PREFACE

In a private communication [9] to I.C. Ross and Frank Harary, N.J. Fine posed the problem of characterizing the graphs that have at least one square root graph and more generally those graphs that have an n-th root for any positive integer n. By using a clique detection procedure [8], Harary and Ross (1960) gave a partial solution to the above problem. They succeeded in characterizing those graphs for which tree square roots exist. In fact, they constructed an algorithm to find a tree square root. In 1967 A. Mukhopadhyay gave a set of necessary and sufficient conditions for the existence of a square root of a graph. Dennis P. Geller (1968) extended this result to digraphs. The results of Mukhopadhyay and Geller were further extended to the case of n-th roots of graphs and digraphs by F. Escalante, L. Montejano and T. Rojano (1974). In 1980 Hamada Takashi gave an algorithm for determining the tree n-th root of a graph. Also, Basangova E.O. (1982) has constructed an algorithm for finding a square root of a graph. In this thesis we make a detailed study of properties of graphs and digraphs with particular types of roots. We also discuss the problem of describing algorithms for root extractions.

Our terminology and notations are as in [3], [7], [8], [11], [13], [15]. We consider finite, simple and connected graphs and digraphs. The thesis consists of four chapters and each chapter is divided in two sections.
First Chapter begins with basic definitions and terminology of Graph Theory. We give some preliminary results which are needed further in the subsequent chapters. Further, we briefly discuss some known results on n-th roots of graphs \((n \geq 2)\) which are used in the later chapter \(([2], [5], [6], [9], [11], [14])\).

Second Chapter consists of properties of graphs and digraphs having square roots. First section deals with various properties of graphs having square roots in terms of paths, connectedness, cut-vertex, bridge, triangulation, pendant vertex, block, vertex connectivity, line connectivity, minimum degree, girth, orientation, diameter etc. Properties of graphs having tree square roots are given in terms of clique, cutset, stable set, stability number, chromatic number, clique number, maximum cardinality of complete subgraphs, simplicial vertex etc. Lower and upper bounds on the number of edges in a graph with square root are given. We can also decide as to which square roots are minimal.

In second section we investigate various properties of digraphs having square roots similar to those in first section. In addition we give some properties of digraphs having square roots in terms of forbidden sets and symmetric triangles. Counter examples are given whenever necessary.

Third Chapter deals with the square roots of graphs and digraphs. In first section we present various characterizations (similar to A. Mukhopadhyay's [11] result) for the existence of (i) Hamiltonian square root (ii) Bipartite square root (iii)
Eulerian square root and (iv) Regular square root. We also present two different sets of necessary and sufficient conditions for the existence of cyclic square roots. These provide us with two different algorithms for finding cyclic square roots of graphs. Also we give a counter example for Basangova's (2) algorithm for finding a maximal square root of a graph. Illustration for each result is given.

In the second section we see how the problem of existence of square roots for digraph can be translated in the language of matrices. We then study the problem for square roots of transitive digraphs. Finally we investigate the problem of the existence of tree, Directed cyclic, Hamiltonian, Bipartite, Regular and Eulerian square roots of digraphs. Also we give algorithms to find the square roots and directed cyclic square roots of digraphs. Illustration for each result is given.

Lastly in Chapter four we give necessary and sufficient conditions for existance of different types of n-th roots of graphs and digraphs. In first section we deal with the question of existence of (i) Regular n-th roots (ii) Hamiltonian n-th roots (iii) Biapartite n-th roots (iv) Eulerian n-th roots and (v) Cyclic n-th roots of graphs (n ≥ 2). Also we discuss the n-th roots of complete graphs.

In the second section we study the n-th roots of transitive digraphs and the existence of tree, regular, hamiltonian, eulerian, directed cyclic and bipartite n-th roots of digraphs. Illustration for each result is given.

(v)
All lemmas, propositions, theorems, algorithms, illustrations, etc. are numbered serially, sectionwise and the references are listed at the end alphabetically. ■ denotes end of proof.