CHAPTER -2

2.1. Basic equations

The investigation of any fluid motion involves solving a set of non-linear partial differential equations called the fundamental equations of fluid dynamics. The fundamental equations governing any flow phenomena are stated below:

2.1.1 Continuity equation;

In fluid dynamics, the continuity equation is a mathematical statement that, in any steady state process, the rate at which mass enters a system is equal to the rate at which mass leaves the system. The differential form of the continuity equation is:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

Here, \( \rho \) is the density and \( \mathbf{u} \) is the velocity of the fluid.

2.1.2 Equation of motion:

The total force acting on a fluid mass enclosed in an arbitrary volume fixed in space is equal to the time rate of change of linear momentum (Law of conservation of momentum). The general form of Nervier–Stokes equation is

\[
\rho \frac{D\mathbf{u}}{Dt} = \Delta P + \rho f
\]

Where \( \mathbf{u} \) is the velocity, \( \rho \) is density of fluid, \( \frac{D}{Dt} \) is substantive derivative, \( f \) is the body force vector, and \( P \) is a tensor that represents the surface forces applied on a fluid particle.
2.1.3. Equation of energy

Physical principle: The energy added to a closed system increases the internal energy per unit mass of the fluid (Law of conservation of energy). In general energy equation can be defined as

\[ \rho C_v \frac{dT}{dt} = k \Delta^2 T + 2\mu e_{ij} + Q \]

Here \( C_v \) is the specific heat; \( k \) is the thermal conducting heating. According to Fourier law \( Q \) rate of energy generated per unit volume, and \( T \) is the temperature.

2.2. Dimensionless parameters

Dimensionless analysis results in sound, orderly arrangement of the various physical quantities involved in the problem. The magnitude of individual quantities encountered in a physical problem can be assembled into dimensionless groups, using dimensional analysis and the differential equations governing the fluid flow can be recast into dimensionless forms. The dimensionless parameters, which appear in the resulting equations, are the parameters of solutions and are the key factors in determining the qualitative and quantitative nature of the flow phenomenon.

The following dimensionless parameters appearing in the thesis. Their general definitions as follows:
**Hartmann number-** $H_a$

Hartmann number is as the ratio of electromagnetic force to the viscous force and is given by.

$$H_a = \frac{\sigma B^2 h^2}{\mu}$$

Where, $\sigma$ is the electrical conductivity, $\mu$ is the viscosity, $B_0$ is the magnetic field, $h$ the characteristic length scale.

**Reynolds number-** $Re$

Reynolds number is defined as the ratio of inertial force to viscous force and is given by $Re = \frac{\rho h u_0}{\mu}$

**Prandtl number-** $Pr$

Prandtl number is a property of the fluid not only of a particular flow, and plays a significant role especially under constrained system and system with through flow. Physically, it represents the ratio of viscous diffusion rate to the thermal diffusion rate, and is a measure of the relative importance of viscosity and heat conduction in the flow field. Temperature and velocity profile are identical when this parameter is unity. $Pr = \frac{\mu C_p}{k}$

**Eckert number-** $E_c$

Eckert number $E_c$ is a key parameter in determining the relative importance in a heat transfer situation of the kinetic energy of a flow. It is
the ratio of the kinetic energy to the enthalpy (or the dynamic temperature to the temperature) driving force for heat transfer.

\[ Ec = \frac{c^2}{c_p(T_1 - T_0)} \]

2.3. **Some Mathematical models for blood flow:**

In the present work some theoretical models have been suggested for blood flow and an attempt has been made to check their validity through experimental results. Before we deal with the present work, we shall first discuss some of the existing models for dusty fluid.

At low shear rates blood exhibits yield stress (Michael et. al., [49]) & Scott Blair [50] have proposed theoretical models for blood flow by using Casson [51] hypothesis. Charm and Kurland [52] have demonstrated the utility of Casson equations in the rheology of blood and they have shown that this equation can be applied to blood flow over a wide range of hematocrit and shear rates. Some other aspects of blood flow such as flow through tapered tube has been studied in several cases of non-Newtonian fluid specified by particular flow curve. Power law fluid Bingham body and fluid obeying Casson equation, however, the applicability of the Casson fluid model to blood flow has some limitations. One cannot explain size effects by Casson fluid. Blood flow models with high shear rates have been given here. Rathod [53] studied a theoretical model for pulsatile flow of blood with applications to cardio
vascular diseases. It is observed that an increase in $\alpha$ leads to an increase in the axial velocity and phase lag between flow rate and pressure gradient increase as the concentration decreases (i.e., as $\alpha$ increase).

Many researchers have studied the studies of blood in the large vessels, which reveals that blood can be modeled as a Newtonian fluid but it fails to be true in smaller vessels. As blood being a suspension of cells in plasma, blood behaves as a non-Newtonian fluid in narrow tubes at low shear rates. Thus, it is relevant to review mathematical models where blood is assumed to be non-Newtonian. To acquire a more complete understanding of the flow in small vessels, it is important, however, to consider flow not only in straight tubes, but also in other geometries.

The theory of Laminar, steady one-dimensional gravity flow of a non-Newtonian by Asterita et al., [54] Suzuki and Tanaka [55] have carried out some experiments on non-Newtonian fluid along an inclined plane. The flow of Rivilin – Erickson incompressible fluid through an inclined channel with two parallel flat walls under the influence of magnetic field has been studied by Rathod and Shrikanth [56]. The gravity flow of a fluid with couple stress along an inclined plane at an angle with horizontal has been studied by Chaturani and Upadhya [57].

It is known that blood is an electrically conducting fluid, so the flow of blood in human system can be decelerated by applying the magnetic field and in turn it may help in the treatment of certain cardio-
vascular diseases and also in the diseases with accelerated blood circulation such as hyper-tension, hemorrhages and so on Kolin [58] was first to coin the idea of electro-magnetic field in the medical research. The possibility of regulating of the blood movement in human system by applying magnetic field was discussed by Korchevskii and Morochnik [59]. For the prevention and rational therapy of arterial hypertension apparently of no less importance in close study of the hydrodynamic changes associated with the electrically conducting fluids, then one such factor may be a magnetic field. Haldar and Ghosh [60] have discussed the application of MHD principles in medicine, biology and in biomathematics. Electromagnetic field is applied in treating the patients with fractures in conduction of non-union, delayed union, compound fractures of severe nature, infections and fresh closed fractures with commutation and large but limited fracture gaps where the treatment methods available are very limited. The medical practitioners have suggested that, for the lower limb fractures, in the initial stages of the treatment, effective magnetic field are all very much essential in acquiring good results, Mathews [61], Ramachandra Rao and Deshikachar [62], have studied the effect of transverse magnetic field on physiological types of flow in uniform circular pipe. Rathod and Roni [63] have studied pulsatile blood flow through closed rectangular channel with micro-organism. Ghosh et al., [64] have studied the mathematical modeling of transient MHD couple stress fluid flow in a
rotating channel. Rathod and Seema [65, 66] have discussed pulsatile flow of blood/two layer blood flow with volume fraction of micro-organism and gravity effect and pulsatile flow of blood in capillaries of small exponential divergence with volume fraction of micro-organism studied by Rathod and Baderunnisa Begum [67].