CHAPTER-3

UNSTEADY FLOW OF A DUSTY MAGNETIC CONDUCTING COUPLE STRESS FLUID THROUGH A PIPE

3.1. Introduction:

The flow of a dusty and electrically conducting fluid through a pipe in the presence of transverse magnetic field has important applications in Magnetohydrodynamics (MHD) generators, pumps, accelerators, and flow meters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and (or) the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase with the so-called particle-phase viscosity. There have been many articles dealing with theoretical modeling and experimental measurement of the particle-phase viscosity in a dusty fluid [68-71].

The flow of a conducting fluid in a circular pipe has been investigated by many authors. Gadiraju et al., [72] investigated steady two-phase vertical flow in a pipe. Dube and Sharma, [73] and Ritter and Peddieson [74] are reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of particle-phase viscous stresses. Chamkha [75] obtained exact solutions that generalize the results reported in Ref. [74] and [75] by the inclusion of the magnetic and
particle-phase viscous effects. A number of industrially important fluids such as molten plastics, polymers and foods exhibit non-Newtonian fluid behavior [76]. Due to the growing use of these non-Newtonian materials in various manufacturing and processing industries, considerable effort has been directed towards understanding their flow characteristics. Many of the inelastic non-Newtonian fluids, encountered in chemical engineering processes, are known to follow the so-called “power-law model” in which the shear stress varies according to a power function of the strain rate [77].

The aim of this chapter is to study the unsteady flow of a dusty couple stress fluid through a circular pipe is investigated in the presence of a uniform magnetic field. The carrier fluid is assumed as viscous incompressible, and electrically conducting. The particle phase is assumed to be pressure less, incompressible, electrically none conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing nonlinear momentum equations for both the fluid and particle-phases are solved using transform technique. The effects of the magnetic field, the couple stress fluid parameter, and the particle-phase viscosity on the velocity of the fluid and particle-phases are reported.

3.2. Governing equations:

Consider unsteady, laminar axi-symmetric horizontal row of a dusty conducting couple stress fluid through an infinitely long pipe of
radius \( (d) \) driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The magnetic Reynolds number is assumed to be very small and consequently the induced magnetic field is neglected. Ref. [78], both phases behaves as viscous fluids Ref. [79] and also it is assumed that the volume fraction of suspended particle is finite and constant. The governing momentums equations can be written as.

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = \frac{dp}{dz} + \frac{1}{r} \frac{d}{dr} \left( \mu r \frac{\partial \mathbf{v}}{\partial r} \right) + \frac{\rho \phi}{1-\phi} (v_{p} - \mathbf{v}) - \sigma \mathbf{B}_{0}^{2} \mathbf{v} - \nu \nabla^{2} (\nabla^{2} \mathbf{v}) \tag{3.2.1}
\]

\[
\rho_{p} \frac{\partial \mathbf{v}_{p}}{\partial t} = \frac{1}{r} \frac{d}{dr} \left( \mu_{p} r \frac{\partial \mathbf{v}_{p}}{\partial r} \right) + \rho_{p} \mathbf{N} (\mathbf{v} - v_{p}) \tag{3.2.2}
\]

Where \( t \) is the time, \( r \) is the distance in direction, \( \mathbf{v} \) is the fluid phase, \( v_{p} \) is the particle phase velocity; \( \rho \) is the fluid phase density, \( \rho_{p} \) is the particle-phase density, \( \frac{dp}{dz} \) is the fluid pressure gradient, \( \phi \) is the particle phase volume fraction, \( \mathbf{N} \) is a momentum transfer coefficient, \( \alpha^{2} = \frac{a^{2} \mu}{\eta} \) couple stress parameter.

In this present work\( \rho, \rho_{p}, \mu_{p}, \phi \) and Bo all are constant. The particle-phase pressure is assumed negligible and that the particles are being dragged along with the fluid-phase.

The initial and boundary conditions of the problem are given as:

\[
\mathbf{v}(r, 0) = 0, \quad v_{p}(r, 0) = 0 \tag{3.2.3a}
\]

\[
v(d, t) = 0, \quad v_{p}(d, t) = 0 \tag{3.2.3b}
\]

\[
\frac{dv^{(o.t)}}{dt} = 0, \quad \frac{dv_{p}^{(o.t)}}{dt} = 0, \quad \nabla^{2} \mathbf{v} = 0 \quad \text{at} \quad r = d \tag{3.2.3c}
\]
\( \nabla^2 v, \nu, v_p \) are finite at \( r=0, t>0 \).

Where \( r \) is the distance in the radial direction and \( d \) is the radius of the pipe. The constitutive equations (3.2.1), (3.2.2) can be made dimensionless by introducing the flowing dimensionless variables and parameters:

\[
\hat{\nu} = \frac{r}{d}, \quad \hat{\nu} = \frac{t \mu_1}{q d^2}, \quad Go = \frac{-dp}{dz}, \quad K = \frac{p_p Q}{q(1-Q)}, \quad \hat{\phi}^2 = \frac{d^2 \mu}{y}, \quad \hat{\nu}(r, t) = \frac{\nu(r, t)}{Go d^2}
\]

\[
\hat{\nu}(r, t) = \frac{p_p(r, t)}{Go d^2}, \quad \alpha = \frac{\rho N d^2}{\mu}, \quad \beta = \frac{\mu_p}{\mu} \text{ is the viscosity ratio.}
\]

Equation (3.2.1) becomes.

\[
\frac{\partial \hat{\nu}}{\partial t} = 1 + \nabla^2 \nu - \frac{1}{\alpha^2} \nabla^2 (\nabla^2 \nu) + K \alpha (\nu_p - \nu) - H^2 \alpha \nu
\]

Equation (3.2.2) becomes.

\[
\frac{\partial \nu_p}{\partial t} = \beta \nabla^2 \nu_p + \alpha (\nu - \nu_p)
\]

\( \nu(0, 0) = 0, \quad \nu_p(0, 0) = 0 \)

\[
\frac{\partial \nu(0, t)}{\partial t} = 0, \quad \frac{\partial \nu_p(0, t)}{\partial t} = 0
\]

\( \nu(1, t) = 0, \quad \nu_p(1, t) = 0 \)

\( \therefore \nabla^2 \nu = 0, \text{ at } r=d, \quad \nabla^2 \nu \) is finite at \( r=0 \)

We get

\[
\nu = \sum_{n=0}^{\infty} \frac{(-1)^n}{n} \frac{1}{x_2} \left[ 1 + \frac{1}{(x_1^2 - 4x_2)^{\frac{1}{2}}} \left( \alpha_2 e^{\alpha_1 t} - \alpha_1 e^{\alpha_2 t} \right) \right]
\]

\[
\sum_{l=1}^{\infty} \frac{J_0(r \xi l)}{J_1(\xi l)} \cos \left( \frac{2m+1}{2\alpha} \right) \pi \theta
\]
Where \( x_1 = 1 + \beta + x + k\alpha + H^2 a + \frac{1}{a^2} \),

\[
x_2 = \beta + \frac{\alpha}{a^2} + \frac{\beta}{a^2} + \beta (k\alpha + H^2 a) + \alpha.
\]

\[
V_p = 4\sum_{m=0}^{\infty} \alpha \left( \frac{(-1)^m}{n} \frac{1}{x_2} \left[ \frac{1-e^{-(a+\beta)t}}{a+\beta} \right] + \frac{1}{(x_4^2-4x_2)^\beta} \right)
\]

\[
\left[ \alpha_2 \left( e^{a_1 t} - e^{-(a+\beta)t} \right) \alpha_1 + (a+\beta) \right] - \left[ \alpha_1 \left( e^{a_2 t} - e^{-(a+\beta)t} \right) \alpha_2 + (a+\beta) \right] \sum_{l=1}^{\infty} J_0(r\xi_l) \cos \left( \frac{(2m+1)}{2a} \right) \pi \theta
\]

3.2.8

The volumetric flow rates and skin friction coefficient for both the fluid and Particle phased are defined respectively as:

\[
Q = 2\pi \int_0^1 v(r, t) \, dr, \quad Q_p = 2\pi \int_0^1 v_p (r, t) \, dr.
\]

\[
C = \frac{\partial V(r, t)}{\partial r}, \quad C_p = -\beta K \frac{\partial v_p(r, t)}{\partial r}
\]

3.2.9

Equations (3.2.7) and (3.2.8) becomes

\[
Q = 8 \pi \sum_{m=0}^{\infty} \left( \frac{-1}{n} \alpha \frac{1}{x_2} \left[ 1 + \frac{1}{(x_4^2-4x_2)^\beta} \right] \right)
\]

\[
\cos \left( \frac{2m+1}{2a} \right) \pi \theta \sum_{l=1}^{\infty} \frac{1}{(\xi_l)^2} \int_0^{\xi_l} \int_0^{T} dT\]

3.2.10

\[
Q_p = 8 \pi \sum_{m=0}^{\infty} \left( \frac{-1}{n} \alpha \frac{1}{x_2} \left[ 1 - e^{-(a+\beta)t} \frac{1}{(a+\beta)} \right] + \right.
\]

\[
\left. \frac{1}{(x_4^2-4x_2)^\beta} \left[ \alpha_2 \left( e^{a_1 t} - e^{-(a+\beta)t} \right) \alpha_1 + (a+\beta) \right] - \left[ \alpha_1 \left( e^{a_2 t} - e^{-(a+\beta)t} \right) \alpha_2 + (a+\beta) \right] \right) \sum_{l=1}^{\infty} J_0(r\xi_l) \cos \left( \frac{(2m+1)}{2a} \right) \pi \theta
\]

3.2.11

\[
C = -\frac{4}{a} \sum_{m=0}^{\infty} \left( \frac{-1}{n} \alpha \frac{1}{x_2} \left[ 1 + \frac{1}{(x_4^2-4x_2)} \right] \right)
\]

\[
\cos \left( \frac{2m+1}{2a} \right) \pi \theta \sum_{l=1}^{\infty} \frac{J_2(r\xi_l)}{J_1(\xi_l)}
\]

3.2.12
\[ C_p = \beta k \sum_{m=0}^{\infty} \frac{(-1)^m}{n} \frac{1}{x_2} \left[ \frac{1-e^{-(\alpha+\beta)t}}{\alpha+\beta} + \frac{1}{\sqrt{x_1^2-4x_2}} \left[ \frac{(\alpha_2 e^{\alpha_2 t} - e^{-(\alpha+\beta)t})}{\alpha_1 + (\alpha+\beta)} - \frac{\alpha_2 (e^{\alpha_2 t} - e^{-(\alpha+\beta)t})}{\alpha_2 + (\alpha+\beta)} \right] \right] \sum_{l=1}^{\infty} \frac{J_1(\xi l)}{J_1(\xi l)} . \]

\[ \cos \left( \frac{(2m+1)\pi}{2\alpha} \right) \theta \]

3.2.13

3.3. Results and discussions:

Equations (3.2.4) and (3.2.5) represent a coupled system of nonlinear partial differential, equations that are solved numerically under the initial and boundary conditions. (3.2.3a), (3.2.3b), using the transform technique.

Computation have been made for \( G_o = 1 \), \( a = 1 \), and \( k = 10 \). It should be mentioned that results obtained herein reduce to those reposted by Ref. [70] and Ref. [72] for the cases of non magnetic, couple-stress fluids and in viscous particle-phase. These comparisons lend confidence in the accuracy and correctness of the solutions.

Figures (3.3.1) present the time evolution of profiles of the velocity of the fluid \( v \) and dust particle \( v_p \) respectively for various values of couple stress parameter \( \tilde{\alpha}^2 \) and \( N_o = 0.5 \) and \( \beta = 0.5 \). Both \( v \) and \( v_p \) decrease as radius is increases. Figures(3. 3.2 )and(3.3.3) present the time evolution of the profiles of the velocity of the fluid \( v \) and dust particle \( v_p \) respectively for \( N_o = 0.5 \), \( \beta = 0.5 \), \( \tilde{\alpha}^2 = 0.5 \). It is clear that velocity of fluid \( v \) and dust particle \( v_p \) are increase with time and also indicates that effect on its steady-state time can be neglected.
Fig. 3.3.4 present the variation of $v$ and $v_p$, at the centre with time for different values of $\bar{\alpha}^2$. It is clear that both $v$ and $v_p$ increase with time. In fig (3.3.5)-(3.3.8) present the influence of magnetic field parameter $H_a$ on the transient behavior of the fluid-phase volumetric flow rate $Q$, the particle-phase volumetric flow rate $Q_p$, the fluid-phase Skin friction coefficient $C$ and the particle-phase Skin friction coefficient $C_p$ for various values of $\bar{\alpha}^2$ and for $\beta=0.5$. Initially, both phases are slowly increases by application of a constant-pressure gradient, as a result the shear stress at the surface for the pipe increases. These parameters continue to increase until the flow stabilizes and steady state condition area attained. The influence of magnetic field is to retard the flow of both phases causing their average velocities and wall shear stress in the pipe as well as their steady state time to decrease.

Figures (3.3.9) to (3.3.12) present the influence of the particle-phase viscosity $\beta$ on the transient behavior of $C$, $C_p$, $Q$ and $Q_p$ for various values of $\bar{\alpha}^2$ and $H_a = 1$. It is clear from the figures that the inclusion of the particle-phase stresses causes $Q$, $Q_p$ and $C$, to decrease and $C_p$ to increases in fig. 3.3.9. Also, approach to steady-state condition is more accelerated than that for case of $\beta=0.5$, as in clear from $Q$, $Q_p$ and $C$. Figure 3.3.10 shows that increasing $\beta$ increases $C_p$ but decreases its steady-state time.
3.4 Conclusion

The transient MHD flow of a particulate suspension in an electrically conducting couple stress fluid in a circular pipe with an applied uniform transverse magnetic field was studied. The governing non linear partial-differential equations were solved. The effect of the magnetic field parameter $H_a$, the couple stress fluid characteristics $\alpha^2$ and the particle-phase viscosity $\beta$ on the transient behavior of the velocity flow rates, and skin friction coefficients of both fluid and particle-phases were studied. It was found that all these parameters decreases as the strength of magnetic field increases or the couple stress parameter decreases. The particle-phase viscosity has apparent effect on increases the Skin friction of the particle-phase while decreasing the rest of the parameters. The approach to steady-state conditions is greatly decreased when increasing $\beta$ but it’s slightly increase with increases in the couple stress parameter.
Fig 3.3.1(a) Time development of $v$ for various values of $\alpha^2$

Fig 3.3.1(b) Time development of $v_p$ for various values of $\alpha^2$
Fig 3.3.2 Time development of $v$ for various values of $t$

Fig 3.3.3 Time development of $v_p$ for various values of $t$
Fig 3.3.4(a) Time development of $v$ for various values of $\alpha^2$

Fig 3.3.4(b) Time development of $v_p$ for various values of $\alpha^2$
Fig 3.3.5 Time development of C for various values of $H_a$

Fig 3.3.6 Time development of $c_p$ for various values of $H_a$
Fig 3.3.7 Time development of $Q$ for various values of $H_a$

Fig 3.3.8 Time development of $Q_p$ for various values of $H_a$
Fig 3.3.9 Time development of $C$ for various values of $\beta$

Fig 3.3.10 Time development of $c_p$ for various values of $\beta$
Fig 3.3.11 Time development of $Q_p$ for various values of $\beta$

Fig 3.3.12 Time development of $Q$ for various values of $\beta$