APPLICATION OF A MICROTROTON ACCELERATOR

TO THE GENERATION OF MILLIMETER WAVES
The conventional microwave generation methods lose their effectiveness in the ultramicrowave region of the spectrum. The difficulties encountered stem from the reduction in the dimensions of the devices, the increased heat dissipation, and the marked increase in the required current densities in the electron beam\(^1\). In the case of resonant structure devices, requirements on the mechanical tolerances become more and more stringent as the wavelength is reduced, whereas, the devices, in which no resonant structure is used, suffer from equivalent limitations on the periodicity and alignment of the periodic structure. Because of these limitations the use of more unconventional methods for generating radiation in the millimeter region were suggested by Ginzburg\(^2\), Motz\(^3\),\(^4\),\(^5\), and Coleman\(^6\). Of the several schemes proposed by these scientists, the one based on the use of the Cerenkov effect is described below. Results of the theoretical investigation of the effect of an alternating electric field\(^7\) on the Cerenkov radiation are discussed from the point of view of their utilization in the generation of millimeter waves.

5.1: USE OF CERENKOV EFFECT TO GENERATE MICROWAVES

Electromagnetic radiation is emitted whenever a
charged particle moves through a medium other than free space with a uniform velocity greater than the velocity of light in that medium. In other words, in \( n \) is the refractive index of the medium corresponding to frequency \( \omega \), then the particle velocity \( v \) must satisfy the condition

\[
v > \frac{c}{n(\omega)}
\]

For a given velocity \( v \), all the frequencies which satisfy the above condition are emitted. The waves of frequency \( \omega \) are emitted in the direction \( \theta \) given by

\[
\cos \theta = \frac{1}{\sqrt{n(\omega)}}
\]

The emission will also occur if the charged particle is moving in close proximity to a medium \(^8\) provided the distance of the particle from the medium does not exceed the chosen wavelength.

In 1947 V. L. Ginzburg \(^2\), suggested that the above phenomenon, known as the Cerenkov effect, could be used for the generation of microwaves in a range of wavelengths difficult to access by other methods, namely, in the range \( \lambda \sim 0.01 \) to 1 cm (provided the medium is free from absorption bands in this region). However, since the yield of radiation falls with increase in
frequency, as \( \omega d\omega \), it would be difficult to obtain high power. For example, consider an electron of energy 10 KeV travelling a distance \( l \), in a medium of refractive index \( n \approx 7 \). The energy radiated in the frequency interval \( \Delta \omega \) is given by(2)

\[
W = 1.28 \times 10^{-40} \times l \omega \Delta \omega \text{ ergs.}
\]

If we put \( \omega = 2 \times 10^{12} \) (i.e. \( \gamma \sim 1 \text{ mm} \)) and \( l = 20 \text{ cm} \),

\[
W = 10^{-15} \text{ ergs/electron },
\]

and for a beam current of 10 mA

\[
W = 60 \text{ ergs}
\]

\[
= 6 \mu \text{W},
\]

a value which is too small to be of practical interest.

If, however, the electrons are bunched, and the dimensions of the bunches are small compared to the wavelength under consideration, the electrons will radiate coherently. Two important advantages are obtained in this case. First, the effective charge is increased by a factor of \( \gamma \), where \( \gamma \) is the number of electrons in a bunch, and this increases the radiated power by a factor of \( \gamma^2 \). Therefore the energy radiated in the frequency interval \( \Delta \omega \) will increase by a factor of \( \gamma^2 N \), where \( N \) is the number of bunches per second.
and $f$ is the form factor which depends upon the size of the bunch. For a pulse duration of 1 $\mu$sec, an average beam current of 1 $\mu$A corresponds to $N = 10^9$ and $N = 10^4$. If we take $f = 1$, the power radiated in the interval $\Delta \nu = 2 \times 10^{11}$ at $\nu = 2 \times 10^{12}$ will be 60 watts. Second, the continuous spectrum becomes a line spectrum with radiation concentrated at the bunching frequency and its harmonics. This can be seen from an examination of Fig.26, which shows the radiation pattern of a bunched electron beam that moves through a dielectric. We assume that the bunches are point charges. A typical wavefront moves from A to B in the time that the associated bunch moves from A to C. Assume that the wavefront contains frequency components $\Omega_1$ which are radiated coherently. For coherence we require that

$$\Delta t \times \Omega_1 = 2\pi N, \ldots, N = 1, 2, 3, \ldots,$$

where $\Delta t$ is the time required for the wavefront to move from A to B. This time corresponds to the time required for the bunch to move from A to C, i.e. $2\pi/\omega$, where $\omega$ is the bunching frequency. Then, from the coherence requirements we have

$$\frac{2\pi \Omega_1}{\omega} = 2\pi N,$$
and the frequencies radiated coherently are given by

\[ \eta_i = N \omega. \]

Thus by using a source which can give tightly bunched electron streams in MeV range, the yield of radiation can be increased by a very large factor, and it is possible to concentrate the energy into bunching frequency and its harmonics. The tighter the bunch is, the higher are its harmonic contents, and the highest frequency that can be emitted, for a given power output, is limited by the space charge effects.

As explained in the following article, the microtron electron accelerator can give a highly bunched beam and hence is a very useful device for the generation of millimeter waves.

5.2: ELECTRON BUNCHING IN MICROTRON

The bunching of the electron beam in a microtron is due to the energy and phase oscillations of electrons that take place about the resonant electron. After a number of orbits have been completed the particles of the beam are essentially contained in the phase stable region, and the electrons keep oscillating between the limits of this region. In addition to these large non-linear synchrotron oscillations there is a phase compression within the phase stable region.
due to small (linear) synchrotron oscillations around the resonant point (10). These oscillations can be described by the relations (11)

\[ \Psi_k(\alpha) = \frac{\Psi_0}{\cos(\frac{\omega_5}{2})} \left[ (1 - \frac{\alpha}{2 \pi}) \cos \omega_3 (k - \frac{1}{2}) + \frac{\alpha}{2 \pi} \cos \omega_3 (k + \frac{1}{2}) \right] + \frac{2 \pi \Psi_0}{\sin \omega_3} \left[ (1 - \frac{\alpha}{2 \pi}) \sin \omega_3 (k - 1) + \frac{\alpha}{2 \pi} \sin \omega_3 k \right] \] (5.1)

and

\[ \chi_k = -\frac{\sin \omega_3 k}{\omega_3} \tan \phi_{res} \Psi_0 + \frac{\cos \omega_3 (k - \frac{1}{2})}{\cos \omega_3 (\omega_5/2)} \chi_0 \] (5.2)

where

\[ \alpha \] - the angle measured from the centre of the cavity,
\[ k \] - the orbit number,
\[ \Psi = \frac{2 \pi (t-t_{res})}{T} \]
\[ t \] - the time at which an electron passes an orbit point determined by the orbit angle \( \alpha \),
\[ t_{res} \] - the time at which the resonant electron passes that point,
\[ T \] - the period of the rf field oscillations,
\[ \chi = \frac{w - \omega_{res}}{w} \]
W - the total energy of an electron,
W_{res} - the total energy of the resonant electron,
\Delta W - the energy the resonant electron gains in each orbit,
\psi_0, \psi_o - the defined quantities at injection,
\omega_s - \cos^{-1}(1 - \pi \tan \phi_{res}) is the angular frequency of synchrotron oscillations per turn, and
\phi_{res} - the phase at which the resonant electron passes the centre of the cavity.

At injection (k = 0) the energy spread is negligible, hence \psi_0 = 0 and

\psi_k(x) = \frac{\psi_0}{\cos(\omega_s/2)} \left[ (1 - \frac{x}{2\pi}) \cos \omega_s (k - \frac{1}{2}) \right.
+ \left. \frac{x}{2\pi} \cos \omega_s (k + \frac{1}{2}) \right] \tag{5.3}

and

\psi_k = \frac{\psi_0}{\sin \omega_s} \tan \phi_{res} \sin (\omega_s k) \tag{5.4}

The phase compression is possible whenever \psi_k(x) = 0, i.e. when

\frac{x}{2\pi} = - \frac{\cos \omega_s (k - \frac{1}{2})}{\cos \omega_s (k + \frac{1}{2}) - \cos \omega_s (k - \frac{1}{2})}
The synchrotron frequency $\omega_s$ can be changed by varying the resonant phase, which in turn means varying the amplitude of the voltage across the cavity. The possible range of $\omega_s$ is

$$0 \leq \omega_s \leq \pi .$$

Eqn. (5.5) can thus be satisfied for any orbit angle $\chi$ of a given orbit by properly choosing $\omega_s$. For $\omega_s = \pi/2$ ($\phi_{res} = 0.3082$), phase compression at $\chi = \pi$, the point diametrically opposite the cavity, occurs in all odd numbered orbits. This is because for $\chi = \pi$, and from eqn. (5.5), one gets

$$\cos \omega_s (k + 1/2) = \cos \omega_s (k - 1/2) ,$$

which is satisfied if

$$\omega_s = \frac{-2i+1-\pi}{2k} \ldots i=0, 1, 2, \ldots k-1 .$$

As shown by Brannen et al. (11), in the third orbit electrons with initial phases $-0.05 \leq \phi_0 \leq 0.37$ are compressed into a region 0.03 radians wide. For a frequency of 2980 MHz this corresponds to a bunch length of 0.5 mm.
Thus a properly designed microtron accelerator can give usable harmonic beam currents up to the 50th order, which can be maintained for a distance of at least 10 to 20 cm (electron energy ~ 10 MeV) through a coupling structure. Frequencies generated can therefore range from 150 GHz, i.e. $\lambda \sim 2$ mm (using S band microtron) to 450 GHz, i.e. $\lambda \sim 0.06$ mm (using X band microtron).

The experimental setup to extract the Cerenkov power from a dielectric radiator is shown in Fig. 27. A beam of relativistic electrons from the microtron accelerator passes through a hole in the dielectric radiator. The radiation emitted at the characteristic Cerenkov angle $\Theta$ is internally reflected from surface 1 to surface 2 where it strikes at the Brewster angle and emerges totally in a direction parallel to the axis. The radiation is then collected by a coaxial horn (12) and detected by means of a wide-band crystal detector (13). A set of cut-off waveguides or a frequency meter can be used to estimate the spectral distribution of the radiated power.

5.3: EFFECT OF ALTERNATING ELECTRIC FIELD ON CERENKOV RADIATION

As explained in the previous article, the Cerenkov
effect can be used to generate millimeter waves. The electromagnetic radiation is, however, concentrated in frequencies which are integral multiples of the bunching frequency. If, for some reason, it is necessary to generate a frequency which is somewhere between two harmonics, the only possibility is to change the bunching frequency. But the bunching frequency is the same as the frequency of the accelerating field, and the latter cannot be changed arbitrarily, since it is decided from the synchronisation conditions of the microtron accelerator. It is possible to generate intermediate wavelengths if we place the Cerenkov radiator in an oscillating electric field. As shown by Tavdgiridge and Diasamidze(14), the presence of an alternating electric field of frequency \( \omega_0 \), reduces the intensity of Cerenkov radiation and leads to the appearance of Doppler radiation with a frequency greater than \( \omega_0 \). The frequencies emitted through Doppler radiation depend upon the field frequency \( \omega_0 \), the velocity of the incident particle \( v = \beta c \), and the angle of observation \( \Theta \) as

\[
\omega(\Theta) = \left| \frac{l \omega_0}{1 - \beta n(\omega) \cos \Theta} \right| \quad ...l = \pm 1, \pm 2, \ldots (5.6)
\]
For $l < 0$, we have the normal Doppler radiation

$$\omega_N(\theta) = \frac{|lw_o|}{1 - \beta n \cos \theta} \quad \beta n \cos \theta < 1 \quad (5.7)$$

and for $l > 0$, we have anomalous Doppler radiation

$$\omega_A(\theta) = \frac{lw_o}{\beta n \cos \theta - 1} \quad \beta n \cos \theta > 1 \quad (5.8)$$

Thus for a given bunching frequency $\omega_o$, and a given Cerenkov radiator, any frequency and its harmonics can be emitted by choosing a proper value of $\beta$.

Moreover, for the purpose of tuning the radiator exactly to the given frequency, the value of $\beta$ can be adjusted to a certain extent by varying the energy of the incident particle. In doing this it should be remembered that for a given length of the radiator, and hence for a given output power, one cannot decrease $\beta$ beyond a particular limit. The reason for this is that for a given energy spread of the beam, the velocity spread decreases as the beam energy increases, and the bunch can travel a long distance without any appreciable change in its size.

The effect of an alternating electric field on
Cerenkov radiation has been investigated in detail theoretically by Risbud and Takwale (7). The two cases considered are

1) electric field applied parallel to the direction of the motion of the charged particle, and

2) electric field applied perpendicular to the direction of motion of the charged particle.

The results of their investigation are presented below.

Consider an electron moving with a velocity \( \mathbf{v}_0 \) in a medium characterized by dielectric constant \( \varepsilon \) and magnetic permeability \( \mu \). An alternating electric field \( \mathbf{E} = E_0 \sin(\omega_0 t) \) is applied parallel to \( \mathbf{v}_0 \). The intensities of Cerenkov and Doppler radiation are given by

\[
I_{\text{cer}} = I_c \left[ J_0^2(\gamma \omega_m) + J_1^2(\gamma \omega_m) \right] \tag{5.9}
\]

and

\[
I_{\text{Dop}} = 2 I_c \sum_{l=1}^{\infty} \left[ J_l^2(\gamma \omega_m) - J_{l-1}(\gamma \omega_m) J_{l+1}(\gamma \omega_m) \right] \tag{5.10}
\]
where

\[ I_c = \frac{\mu e^2}{c^2} \mathcal{J}_0 \left( \frac{\omega_m^2}{c^2} \right) \left( \frac{\omega_0}{\nu} - \frac{1}{\epsilon \mu \beta^2} \right) \frac{\omega_m^2}{2} , \]

\[ J_0, J_1 \] are the Bessel functions, and

\[ \omega_m \] is the maximum frequency up to which the Cerenkov condition is satisfied.

When the field \( E = E_0 \sin(\omega_0 t) \) is applied perpendicular to \( \nu_0 \), the intensities are given by (15)

\[ I_{\text{cer}} = \frac{I_0}{\gamma} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)} \left( \frac{\omega_0}{\nu} \right)^{2n} \left( \frac{\lambda \omega_m}{2} \right)^{2n} \left( \frac{\lambda \omega_m}{2} \right)^{2n+2} \]  (5.11)

and

\[ I_{\text{Dop}} = \frac{2I_0}{\gamma} \sum_{n=0}^{\infty} \sum_{l=1}^{\infty} \frac{(-1)^n}{n!} \left( \frac{\omega_0}{\nu} \right)^{2n+2} \left( \frac{\lambda \omega_m}{2} \right)^{2n+2} \left( \frac{\lambda \omega_m}{2} \right)^{2n+2} \]  (5.12)

where

\[ I_c' = \frac{\mu e^2}{c^2} \mathcal{J}_0 \left( \frac{\omega_m^2}{c^2} \right) \left( 1 - \frac{1}{\epsilon \mu \beta^2} \right) \]

and

\[ \alpha = \sqrt{\epsilon \mu \beta_0^2} - 1 \]
From eqns. (5.9) and (5.11) one can study the variation of the intensity of Cerenkov radiation with $E_0$ and $\omega_0$. The results of calculations for a particular case of $\beta = 0.998$ and $n = 1.221$ are shown in Fig. 28. In the absence of field, both the equations reduce to the familiar Frank and Tamm result (16). From the Fig. 28, it is seen that the presence of an alternating electric field reduces the intensity of the Cerenkov radiation. At small amplitudes the Cerenkov radiation is reduced slightly, while for large amplitudes there is a marked reduction in the intensity. Also when the values of the field parameters are the same, the parallel field reduces the Cerenkov radiation intensity more effectively than the perpendicular one.

The experimental arrangement shown in Fig. 29 can be used to verify these effects. An S-band magnetron (frequency-2700 to 2900 MHz) delivers power to the microwave cavity through a ferrite isolator. A small cylindrical rod of perspex through which passes an electron beam of energy 2 MeV is placed along the axis of the cavity. Cerenkov radiation emitted in the forward direction is detected by a 56 AVP photomultiplier. As explained in chapter 3, for a given input power the voltage across the cavity depends upon the Q of the cavity.
This can be changed by placing some resistive material inside the cavity, and the curves of Fig. 28 can be verified over a limited range of $\lambda \omega_m$.

To get an idea of the frequencies emitted through Doppler radiation and the variation of the intensity of the radiation with field parameters $E_0$ and $\omega_0$, let us consider an example. For $\beta = 0.998$ and $n = 1.495$, the Cerenkov angle $\Theta$, will be $34.97^\circ$. If we take $\omega_0 = 2 \pi x 2.8 \times 10^9$, eqn. (5.6) gives the Doppler frequencies. The variation of the intensities of the first five Doppler modes ($\lambda = 1, 2, 3, 4,$ and $5$) for the field applied parallel to the velocity of the particle is shown in Fig. 30. Here the Doppler frequencies are observed at $\Theta = 34^\circ$ (anomalous Doppler frequencies) and $\Theta = 40^\circ$ (normal Doppler frequencies). Table IV gives the values of these frequencies $\omega_A$ and $\omega_N$, and their maximum possible intensities of emission with the corresponding values of $\lambda \omega_m$ for both the parallel as well as the perpendicular cases.
<table>
<thead>
<tr>
<th>A \times 10^{11} Hz</th>
<th>N \times 10^{11} Hz</th>
<th>\frac{I}{2I_c} \text{ max } \gamma \omega_m</th>
<th>\frac{I}{2I_c} \text{ max } \gamma \omega_m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 2.4058</td>
<td>0.4293</td>
<td>0.1348</td>
<td>2.4</td>
</tr>
<tr>
<td>2.0 4.8116</td>
<td>0.8585</td>
<td>0.0805</td>
<td>4.0</td>
</tr>
<tr>
<td>3.0 7.2174</td>
<td>1.2878</td>
<td>0.0575</td>
<td>5.0</td>
</tr>
<tr>
<td>4.0 9.6232</td>
<td>1.7170</td>
<td>0.0432</td>
<td>6.0</td>
</tr>
<tr>
<td>5.0 12.0290</td>
<td>2.1463</td>
<td>0.0351</td>
<td>8.0</td>
</tr>
</tbody>
</table>

---

**TABLE IV**
Fig-26 - Radiation pattern of a bunched electron beam moving through a dielectric material.

Fig-29 - Diagram of electron beam and microwave cavity with a dielectric rod along the axis, ferrite isolator, coaxial to waveguide transition, and magnetron valve.
1) Electron beam from the microtron accelerator.
2) Beam current monitor. 3) Spacer.
4) Radiator. 5) Coaxial horn. 6) 90° Bend.
7) Set of cut-off waveguides.
8) Crystal detector. 9) Amplifier.
10) Oscilloscope.

Fig-27-Schematic of the experimental setup.
Fig-28 - Variation of the intensity of cerenkov radiation with $\lambda \omega_m = \frac{eE_0 \omega_m}{m \omega_0^2 v_0}$

Curve A $\rightarrow$ $E \perp V_0$

Curve A $\rightarrow$ $E \parallel V_0$
VARIATION OF THE INTENSITY OF DIFFERENT MODES OF DOPPLER RADIATION \((i = 1, 2, \ldots, 5)\) WITH \(x\), FOR \(\overline{E} \parallel \overline{B}\).
REFERENCES

23 (1952) 812.

56 (1947) 253.

22 (1951) 527.

 and R.N.Whitehurst 24 (1953) 826.

 K.B.Mallory 26 (1955) 1384.

 M.D.Sirkis 28 (1957) 944.

7) A.A.Risbud, and Journ.Phys.(A)
 R.G.Takwale 10 (1977) 2181.

56 (1947) 145.

9) J.V.Jelly "Cerenkov Radiation and its

 P.D.Coleman 27 (1956) 1250.
11) H. Froleich, and E. Brannen

12) J. W. Dees, and A. P. Sheppard

13) R. J. Batt, and D. J. Harris

14) Zh. M. Dasamidze, and T. L. Tavdgiridze

15) A. A. Risbud, and R. G. Takwale

16) I. E. Tamm, and I. M. Frank


Journ. Phys. (A) (Accepted for publication as a note).