand (MW)</th>
<th>GA Method (Rs/h) [82]</th>
<th>Proposed PSO Method (Rs/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>680</td>
<td>6639.47</td>
<td>6588.3</td>
</tr>
<tr>
<td>2.</td>
<td>750</td>
<td>7267.93</td>
<td>7235.1</td>
</tr>
<tr>
<td>3.</td>
<td>869</td>
<td>8398.07</td>
<td>8346.8</td>
</tr>
</tbody>
</table>

2.2.6.3. Test Case 3: Six-Unit Thermal System

The third case of a six unit thermal system is solved by the PSO method and the optimal scheduling of generators for the load demands of 700 MW and 800 MW is tabulated in Table 2.8. The results obtained by the proposed algorithm are compared with other evolutionary computing techniques and are given in Table 2.9.

Table 2.8. Optimal scheduling of generators – 6-unit system

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load Demand (MW)</th>
<th>P1 (MW)</th>
<th>P2 (MW)</th>
<th>P3 (MW)</th>
<th>P4 (MW)</th>
<th>P5 (MW)</th>
<th>P6 (MW)</th>
<th>F (Rs/h)</th>
<th>Loss PL (MW)</th>
<th>Execution Time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>700</td>
<td>16.56</td>
<td>24.73</td>
<td>138.2</td>
<td>116.6</td>
<td>208.4</td>
<td>214.2</td>
<td>36987</td>
<td>18.8</td>
<td>1.172</td>
</tr>
<tr>
<td>2.</td>
<td>800</td>
<td>25</td>
<td>12</td>
<td>116</td>
<td>182</td>
<td>287</td>
<td>203</td>
<td>42114</td>
<td>26</td>
<td>1.609</td>
</tr>
</tbody>
</table>
Table 2.9. Solution of different methods – 6-unit system

<table>
<thead>
<tr>
<th>Method</th>
<th>Fuel Cost (Rs/h)</th>
<th>Loss $P_L$ (MW)</th>
<th>Execution Time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional method [83]</td>
<td>37288.70</td>
<td>26.57</td>
<td>0.25</td>
</tr>
<tr>
<td>Hybrid GA method [83]</td>
<td>37137.96</td>
<td>23.124</td>
<td>1.21</td>
</tr>
<tr>
<td>Proposed PSO method</td>
<td>36987</td>
<td>18.8447</td>
<td>1.17</td>
</tr>
</tbody>
</table>

From the comparison of results for the test cases, it is demonstrated that the proposed algorithm performs better than the conventional, genetic algorithm and refined genetic based algorithm methods in the aspect of reduction of fuel cost as well as real power loss.

2.3. COMBINED ECONOMIC EMISSION DISPATCH PROBLEM

2.3.1. Problem Description

The optimum economic dispatch may not be the best in terms of the environmental criteria. Harmful ecological effects by the emission of gaseous pollutants from fossil fuel power plants can be reduced by proper load allocation among the various generating units of the plants. But this load allocation may lead to increase in the operating cost of the generating units. So, it is necessary to find out a solution which gives a balanced result between emission and cost. This is achieved by combined economic emission dispatch problem. This dual-objective CEED problem is converted into a single objective function using a price penalty factor approach [46].
2.3.2. Objective Function

Optimization of generation cost has been formulated based on classical ELD with emission and line flow constraints. The detailed problem is given [46] as follows.

\[
F = \text{Min} \sum_{i=1}^{d} f_i(FC, EC) \quad (2.15)
\]

where \( F \) is the optimal cost of generation, \( FC \) and \( EC \) are total fuel cost and emission costs of generators, respectively, \( d \) represents the number of generators connected in the network.

The minimum value of the above objective function has to be found out subject to constraints given by Eqs (2.3) and (2.5).

The power flow equation of the power network [46]

\[
g(|V|, \delta) = 0 \quad (2.16)
\]

where \( g(|V|, \delta) = \begin{bmatrix} P_i \left( |V|, \delta \right) - P_{i, \text{net}} \\ Q_i \left( |V|, \delta \right) - Q_{i, \text{net}} \\ P_{\text{cal}, m} \left( |V|, \delta \right) - P_{\text{spec}, m} \end{bmatrix} \quad (2.17)
\]

where \( P_i \) and \( Q_i \) are calculated real and reactive power for PQ bus \( i \), respectively;

\( P_{i, \text{net}} \) and \( Q_{i, \text{net}} \) are specified real and reactive power for PQ bus \( i \), respectively;

\( P_{\text{cal}, m} \) and \( P_{\text{spec}, m} \) are calculated and specified real power for PV bus \( m \), respectively;

\(|V|\) and \( \delta \) are voltage magnitudes and phase angles of different buses.

The inequality constraint on voltage of each PQ bus [46]

\[
V_{\text{min}} (i) \leq V_i \leq V_{\text{max}} (i) \quad (2.18)
\]
where \( V_{\text{min}}(i) \) and \( V_{\text{max}}(i) \) are minimum and maximum voltage at bus \( i \), respectively.

The maximum power limit on transmission line [46] is given by

\[
L_{f_{\text{calMVA}}} \leq L_{f_{\text{ratedMVA}}}
\]  
(2.19)

where \( n_{l} \) represents number of lines,

\( L_{f_{\text{calMVA}}} \) is the calculated line flow of each transmission line,

\( L_{f_{\text{ratedMVA}}} \) is the rated line flow of each transmission line.

Total fuel cost of generation \( FC \) in terms of control variables generator powers can be expressed [46] as follows.

\[
FC (P) = \sum_{i=1}^{d} (a_{i} P_{i}^{2} + b_{i} P_{i} + c_{i}) \quad \text{$/h}$  
(2.20)

where \( P_{i} \) is the real power output of an \( i^{th} \) generator in MW, 

\( i \) represents the corresponding generator,

\( a_{i}, b_{i}, c_{i} \) are the fuel cost coefficients of generators.

The total emission release can be expressed [46] as

\[
EC(P) = \sum_{i=1}^{d} (\alpha_{i} P_{i}^{2} + \beta_{i} P_{i} + \gamma_{i}) \quad \text{kg/h}$  
(2.21)

where \( \alpha_{i}, \beta_{i}, \gamma_{i} \) are emission coefficients of generators.

The dual-objective combined economic emission dispatch problem is converted into single optimization problem by introducing a price penalty factor \( h \) [92] as follows.

\[
\text{Minimize } \Phi_{i} = FC + h \times EC \quad \text{$/h}$  
(2.22)
subjected to the power flow constraints of equation (2.3, 2.5, 2.17, 2.18 and 2.19). The price penalty factor \( h \) blends the emission with fuel cost and \( \Phi_1 \) is the total operating cost in \$/h.

The price penalty factor \( h_i \) is the ratio between the maximum fuel cost and maximum emission of corresponding generator.

\[
h_i = \frac{FC(P_i^{\text{max}})}{EC(P_i^{\text{max}})} \quad \text{$/kg} \quad i = 1, 2, \ldots, d
\]

(2.23)

The following steps are used to find the price penalty factor for a particular load demand:

1. Find the ratio between maximum fuel cost and maximum emission of each generator
2. Arrange the values of price penalty factor in ascending order.
3. Add the maximum capacity of each unit \( (P_i^{\text{max}}) \) one at a time, starting from the smallest \( h_i \) until \( \sum P_i^{\text{max}} \geq P_D \).
4. At this stage, \( h_i \) associated with the last unit in the process is the price penalty factor \( h \) for the given load.

The above procedure gives the approximate value of price penalty factor computation for the corresponding load demand. Hence a modified price penalty factor \( (h_m) \) is used to give the exact value for the particular load demand. The first two steps of \( h \) computation remain the same for the calculation of modified price penalty factor. Then it is calculated by interpolating the values of \( h_i \) corresponding to their load demand values.

2.3.3. Step-by-Step algorithm

The step-by-step algorithm for the proposed method is explained as follows:

1. Specify the maximum and minimum limits of generation power of each generating unit, maximum number of iterations to be performed and fuel cost coefficient of each unit.
2. Initialize randomly the individuals of the population according to the limit of each unit including individual dimensions, searching points, and velocities. These initial individuals must be feasible candidate solutions that satisfy the practical operation constraints.

3. To each chromosome of the population the dependent unit output \( P_d \) will be calculated from the power balance equation and B-coefficient loss formula is employed to calculate the transmission loss \( P_t \).

4. Calculate the evaluation value of each population \( P_k \) using the evaluation equations (2.20) and (2.21).

5. Calculate the price penalty factor using the equation (2.24).

6. Compute the new evaluation function using the equation (2.22).

7. Compare each population's evaluation value with its pbest. The best evaluation value among the pbest is denoted as gbest.

8. Modify the member velocity \( V \) of each individual \( P_k \) according to the equation (2.12).

9. Check the velocity components constraint occurring in the limits using the conditions (2.13).

10. Modify the member position of each individual \( P_k \) according to the equation (2.14). If \( P_{k, \text{d} (t+1)} \) must satisfy the constraints, namely the generating limits, described by (2.5). If \( P_{k, \text{d} (t+1)} \) violates the constraints, then \( P_{k, \text{d} (t+1)} \) must be modified towards the near margin of the feasible solution.

11. If the evaluation value of each population is better than the previous pbest the current value is set to be pbest. If the best pbest is better than the gbest the value is set to be gbest.

12. If the number of iterations reaches the maximum then go to step 13, otherwise go to step 3.

13. The individual that generates the latest gbest is the optimal generation power of each unit.

14. After obtaining the global optimum solution, power flow is computed using Newton-Raphson method and the calculated MVA of line flow is compared with the rated MVA of line flow.
15. If the line is found to be overloaded previous gbest value is chosen as the global optimum solution.

16. Stop

2.3.4. Simulation Results

The proposed algorithm is applied to an IEEE-30 bus system. The total system load demand is 283.4 MW whose data has been given in Appendix-A. In the proposed approach minimum generation cost of the generating units was obtained using PSO based ELD in CEED environment. The line flows in the system was compared using Newton-Raphson method. The simulation studies were carried out using a P-IV 2.4 GHz, 512 MB DDR RAM system in MATLAB environment. Table 2.10. provides the simulation parameters of the proposed PSO algorithm. The execution time for the PSO based method is obtained as 79.9220 seconds. Fig. 2.7. shows the convergence characteristics of PSO based ELD algorithm. The maximum, average and minimum cost of generation are presented for an IEEE-30 bus system. Table 2.11. summarizes the minimum solution obtained by particle swarm optimization based economic load dispatch with line flow constraints for the bus system. The minimum solution includes optimum generations, total loss, total fuel cost for IEEE-30 bus system. Table 2.12. shows the comparison of optimal generation schedule obtained by the PSO based method with other evolutionary techniques. The best generation of IEEE-30 bus generating units obtained from PSO-CEED algorithm and is presented in the Fig. 2.8.

Table 2.10. Parameters used in PSO method – IEEE-30 bus system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>50</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>100</td>
</tr>
<tr>
<td>( W_{\text{max}} )</td>
<td>0.9</td>
</tr>
<tr>
<td>( W_{\text{min}} )</td>
<td>0.4</td>
</tr>
<tr>
<td>Acceleration coefficients, ( c_1, c_2 )</td>
<td>2.0, 2.0</td>
</tr>
</tbody>
</table>
Table 2.11. ELD results obtained by various methods – IEEE-30 bus system

<table>
<thead>
<tr>
<th>Method</th>
<th>Overall cost ($/h)</th>
<th>Loss (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ED-LF [85]</td>
<td>805.4500</td>
<td>11.0400</td>
</tr>
<tr>
<td>SA [85]</td>
<td>804.4300</td>
<td>10.5800</td>
</tr>
<tr>
<td>Proposed PSO</td>
<td>801.7708</td>
<td>6.2326</td>
</tr>
</tbody>
</table>

Table 2.12. Minimum power dispatch results by various methods – IEEE-30 bus system

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>192.65</td>
<td>188.02</td>
<td>173.848</td>
<td>155.6326</td>
</tr>
<tr>
<td>P2</td>
<td>48.92</td>
<td>47.45</td>
<td>49.998</td>
<td>20.0000</td>
</tr>
<tr>
<td>P3</td>
<td>19.26</td>
<td>19.77</td>
<td>21.386</td>
<td>42.0000</td>
</tr>
<tr>
<td>P4</td>
<td>10.58</td>
<td>13.40</td>
<td>22.630</td>
<td>35.0000</td>
</tr>
<tr>
<td>P5</td>
<td>10.79</td>
<td>11.25</td>
<td>12.928</td>
<td>25.0000</td>
</tr>
<tr>
<td>P6</td>
<td>12.24</td>
<td>14.09</td>
<td>12.000</td>
<td>12.0000</td>
</tr>
<tr>
<td>P7</td>
<td>11.04</td>
<td>10.58</td>
<td>9.390</td>
<td>6.2326</td>
</tr>
</tbody>
</table>

Fig. 2.7. PSO-based ELD convergence characteristics – IEEE-30 bus system
Fig. 2.8. Best generator settings of PSO-based CEED method – IEEE-30 bus system

The line flows in MVA for the combined economic emission dispatch corresponding to the best generation schedule for IEEE-30 bus system are shown in Table 2.13. The lines were not overloaded with the economic scheduling of generators.

**Table 2.13. Line flows with line flow constraints – IEEE-30 bus system**

<table>
<thead>
<tr>
<th>From Bus</th>
<th>To Bus</th>
<th>Line flow in MVA</th>
<th>Rated MVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>80 5936</td>
<td>130</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>40 1548</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>25 4572</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>37 2538</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>53 1516</td>
<td>130</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>30 6299</td>
<td>65</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>32 2271</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>15 9236</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>37 2409</td>
<td>130</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>17 5155</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>8 7433</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>8 9229</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>32 8083</td>
<td>65</td>
</tr>
</tbody>
</table>
The computational procedure of price penalty factor for IEEE-30 bus system is explained as follows. The ratio between the maximum fuel cost and minimum emission of six generating units are found and arranged in ascending order.

\[ h_i = [h_3, h_4, h_6, h_3, h_2, h_1] \]

\[ h_i = [1.7707, 1.7916, 2.0534, 2.2198, 2.3310, 2.3436] \]
The corresponding maximum limits of generating units are given by:

\[ P_{i,\text{max}} = [30 \ 35 \ 40 \ 50 \ 80 \ 200] \]

For a load of \( P_D \) MW starting from the lowest \( h_i \) value, the maximum capacity of the units is added one by one (\( m \)) and when this total equals or exceeds the load, \( h_i \) associated with the last unit in the process is price penalty factor [92].

\[ m = [30 \ 65 \ 105 \ 155 \ 235 \ 435] \]

For \( P_D = 283.4 \) MW, \((30+65+105+155+235+435)\) MW > 283.4 MW.

Hence price penalty factor (\( h \)) is determined as 2.3436 for IEEE-30 bus system. Even though the price penalty factor was computed for 283.4 MW but it gives the value up to a load demand of 435 MW. So the modified price penalty factor [46] is computed by interpolating the values of \( h_i \) for the last two units by satisfying the corresponding load demand.

\[
h_m = h_{i1} + \left( \frac{h_{i2} - h_{i1}}{P_{\text{max}2} - P_{\text{max}1}} \right) \times (P_D - P_{\text{max}1}) \quad (2.24)
\]

where \( h_m \) is the modified price penalty factor in \$/kg.

- \( h_{i1} \) is the price penalty factor associated with the last unit in \$/kg.
- \( h_{i2} \) is the price penalty factor associated with the current unit in \$/kg.
- \( P_{\text{max}1} \) is the Maximum power associated with the last unit in MW.
- \( P_{\text{max}2} \) is the Maximum power associated with the current unit in MW.

\[
h_i = 2.3310 + \left( \frac{2.3436 - 2.3310}{425 - 235} \right) \times (283.4 - 235)
\]

\[
h_i = 2.3340 \ \$/kg
\]

By following the above procedure, the minimum solution was obtained by the proposed PSO method for IEEE-30 bus system as given in Table 2.14. By incorporating modified price penalty factor approach, the total operating cost of 1624 \$/h was obtained. The convergence characteristics for the PSO method are shown in Fig. 2.9.
Table 2.14. CEED results – IEEE-30 bus system

<table>
<thead>
<tr>
<th>Method</th>
<th>Price Penalty Factor $h_1$ ($$/Kg$)</th>
<th>Fuel Cost $FC$ ($$/h$)</th>
<th>Emission Output $EC$ (Kg/h)</th>
<th>Total Operating Cost $\Phi_t$ ($$/h$)</th>
<th>Total Loss $P_L$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>2.3340</td>
<td>835.5655</td>
<td>377.2407</td>
<td>1624</td>
<td>5.664</td>
</tr>
</tbody>
</table>

Fig. 2.9. PSO-based CEED convergence characteristics – IEEE-30 bus system

2.4. ECONOMIC DISPATCH PROBLEM WITH PROHIBITED OPERATING ZONES

2.4.1. Problem Description

Economic dispatch is an important daily optimization task in the power system operation. Large modern generating units with multivalve steam turbines exhibit a large variation in the input-output characteristic functions. Thus, the practical ED planning must perform optimal generation dispatch among the generating units to
satisfy the system load demand and practical operation constraints of generators that include the ramp rate limits and the prohibited operating zones.

2.4.2. Objective Function

The objective of ED problem is to minimize the total fuel cost of power plants subjected to the operating constraints of a power system. Generally, it can be formulated with an objective function, subject to the practical operation constraints of generator [42].

Minimize \( F_i = \sum_{i=1}^{d} F_i(P_i) \)  \hspace{1cm} (2.25)

\( F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \)  \hspace{1cm} (2.26)

where \( F_i \) is the total generation cost,

\( F_i(P_i) \) is the cost function of \( i^{th} \) generator,

\( a_i, b_i, c_i \) are the cost coefficients of \( i^{th} \) generator,

\( P_i \) is the real power generation of \( i^{th} \) generator,

\( d \) represents the number of generators connected in the network.

The minimum value of the above objective function has to be found out by satisfying the following constraints.

- Power balance Constraint, in which

The cost is optimized with the following power system constraint [42]

\[ \sum_{i=1}^{d} P_i = P_D + P_L \]  \hspace{1cm} (2.27)

where \( P_D \) is the total load of the system and

\( P_L \) is the transmission power loss of the system.

- Ramp rate limit constraint

In ELD research, a number of studies have focused upon the economical aspects of the problem under the assumption that unit generation output can be
adjusted instantaneously. Even though this assumption simplifies the problem, it does not reflect the actual operating processes of the generating unit. The operating range of all on-line units is restricted by their ramp rate limits [50]. Fig. 2.10 shows three possible situations when a unit is on-line from hour t-1 to hour t. Fig 2.10(a) shows that the unit is in a steady operating status. Fig 2.10(b) shows that the unit is in an increasing power generation status. Fig. 2.10(c) shows that the unit is in a decreasing power generation status.

Fig. 2.10. Three possible operating conditions of a generating unit

(i) as generation increases [42]

\[ P_i - P_i^0 \leq UR_i \]  \hspace{1cm} (2.28)

(ii) as generation decreases [42]

\[ P_i^0 - P_i \leq DR_i \]  \hspace{1cm} (2.29)

where \( P_i \) is the power generation of unit i,

\( P_i^0 \) is the power generation of unit i at previous hour,

\( UR_i \) is the ramp rate limit of unit i as power generation increases and

\( DR_i \) is the ramp rate limit of unit i as power generation decreases

The ramp rate constraints restrict the operating range of the physical lower and upper limit to the effective lower limit \( \overline{P_i}^{\text{min}} \) and effective upper limit \( \overline{P_i}^{\text{max}} \), respectively [42].

\[ \overline{P_i}^{\text{min}} = \text{Max} ( P_i^{\text{min}}, P_i^0 - DR_i ) \]

\[ \overline{P_i}^{\text{max}} = \text{Min} ( P_i^{\text{max}}, P_i^0 + UR_i ) \]  \hspace{1cm} (2.30)

Hence the ramp rate constraint is stated [42] as

\[ \overline{P_i}^{\text{min}} \leq P_i \leq \overline{P_i}^{\text{max}} \]  \hspace{1cm} (2.31)

- Prohibited Operating Zone Constraint, in which references [42] have shown the input-output performance curve for a typical thermal unit with many valve
points. These valve points generate many prohibited zones. For a unit with prohibited operating zones, the zones divide the operating region between the minimum generation limit ($P_{\text{mm}}$) and the maximum generation limit ($P_{\text{max}}$). These prohibited operating zones are due to physical limitations of power plant components such as vibrations in a shaft bearing which amplified in a certain operating region. For a prohibited zone, the unit can only operate above or below the zone [42].

\[
\begin{align*}
P_i & \in \begin{cases} 
    P_{\text{mm}}^i \leq P_i \leq P_{\text{th}}^i \\
    P_{\text{l}_i}^i \leq P_i \leq P_{\text{u}_i}^i & \text{for } z = 2,3,\ldots,m_i, \\
    i = 1,2,\ldots,d \\
    P_{\text{l}_i}^\text{ms} \leq P_i \leq P_{\text{max}}^i 
\end{cases}
\end{align*}
\]  

(2.32)

where $z$ is the number of prohibited zones of a unit,

$P_{\text{th}}^i, P_{\text{th}}^i$ are the Lower and Upper limits of $z$ prohibited zones of unit $i$ (MW),

$l_i$ is the lower limit of the prohibited operating zone of unit $i$,

$u_i$ is the upper limit of the prohibited operating zone of unit $i$,

$d$ is the number of generating units,

$P_{\text{mm}}^i$ is the effective lower limit of $i^{th}$ unit with ramp rate constraint,

$P_{\text{max}}^i$ is the effective upper limit of $i^{th}$ unit with ramp rate constraint.

The total transmission network losses is a function of unit power outputs that can be represented using B coefficients [42]:

\[
\begin{align*}
P_{\text{t}} &= \sum_{i=1}^{d} \sum_{j=1}^{d} P_i B_{i,j} B_j + \sum B_{\infty} P_j + B_{\infty}
\end{align*}
\]  

(2.33)

where $P_i$ and $P_j$ are the real power injections at $i^{th}$ and $j^{th}$ buses, respectively,

$B_{ij}$ are the B-coefficients of transmission loss formula,

$B_{\infty}$ is the vector of same length as generators,

$B_{00}$ is a constant.
2.4.3. Features of Genetic Algorithms

A global optimization technique known as genetic algorithm (GA), a probabilistic and heuristic approach is used to solve power system optimization problems. Genetic algorithm, unlike strict mathematical methods, has the apparent ability to adapt to nonlinearities and discontinuities commonly found in the optimization problems. Genetic algorithms are attractive and serve as an alternative tool for solving combinational optimization problems and they are found to be superior in their parallel search ability that climbs many peaks in parallel.

Genetic algorithms are adaptive heuristic search algorithms premised on the evolutionary ideas of natural selection and genetics [4]. The basic concepts of GAs are designed to simulate processes in natural system, necessary for evolution, specifically those that follow the principles first laid down by Charles Darwin of survival of the fittest. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem.

First pioneered by John Holland in 1960, Genetic algorithms have been widely studied, experimented and applied in many fields in engineering world. Not only does GAs provide alternate methods for solving problems but it consistently outperforms other traditional methods in most of the problem links. Many of the real world problems involved in finding optimal parameters, which might prove difficult for traditional methods are ideal for GA’s [49]. They can solve problems that do not have a precisely defined solving methods, or if they do, when following the exact solving method would take far too much time. The genetic algorithm on the other hand, works only with the objective function information in a search space for an optimal parameter set.

2.4.3.1 Components of Genetic Algorithms

GAs are derived from a simple model of population genetics. They have the following five components [86]:

(i) String representation of the control variables.
(ii) An initial population of strings.

(iii) An evaluation function that plays the role of the environment, rating the strings in terms of their fitness i.e., their ability to survive.

(iv) Genetic operators determine the composition of a new population generated from the previous one by reproduction, crossover and mutation.

(v) Value of the parameters that the GAs use.

Since GAs are based on natural genetics, there exists strong analogies between genetic algorithm and natural genetics. The strings are similar to chromosomes in biological systems, where the chromosomes are composed of genes, which may take any of several forms called “alleles” [87]. If the control variables are represented in binary bits and concatenated to form a string, then it is called as binary coded GA and if the control variables are represented in real numbers, then it is called as real coded GA. GAs do not work with a single string but with a population of strings, which evolves iteratively by generating new strings taking the place of their parents. GAs treat the problem, as a black box in which the input is the strings and the output is their fitness.

Three basic operators comprise a GA. They are reproduction, crossover and mutation. Reproduction is the mechanism by which the most highly fit members in a population are selected to pass on information to the next population of members. It effectively selects the fittest of the strings in the current population to be used in generating the next population. In this way, relevant information concerning the fitness of a string is passed along to successive generations. It can be shown that GAs actually allocate exponentially increasing trials to the most fit of these strings. Crossover serves as a mechanism by which strings can exchange information, possibly creating more highly fit strings in the process and allowing the exploration of new regions of the search space [87]. Many types of crossovers are available like single point crossover, multipoint crossover, uniform crossover and window crossover. The last of the GA operators is mutation, and is generally considered as a secondary operator. Mutation ensures that a string position will never be fixed at a certain value for all time. Like other stochastic methods, GAs require a number of parameters, which are population size, probability of crossover, probability of mutation. Usually small population size, high crossover probability and low mutation probability are recommended [88].
The GA's can be distinguished from other optimization methods in four different ways as follows:

(i) GA's use objective function information to guide the search, not the derivatives or other auxiliary information.

(ii) GA's use a coding of the parameters used to calculate the objective function in guiding the search, not the parameter themselves.

(iii) GA's search through many points in the solution space at one time, not a single point.

(iv) GA's use probabilistic rules, not deterministic rules, in moving from one set of solutions (a population) to the next.

The soft computing techniques for optimization are mainly based on GA. Though the GA methods have been employed successfully to solve complex optimization problems, recent research has identified some deficiencies in GA performance. This degradation in efficiency is apparent in applications with highly epistatic objective functions (i.e., where the parameters being optimized are highly correlated) [the crossover and mutation operations cannot ensure better fitness of offspring because chromosomes in the population have similar structures and their average fitness is high towards the end of the evolutionary process] [89], [90]. Moreover, the premature convergence of GA degrades its performance and reduces its search capability that leads to a higher probability toward obtaining a local optimum [89].

2.4.4. GA and PSO Combined Hybrid Method

2.4.4.1. Description of the proposed method

In this work, a hybrid genetic-PSO search (hybrid GA-PSO) algorithm is proposed which utilizes Genetic algorithm to explore the high performance region in solution space and PSO algorithm to exploit the solution space for locating the optimal solution. Thus, the GA guides PSO for better performance in the complex solution space. In this work, the constrained economic dispatch problem is solved by the integrated GA-PSO algorithm and a high quality solution is obtained for a practical power system operation. The GA-PSO algorithm is utilized to determine the
optimal generation power of each unit that was used in operation at the specific period, thus minimizing the total generation cost.

2.4.4.2. Evaluation Function

The evaluation function $f$ (it is called fitness in GA) must be defined for evaluating the fitness of each individual in the population. For emphasizing the best chromosome and faster convergence of the iteration process, the evaluation value is normalized in the range between 0 and 1. The evaluation function $f$ is given in equation (2.34) which is the reciprocal of the summation of generation cost function $F_{\text{cost}}$ and power balance constraint $P_{\text{pbc}}$ [42].

$$f = \frac{1}{F_{\text{cost}} + P_{\text{pbc}}}$$  \hspace{1cm} (2.34)

where

$$F_{\text{cost}} = 1 + \text{abs} \left( \frac{\sum_{i=1}^{d} F_i(P_i) - F_{\text{min}}}{F_{\text{max}} - F_{\text{min}}} \right)$$  \hspace{1cm} (2.35)

$$P_{\text{pbc}} = 1 + \left( \sum_{i=1}^{d} P_i - (P_D + P_I) \right)^2$$  \hspace{1cm} (2.36)

$F_{\text{max}}$ and $F_{\text{min}}$ are the maximum and minimum generation cost among the individuals in the initial population.

2.4.4.3. Application of GA-PSO Algorithm

The sequential steps of the proposed hybrid GA-PSO algorithm are shown as below:

1. Initialize randomly the individuals of the population according to the limit of each unit including individual dimensions, searching points, and velocities. These initial individuals must be feasible candidate solutions that satisfy the operation constraints.
2. To each chromosome of the population the dependent unit output $P_d$ will be calculated from the power balance equation and $B$ coefficient matrix.

3. Calculate the evaluation value of each individual $P_{\text{vi}}$ in the population using the evaluation function $f$ given by equation (2.34).

4. Compare each individual's evaluation value with its pbest. The best evaluation value among the pbest's is denoted as gbest.

5. Modify the member velocity $V$ of each individual $P_{\text{vi}}$ according to the equation (2.12).

6. Check the velocity components constraint occurring in the limits using the conditions (2.13).

7. Modify the member position of each individual $P_{\text{vi}}$ using (2.14), if $P_{\text{vi}}^{(n+1)}$ violates the constraints, then $P_{\text{vi}}^{(n+1)}$ must be modified toward the near margin of the feasible solution.

8. Apply the genetic operators selection, crossover and mutation to the above population and generate offspring. Now compare the parents and offspring to select the fittest chromosomes for the next step.

9. If the evaluation value of each individual is better than previous pbest, the current value is set to be pbest. If the best pbest is better than gbest, the value is set to be gbest.

10. If the number of iterations reaches the maximum, then go to step 11. Otherwise, go to step 2.

11. The individual that generates the latest gbest is the optimal generation power of each unit with the minimum total generation cost.

2.4.5. Simulation Results

To verify the feasibility of the proposed hybrid algorithm, the 6-, 15- and 40-unit systems are tested. The ramp rate limits and prohibited operating zones of the units were taken into consideration. The proposed hybrid GA-PSO algorithm is compared with the GA and the PSO methods. Under each sample system, 50 trials were performed using the same evaluation function with the proposed algorithm. This gives a fair comparison of the proposed GA-PSO method with the aspects of computational efficiency and a qualitative solution. An optimal range of inertia
weight and acceleration factors for the PSO algorithm is estimated for 6-, 15- and 40-
unit test cases in this research. On comparison of the results, it has been demonstrated
that the proposed algorithm is capable of obtaining higher quality of solution
efficiently for economic dispatch problem covering prohibited operating zones.

After many experiments, the following parameters have been selected for the
proposed hybrid GA-PSO algorithm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>50</td>
</tr>
<tr>
<td>Generations</td>
<td>100</td>
</tr>
<tr>
<td>(W_{\text{max}})</td>
<td>0.9</td>
</tr>
<tr>
<td>(W_{\text{mm}})</td>
<td>0.4</td>
</tr>
<tr>
<td>Acceleration coefficients (c_1, c_2)</td>
<td>2.0, 2.0</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>0.55</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Table 2.15. Parameters used in GA-PSO method – 6-unit, 15-unit, 40-unit systems**

2.4.5.1. Six-Unit System

The system contains six thermal units, 26 buses and 46 transmission lines and
the data were taken from [42]. The cost coefficients, generating unit capacity limits,
ramp rate, prohibited operating zones and loss coefficients are provided in Appendix-B.

The load demand is 1263 MW. To simulate this system, each individual \(P_g\) contains
six generator power outputs. Since one unit is considered as a dependent unit, each
individual in the population contains five generator power outputs. The dimension of
the population is equal to 50\times 5. Table 2.16 shows the best solution obtained by the
proposed algorithm and its comparison with the GA and PSO methods. Table 2.17
shows the comparison of average generation cost and average CPU time of different
methods.
Table 2.16. Optimal generator dispatch solution by various methods – 6-unit system

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P_1</td>
<td>474.8066</td>
<td>447.4970</td>
<td>446.3740</td>
</tr>
<tr>
<td>P_2</td>
<td>178.6363</td>
<td>173.3221</td>
<td>174.1230</td>
</tr>
<tr>
<td>P_3</td>
<td>262.2089</td>
<td>263.4745</td>
<td>264.5975</td>
</tr>
<tr>
<td>P_4</td>
<td>134.2826</td>
<td>139.0594</td>
<td>137.8770</td>
</tr>
<tr>
<td>P_5</td>
<td>151.9039</td>
<td>165.4761</td>
<td>163.2020</td>
</tr>
<tr>
<td>P_6</td>
<td>74.1812</td>
<td>87.1280</td>
<td>89.4021</td>
</tr>
<tr>
<td>Total power output (MW)</td>
<td>1276.03</td>
<td>1276.01</td>
<td>1275.5756</td>
</tr>
<tr>
<td>Power loss (MW)</td>
<td>13.0217</td>
<td>12.9584</td>
<td>12.5755</td>
</tr>
<tr>
<td>Total generation cost ($/h)</td>
<td>15,459</td>
<td>15,450</td>
<td>15,444</td>
</tr>
</tbody>
</table>

Table 2.17. Comparison of solution quality – 6-unit system

<table>
<thead>
<tr>
<th>Method</th>
<th>Generation Cost ($/h)</th>
<th>Average CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>GA method [42]</td>
<td>15,459</td>
<td>15,524</td>
</tr>
<tr>
<td>PSO method [42]</td>
<td>15,450</td>
<td>15,492</td>
</tr>
<tr>
<td>Hybrid GA-PSO method</td>
<td>15,444</td>
<td>15,488</td>
</tr>
</tbody>
</table>

The comparison of the performance of proposed algorithm with other methods demonstrates that the solution quality and computation efficiency is good for the hybrid algorithm.
2.4.5.2. Fifteen-Unit System

This system contains 15 thermal generating units whose characteristics and transmission loss (B loss) coefficients are taken from [42] and are provided in Appendix–C. The total power demand of the system is 2630 MW. To simulate this system, each individual $P_g$ contains 15 generator power outputs. Since one unit is considered as a dependent unit, each individual in the population contains 14 generator power outputs. The dimension of the population is equal to $50 \times 14$. The simulation results are shown in Tables 2.18 and 2.19.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Power Output (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>415.3108</td>
</tr>
<tr>
<td>$P_2$</td>
<td>359.7206</td>
</tr>
<tr>
<td>$P_3$</td>
<td>104.4250</td>
</tr>
<tr>
<td>$P_4$</td>
<td>74.9853</td>
</tr>
<tr>
<td>$P_5$</td>
<td>380.2844</td>
</tr>
<tr>
<td>$P_6$</td>
<td>426.7902</td>
</tr>
<tr>
<td>$P_7$</td>
<td>341.3164</td>
</tr>
<tr>
<td>$P_8$</td>
<td>124.7867</td>
</tr>
<tr>
<td>$P_9$</td>
<td>133.1445</td>
</tr>
<tr>
<td>$P_{10}$</td>
<td>89.2567</td>
</tr>
<tr>
<td>$P_{11}$</td>
<td>60.0572</td>
</tr>
<tr>
<td>$P_{12}$</td>
<td>49.9998</td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>38.7713</td>
</tr>
<tr>
<td>$P_{14}$</td>
<td>41.9425</td>
</tr>
<tr>
<td>$P_{15}$</td>
<td>22.6445</td>
</tr>
<tr>
<td>Total power output (MW)</td>
<td>2668.4</td>
</tr>
<tr>
<td>Power loss (MW)</td>
<td>38.2782</td>
</tr>
<tr>
<td>Total generation cost ($/h)</td>
<td>33,113</td>
</tr>
</tbody>
</table>
Table 2.19. Comparison of solution quality – 15-unit system

<table>
<thead>
<tr>
<th>Method</th>
<th>Generation Cost ($/h)</th>
<th>Average execution time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>GA method [42]</td>
<td>33,113</td>
<td>33,337</td>
</tr>
<tr>
<td>PSO method [42]</td>
<td>32,858</td>
<td>33,331</td>
</tr>
<tr>
<td>Hybrid GA-PSO method</td>
<td>32,724</td>
<td>33,188</td>
</tr>
</tbody>
</table>

For a power demand of 2630 MW, the total transmission losses are 31.75 MW the optimal dispatch is obtained by the proposed algorithm. Since the total power output from all the 15 units are 2661.75 MW, the power balance equation is exactly satisfied. The total generation cost as well the power losses are less for the proposed algorithm compared with GA and PSO methods. By comparing the results obtained by various methods, it is found that the proposed algorithm is capable of providing optimal solution.

2.4.5.3. Forty-Unit System

The system consists of 40 units in the realistic Taipower system which is a large-scale and mixed-generating system with coal-fired, oil-fired, gas-fired, diesel and combined cycle cogeneration units [50]. The cost coefficients of Taipower 40-unit are shown in Appendix–D. The system load demand is 8550 MW. Since one unit is considered as a dependent unit, each individual in the population contains 39 generating unit outputs. The dimension of the population is 50 × 39. The simulation results and a comparison of performance are given in Tables 2.20 and 2.21.
Table 2.20. Test results of the proposed approach – 40-unit system

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total power output (MW)</td>
<td>8641.08</td>
<td>8637.26</td>
<td>8636.582</td>
</tr>
<tr>
<td>Power loss (MW)</td>
<td>89.76</td>
<td>87.24</td>
<td>86.58</td>
</tr>
<tr>
<td>Total generation cost ($/h)</td>
<td>135,070</td>
<td>130,380</td>
<td>130,255</td>
</tr>
</tbody>
</table>

Table 2.21. Comparison of solution quality – 40-unit system

<table>
<thead>
<tr>
<th>Method</th>
<th>Generation Cost ($/h)</th>
<th>Average CPU Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>GA method [42]</td>
<td>135,070</td>
<td>137,980</td>
</tr>
<tr>
<td>PSO method [42]</td>
<td>130,380</td>
<td>137,740</td>
</tr>
<tr>
<td>Hybrid GA-PSO method</td>
<td>130.255</td>
<td>136,986</td>
</tr>
</tbody>
</table>

- Solution Quality

The developed software package has been executed 50 different runs for the proposed hybrid GA-PSO algorithm and the results are given in Tables 2.16 to 2.20. From the comparison of results shown in the tables, it is evident that the proposed hybrid GA-PSO algorithm for three systems has obtained low generation cost in comparison to the PSO and GA methods. Fig. 2.11 shows the plot of convergence of best solution obtained by the proposed algorithm for a 15-unit system. There is an indication of a better quality of solution obtained by the proposed method when compared to other methods.
- Computational Efficiency

From the comparison of Tables 2.17, 2.19 and 2.21, it can be found that the proposed hybrid GA-PSO has lesser average execution time compared with GA and PSO methods. Hence, the computation efficiency of the proposed algorithm is successfully demonstrated. From the above study it is found that, even though execution time for one iteration is more because of the presence of GA operators, crossover and mutation, the identification of the high performance region and locating the optimal solution is possible within less number of iterations. Hence it finds the optimal solution within lesser average execution time.
2.5. CONCLUSION

This work adapts the particle swarm optimization algorithm and genetic approach based particle swarm optimization algorithm to different types of economic dispatch problems. The test results for the IEEE and the various test systems bring out the advantage of the proposed method. The convergence abilities of the PSO method are better than the classical evolutionary programming method. The PSO method converges to the global or near-global point, irrespective of the shape of the cost function, for example, discontinuities in the cost functions. The better computation efficiency and convergence property of the proposed PSO approach shows that it can be applied to a wide range of optimization problems.