Chapter 1

Introduction

Vector Optimization accredits versatile application areas such as calculus of variations, control theory, convex optimization, decision theory, game theory, linear programming, Markov chains, network analysis, queuing systems etc. In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values within an allowed set and computing the value of the function. More commonly, optimization pertains to finding the best available values of an objective function in a given domain. Various problems encountered in areas of engineering, sciences, management science and economics are based on the fundamental ideas of mathematical formulation. Optimization is a constitutive tool for the formulation and solution of many such problems expressed in the form of minimization or maximization functions.

In this work, we emphasize on applications of Multi Criteria Decision Making (MCDM) problems. Major thrust is on pertinence of previously developed methodologies in decoding decision making problems and to foster advanced approaches to formerly
deviced techniques for procurement of more fitting solutions. The chapter is di-
vided into four sections, namely, Literature Review, Introduction to Multi-Criteria
Decision Making (MCDM) Methods, Organization of the Thesis and Publications.

1.1 Literature Review

The study of decision problems has a long history and in the last few decades,
it has been one of the major research field in optimization discipline. A preem-
nent work has been carried out in 20th century in field of optimization theory.
The mathematical modeling of these decision making problems gained momen-
tum with monumental contributions of various eminent economists and mathe-
maticians. Earlier contributions are reckoned to Jensen(1906), who introduced
convex functions, the idea had already appeared in the work of Hadamard(1902).
Cantor(1915) made many fundamental contributions to optimization techniques.
These contributions are also the foundations of the mathematical concepts used
in modern MCDM. Since 1992, the International Society on Multiple Criteria
Decision Making has been giving out Georg Cantor awards. Neumann & Morgen-
stern(1944; 1947) were two eminent economists, who propounded Games Theory,
an integral part of decision analysis. Historically, the first term for optimization is
‘linear programming’, which is due to Dantzig(1947), who formulated the Simplex
algorithm, which was published later in 1951. Dantzig(1953) exclusively wrote a
book on linear programming successively, although much of the theory had been
introduced by Kantorovich(1940), more than a decade prior to this achievement.
In early 1950s, Koopman(1951) worked on MCDM problems considering multiple
conflicting criteria or goals. Optimality conditions for nonlinear problems were proposed by Kuhn & Tucker (1951) and Kuhn (1955) also worked on the Hungarian method for assignment problem. Parallely, Markowitz (1952) presented his portfolio theory, that is based on quadratic optimization. Later, the Nobel memorial prize in economics was bagged by Markowitz (1991) for his prodigious work in the field. Significant works dealing with the parametric objective functions were published by Gass & Saaty (1954; 1955; 1955), which could be used for generating efficient solutions by varying the weights of an aggregate value function, a popular technique used in early multiple objective linear programming methods. Later in the century, Gass (1985) published a book on MCDM, named ‘Decision Making Models and Algorithms’. Edwards (1954; 1961) published two articles namely, ‘The Theory of Decision Making’ and ‘Behavioral Decision Theory’ which laid milestones in modern decision theory. The papers discussed issues such as how should people make decisions and how they could improve decisions. In mid century, Charnes, Cooper & Ferguson (1955) published an article that accommodated the fundamentals of goal programming, even though the name goal programming was first introduced in a book published by Charnes and Cooper (1961). By the same span, Bellman (1957) presented the optimality principle that brought evolutionary advances to dynamic programming model. A book on sequential decision processes was brought up by Ronald & Kimball (1959), which elaborated various decision methodologies till date. An updated version was brought up by Ronald & James (1990), by the end of 20th century. Zoutendijk (1960) presented the methods of feasible directions to generalize the

In context of MCDM problems, Major advances have been recorded since the 1970s. In November 1972, an international conference on MCDM was organized by Cochrane & Zeleny (1973) in Columbia, South Carolina. The proceedings of this conference was the first major volume on decision analysis and is still heavily cited. Intrigued by the multi-criteria problems, Zionts & Wallenius (1976) developed the Zionts - Wallenius interactive method for solving multiple-objective linear programming problems. A friend and colleague of Wallenius, Korhonenin (1984) joined their collaboration and in the late 1970s, jointly they worked on methods and decision support systems for solving interactive multiple objective mathematical programming problems. Their work was published in early 1980s in management science, a U.S. journal. Keeney & Raiffa (1976) published a book, ‘Decisions with Multiple Objectives: Preference and Value Tradeoffs’, which was instrumental in establishing the theory of decision analysis and MCDM as a discipline.

Eminent work has been carried out in MCDM since 1980s. Saaty (1980) introduced the Analytic Hierarchy Process (AHP), which brought evolutionary advances in MCDM approaches. Saaty being placed in Fortune magazine, is visibly one of the
most successful people in MCDM. AHP, developed by Saaty is one of the most popular techniques used by the researchers and practitioners. Another useful technique, Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was developed by Hwang and Yoon(1981) and it is a popular approach to MCDM problems. In early 1980s, Brans(1982) laid fundamentals of Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), which is a part of outranking methods used as decision-aid in various MCDM problems. Considerable work was carried out for more than a decade to bring out various versions of PROMETHEE by Brans et al.(1982; 1984; 1988; 1992; 1994; 1996).

Saaty & Takizawa(1986) revised AHP in their study so that it could handle the non-linear hierarchies. Edwards & Barron(1994) first propounded Rank Order Centroids (ROC) technique to assign attributes’ weights. Carlsson & Fuller(1995) in mid 1990s introduced interdependency concept into MCDM which later on was improved by Ostermark(1997). Saaty(1996; 1999) developed the Analytical Network Process (ANP) by the end of 20th century, which is a more general form of AHP. Numerous other credible approaches have been cultivated and fostered till date, by eminent researchers in the specified field.

1.2 Introduction to Multi Criteria Decision Making (MCDM) Methods

Decision making is one of the perpetual challenges in our day to day lives. In the recent decades, researchers have emphasized the use of MCDM models for
complex decisions making scenarios.

MCDM aims at improving the quality of decisions by making the process more explicit, rational and efficient. It is concerned with structuring and solving decision making problems involving multiple criteria (Figure 1.1).

![Figure 1.1: MCDM Problem](image)

The use of decision making tools under a multi-criteria approach are intended to aid the decision maker in correlating various alternatives in terms of evaluation criteria. A decision support system can be defined as an interactive system that is able to produce data and information and even promote understanding related to a given application domain in order to give useful assistance in resolving complex and ambiguous problems. Decision making processes are analyzed from different viewpoints not only involving the implementation of analytical methods and supporting tools but also taking into consideration the organizational structure of the problem and the dynamics of the decision makers involved.
Hwang (1979; 1981) is the person behind the development of keynote techniques of modern MCDM. His two references viz. Hwang & Masud (1979) and Hwang & Yoon (1981) list various techniques, grouped in two classes; the Multi - Attribute Decision Making (MADM) and Multi - Objective Decision Making (MODM) techniques.

According to Hwang & Masud (1979), MADM problems involve the selection of the best alternative from a pool of preselected alternatives described in terms of their attributes. Attributes are generally defined as characteristics that describe in part, the state of a product or system. Works of Dong & Kang (2000), and Zopounidis & Doumpos (2000) use MADM models for decision problems.

MODM problems, as accorded by Hwang & Yoon (1981), involve the design of alternatives which optimize or best satisfy the objectives (usually more than one) of the decision maker. MODM has been widely studied with mathematical programming methods, which have a well-formulated theoretical frame in which optimization problem can be studied making different assumptions on the variables as well as on functions that define the model and constraints. More information on MODM can be found in works of Keeney & Raiffa (1976), Wierzbicki (1980), Slowinski & Teghem (1990), Lai & Hwang (1996), Kirkwood (1997), and Ehrgott & Gandibleux (2002). Many evolutionary algorithms have been applied to MODM and they seem more appropriate to deal with problems with multiple solutions than conventional optimization techniques as described by Fonseca & Fleming (1995).

Together all the techniques for solving both type of problems can be classified as Multi - Criteria Decision Making (MCDM) techniques. While criteria typi-
cally describe the standards of judgment or rules to test acceptability, here they simply indicate attributes or objectives. Multi - Criteria Decision Aid (MCDA), an advanced field of operations research, provides decision makers and analysts a wide range of methodologies, which are well suited to the various decision problems. MCDA intends to provide tools that allow the decision-maker to capture, analyze and understand these view-points, in which the decision process must be handled. This is not an easy task because often these judgments are contradictory. Consequently, it is not always possible to find a unique solution that is the best for all the opinions. The apparent idea at the core of a classic decision making tool is that, for any given problem there does not exist a unique optimal solution, and it is necessary to use decision maker’s preferences to get prioritized solutions. Thus some trade-off must be done among the different point of views to determine an acceptable solution. Since the beginning of the MCDA research field, many different methods have been proposed. Each methodology has its own characteristics and there are many ways, one can classify them. For example, we can separate methods with a single decision maker and methods with a group of decision makers. Some of the approaches of MCDA as adopted for single decision maker are Yu(1985), Roy & Vanderpooten(1996), and Triantaphyllou(2000). The methods involving more than one decision maker are included in the research field of Group Decision Making and Negotiation, an introduction to the field can be found in works of DeSanctis & Gallupe(1987), Jelassi, Kersten & Zionts(1990), and Basak & Saaty(1993).

With regard to the subject under consideration, the problems of decision making
in this research cover both MADM and MODM. By considering the various techniques of optimization, some of the MCDA methods used in the present work are: Analytical Hierarchy Process (AHP), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), Simple Additive Weighting (SAW), Rank Order Centroids (ROC), Ratio Method, Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Lexicographic Approach etc. These techniques will be discussed in the next parts.

1.2.1 Analytical Hierarchy Process (AHP)

The formulation of AHP given by Saaty(1980) emerged as a paradigm for decision making scenarios categorized under MADM. It is based on the well-defined mathematical structure of consistent matrices and their associated eigenvector’s ability to generate true or approximate weights. AHP can be considered to be both a descriptive and prescriptive model of decision making and it is perhaps, the most widely used decision making approach in the world today. Its validity is based on the many thousands of actual applications in which the AHP results are accepted and used by the cognizant decision makers such as Byun(2001), Hummel(2001), Bhushan & Ria(2004) and many more. AHP is an approach to decision making that involves structuring multiple judgment criteria into a hierarchy, assessing the relative importance of these criteria, comparing alternatives for each criterion, and determining an overall ranking of the alternatives. The outcome of AHP is a prioritized weighting of each decision alternative. The AHP converts these evaluations to numerical values that can be
processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. The first step is to model the problem as a hierarchy. The hierarchy (Figure 1.1) is a structured means of describing the problem at hand. It consists of an overall goal at the top level, a group of options or alternatives for reaching the goal and a group of factors or criteria that relate the alternatives to the goal. In most cases the criteria are further broken down into sub criteria in many levels as per the requirement of the problem. Once the hierarchy has been constructed, the participants use the AHP to establish priorities for all its nodes. For this, the elements of a problem are compared in pairs with respect to their relative impact on a property they share in common. The pair wise comparison is quantified in a matrix form by using the scale of Relative Importance developed by Saaty(1980) as shown in Table 1.1.

During the elicitation process, a positive reciprocal matrix is formed in which the $(i, j)^{th}$ element $a_{ij}$ is filled by the corresponding number from the Table 1.1 and the number is chosen according to the following criterion.

$$
\begin{align*}
 a_{ij}, & \quad \text{if } x_i \text{ dominates } x_j; \\
 \frac{1}{a_{ij}}, & \quad \text{if } x_j \text{ dominates } x_i; \\
 1, & \quad \text{if } x_i \text{ and } x_j \text{ do not dominate over one another}.
\end{align*}
$$

This scale has been validated for effectiveness, not only in many applications, but also through theoretical comparison with a large number of other scales.
<table>
<thead>
<tr>
<th>Reciprocal measure of importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two activities contribute equally to the objective</td>
</tr>
<tr>
<td>3</td>
<td>Weak importance of one over another</td>
<td>Experience and judgment slightly favor one activity over another</td>
</tr>
<tr>
<td>5</td>
<td>Moderate importance</td>
<td>Experience and judgments moderately favor one activity over another</td>
</tr>
<tr>
<td>7</td>
<td>Strong Importance</td>
<td>An activity is strongly favored and its dominance is demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Absolute Importance</td>
<td>The evidence favoring one activity over another is of the highest possible order of affirmation</td>
</tr>
<tr>
<td>2,4,6,8</td>
<td>Intermediate values between two adjacent judgments</td>
<td>When compromise is needed</td>
</tr>
</tbody>
</table>

Let $A_{n \times n}$ be a typical pairwise comparison matrix of $n$ alternatives representing the intensities of the expert’s preference between individual pairs of alternatives $A_i$ versus $A_j$, for all $i, j = (1, 2, \ldots, n)$.

$$A = [a_{ij}] = \begin{pmatrix}
1 & a_{12} & \cdots & a_{1n} \\
1/a_{12} & 1 & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
1/a_{1n} & 1/a_{2n} & \cdots & 1
\end{pmatrix}$$

The matrix so formed is called the reciprocal matrix. This reciprocal matrix is used to calculate the local priority weight of each criterion. The local priority
weight \((w)\) is the normalized eigen vector of the priority matrix corresponding to
the maximum eigen value of the matrix. For detailed reasoning of this account
we refer to Saaty(1980), Saaty & Vargas(1984), Cogger & Yu(1985), Saaty(1990),
Lunging(1992), Ball & Noel(1994), and Bryson & Mobolurin(1994). An interesting
property of the priority matrix is that if in addition its elements are such that

\[
a_{ij}a_{jk} = a_{ik}, \quad \forall \ i \leq j \leq k
\]

(1.1)

then the derived priority vector \(w\) satisfies

\[
w_i / w_j = a_{ij}, \quad \forall \ i < j.
\]

(1.2)

Any reciprocal matrix satisfying (1.1) is called consistent. However in practice, the
priority matrix seldom satisfies (1.1), thereby making it more important to define
some relaxed measuring of consistency check. Saaty(1980) introduced the concept
of Consistency Index \((CI)\) of a reciprocal matrix as the ratio

\[
\frac{\lambda_{\text{max}} - n}{n-1}
\]

where \(\lambda_{\text{max}}\) and \(n\) respectively stand for the maximum eigen value and order of the reciprocal
matrix.

Saaty(1980) has also shown that if a matrix \(A\) is absolutely consistent then \(\lambda_{\text{max}} = n\) and hence \(CI = 0\). If the judgments are not absolutely consistent then \(\lambda_{\text{max}} > n\),
and we need to measure this level of inconsistency. The obtained \(CI\) value is
compared with the Random Index \((RI)\) given in Table 1.2.

| Table 1.2: Random Consistency Index \((RI)\) |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| N     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
| RI    | 0     | 0     | 0.58  | 0.9   | 1.12  | 1.24  | 1.32  | 1.41  | 1.45  | 1.49  |

It had been calculated as an average of \(CI's\) of many thousands matrices of the
same order whose entries were generated randomly from the scale 1 to 9 with
reciprocal effect. The simulation results of RI for matrices of size 1 to 10 had been
developed by Saaty(1980) and Forman(1990), and are given in Table 1.2.
The ratio of $CI$ and $RI$ for the same order matrix is called Consistency Ra-
tio ($CR$). In general, a consistency ratio of 10% or less is considered very good. If
inconsistency of judgments within the matrix has occurred then evaluation process
should be reviewed and improved upon.

1.2.2 Preference Ranking Organization Method for En-
richment Evaluation (PROMETHEE)

Another useful tool for solving discrete MCDM problems is Preference Ranking
Organization Method for Enrichment Evaluations(PROMETHEE), a new class of
outranking methods developed by Brans(1982). At that time he proposed only
the basic PROMETHEE I (partial ranking) and PROMETHEE II (complete
ranking) versions. Soon thereafter Brans, Mareschal & Vincke(1984) presented
PROMETHEE III (ranking based on intervals) and PROMETHEE IV (continu-
ous case) extensions setting new standards in outranking methods.
Brans & Mareschal(1988; 1994) proposed Graphical Analysis for Interactive Assis-
tance(GAIA) method as a descriptive extension of PROMETHEE which provides
a graphical representation, supporting the PROMETHEE methodology. PROM-
CALC(later PROMCALC-GAIA) was one of the first truly interactive MCDA
software with a strong emphasis on user interface, graphical representations and
sensitivity analysis.
A nice extension namely PROMETHEE V (MCDA including segmentation constraints) was developed by Brans & Marshal (1992) in which the problem is converted to a 0–1 goal programming problem. This method is useful for resource allocation and project ranking type of problems. Soon thereafter, PROMETHEE VI (representation of the human brain) was given by Brans (1996) which provides the decision-maker with additional information on his own personal view of his multi-criteria problem. It allows to appreciate whether the problem is hard or soft according to his personal opinion. As soon as the weights are fixed, a final ranking is proposed by PROMETHEE II.

PROMETHEE methods have taken an important place among the existing outranking multiple criteria methods. This methodology is known for its applicability and efficiency in solving MCDM problems almost in all important fields. There are also some other studies to develop hybrid methodologies which are the combinations of the unique and specific tools. AHP and PROMETHEE methods are analyzed and discussed together thoroughly in the study of Macharis, Springael, Brucker & Verbeke (2003). They argue that AHP could be used during the weight determination stage of PROMETHEE method, in which no particular weighing approach was suggested. Similarly, Wang & Yang (2007) combined AHP and PROMETHEE II to form a hybrid method to rank alternatives. They used AHP for determination of the weights of the criteria and to understand the structure of the problem whereas PROMETHEE II for the final ranking.

Specifically for resolving a MCDA problem using PROMETHEE II, which is known for complete ranking analysis, consider multi-criteria decision problem having $m$
alternatives, \( \{A_1, A_2, \ldots, A_m\} \) and \( n \) evaluation criteria, \( \{C_1, C_2, \ldots, C_n\} \). Also let \( \{W_1, W_2, \ldots, W_n\} \) be the associated criteria weights as shown in Table 1.3.

<table>
<thead>
<tr>
<th>((A_i))</th>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(C_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \downarrow )</td>
<td>( W_1 )</td>
<td>( W_2 )</td>
<td>( W_n )</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( f_j(A_i) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_m )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Furthermore criteria weights \( W_j \) are normalized and have been determined by an appropriate method. Without loss of generality, we may assume that these criteria weights are to be maximized. The procedural steps as involved in PROMETHEEII method are enlisted below.

**Step 1:** Obtain an evaluation \( x_{ij} \) corresponding to every \( f_j(A_i) \) as crisp score for each alternative corresponding to the respective criterion. Here \( x_{ij} \) is the performance measure of \( i^{th} \) alternative with respect to \( j^{th} \) criterion.

**Step 2:** Normalize the decision matrix using the following equation:

\[
r_{ij} = \frac{[x_{ij} - \min_j(x_{ij})]}{[\max_j(x_{ij}) - \min_j(x_{ij})]} \quad (i = 1, \ldots, m), \quad \forall j
\]  

(1.3)

For non-beneficial criteria, Eqn. (1.3) can be rewritten as follows:

\[
r_{ij} = \frac{[\max_j(x_{ij}) - x_{ij}]}{[\max_j(x_{ij}) - \min_j(x_{ij})]} \quad (i = 1, \ldots, m), \quad \forall j
\]  

(1.4)
Step 3: Now to find the evaluative differences of \(i^{th}\) alternative with respect to other alternatives, calculate the preference function, \(P_j(A_i, A_i')\), where \(A_i\) and \(A_i'\) denote different alternatives, given in \(i^{th}\) and \(i^{'}\) \(th\) rows. This step involves the calculation of differences in criteria values between different alternatives pair-wise. There are mainly six types of generalized preference functions as proposed by Brans (1982) and Mareschal & Brans (1991). But these preference functions require the definition of some preferential parameters, such as the preference and indifference thresholds. However, in real time applications, it may be difficult for the decision maker to specify which specific form of preference function is suitable for each criterion and also to determine the parameters involved. To avoid this problem, the following simplified preference function is adopted here:

\[
P_j(A_i, A_i') = 0, \quad \text{if} \quad r_{ij} \leq r_{i'j} \tag{1.5}
\]

\[
P_j(A_i, A_i') = r_{ij} - r_{i'j}, \quad \text{if} \quad r_{ij} > r_{i'j} \tag{1.6}
\]

\(0 \leq P_j(A_i, A_i') \leq 1\) and

\(P_j(A_i, A_i') = 0\) means no preference or indifference,

\(P_j(A_i, A_i') \approx 0\) means weak preference,

\(P_j(A_i, A_i') \approx 1\) means strong preference, and

\(P_j(A_i, A_i') = 1\) means strict preference.

The simplicity is the main advantage of these preferences functions. There are not more than two parameters at a time, each having a clear chronological significance.

Step 4: A multi criteria preference index \(\pi(A_i, A_i')\) of \(A_i\) over \(A_i'\) can then be defined considering all the criteria:
\[ \pi(A_i, A_i') = \sum_{j=1}^{n} W_j P_j(A_i, A_i') \]  

(1.7)

where \( W_j \) is the relative importance (weight) of \( j^{th} \) criterion.

This index also takes values between 0 and 1, and represents the global intensity of preference between the couples of alternatives.

**Step 5:** Determine the leaving (or positive) and entering (or negative) outranking flows as follows:

\[ \phi^+(A_i) = \frac{1}{m-1} \sum_{i=1}^{m} \pi(A_i, A_i'), \quad (i \neq i') \]  

(1.8)

\[ \phi^-(A_i) = \frac{1}{m-1} \sum_{i=1}^{m} \pi(A_i', A_i), \quad (i \neq i') \]  

(1.9)

where \( m \) denotes number of alternatives. \( \phi^+(A_i) \) and \( \phi^-(A_i) \) denote positive and negative outranking flows respectively. Here, each alternative faces \((m - 1)\) number of other alternatives. The leaving flow expresses how much an alternative dominates the other alternatives, while the entering flow denotes how much an alternative is dominated by the other alternatives.

**Step 6:** Calculate the net outranking flow for each alternative.

\[ \phi(A_i) = \phi^+(A_i) - \phi^-(A_i) \]  

(1.10)

**Step 7:** Determine the ranking of all the considered alternatives depending on the values of \( \phi(A_i) \). The higher value of \( \phi(A_i) \), the better is the alternative. It is a number between \(-1\) and \(+1\). The best alternative correspond to large positive values of \( \phi(A_i) \) and the worst to large negative value. Moreover the net flow can be interpreted as a centered score:

\[ \sum_{i=1}^{m} \phi(A_i) = 1 \]
The PROMETHEE method can classify the alternatives which are difficult to be compared because of a trade-off relation of evaluation standards as non comparable alternatives. It is noteworthy to mention here that PROMETHEE methodology may only be applied if the decision maker can express the importance of various criteria on a ratio scale. Therefore a decision maker must be able to supply such quantitative criterion importances with the necessary accurateness. A considerable number of successful applications have been accomplished by the PROMETHEE methodology in various fields such as Banking, Industrial Location, Manpower planning, Water resources, Investments, Medicine, Chemistry, Health care, Tourism etc. The application of the methodology is basically due to its mathematical properties and its friendliness of use.

1.2.3 Simple Additive Weighting (SAW)

The SAW method is probably the simplest and most widely used MCDM method. It is intuitive and easy. A score in the SAW method is obtained by contributions from each criteria. Since two items with different measurements cannot be added, a common numerical scaling system such as normalization is required to permit addition among criteria values. The total score for each alternative can be computed by multiplying the comparable ratings for each alternative with its respective criterion weight and then adding these products over all the criteria.

In general, suppose that a given MCDM problem is defined on \( m \) alternatives and \( n \) decision criteria. Furthermore, let us assume that all the criteria are beneficial criteria. That is, the higher the values are, the better it is. Next suppose that \( W_j \)
denotes the relative weight of importance of the criterion $C_j$ and $r_{ij}$ is the normalized performance value of alternative $A_i$ when it is evaluated in terms of criterion $C_j$. Then the total importance of alternative $A_i$, denoted as $S_i$ is defined as follows:

$$S_i = \sum_{j=1}^{n} r_{ij} W_j, \quad \forall i$$  \hspace{1cm} (1.11)

The alternative with the highest score is selected as the preferred one.

1.2.4 Rank Order Centroids (ROC)

Proposed by McCaffrey (2005; 2006), ROC is a MADM technique based on a hierarchical decomposition (Figure 1.1) of comparison attributes and rating assignment using rank order centroids. It has features similar to AHP and is used to assign a single, overall measure of quality to each member of a system having arbitrary number of comparison attributes. The decision-makers usually can rank items much more easily than giving weight to them. This method takes those ranks as inputs and converts them to weights for each of the items. The technique uses rank order centroids to assign normalized numeric weights to the comparison attributes as an overall measure of quality with reference to a set of evaluation criteria. The term `Rank Order Centroid’ was coined by Barron & Barrett (1996), who also argued for its use in multi-attribute decision problems. The idea is to convert ranks ($1^{st}$, $2^{nd}$, $3^{rd}$, ... $n^{th}$) into values that are normalized on a 0.0 to 1.0 interval scale. Ranks hypothetically emphasize on rating data sequentially as: $1^{st}/n$, $2^{nd}/n$, $3^{rd}/n$, ... $n^{th}/n$, but it is credible from elementary statistics that rate data is best handled using harmonic techniques. For example the aver-

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age of 30 mph and 60 mph over a fixed distance is not \((30 + 60) / 2 = 45\) mph, but rather \(2 / (1/30 + 1/60) = 40\) mph. Notice that calculations for ROCs are conceptualized on the similar pattern emphasized above. Crain(2003) used ROCs for multi-attribute weight determination in his dissertation.

This methodology was originally developed to validate the results of AHP by adopting a comparatively simplified course of action and the technique has produced highly useful results in practice. This technique is very general and can be applied to virtually any type of system including traditional applications.

As proposed by Edwards & Barron(1994), after structuring a problem into hierarchical form, the next step is to determine the relative weights of each of the comparison attributes. In most situations, certain attributes are more important than others. The criteria are then rank-ordered with regard to their priority relative to the decision goal. These rank orders are then converted into numeric scales using rank order centroids, a value that estimates the distance between adjacent ranks on a normalized scale running from 0 to 1. Rank order centroids are fairly easy to calculate as illustrated:

Assume that there are \(k\) items that need to be weighted and that they are rank-ordered so that \(w_1 > w_2 > \ldots > w_k\). The rank order centroid of \(i^{th}\) item is given by:

\[
 w_i = \frac{1}{k} \sum_{n=i}^{k} \frac{1}{n}
\]

as \(n\) goes from \(i\) (criterion number) to \(k\) (the number of criteria to be ranked at a time).

When there are ties, the average of the weights for the tied places is used. For example, if there is a tie for the second best item, the mean of the weights for ranks 2 and 3 is used for both of the tied items. We can convert the rankings into
weights by using the table below. Table 1.4 shows the Rank Order Centroid (ROC) methodology developed by Edwards & Barron(1994) and depicts the weights up to 12 criteria.

<table>
<thead>
<tr>
<th>Rating Of</th>
<th>Number Of Criteria →</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria</td>
<td>2</td>
</tr>
<tr>
<td>1st</td>
<td>.75</td>
</tr>
<tr>
<td>2nd</td>
<td>.25</td>
</tr>
<tr>
<td>3rd</td>
<td>.111</td>
</tr>
<tr>
<td>4th</td>
<td></td>
</tr>
<tr>
<td>5th</td>
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<td>6th</td>
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<td>8th</td>
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<tr>
<td>10th</td>
<td></td>
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<tr>
<td>11th</td>
<td></td>
</tr>
<tr>
<td>12th</td>
<td></td>
</tr>
</tbody>
</table>

The nodes at each level are compared pairwise with respect to their contribution to the nodes above them to find their respective global weights. We rank each of the criteria in the final set by evaluating it with respect to upper level attributes separately. The evaluation process finally generates the global weights for each requisite criterion of interest. In a realistic scenario, this technique is very adaptable and can handle any number of attributes in a system.
Rank order centroids are a more methodical approach compared to the arbitrariness of the weighted summation. This simplification can reduce the calculation effort for the weights significantly, especially when judgment criteria are large in number and pairwise comparisons are difficult to be accomplished.

An outright code of the above methodology is given in APPENDIX A.

### 1.2.5 Ratio Method

The Ratio Method is another simple way of calculating weights for a number of critical factors. A decision-maker should first rank all the items according to their importance in the preferred domain. The next step is giving weight to each item based on its rank in the interval [10, 90]. Here lowest ranked item will be given a weight of 10 and rests of the items are rated in multiples of 10 based on the preferences given by decision maker. For example if item $II$ is five times more important to item $I$ (the lowest ranked item), then item $II$ is provided with a rating 50. The last step is normalizing these raw weights as proposed by Weber & Borcherding (1993). This process is shown in the example below.

Note that the weights should not necessarily jump 10 points from one item to the next. Any increase in the weight is based on the subjective judgment of the decision-maker and reflects the difference between the importances of the items. Ranking the items in the first step helps in assigning more accurate weights. For example, if there are four items ranked successively with the priorities 50, 40, 20 and 10 respectively, then the normalized respective weighted score of each item will be 0.417, 0.333, 0.167, and 0.083 respectively. Normalized weights are simply
calculated by dividing the raw weight of each item over the sum of the weights for all items. For example, normalized weight for the first item is calculated as $50/(50 + 40 + 20 + 10) = 0.417$. Ratio method can be easily applied in single and multi-dimensional MCDM problems. An advantage of this method is that instead of the actual values, it can use relative ones.

It is conspicuous to note here that Ratio Method entrusts more power in the hands of decision maker to prioritize the attributes quantitatively. The integrated code of the methodology is given in APPENDIX B.

### 1.2.6 Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS is a MADM method to identify solutions from a finite set of alternatives. As suggested by Hwang & Yoon (1981), a MADM problem may be viewed as a geometric system in which the $m$ alternatives that are evaluated by $n$ attributes are similar to $m$ points in a $n$ dimensional space. Therefore, the most preferable alternative should be the point in that space that is nearest to the ideal solution and farthest from the worst solution. According to Shih, Syur & Lee (2007), some advantages of TOPSIS are as follows:

- A sound logic that embodies the rational of human choice,
- A simple computation process that can be easily programmed into a spreadsheet,
- A scalar value that accounts for both the best and worst alternative at the same time.

Let $A_i, i = (1,\ldots,m)$ be a set of $m$ alternatives, $C_j, j = (1,\ldots,n)$ be $n$ evalu-
ation criteria and $W_j$, $j = (1, ..., n)$ be the associated criteria weights as shown in Table 1.5. It is also assumed that the weights $W_j$ of the criteria have been determined by an appropriate method (this is not a part of the TOPSIS methodology). Furthermore $x_{ij}$ is the performance measure of alternative $A_i$ with respect to criterion $C_j$.

**Table 1.5: Decision Matrix**

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_j$</th>
<th>$C_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i$</td>
<td>$W_1$</td>
<td>$W_j$</td>
<td>$W_n$</td>
</tr>
<tr>
<td>$A_1$</td>
<td>$x_{11}$</td>
<td>$x_{1n}$</td>
<td></td>
</tr>
<tr>
<td>$A_i$</td>
<td>$x_{ij}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_m$</td>
<td>$x_{m1}$</td>
<td>$x_{mn}$</td>
<td></td>
</tr>
</tbody>
</table>

The procedural steps of TOPSIS are summarized as follows:

**Step 1:** Obtain an evaluation $x_{ij}$ as crisp score for each alternative corresponding to the respective criterion. Here $x_{ij}$ is the performance measure of $i^{th}$ alternative with respect to $j^{th}$ criterion.

**Step 2:** Normalize the decision matrix using the following equation:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad (i = 1, \ldots, m), \quad (j = 1, \ldots, n)$$  \hspace{1cm} (1.12)

Normalization of the decision matrix transforms the various attribute dimensions into non-dimensional attributes, which allows comparison across the attributes.
Step 3: Form the weighted normalized decision matrix using the formula

$$v_{ij} = r_{ij}w_j \quad (i = 1, \ldots, m), \quad (j = 1, \ldots, n)$$  \hspace{1cm} (1.13)

Step 4: Determine the ideal and negative-ideal solutions.

$$A^+ = [(\max_i(v_{ij}) : j \in J), (\min_i(v_{ij}) : j \in J), \quad (i = 1, \ldots, m)]$$

$$A^+ = [v^+_1, v^+_2, \ldots, +v^+_j, \ldots, +v^+_n]$$  \hspace{1cm} (1.14)

$$A^- = [(\min_i(v_{ij}) : j \in J), (\max_i(v_{ij}) : j \in J), \quad (i = 1, \ldots, m)]$$

$$A^- = [v^-_1, v^-_2, \ldots, -v^-_j, \ldots, -v^-_n]$$  \hspace{1cm} (1.15)

- \( J \) be the set of benefit attributes or criteria (more is better).

- \( J' \) be the set of negative attributes or criteria (less is better)

Step 5: Calculate the separation measures for each alternative.

Positive Ideal Separation

$$S^+_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^+_j)^2} \quad (i = 1, \ldots, m)$$  \hspace{1cm} (1.16)

Negative Ideal Separation

$$S^-_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^-_j)^2} \quad (i = 1, \ldots, m)$$  \hspace{1cm} (1.17)

Step 6: Calculate the relative closeness to the ideal solution.

$$C^*_i = \frac{S^-_i}{S^+_i + S^-_i} \quad 0 < C^*_i < 1, \quad (i = 1, \ldots, m)$$  \hspace{1cm} (1.18)
• $C_i^* = 1$, if $A_i = A_i^+$

• $C_i^* = 0$, if $A_i = A_i^-$

Step 7: A set of alternatives can now be preference ranked according to the descending order of $C_i^*$

1.2.7 Lexicographic Approach

Multi-objective optimization consists of optimizing a number objectives that are usually conflicting. One way to tackle MODM problems is the lexicographic method. Here each objective is optimized one at a time subject to a pre-defined ordering established by the decision makers. It is important that the decision-maker must express preferences in order to establish the ordering and the performance of the method is vulnerable to the priorities given to various objectives.

A k-objective optimization problem can be formally described as:

$$\text{Minimize } Z(X) = (f_1(x), f_2(x), \ldots, f_k(x))$$

subject to:

$$g_i(x) \leq 0 \quad (i = 1, \ldots, m) \quad (1.19)$$

The single objective problem is solved consisting of most preferred desideratum (say $f_1(x)$) as the objective function subject to given constraints by usual methods ignoring rest of the objectives. The problem is reconstituted taking the next prioritized objective (say $f_2(x)$) with the previous solution as an added constraint. The process is repeated until all the $k$ objectives are dealt one by one adding the previous solutions in the constraint inequations. The approach is useful while
dealing with few objectives (two or three) but the procedure may be lengthy when number of objectives exceed.

### 1.2.8 Weighted Penalty Method

While solving the MODM problems for a variety of parameters, scalarization is most practical and feasible approach. Several possible solutions are generated depending on the priorities provided to various objectives. In the last decades the main focus was on finding one optimal solution to such problems by interactive methods, for ready reference, see Schwefel(1995) and Miettinen(1999) but now due to availability of much advanced application software, it is possible to represent the whole efficient set without much manual efforts. The decision maker gets a better insight in the problem structure by visualizing the whole solution set at a stretch. A wide variety of scalarization methods exist based on which one can assemble a MODM problem into prioritized single objective problem. In this work, a Weighted Penalty Cost Approach to put restrictions on assignments, apart from achieving a prioritized efficient solution by exercising priorities to preemptive factors, has been employed.

### 1.3 Organization of the Thesis

The thesis is organized as follows:

In earlier two chapters, both MADM and MODM methods are explored simultaneously and rest of the chapters focus on applications of MADM methods in real life situations. Chapter 2 propounds the effective resolution of parking problems
bumping into residential societies by framing a bi-objective problem to minimize the weighted cost and maximum distances traveled by the residents. All the aspects of sharing of parking slots are visualized subject to land availability. AHP is used to compute weighted set up costs of various parking slots. The quandary is scalarized and subsequently solved using Hungarian Method to assignment problem and integer programming.

Chapter 3 extends the study conducted in Chapter 2 by analyzing a three objective problem in relevance to site selection model. The three objective problem seeking to maximize the profit, minimize the set up cost born by the investors and to maximize qualitative standards is untangled using Lexicographic approach and thereafter solved using integer programming. A comparative solution is projected using scalarization method.

Chapter 4 proposes the use of improved PROMETHEE methodology for candidate selection eligibility and has the potential to re-shape and influence the way the general public perceive social justice for a rational decision.

Chapter 5 critically analyzes the rural market opportunities for the investors so as to formulate better merchandise programs. TOPSIS together with ROC was used to explore anticipation in rural marketing due to its simplicity and feasibility in application specifically due to low literacy level in these areas.

Chapter 6 provides comparative ranking of educational software for scholastic programs scouting both technical and non technical aspects. Work was conducted to help educational institutions gain a deeper understanding of professional decisions they face and reduce their initial state of uncertainty about the best course of
action. Rank Order Centroid (ROC) methodology and Ratio Method are used to accomplish the results.

Chapter 7 designs a new MADM method which is independent enough not to require a deep understanding of the process by the decision maker. The developed methodology considerably improves the consistencies of pairwise comparison matrices with minimized computational efforts.

Chapter 8 summarizes the conclusions of this study and projects future plan of action for further research.

### 1.4 Publications

The subject matter of the thesis is published / under publication in the form of following research papers worked out by the author.


