Chapter 3

ANALYTICAL INVESTIGATION

3.1 Background and objectives

Researches show that several parameters affect the penetration of unitary and segmented projectile on multi-layered and monolithic targets. The objective of the thesis was to numerically investigate (FE) the effect of several factors such as impact velocity, materials properties, thickness of the target, nose shape, number of layers and distance between the layers, oblique angle of target on the quality and quantity of impact damage. On the basis of numerical results the work optimized the parameters on the basis of high performance of penetrator.

✔ Optimize the performance of monolithic and segmented penetrator used as ammunition.
✔ Compare the performance of monolithic and segmented penetrator of two different nose shapes i.e. concave and ogive, for the projectile in crash analysis software
✔ To review the literature work done on the impact analysis.
✔ To review the literature on the finite element design verification approaches on the impact analysis.
✔ To carry out the stress distribution and damage caused around the target and projectile with different nose (blunt, hemi, cone) head and segmented projectiles.

Ballistic perforation and numerical analysis of unitary projectiles with the nose head blunt, conical, and hemispherical type and segmented 3:2 projectile on the vertical plate and inclined angles of 45° and 55° for impact velocities of 300m/s, 350m/s, and 400m/s, are performed. The material used for the unitary projectile is tungsten and for the segmented type a combination of tungsten and titanium is used. A segmented projectile is a cylindrical rod where the main material is separated by spacers, which are usually made of another material. In the present analysis the main material is tungsten and the spacers are titanium (here 3 tungsten materials are separated by 2 titanium spacers) and target is RHA steel, which is used for perforation.
3.2 Analytical Method

The impact of the penetrator into a target of known material property is governed by the impact dynamics. The problem is to determine the optimum nose shape of the penetrator with smoothly increasing concave shape function that is governed by impact dynamics. Mathematically similar problem is known to be established in hypersonic aerodynamics. The projectile target interaction for maximum penetrator efficiency is the function of the nose shape. The 'dynamical' work during impact is represented by Thompson's formula in the following form

\[ W_d = \pi \rho b v^2 \int_0^L \left( \phi \phi_{xx} + 2 \phi_x^2 \right) \phi_x \, dx \]

where

\( \rho \) is density of the target material,

\( b \) is the thickness of the target,

\( v \) is the initial velocity during impact,

\( \phi \) is the function describing the shape of the penetrator,

\[ \phi_{xx} = \frac{d^2 y}{dx^2} \quad \text{and} \quad \phi_x = \frac{dy}{dx}. \]

For the derivation of equation (3-1) which is obtained from the first principle.

To express equation (3-1) into dimensionless form, the variables are expressed in the dimensionless form as

\[ \bar{x} = \frac{x}{L} \quad \text{and} \quad \bar{y} = \frac{\phi}{R}, \]

\[ A = \frac{R^4}{L^2}. \]

Where \( L \) is the axial length of the penetrator, \( R \) is the maximum radius at the base.

With the assumption of dimensionless variables equation gets transformed to
\[ W_d = \pi \rho b v^2 AF \left[ \frac{\bar{y}(\bar{x})}{x} \right] \]  

(3-2)

for convenience \( \bar{x} \) and \( \bar{y} \) are represented by \( x \) and \( y \). Hence the functional

\[ F = \int \left[ f \left( y, \frac{dy}{dx}, \frac{d^2y}{dx^2} \right) \right] dx \]  

(3-3)

where \( f \) is the functional given by

\[ f = \left[ y, \frac{d^2y}{dx^2} + 2 \left( \frac{dy}{dx} \right)^2 \right] \frac{dy}{dx} \]  

(3-4)

Equation (3-4) is a non-linear differential equation to be solved to express \( y \) in terms of \( x \), considering the optimality solution.

### 3.3 Optimum shape

The variables describing the shape of the impactor and the target interacting with the penetrator are given in Fig. 3.1. (An arbitrary shape is initially assumed, which is contradicted by the obtained numerical shape which is concavely parabolic). The \( 'x' \) \( \text{co-ordinate} \) is directed along the length and \( 'y' \) \( \text{co-ordinate} \) is radially directed to represent the shape \( \phi \). The parabolic shape is assumed for the function \( \phi \) which should be obtained such that the functional \( F \) is minimum. This is satisfied by the minima condition given as

\[ \frac{dF}{dx} = 0 \]  

(3-5)
The minimization condition increases the efficiency of penetration as it minimizes drag of the penetrator into the finite target. This leads to the formulation in

\[
y \frac{d^2y}{dx^2} + 2 \left( \frac{dy}{dx} \right)^2 = 0
\]

or in simple compact form

\[
y.y'' + 2(y') = 0 \quad (3-6)
\]

Equation (3-6) is a non-linear equation that is solved for \(y\) in-line with the non-linear optimization. Equation (3-6) is written in the more useful form as

\[
Z_1 + 2Z_2 = 0 \quad (3-7)
\]

where \(Z_1 = y.y'' \quad and \quad Z_2 = (y')^2\).

Equation (3-7) has the form equivalent to equation (A1) of Appendix-A.

By applying the Interface Theory

\[
Z_1 = \frac{1}{2} \quad and \quad Z_2 = -\frac{1}{2} \left( \frac{1}{2} \right) \quad (3-8)
\]

It may be noted that the co-efficients of equation (7) are arranged in the ascending order.

From solution (3-8)

\[
y.y'' = \frac{1}{2} \quad and \quad y' = \sqrt{-\frac{1}{4}}
\]

The second solution is infeasible as it generates a complex number, and is hence neglected.

From the first solution, \(y = \frac{1}{2} \left( \frac{1}{y''} \right)\), which is linearized to

\[
-\frac{1}{2}y'' + y = 0 \quad (3-9)
\]
Applying the Interface Theory

\[ \frac{1}{y''} = 1 \quad \text{and} \quad y = \frac{1}{2} \ \text{or} \ y'' = 1 \]

On integration

\[ y = \frac{x^2}{2} + C_2 x + C_1 \] (3-10)

From the boundary conditions,

\[ y(0) = 0 \quad \text{and} \quad y(1) = 1 \] The constants of integration are evaluated to be

\[ C_1 = 0 \quad \text{and} \quad C_2 = \frac{1}{2} \]

Hence the solution to non-linear equation (3-6) is

\[ y = \frac{x}{2} (x+1) \] (3-11)

The functional \( F \) is given by

\[ F = \int_0^1 \left[ \frac{x}{2} (x+1) + (2x+1) \right] \cdot \frac{x}{2} (x+1) \left(2x+1\right) \, dx \] (3-12)

Hence \( F \) gets evaluated to \( F = 3.15 \) for \( x = 1 \). The results for various values of \( x \) are tabulated in Table 3.1.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0.055</td>
<td>0.12</td>
<td>0.195</td>
<td>0.28</td>
<td>0.375</td>
<td>0.48</td>
<td>0.595</td>
<td>0.72</td>
<td>0.855</td>
<td>1.00</td>
</tr>
<tr>
<td>( F )</td>
<td>0.00355</td>
<td>0.0195</td>
<td>0.0593</td>
<td>0.138</td>
<td>0.277</td>
<td>0.504</td>
<td>0.718</td>
<td>1.376</td>
<td>2.119</td>
<td>3.15</td>
</tr>
</tbody>
</table>
The computed results of normalized shape and the functional are listed, which are obtained from equations (3-11) and (3-12), in Table 3.1. The values of $\frac{\phi}{R}$ and $F$ are plotted in Fig. 3.2 and 3.3 respectively. Fig. 3.2 shows the optimal shape of the impactor for maximum efficiency of penetration. The shape of the penetrator is a concave paraboloid which dictates the practical feasibility of efficient and maximum penetration. The functional, $F$ plotted in Fig. 3.3 is the indicative of the drag force varying with the axial distance along the penetrator. The drag force is given by

$$D = (\pi \rho b A) F$$

(3-13)

![Fig. 3.2 Plot of the Optimal Shape](image1)

![Fig. 3.3: Plot of the Functional (F) vs Distance (X/L)](image2)

The penetrator of given dimension, for a given armour material of finite thickness, $b$, experiences the minimum drag force that varies directly with the functional that shows variation along the length of the impactor as shown by Fig. 3.3.

### 3.4 Computational methodology

The objective of the work was to investigate the effect of nose eccentricity on performance such as speed variation on different location, pressure variation, air density and temperature of penetrator using computational fluid dynamics. A 3D model of a conical penetrator is shown in Fig. 4.4.
3.4.1 Model geometry specifications

The geometrical model consisted of sharp cone, cylindrical fuselage and fins. The outer diameter (D) of the cylindrical fuselage is 50 mm and its length is 6D. The length of cone and fins is 4D and 2D respectively. Commercial CFD software CFX is used for simulation. The software has four major modules: a) CFX Build – imports the computer-aided design (CAD) geometry or creates geometry and generates unstructured volume meshing based on the user input, b) pre-processor – sets up the boundary condition and initial field condition, c) solver manager – solves the flow field based on the grid and boundary condition, and d) post-processor – visualizes and extracts the results.

As the free stream flow is supersonic, the inlet and outlet flow domains are taken as 5 times the total length of the projectile from the nose tip and tail end respectively. The upper, lower, left and right flow domains are taken 5 times the diameter of the cylindrical fuselage above and below the centre of the penetrator.

Generation of coarse mesh and fine mesh around the body of the conical penetrator specifies the surfaces for which meshing parameters are to be defined. Grids are made very fine in the nose of the conical region to capture the complex flow features of the flow field. In the simulation, x-axis is taken along the longitudinal direction from the nose of the conical penetrator, while the y-axis is along the normal direction of the penetrator and z-axis is along the lateral direction of the penetrator as shown in Fig. 3.5.
3.4.2 Computational Grid

The geometry and unstructured mesh were generated using Ansys ICEM-CFD. A three-dimensional (3D) mesh was required to simulate the effect of aerodynamic properties. The unstructured tetrahedral Octree Mesh generated around the conical shape model with total 1 million of elements. Fine mesh was generated on inlet and outlet of the domain. Penetrator eccentricity has a minimum size of 0.1mm, at the defect region having a total number of elements 12,617. For the penetrator with the maximum element size 4mm, This Octree Mesh ensures refinement of the mesh where necessary, but maintains larger elements where possible, around the penetrator from inlet to outlet has maximum sizes.

A hybrid mesh was generated by creating prism layers around the penetrator. Three layers with 1mm thick of 3 layers of prism generated. The prism layers are very essential for outer flow domains to calculate near wall functions. The computational final mesh domain is shown if Fig. 3.6.

Tetra meshing is not efficient for capturing shear or boundary layer physics. The prism mesh efficiently captures these effects near the surface to determine the aerodynamic properties over the penetrator. Three prism layers of fine grids with 1mm spacing liner growth near the wall and unstructured tetrahedral mesh in the outer region are generated in the computational domain. This efficiently allows for better resolution of the solution normal to the surface, without increasing the number of elements along the surface. This gives a
quicker and more accurate solution than a very fine tetra mesh. Importing the meshed component into the Ansys-CFX Pre-processor, creating the domain type fluid domain, material air, ideal gas because of supersonic flow, fluid model has heat transfer total energy, K-Omega turbulence model and standard K-Omega model results are used for determining the aerodynamic properties. The inlet velocity is of supersonic flow varied from 1.2 to 1.8 Mach number. The problem was setup for the standard atmospheric conditions with static temperature 288 K and the solver control has basic setting of advection upwind scheme for forward difference (first-order upwind is available in the pressure-based and density-based solvers), convergence criteria are of residual type (RMS). Post-processor visualizes of the velocity contours and extracts the aerodynamic properties. Simulations are carried out for eccentricity (a probable defect) from 0.3 to 0.9 mm.

3.5 Results and discussion

Fig. 3.8 shows the effect of aerodynamic velocity on the eccentric penetrator performance at different locations for 400, 500, and 600 m/s.

The conical eccentric shape penetrator is almost (linear velocity) equal to wind velocity at the tip of the penetrator and maximum at fillet joining between slope of geometry and cylindrical rod and at the presence of defects like eccentricity. On cylindrical rod subjected uniform velocities (400, 500, 600m/s). Similar ratio can be seen for other velocities of penetrator.

Fig. 3.8: Velocity contours at 400,500 and 600
Fig. 3.9 shows the result of aerodynamic velocity for 0 (symmetric), 0.3, 0.6 and 0.9 mm eccentricity. The simulation summary of velocity distribution along different location can be classified into two groups a) velocity distribution is almost the same from section 1 to section 7. In the second region the sudden drop of velocity can be observed. All graphs show the same trend but speed of air on different location is changes as per velocity of the air. In all cases the velocity of air trend can be seen to be the same in nature.

Fig. 3.9: Comparison of velocity distribution at different locations for 0.0, 0.3, 0.6 and 0.9 mm eccentricity.

Fig. 3.10: Comparison of pressure distribution at different locations for 0.0, 0.3, 0.6 and 0.9 mm eccentricity.
Fig. 3.10 shows the result of aerodynamic pressure for 0 (symmetric), 0.3, 0.6 and 0.9 mm eccentricity. The simulation outline of pressure distribution along different location can be classified in five groups.

There is a sudden increase in pressure distribution from section 1 to section 2 in all cases, but the maximum variation can be observed in 0.3mm eccentricity for 400, 500, and 600m/s. In the second region the sudden drop of pressure can be observed for eccentric geometries compared to the symmetry penetrator for 400m/s and a slight increase in pressure for 500, and 600m/s until at location 4. The sudden increase in pressure (from cylindrical portion to the starting point of fins) from location 4–5 reaches the peak value for 500m/s and 600m/s, but a slight increase in pressure can be observed for 400m/s. There is a linear and sudden decreasing in pressure distribution from extended fins to the back end of penetrator for all the velocities. All graphs show the same trend but aerodynamic pressure on different locations changes as per velocity of the air. In all cases the velocity of air trend can be seen to be slightly different in nature.

Fig. 3.11 shows the result of aerodynamic density for 0 (symmetric), 0.3, 0.6 and 0.9 mm eccentricity. The simulation summary of density distribution along different location can be classified into five groups.

Fig. 3.11: Comparison of density distribution at different locations for 0.0, 0.3, 0.6 and 0.9 mm eccentricity.
There is a sudden increase in density distribution from section 1 to section 2 for the geometry but the maximum variation can be observed in 0.3mm eccentricity for 400, 500, and 600m/s. In the second region the sudden drop of density can be observed for eccentric geometry compared to symmetry penetrator for 400m/s and remains unchanged for 500, and 600m/s until at location 4. Sudden increase in density (from cylindrical portion to starting point of fins) from location 4–5 reaches maximum value for 500m/s and 600m/s, but for 400m/s it retains constant density. There is a linear and sudden decrease in density distribution from extended fins to the back end of the penetrator for all the velocities. The all graphs show the same trend but aerodynamic density on different location changes as per velocity of air. In all cases the velocity of air trend can be seen to be slightly different.

Fig. 3.12: Comparison of temperature distribution at different locations for 0.0, 0.3, 0.6 and 0.9 mm eccentricity.

There is a sudden increase in density distribution from section 1 to section 2 for the geometry but the maximum variation can be observed in 0.3mm eccentricity for 400, 500, and 600m/s. In the second region the sudden drop of density can be observed for eccentric geometry compared to symmetry penetrator for 400m/s and remains unchanged for 500, and 600m/s until at location 4. Sudden increase in density (from cylindrical portion to starting point of fins) from location 4–5 reaches maximum value for 500m/s and 600m/s, but for 400m/s it retains constant density. There is a linear and sudden decrease in density distribution from extended fins to the back end of the penetrator for all the velocities. The all graphs show the same trend
but aerodynamic density on different location changes as per velocity of air. In all cases the velocity of air trend can be seen to be slightly different.

Fig. 3.12 shows the result of aerodynamic temperature for 0 (symmetric), 0.3, 0.6 and 0.9 mm eccentricity. The simulation review of temperature distribution along different location can be classified in two groups. a) The temperature distribution is almost same from section 1 to section 7. In the second region the sudden increase in temperature can be observed. All the graphs show the same trend but temperature on different location changes according to the velocity of air. In all cases the velocity of air trend can be seen to be same in nature. At the tip of the conical shape of the penetrator it has a temperature less than the atmospheric temperature. In the above simulation, it shows not much variation in temperature for symmetric and eccentric geometry penetrators.

Fig. 3.13 and 3.14 compares the results of effect of aerodynamic velocity and densities on the eccentric penetrator for 0.0 (symmetric), and 0.3 to 0.9 mm with varying steps of 0.1mm eccentricity performance at different locations for 400, 500 and 600 m/s.