CHAPTER 4

ALAMOUTI CODED MIMO-OFDM WITH LDPC CODES

It has been observed in previous chapters that MIMO communication systems along with OFDM play a key role in designing of next generation broadband wireless systems. To further improve system robustness and feasibility, coding gain advantage is added by concatenating MIMO-OFDM systems with error correcting codes like convolutional codes (CC) [49,74], turbo codes [75-78] and low density parity check codes (LDPC) codes [79-84]. CC codes are not preferred due to their large capacity gap with compared to Shannon's boundaries [85]. LDPC codes can be used to bridge this capacity gap. Most of the research related to this area is focused on the impact of increase in code rate and diversity of the system but not on increase in coding gain. In this chapter, we analyzed the impact of increased coding gain. The performance analysis of serially concatenated regular and irregular LDPC codes with Alamouti STBC and SFBC MIMO-OFDM systems for high data rate wireless transmission is investigated. The performance is analyzed using density evolution (DE) [77, 86] techniques under Rayleigh frequency flat channels mentioned in digital video broadcast-next generation handheld (DVB-NGH) standard [87]. DE tool is used to estimate pdf of information that is in log likelihood form (LLR) passed between check nodes and bit nodes of Tanner graph [88] used during decoding process. The pdf's are the function of SNR as well as iteration number and is used for probability of error computation after every iteration. The conventional iterative receiver [80] used to decode above system consists of maximum posteriori (MAP) soft input soft output decoder called soft demapper along with LDPC decoder [82-83]. The mentioned iterative receiver suffers from computational complexity compared to CC (Viterbi receiver). Efforts are made to reduce this decoder complexity by using bypass decoder [89] and semi iterative receiver [90] in AWGN and Rayleigh channels. In this section, we combine optimized LDPC codes with Alamouti ST and SF codes in order to get diversity as well as coding gain advantage along with reduced decoder complexity. The decoder complexity is reduced due to linear combination of receiver estimates by ML method as suggested by Alamouti. BER results are compared with existing designs in
terms of $\text{SNR}_{\text{min-op}}$ and complexity. $\text{SNR}_{\text{min-op}}[80]$ is defined as the minimum operational SNR required for which probability of error tends to zero. The generalized schematic of LDPC coded $M_{\text{Tx}} \times M_{\text{Rx}}$ Alamouti MIMO-OFDM system is shown in Figure 4.1.

![Diagram of LDPC coded MIMO-OFDM system](image)

**Figure 4.1**: Transceiver structure for LDPC codes concatenated with Alamouti coded MIMO-OFDM system

Initially, a block of $k$ bits of incoming data stream is encoded by a rate $R_c = k/n_{\text{out}}$ regular or irregular LDPC code. The LDPC encoding is done to introduce redundancy in original information, which enables the receiver to correct errors. The resultant $n$ coded bits are mapped into data symbols via M-PSK modulation technique, which leads to block of $n_{\text{out}}/\log_2 M$ symbols. In each OFDM time slot, only $N_{\text{C}}M_{\text{Tx}}$ out of $n_{\text{out}}/\log_2 M$ symbols can be transmitted simultaneously through $N_{\text{C}}$ OFDM subcarriers and $M_{\text{Tx}}$ transmitting antennas. This would results into $(n_{\text{out}}/\log_2 M)/N_{\text{C}}M_{\text{Tx}} = T$ OFDM time slots. To improve diversity performance, data symbols are further encoded by Alamouti STBC and SFBC transmit diversity scheme. OFDM symbols are produced by transforming frequency domain signals to time domain by applying IFFT. Each OFDM block is then appended by CP whose length is considered to be longer than channel delay spread. These symbols are transmitted simultaneously through $M_{\text{Tx}}$ transmitting antennas.
information is then passed through Rayleigh i.i.d. channel that has been generated based upon digital video broadcast (DVB)-next generation handheld (NGH) standard [87].

4.1 LOW DENSITY PARITY CHECK (LDPC) ENCODING

Since the invention of performance bounds derived from Shannon [85] on error-correction coding, many practical error-correction schemes have been proposed, but none of them achieve performances close to the ideal until the turbo coding was proposed. In year 1996, MacKay and Neal [85] rediscovered a class of codes first introduced by Gallager in 1962 [91] that have near-ideal performance. These codes, now widely known as LDPC codes, are linear block codes that are constructed by designing a sparse parity check matrix $H$. The matrix $H$ contains relatively few 1’s spread among many 0’s. Gallager proposed an iterative method of decoding the LDPC codes, which was capable of achieving excellent performance. However, the complexity of the iterative decoding algorithm was beyond the capabilities of the electronic processors available then and that is why the codes were forgotten until 1996. The first construction method proposed for the design of the sparse parity check matrix $H$ associated with these codes involves the use of a fixed number of 1’s per row and column of that matrix. In this case LDPC codes are said to be regular. However, the number of 1’s per row and column can be varied, leading to the so called irregular LDPC codes. The BER performance of LDPC codes is close to that of the turbo codes.

A systematic linear block code $C_b(n_{\text{out}}, k)$ is uniquely specified by its generator matrix, which is of the form

$$
G = \begin{bmatrix}
g_0 \\
g_1 \\
. \\
. \\
g_{k-1}
\end{bmatrix} = \begin{bmatrix}
p_{00}^c & p_{01}^c & \ldots & p_{0,n_{\text{out}}-k-1}^c & 1 & 0 & 0 & \ldots & 0 \\
p_{10}^c & p_{11}^c & \ldots & p_{1,n_{\text{out}}-k-1}^c & 0 & 1 & 0 & \ldots & 0 \\
. & . & \ldots & . & . & \ldots & . & \ldots & . \\
. & . & \ldots & . & . & \ldots & . & \ldots & . \\
p_{k-1,0}^c & p_{k-1,1}^c & \ldots & p_{k-1,n_{\text{out}}-k-1}^c & 0 & 0 & 0 & \ldots & 1
\end{bmatrix}
$$

(4.1)

Equation (4.1) can be represented in shorter form as
\[ G = \begin{bmatrix} p^c & I_k \end{bmatrix} \]  

(4.2)

where the message bits appear at the end of the code vector. The systematic form of the parity check matrix \( H \) of the code \( C_b \) is given by

\[
H = \begin{bmatrix}
1 & 0 & \cdots & 0 & p^c_{00} & p^c_{01} & \cdots & p^c_{k-1,0} \\
0 & 1 & \cdots & 0 & p^c_{01} & p^c_{11} & \cdots & p^c_{k-1,1} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & p^c_{0,n_{out}-k+1} & p^c_{1,n_{out}-k+1} & \cdots & p^c_{k,n_{out}-k+1}
\end{bmatrix} = \begin{bmatrix} I_{n_{out}-k} & p^c \end{bmatrix} \]  

(4.3)

In \( H \) the inner product of a row vector \( g_i \) of the generator matrix \( G \) and a row vector \( h_j \) of the parity check matrix \( H \) is zero i.e. \( g_i \) and \( h_j \) are orthogonal. Therefore

\[ G \circ H^T = 0 \]  

(4.4)

and therefore

\[ c \circ H^T = m \circ G \circ H^T = 0 \]  

(4.5)

Hence, the design of an LDPC code starts with the construction of parity check matrix \( H \), from which an equivalent systematic parity check matrix is obtained leading to formulation of the generator matrix \( G \) of the code. The syndrome equation for a block code can be described in terms of matrix \( H \) instead of its transpose. The code vector is generated from

\[ c = G^T \circ m \]  

(4.6)

Since \( G^T \) is a generator matrix of dimension \( n_{out} \times k \), and the message vector \( m \) is of dimension \( k \times 1 \), results in code vector \( c \) of dimension \( n_{out} \times 1 \), which is a column vector. If the code vector is generated using expression (4.6), then its corresponding syndrome decoding is based on syndrome vector \( S \) of the form

\[ S = H \circ c \]  

(4.7)

Which means every code vector would satisfies the condition
\[ H \circ c = 0 \]  \hspace{1cm} (4.8)

and (4.4) can be written in the form
\[ H \circ G^T = 0 \]  \hspace{1cm} (4.9)

Since the parity check matrix \( H \) is of dimension \((n_{\text{out}} - k) \times n_{\text{out}}\), and the generator matrix \( G^T \) is of dimension \( n \times k \), the syndrome condition is represented by a matrix of dimension \((n_{\text{out}} - k) \times k\) with all its elements equal to zero. This alternative way of encoding and decoding a block code indicates that the parity check matrix \( H \) contains all the necessary information to completely describe the block code. On the other hand, (4.9) is useful for designing an iterative decoder. LDPC codes are usually designed to be linear and binary block codes. In this case there is a generator matrix \( G \) that converts a message vector \( m \) into a code vector \( c \) by means of a matrix multiplication. The corresponding parity check matrix \( H \) has the property that it is constructed with linearly independent rows which means that every code vector satisfies the condition \( H \circ c = 0 \).

LDPC codes are designed by an appropriate construction of the corresponding parity check matrix \( H \). According to Gallager [91], an LDPC code is denoted by \( C_{\text{LDPC}}(n_{\text{out}}, k, s, t) \), where \( n_{\text{out}} \) and \( k \) are output and input bits with \( s \) number of 1’s in each column and \( t \) number of 1’s in each row. Usually \( t > s \), and \( s < n \). The 1’s are placed at random positions in a parity check matrix of size \((n_{\text{out}} - k) \times n\). These codes are of two types namely regular and irregular depending upon whether number of 1’s is fixed or varying. Main characteristics of LDPC codes includes (i) the error probability decreases exponentially with increasing code block length and (ii) the minimum distance of the code increases with increasing code length. Tanner [88] generalized the Gallager construction by defining bipartite graphs, where the equations related to the graph are generalized to be independent equations for each graph condition, instead of being simply stated as parity check conditions. The construction method proposed by Gallager consists of forming a sparse parity check matrix \( H \) by random determination of the positions of 1’s, with a fixed number of ones 1’s per column and per row, thus generating a regular LDPC code. The condition on the number of 1’s can be relaxed, provided that the number of 1’s per column \( s \) satisfies \( s > 2 \). In this case, the LDPC code is said to be irregular. The
conditions to be satisfied in the construction of the parity check matrix $H$ of a binary regular LDPC code are

- Parity check matrix $H$ should have a fixed number $t$ of 1’s per row.
- $H$ should have a fixed number $s$ of 1’s per column.
- The overlapping of 1’s per column and row should be at the most equal to one. This is a necessary condition for avoiding the presence of cycles in the corresponding bipartite graph.
- The parameters $s$ and $t$ should be small compared with the code length.

For good LDPC code construction, it is difficult to fulfill third condition because cycles are unavoidable in the bipartite graph [88]. The construction does not normally lead to the design of a sparse parity check matrix $H$ in the systematic form, and therefore it is necessary to utilize Gaussian elimination to convert resulting matrix into a systematic parity check matrix $H’ = \begin{bmatrix} I_{n_{out} - k} & P^T \end{bmatrix}$, where $I_{n_{out} - k}$ is the identity submatrix of dimension $(n_{out} - k) \times (n_{out} - k)$.

### 4.2 LDPC Decoding

The code vector generated using (4.6) is affected by the channel noise. Therefore the received signal vector is $Y = c + n$. The received signal will act as input for decoders of block codes. The syndrome vector is given by

$$S = H \circ Y$$

$$= H \circ (G^T \circ m + n)$$

$$= H \circ n \quad (4.10)$$

The decoding algorithm for LDPC codes is known as sum–product algorithm, or belief propagation algorithm. This algorithm determines the a posteriori probability of each message symbol as a function of the received signal. This algorithm is conveniently described over a bipartite graph [88] as shown in Figure 4.2. It is described by the parity equations described in the corresponding parity check matrix $H$. The bipartite graph depicts the relationship between two types of nodes, the symbol nodes (or variable nodes)
which represent the transmitted symbols or bits, and the parity check nodes \( h \) which represent the parity equations. Rows of the parity check matrix \( H \) identify symbols involved in each parity equation. Therefore a given row describes parity check equation and 1’s position determines the symbols involved in that parity check equation. For binary LDPC codes, if the entry \( \{ i, j \} \) of the sparse parity check matrix \( H \) is equal to one i.e. \( H_{i,j} = 1 \), there exists a connection between the symbol node \( d_j^i \) and the check node \( h_i \) in the bipartite graph otherwise the connection is not present. The state of a parity check node depends on the values of the symbol nodes connected to it. In general, the parity check nodes connected to a given symbol node are said to be the children nodes of that symbol node.

\[ \lambda(x) = \sum_{i=1}^{d^v} \lambda_i x^{i-1} \]  

\[ \rho(x) = \sum_{i=1}^{d^c} \rho_i x^{i-1} \]

where \( \lambda_i \) is the fraction of edges in bipartite graph that are connected to variable nodes of degree \( i \), and \( \rho_i \) is the fraction of edges in the same graph that are connected to check nodes of degree \( i \). Identifiers \( d^v \) and \( d^c \) are maximum degrees of variable nodes and check
nodes respectively. In Figure 4.3, the output of Alamouti decoder is mapped to soft demodulator. A serial concatenated turbo iterative receiver [80] is then employed which consists of combination of soft demodulator and LDPC decoder. Iterative receiver is based on passing extrinsic information between soft demodulator and LDPC decoder. Extrinsic information is in LLR form and is denoted by $L$. This information is iteratively passed along the edges in the bipartite graph during $p^{th}$ round of inner iteration within LDPC decoder and $q^{th}$ round of outer iteration between the LDPC decoder and soft demodulator. The extrinsic information of LDPC coded bit $b_i$ is given as

$$L_{S, \text{Demod} \rightarrow L, \text{Dec}}^q(b_{i,j}) = \log \frac{P(b_{i,j} = +1/Y)}{P(b_{i,j} = -1/Y)} = L_{S, \text{Demod} \rightarrow L, \text{Dec}}^{q-1}(b_{i,j})$$

where $L_{S, \text{Demod} \rightarrow L, \text{Dec}}^q$ is the extrinsic information sent from soft demodulator to LDPC decoder during $q^{th}$ iteration and $L_{S, \text{Demod} \rightarrow L, \text{Dec}}^{q-1}(b_{i,j})$ is sent from LDPC decoder to soft demodulator during $(q-1)^{th}$ iteration assuming it to be zero at first turbo iteration, $Y$ is received signal matrix which contains signals from all receiving antennas i.e. $Y=[y_1, y_2, \ldots, y_{M_{\text{r}}}]$. For a particular subcarrier and time slot, total $M_{\text{r}} \log_2 M$ bits are transmitted from $M_{\text{r}}$ antennas. Thus, $L_{S, \text{Demod} \rightarrow L, \text{Dec}}^q$ is computed for $i=1,2,\ldots,M_{\text{r}}\log_2 M$ bits as shown below.

$$L_{S, \text{Demod} \rightarrow L, \text{Dec}}^q(b_{i,j}) = \log \frac{\sum_{X \in C_i^+} P(X = X^+ / Y)}{\sum_{X \in C_i^-} P(X = X^- / Y)} - L_{S, \text{Demod} \rightarrow L, \text{Dec}}^{q-1}(b_{i,j})$$

where $C_i^+$ and $C_i^-$ are set of values of $X$ (transmission matrix for $M_{\text{r}}$ transmitting antenna) for which LDPC coded bit is $+1$ or $-1$. In a MAP MIMO-OFDM demodulator, $(4.14)$ can be rewritten as

$$L_{S, \text{Demod} \rightarrow L, \text{Dec}}^q(b_{i,j}) = \log \frac{\sum_{X \in C_i^+} P(y|x = x^+) P(x = x^+)}{\sum_{X \in C_i^-} P(y|x = x^-) P(x = x^-)} - L_{S, \text{Demod} \rightarrow L, \text{Dec}}^{q-1}(b_{i,j})$$

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\[
\sum_{x^+ \in C_1^+} \exp \left( -\frac{1}{2} \left( Y - \sqrt{\frac{\text{SNR}}{N_0}} H x^+ \right)^2 \right) + \sum_{j=1}^{M_1 \log_2(M)} X_j^+ \frac{L_{8, \text{De-mod} \rightarrow \text{L-Dec}}^{L-1} (b_{k,j})}{2} \right) \\
\sum_{x^- \in C_1^-} \exp \left( -\frac{1}{2} \left( Y - \sqrt{\frac{\text{SNR}}{N_0}} H x^- \right)^2 \right) + \sum_{j=1}^{M_1 \log_2(M)} X_j^- \frac{L_{8, \text{De-mod} \rightarrow \text{L-Dec}}^{L-1} (b_{k,j})}{2} \right) \\
- L_{S, \text{De-mod} \rightarrow \text{L-Dec}}^{L-1} (b_{i,j}) 
\]

(4.16)

\(X_j^+, X_j^-\) represents \(j^{th}\) bit in symbol \(X_j^+\) and \(X_j^-\). The message passing or belief propagation [82] method is used to describe inner iterations within LDPC decoder. Message passing between variable and check nodes is described using LLR’s which is denoted as \(L_{d^s \rightarrow h}^{p,q} (\rho_{i,m}^{d^s})\), where \(d^s \rightarrow h\) represents quantities passed from variable node to check nodes of the LDPC code and vice versa. Thus, \(L_{d^s \rightarrow h}^{p,q} (\rho_{i,m}^{d^s})\) denotes extrinsic message passed from variable node to check node along \(m^{th}\) edge connected to \(i^{th}\) variable node during \(p^{th}\) iteration in LDPC decoder and \(q^{th}\) iteration between LDPC decoder and soft demodulator. The message passing algorithm consists of following steps.

(i) Updating variable and check nodes

LLR passed between variable nodes and check nodes are updated with every iteration. These can be represented as follows.

\[
L_{d^s \rightarrow h}^{p,q} (\rho_{i,m}^{d^s}) = L_{8, \text{De-mod} \rightarrow \text{L-Dec}}^q (b_i) + \sum_{k=1, k \neq m}^{\text{deg}(d^s)} L_{d^s \rightarrow h}^{p-1,q} (\rho_{i,k}^{d^s}) 
\]

(4.17)

\[
L_{d^s \rightarrow h}^{p,q} (\rho_{i,m}^{d^s}) = 2 \tanh^{-1} \left[ \prod_{k=1, k \neq m}^{\text{deg}(d^s)} \tanh \left( \frac{L_{d^s \rightarrow h}^{p,q} (\rho_{i,k}^{d^s})}{2} \right) \right] 
\]

(4.18)

(ii) Compute extrinsic messages

Based upon above updates, the extrinsic messages are computed and sent back from LDPC decoder to soft demodulator as shown below.
\[ L_q^{\text{L_L_{Dec}}} (b_i) = \sum_{k=1}^{d_i} L_{d_i}^{p,q} \left( \text{deg} (d_i^q) \right) \]  \hspace{1cm} (4.19) 

(iii) Store LLR for next iteration

Inner iterative LLR, calculated in (4.19) can be stored and used as basis for next outer iteration.

\[ L_{d_i \rightarrow h}^{0,q+1} (\rho) = L_{d_i \rightarrow h}^{p,q} (\rho) \]  \hspace{1cm} (4.20) 

(iv) Hard decision decoding

Finally, hard decisions are made for information and parity bits but after sufficient Q turbo iterations as shown below.

\[ \hat{b_i} = \text{sign} \left[ L_{d_i \rightarrow h}^{\text{L_L_{Dec}}} (b_i) + L_{d_i \rightarrow h}^{\text{L_L_{Dec}}} (b_i) \right] \]  \hspace{1cm} (4.21) 

4.3 DENSITY EVOLUTION (DE) ANALYSIS FOR LDPC CODES

In this section, we use DE [77, 86] as a tool for analyzing system performance based upon computing average probability of incorrect bits. This can be done by estimating the pdf of extrinsic messages passed between variable nodes and check nodes during each iteration of decoding algorithm. When the number of codewords is infinite, extrinsic information between each variable and check nodes is considered to be independent random variables. Initially, the LLR from variable node to check node is given by 

\[ L_{d_i \rightarrow h}^{0.0} (b_i) = \frac{2}{\sigma^2} Y \]  and its pdf for Rayleigh i.i.d channel can be represented as

\[ P_{d_i \rightarrow h}^{0.0} (X) = \frac{X}{4/\sigma^2} e^{-X/4/\sigma^2} \]  \hspace{1cm} (4.22) 

The distribution in (4.22) represents Rayleigh distribution with mean \( \sqrt{2} \sigma \) and variance \( \frac{4 - \pi}{2} \left( \frac{4}{\sigma^2} \right) \). For regular LDPC codes, the pdf’s from variable nodes to check nodes and vice versa after \( p^{th} \) inner and \( q^{th} \) outer iterations [79] are given by
\[ P_{d^0 \rightarrow h}^p (X) = P_{d^q \rightarrow h}^0 (X) \otimes \left( P_{d^q \rightarrow h}^p (X) \right)^{\otimes (d^q - 1)} \] (4.23)

\[ P_{d^q \rightarrow h}^p (X) = \Gamma^{-1} \left( \left[ \Gamma \left( P_{d^q \rightarrow h}^{p-1, q} (X) \right) \right]^{\otimes (d^q - 1)} \right) \] (4.24)

where \( \otimes \) represents for convolution operator and \( \Gamma \) is the probability transformation function generated from other function \( Y(X) \in GF(2) \times [0, +\infty] \), which is given by

\[ Y(X) = \left( \text{sgn}(X), -\ln \tanh \left( \frac{|X|}{2} \right) \right) \] (4.25)

where \( \text{sgn}(x) = 0 \) if \( x \geq 0 \) and \( \text{sgn}(x) = 1 \) if \( x < 1 \), GF(2) is Galois or finite field of two elements that are nearly 0 and 1. Thus, the average probability of error after \( p \) inner and \( q \) outer iterations can be defined as

\[ P_{e}^{p, q}(X) = \int_{-\infty}^{\infty} P_{d^q \rightarrow h}^p (X) \, dx \] (4.26)

The expressions in (4.23) and (4.24) can be extended for irregular LDPC codes as follows.

\[ P_{d^0 \rightarrow h}^p (X) = P_{d^q \rightarrow h}^0 (X) \otimes \sum_{i=2}^{\lambda_1} \lambda_i \left( P_{d^q \rightarrow h}^p (X) \right)^{\otimes (i-1)} \] (4.27)

\[ P_{d^q \rightarrow h}^p (X) = \Gamma^{-1} \left( \sum_{i=2}^{\lambda_1} \left[ \Gamma \left( P_{d^q \rightarrow h}^{p-1, q} (X) \right) \right]^{\otimes (i-1)} \right) \] (4.28)

The expression of probability of error for irregular codes will be similar to (4.26). To explore DE for Alamouti coded LDPC codes, we have to derive pdf of initial LLR and then pdf after \( p \) inner and \( q \) outer iterations.

### 4.3.1 Alamouti Coded LDPC with \( M_{1x} = 2 \) and \( M_{R_r} = 1 \)

In the previous work [84], analysis was done under AWGN channels. In this section, we derived the pdf for concatenation of Alamouti coding with LDPC codes in Rayleigh i.i.d channels. From (2.11) we have
\[ Y = \hat{x}_1 = h_1^* y_1 + h_2^* y_2 = (\alpha_1^2 + \alpha_2^2) x_1 + h_1^* n_1 + h_2^* n_2 \]

\[ = (\alpha_1^2 + \alpha_2^2) x + \alpha_1 n_1 + \alpha_2 n_2 \]  \hspace{1cm} (4.29)

where \(n_1\) and \(n_2\) are Gaussian random variables with mean 0 and variance \(\sigma^2\). Considering \(\alpha = \alpha_1^2 + \alpha_2^2\), \(y\) can be represented as Gaussian variable with mean \(\alpha\) and variance \(\alpha \sigma^2\). The input to LDPC decoder at time \(t+T\) give the same result as in (4.29). Thus, the pdf of \(Y\) and its initial LLR is given as following.

\[ P(Y / X, \alpha) = \frac{1}{\sqrt{2\pi\alpha\sigma^2}} e^{-\frac{(Y-\alpha X)^2}{2\alpha \sigma^2}} \]  \hspace{1cm} (4.30)

\[ L_{d \rightarrow b}^{0,0}(b_1) = \ln \frac{P(Y / X = 1, \alpha)}{P(Y / X = -1, \alpha)} = \frac{2}{\sigma^2} Y \]  \hspace{1cm} (4.31)

Equation (4.31) indicates that initial LLR is a Gaussian random variable with mean \(\frac{2\alpha}{\sigma^2}\) and variance \(\frac{4\alpha}{\sigma^2}\). To compute pdf of initial LLR, we need to derive pdf of \(\alpha\). We considered Rayleigh i.i.d. channel and hence \(\alpha_1\) and \(\alpha_2\) are \(X_1^2 + X_2^2\) and \(X_3^2 + X_4^2\) respectively, collectively

\[ \alpha = \sum_{i=1}^{4} X_i^2 \]  \hspace{1cm} (4.32)

where \(X\) is Gaussian random variable with mean 0 and variance 1/2. Thus the pdf of \(\alpha\) for \(m\) such samples is given by

\[ P(\alpha) = \frac{1}{\sigma^m 2^{m/2} \Gamma_f \left( \frac{m}{2} \right)} \alpha^{m/2 - 1} e^{-\alpha / 2\sigma^2} \]  \hspace{1cm} (4.33)

where \(\Gamma_f\) is a Gamma function. Putting \(m=4\) in (4.33), pdf of \(\alpha\) is given by \(P(\alpha) = \alpha e^{-\alpha}\). Finally, the pdf of initial LLR is given by
\[ P_{d^s \rightarrow h}^{0,0} (X) = \frac{1}{\sqrt{2\pi(4\alpha/\sigma^2)}} e^{-\frac{(x - 2\alpha/\sigma^2)^2}{8\alpha/\sigma^2}} \alpha e^{-\alpha} d\alpha \] (4.34)

Thereafter pdf after p inner and q outer iterations are obtained by using (4.27) and (4.28).

4.3.2 Alamouti Coded LDPC with \( M_{Tx} = 2 \) and \( M_{Rx} = 2 \)

In this section, we derive the pdf for concatenation of Alamouti and LDPC codes with two transmitting and two receiving antennas. From (2.17) we have

\[ Y = \hat{x}_i = (\alpha_{11}^2 + \alpha_{21}^2 + \alpha_{12}^2 + \alpha_{22}^2) x_i + h_{11}^* n_{11} + h_{21}^* n_{12} + h_{12}^* n_{21} + h_{22}^* n_{22} \]

\[ = (\alpha_{11}^2 + \alpha_{21}^2 + \alpha_{12}^2 + \alpha_{22}^2) x_i + \alpha_{11} n_{11}^* + \alpha_{12} n_{12}^* + \alpha_{21} n_{21}^* + \alpha_{22} n_{22}^* \] (4.35)

where \( n_{11}^* \), \( n_{12}^* \), \( n_{21}^* \) and \( n_{22}^* \) are Gaussian random variables with mean 0 and variance \( \sigma^2 \). As per discussion in section 5.3.1, let \( \chi = [\alpha_{11}^2 + \alpha_{12}^2 + \alpha_{21}^2 + \alpha_{22}^2] \), \( Y \) is a Gaussian variable with mean \( \chi \) and variance \( \chi \sigma^2 \). The output \( Y \) at \( t+T \) is same as (4.35), thus its corresponding pdf and initial LLR can be defined as

\[ P(Y/X, \chi) = \frac{1}{\sqrt{2\pi \chi \sigma^2}} e^{-\frac{(Y - \chi x)^2}{2\chi \sigma^2}} \] (4.36)

\[ P_{d^s \rightarrow h}^{0,0} (b_i) = \ln \frac{P(Y/X = 1, \chi)}{P(Y/X = -1, \chi)} = \frac{2}{\sigma^2} Y \] (4.37)

Initial LLR mentioned above is a Gaussian random variable with mean \( \frac{2\chi}{\sigma^2} \) and variance \( \frac{4\chi}{\sigma^2} \). Further, \( \chi \) can be written as \( \chi = \sum_{i=1}^{8} X_i \). If we put \( m = 8 \) in (4.33), pdf of \( \chi \) becomes \( P(\chi) = \frac{1}{6} \chi^3 e^{-\chi} \). Thus the pdf of Initial LLR can be written as

\[ P(\chi) = \frac{1}{6} \chi^3 e^{-\chi} \]
\[ p_{\theta \rightarrow h}^{0,0}(X) = \frac{1}{\sqrt{2\pi(4\chi/\sigma^2)^2}} e^{-\frac{(X-2\chi)^2}{8\chi/\sigma^2}} \frac{1}{6} \chi e^{-\chi} d\chi \] (4.38)

Thereafter, we compute the pdf of LLR’s after \( p \) inner and \( q \) outer iterations using (4.27) and (4.28).

4.4 SIMULATION RESULTS

In this section, simulation results are shown for LDPC coded Alamouti MIMO-OFDM systems under Rayleigh i.i.d channels. The list of simulation parameters used is shown in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total bandwidth</td>
<td>8MHz</td>
</tr>
<tr>
<td>Number of transmitting antenna</td>
<td>2</td>
</tr>
<tr>
<td>Number of receiving antenna</td>
<td>1 and 2</td>
</tr>
<tr>
<td>FFT Size</td>
<td>4×1024</td>
</tr>
<tr>
<td>Carrier modulation used</td>
<td>QPSK</td>
</tr>
<tr>
<td>Guard period type</td>
<td>Cyclic extension</td>
</tr>
<tr>
<td>Cyclic prefix length</td>
<td>1024</td>
</tr>
<tr>
<td>Window type</td>
<td>No windowing used</td>
</tr>
<tr>
<td>Channel coding</td>
<td>Regular and irregular LDPC codes</td>
</tr>
<tr>
<td>LDPC code rate</td>
<td>1/2</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>3000</td>
</tr>
<tr>
<td>Number of 1’s per column (s)</td>
<td>3</td>
</tr>
<tr>
<td>Number of 1’s per row (t)</td>
<td>6</td>
</tr>
<tr>
<td>Channel model used [87]</td>
<td>Rayleigh i.i.d DVB-NGH outdoor</td>
</tr>
<tr>
<td>Channel fading</td>
<td>Rayleigh independent and identically distributed</td>
</tr>
<tr>
<td>Number of channel taps</td>
<td>8</td>
</tr>
<tr>
<td>Path delays (in ( \mu )sec)</td>
<td>0.02, 0.1094, 0.2188, 0.6094, 1.109, 2.109, 4.109, 8.109</td>
</tr>
<tr>
<td>Channel estimation</td>
<td>Ideal</td>
</tr>
<tr>
<td>Doppler shift</td>
<td>33.3Hz</td>
</tr>
</tbody>
</table>
For simulation, (3, 6) regular LDPC code of rate 1/2 is used. It consists of 10 variable and 5 check nodes as shown in Figure 4.3.

![Diagram of (3, 6) regular LDPC code](image)

**Figure 4.3**: Bipartite graph for (3, 6) regular LDPC codes

In regular LDPC codes, all the nodes of same type have same degree as shown in Figure 4.3, where all the variable and check nodes are of degree 3 and 6 respectively. For irregular codes, the degrees of each set of nodes are chosen according to optimized distribution. The optimized distribution [80] for irregular LDPC codes is given by

\[
\lambda(x) = 0.269052 x + 0.135031 x^2 + 0.024564 x^4 + 0.028685 x^5 + 0.075819 x^6 \\
+ 0.033661 x^7 + 0.024360x^8 + 0.020951x^9 + 0.018975x^{10} + 0.014373x^{12} + 0.035585x^{13} \\
+ 0.015569x^{14} + 0.013611x^{16} + 0.289765x^{19}
\]

(4.39)

and \( \rho(x) = 0.307710x^7 + 0.692290x^8 \) for spatially uncorrelated MIMO-OFDM systems. To support analytical results, simulation results are obtained for both MIMO and MIMIO-OFDM configurations. Figure 4.4 and 4.5 shows BER performance of 2\times1 and 2\times2 MIMO systems with and without LDPC codes. Figure 4.4 and 4.5 consists of 11 different cases, the first 2 cases corresponds to Alamouti STBC and SFBC without LDPC, and next 8 cases corresponds to Alamouti STBC and SFBC with regular and irregular LDPC codes along with results computed from DE analysis. The results are also compared with best available results in [83]. Figure 4.4 and 4.5 shows that BER is monotonically decreasing function of SNR. In Figure 4.4, BER decreases up to 3.2\times10^{-6}, 5.8\times10^{-6}, 5\times10^{-6}, 6.1\times10^{-6}, 1.5\times10^{-5} and 3.1\times10^{-6} at SNR of around 20dB, 19dB, 18dB, 16dB, 14dB and 14dB respectively from top to bottom plot. In Figure 4.5, BER decreases up to 1\times10^{-6}, 1\times10^{-5}, 2.8\times10^{-6}, 5.1\times10^{-6}, 1.5\times10^{-6} and 3.1\times10^{-6} at SNR of around 18dB, 16dB, 16dB, 14dB, 14dB and 12dB respectively from top to bottom plot.
Figure 4.4: BER comparison for regular and irregular LDPC codes for $2 \times 1$ MIMO system

Figure 4.5: BER comparison for regular and irregular LDPC codes for $2 \times 2$ MIMO system
The results in Figure 4.4 clearly depicted that Alamouti SFBC with irregular LDPC outperforms remaining combinations due to its coding gain advantage and it also outperforms [83]. Comparing Figure 4.4 with 4.5, it can be concluded that slope of BER curve is more in Figure 4.5 due to its higher diversity order. Further, the $\text{SNR}_{\text{min-op}}$ values calculated from Figure 4.4 and 4.5 are given in Table 4.2.

![BER comparison for regular and irregular LDPC codes for 2×1 MIMO-OFDM system](image)

**Figure 4.6:** BER comparison for regular and irregular LDPC codes for 2×1 MIMO-OFDM system

Figure 4.6 and 4.7 shows the results with 2×1 and 2×2 MIMO-OFDM systems. Results are of same pattern like Figure 4.4 and 4.5. Results are also compared with best available codes in [89] and [90]. Among all the combination of codes in Figures 4.4, 4.5, 4.6 and 4.7, the BER performance of Alamouti SFBC with irregular LDPC in 2×2 MIMO-OFDM system is best due to its large coding gain and high diversity. Moreover, it also exhibit better mitigation to fading due to combination with OFDM. $\text{SNR}_{\text{min-op}}$ values corresponding to Figures 4.6 and 4.7 are shown in Table 4.2.
Figure 4.7: BER comparison for regular and irregular LDPC codes for 2×2 MIMO-OFDM system

Table 4.2: $\text{SNR}_{\text{min-op}}$ (in dB) values for various code combinations and system configurations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MIMO</th>
<th>MIMO-OFDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antennas</td>
<td>$M_{\text{Tx}}=2, M_{\text{Rx}}=1$</td>
<td>$M_{\text{Tx}}=2, M_{\text{Rx}}=1$</td>
</tr>
<tr>
<td>Codes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reg-LDPC-STBC</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Irreg.LDPC-STBC</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Reg-LDPC-SFBC</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Irreg.LDPC-SFBC</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4.2 shows that irregular LDPC codes along with Alamouti SFBC under 2×2 MIMO-OFDM system exhibits minimum value of $\text{SNR}_{\text{min-op}}$. In this section, we considered the performance analysis of serially concatenated regular and irregular LDPC codes with Alamouti STBC and SFBC MIMO-OFDM systems for high data rate transmission. The DE tools based on pdf approach have been used for analyzing and
optimizing LDPC codes. We have evaluated BER results of irregular and regular LDPC codes with Alamouti STBC and SFBC under MIMO and MIMO-OFDM system configurations. From simulation results, we showed that optimized irregular LDPC codes concatenated with Alamouti SFBC in 2×2 MIMO-OFDM systems achieved best BER results. Comparing the results with existing methods, it can be concluded that the mentioned combination outperform the existing schemes in terms of complexity and minimum operational SNR. Also the simulation and DE analysis results are almost identical.

Above analysis is based upon pdf approach, which is computationally expensive. Thus, it is difficult to predict error performance using this approach. In the next section, we derived moment generating function (MGF) based PEP bounds for the performance of concatenated LDPC codes with Alamouti coded MIMO-OFDM in various fading conditions. It is well known fact that LDPC codes achieved excellent error correction capabilities in a region close to Shannon’s limit [85], but the loose error bound in this region is undesirable. We used MGF approach instead of pdf approach to develop these bounds because it is computationally inexpensive and helps in developing tight bounds. We analyzed the LDPC codes concatenated with Alamouti coded MIMO-OFDM mentioned in Figure 4.1. The results are obtained for various fading conditions like spatially independent, spatially correlated, spatial temporal correlated and spatial time frequency correlated quasi-static Rayleigh fading channels. The analysis quantifies the diversity and coding gain losses occurred due to correlation. The MGF approach is also used to derive BER expressions for mentioned system with M-PAM and M-PSK constellations.

A large number of papers exist in literature for above mentioned concatenated schemes with different channel codes, system models and fading conditions. In papers [75-78] the design issues and performance analysis of turbo codes in quasi-static fading channels have been discussed. The upper bounds are given to predict the performance of turbo codes but the bounds are not tight. In [92], the analytical evaluation of error probability and its upper bound is given for space time codes under quasi-static channels without consideration of block fading and correlative channels. In [93-95], space time
coding in spatial and spatial time correlation Rayleigh channels is analyzed, the performance bounds are obtained in [96] and [97]. Papers [98-100] derive exact BER expressions for OSTBC codes in generalized Rayleigh and Rician fading channels in MIMO correlation channels. No concatenation with channel coding techniques is investigated. In [74] the MGF approach is used to analyze the PEP performance bounds for various channel coding techniques like convolutional codes, trellis codes etc concatenated with Alamouti STBC. Hence the impact of LDPC code and OFDM modulation is not considered. Paper [101] gives the analysis for concatenation of convolutional and Alamouti code with different receivers. The performance of OSTBC code with OFDM is analyzed in [102]. The performance of MIMO system is evaluated in [103-104] under various fading conditions but without OFDM, OSTBC and channel codes. The space frequency correlation channel model for MIMO systems is presented in [105]. The PEP bounds and capacity of MIMO-OFDM systems is analyzed in [106-108] in independent and spatio temporal correlate Rayleigh channels without concatenation with channel codes. Paper [109] analyze the performance of convolutionally coded MIMO-OFDM system without correlation channel models. In [83] and [110], the performance of LDPC codes over quasi-static MIMO Rayleigh channels with pdf approach is analyzed. The BER expressions with different M-PAM and M-PSK constellations are derived in [111] and [112].

The concatenation scheme of LDPC codes with Alamouti coded MIMO-OFDM system is similar to the one given in Figure 4.1 but with the difference in structure of iterative receiver. The output of Alamouti decoder is mapped to iterative receiver. The iterative receiver consists of combination of soft demodulator and LDPC decoder as shown in Figure 4.8. Iterative receiver [113] processes each coded bit. It uses priori information $L_{A1}$ to compute posteriori information $L_{D1}$ on each coded bit. The information available in $L_{A1}$ and $L_{D1}$ is in LLR form. The difference between priori and posteriori information gives extrinsic message $L_{E1}$ that becomes priori information $L_{A2}$ for LDPC decoder. Then the LDPC decoder generates posteriori information $L_{D2}$ based upon $L_{A2}$. The difference of $L_{E1}$ and $L_{D2}$ gives extrinsic message $L_{E2}$ and is back as a
priori knowledge to the detector. Therefore these messages can be calculated depending upon number of iterations.

![Diagram of Iterative Detection and Decoding Algorithm](image)

Figure 4.8: Iterative detection and decoding algorithm

4.5 LDPC PERFORMANCE ANALYSIS

For a linear block code $C(n, k)$ the BER and frame error rate (FER) bounds are

$$P_b \leq \sum_{i=0}^{k} \sum_{d_H=0}^{n_{out}} \frac{1}{n_{out}} A_{i,w} P_e(d_H)$$

and

$$P_f \leq \sum_{i=0}^{k} \sum_{d_H=0}^{n_{out}} A_{i,w} P_e(d_H)$$

respectively. The coefficients $A_{i,w}$ forms the input output weight enumerating function (IOWEF) for a given block code.

To generalize IOWEF, we need to determine how error bits are distributed among different fading blocks. Histogram can be built to characterize these distributions. The average PEP based upon these patterns is given as $P_e(d_H) = E[P_e(d_H / f)]$, where $f = (f_0, \ldots, f_w)$ is an error pattern vector and $d_H = \sum w f_w$ is Hamming weight respectively.

4.5.1 PEP Analysis Based Upon MGF

In wireless communication, the PEP calculation assumes all-zero codeword transmission. The PEP with weight $d_H$ for an error pattern $f$ is given by

$$P_e(d_H / f, \gamma) = Q\left(\sqrt{\frac{2}{\gamma} \sum_{w=1}^{W} \sum_{i=1}^{F_w} \gamma_{w,i}}\right)$$

(4.40)

where $W_i = \min(\ell, d_H)$ and $\ell$ is length of fading block. Using Craig’s definition [114] of standard $Q$ function i.e. $Q(x) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} \exp\left(-\frac{x^2}{\sin^2 \theta}\right) d\theta$, equation (4.40) can be rewritten and averaged over SNR as follows.
\[ P_e(d_H / f) = E[P_e(d_H / f, \gamma)] \]

\[ = \frac{1}{\pi} \prod_{0 \leq w \leq W} \prod_{i=1}^{W_i} \frac{1}{\theta} \exp \left( -\frac{w \gamma_{w,i}}{\sin^2 \theta} \right) P(\gamma_{w,i}) d\gamma_{w,i} d\theta \quad (4.41) \]

The inner integral is an MGF of \( \gamma \) evaluated at \( t = \frac{-w}{\sin^2 \theta} \), i.e. \( M_{\gamma}(t) = E(e^{\gamma t}) \).

Therefore (4.41) can be rewritten as

\[ P_e(d_H / f) = \frac{1}{\pi^2} \prod_{0 \leq w \leq W} M_{\gamma}(t)_{f_w} d\theta \quad (4.42) \]

It is assumed that \( \gamma_{w,i} \) has the same distribution in all blocks.

4.5.2 PEP Analysis Based upon Spatially Uncorrelated Fading

In this section, it is assumed that the channel matrix is of full rank, i.e. the fading experienced by different paths is independent. The SNR follows chi-square distribution, and the MGF for such a pdf is given by

\[ M_{\gamma}(t) = (1 - t \bar{\gamma})^{-D} \quad (4.43) \]

where \( D \) is the full diversity factor. In spatially independent fading, the full diversity is \( D = M_{T_R} M_{R_S} \). Equation (4.43) can be rewritten as

\[ P_e(d_H / f) = \frac{1}{\pi^2} \prod_{0 \leq w \leq W} [(1 - t \bar{\gamma})^{-D}]_{f_w} d\theta \leq \frac{1}{2} \prod_{0 \leq w \leq W} [(1 + w \bar{\gamma})^{-D}]_{f_w} d\theta \quad (4.44) \]

where the last inequality is Chernoff bound [96] that is used to remove the integral operation appearing in (4.44). For quasi static channels (4.44) can be written as

\[ P_e(d_H / f) = \frac{1}{\pi} \left( 1 + \frac{d_{H_R} \bar{\gamma}}{\sin^2 \theta} \right)^{-D} d\theta \leq \frac{1}{2} \left( 1 + d_{H_R} \bar{\gamma} \right)^{-D} \quad (4.45) \]

For high SNR region, PEP based on Chernoff bound [96] can be expressed as
\[ P_e(d_H / f) \leq \frac{1}{2} (d_H / f)^{-D} \]  

(4.46)

Above equations indicates that in order to achieve high coding gain and diversity, one must use channel code or outer code of high Hamming distance. Also the blocks with non-zero weight should be maximized.

4.5.3 PEP Based on Spatially Correlated Fading

If the antenna elements on either side are not far enough or there are unfavorable scattering conditions to encounter independent fading the correlation may be present among channel fading coefficients \( (h_{i,j}) \) [74]. To characterize correlation among fading coefficients, the channel matrix can be decomposed and diagonalized as \( H = \mathcal{R}_{R_x}^{1/2} \tilde{H} \mathcal{R}_{T_x}^{1/2} \). Thus, Frobenius norm of \( H \) is given by

\[ \|H\|^2 = \text{vec}(H) ^{H} \Lambda \text{vec}(\tilde{H}) \]  

(4.47)

where \( \Lambda = \Lambda_{T_x} \otimes \Lambda_{R_x} \), and \( \Lambda_{T_x} (\Lambda_{R_x} ) \) is the diagonal matrix whose elements are the eigen values of \( \mathcal{R}_{T_x} (\mathcal{R}_{R_x} ) \). Equation (4.47) can be written as

\[ \|H\|^2 = \sum_{i=1}^{M_{T_x}} \sum_{j=1}^{M_{R_x}} \lambda_{T_x}^{i} \lambda_{R_x}^{j} \left| \tilde{h}_{i,j} \right|^2 \]  

(4.48)

where \( \lambda_{T_x}^{i}, \lambda_{R_x}^{j} \) are the eigen values of \( \mathcal{R}_{T_x} \) and \( \mathcal{R}_{R_x} \), and \( \tilde{h}_{i,j} \) is channel between transmitting antenna \( i \) and receiving antenna \( j \) at particular error event in time domain. Equation (4.48) represents the MIMO channel as an equivalent of SISO channel. Thus the SNR can be represented as

\[ \gamma = \frac{1}{\gamma} \|H\|^2 = \frac{1}{\gamma} \sum_{i=1}^{M_{T_x}} \sum_{j=1}^{M_{R_x}} \lambda_{T_x}^{i} \lambda_{R_x}^{j} \left| \tilde{h}_{i,j} \right|^2 \]  

(4.49)

The MGF of \( \gamma \) can be calculated as

\[ M_{\gamma}(t) = \prod_{i=1}^{M_{T_x}} \prod_{j=1}^{M_{R_x}} \left( 1 - t \lambda_{T_x}^{i} \lambda_{R_x}^{j} \gamma \right)^{-1} \]  

(4.50)

Therefore the PEP based on MGF defined in (4.50) can be calculated as
\[ P_e(d_H/f) = \frac{1}{\pi} \prod_{x=1}^{w} \prod_{y=1}^{w} \prod_{j=1}^{j} \left[ 1 + \frac{w\lambda_{\text{Tx}}^{R_{\text{Tx}}}}{\sin^2 \theta} \right]^{-1} d\theta \quad (4.51) \]

By applying Chernoff bound [96], the integral in (4.51) can be removed and PEP at high SNR [98] is given by

\[ P_e(d_H/f) < \frac{1}{2} (d_H) \left[ \frac{r_{\text{Tx}}}{\prod_{m=1}^{M_{\text{Rx}}} \lambda_{\text{m}}} \left( \frac{r_{\text{Rx}}}{\prod_{m=1}^{M_{\text{Tx}}} \lambda_{\text{m}}} \right) \right]^{-1} \gamma^{-D} \quad (4.52) \]

In (4.52) the diversity factor \( \hat{D} \) is the multiplication of \( r_{\text{Tx}} \) and \( r_{\text{Rx}} \), where \( r_{\text{Tx}} \) and \( r_{\text{Rx}} \) are ranks of matrices \( R_{\text{Tx}} \) and \( R_{\text{Rx}} \). Comparing (4.46) and (4.52) it can be observed that there is a loss of coding gain due to correlation on transmitter and receiver side that can be quantified as

\[ \left[ \frac{r_{\text{Tx}}}{\prod_{m=1}^{M_{\text{Rx}}} \lambda_{\text{m}}} \left( \frac{r_{\text{Rx}}}{\prod_{m=1}^{M_{\text{Tx}}} \lambda_{\text{m}}} \right) \right]^{-1}. \]

### 4.5.4 PEP Based on Combination of Spatial and Time Correlated Fading

In this section MGF and PEP are computed considering both spatial and time correlation [74] fading. Time correlation is modeled by channel matrix observation at different time instants. As per discussion given in section 4.5.3 the equivalent channel matrix can be expressed as

\[ H = R_s^{1/2} H T^{1/2} \quad (4.53) \]

where \( R_s = R_{\text{Tx}} \otimes R_{\text{Rx}} \). The correlation matrix \( R \) becomes \( R = R_{\text{Tx}} \otimes R_{\text{Rx}} \otimes R_T \). The matrix \( R \) is also called covariance matrix of channel \( H \). The MGF of \( \gamma \) based on (4.50) can be modified as

\[ M_{\gamma}(t) = \prod_{j=1}^{J} \prod_{i=1}^{I} \prod_{y=1}^{Y} \left( 1 - \lambda^T \lambda_{\text{T}} \lambda_{\text{Rx}} \gamma^{-1} \right)^{-1} \quad (4.54) \]

where \( \lambda^T \) is the eigen value of time correlation matrix \( R_T \). Also the PEP based on above MGF is given by
\[ P_e(d_H / f) = \frac{1}{\pi} \prod_{l=1}^{2} \prod_{i=1}^{M_{Tx}} \prod_{j=1}^{M_{Rx}} \left[ 1 + \frac{\lambda_i^T \lambda_j^T \lambda_i^R \lambda_j^R}{\sin^2 \theta} \right]^{-1} d\theta \]  

(4.55)

Similar to (4.52), the Chernoff bound [96] can be applied to (4.55). The PEP at high SNR is given by

\[ P_e(d_H / f) \leq \frac{1}{2} r_i^D \prod_{l=1}^{2} \prod_{i=1}^{M_{Tx}} \prod_{j=1}^{M_{Rx}} \left( \lambda_i^T \lambda_j^T \lambda_i^R \lambda_j^R \right)^{-1} \]  

(4.56)

where \( r_i \) is the rank of \( \mathcal{R}_C \). It is observed in (4.56) that while the spatial diversity depends upon the rank of \( \mathcal{R}_{Tx} \) and \( \mathcal{R}_{Rx} \), the time diversity depends upon the rank of \( \mathcal{R}_T \). If all these correlation matrices are full rank it means there is no loss of diversity. Loss in coding gain can be expressed as given in previous section.

4.5.5 PEP Based on Joint Spatial, Time and Frequency Correlated Fading

This section considers the impact of combination of spatial, time and frequency correlation on diversity and coding gain. Frequency correlation \( \mathcal{R}_F \) occurs between different subcarriers in one OFDM symbol, and can be represented as \( \mathcal{R}_F = \mathbb{E}(H_{i,j} H_{i,j}^H) \), where \( H_{i,j} \) is a vector of \( N_C \) subcarriers. The matrix \( \mathbf{H} \) can be diagonalized similar to the one given in (4.53). The correlation matrix \( \mathcal{R} \) in this case becomes

\[ \mathcal{R} = \mathcal{R}_{Tx} \otimes \mathcal{R}_{Rx} \otimes \mathcal{R}_T \otimes \mathcal{R}_F \] .

Also MGF \( M(t) \) is given by

\[ M(t) = \prod_{n=0}^{N_C} \prod_{i=1}^{M_{Tx}} \prod_{j=1}^{M_{Rx}} \left( 1 - t \lambda_i^F \lambda_j^T \lambda_i^R \lambda_j^R \right)^{-1} \]  

(4.57)

where \( \lambda_i^F \) is eigen value of frequency correlation matrix \( \mathcal{R}_F \). Thus, the PEP can be computed based on the MGF defined above and is given by

\[ P_e(d_H / f) = \frac{1}{\pi} \prod_{n=0}^{N_C} \prod_{l=1}^{2} \prod_{i=1}^{M_{Tx}} \prod_{j=1}^{M_{Rx}} \left[ 1 + \frac{\lambda_i^F \lambda_j^T \lambda_i^R \lambda_j^R}{\sin^2 \theta} \right]^{-1} d\theta \]  

(4.58)

After applying Chernoff bound on (4.58), the PEP at high SNR is given as
\[ P_f(d_{H}/f) \leq \frac{1}{2} \tau^{-\frac{3}{2} \hat{D}} \prod_{n=0}^{N_c} \prod_{i=1}^{M_{\text{TX}}} \prod_{j=1}^{M_{\text{RX}}} \left[ \lambda_n \lambda_i \lambda_j \right]^{-1} \] (4.59)

where \( r_f \) is rank of \( R_T \otimes R_F \). It can be observed that spatial diversity decreases if one of the correlation matrices has zero Eigen value. The diversity loss can be quantified as \( \hat{D}(d_{H} - r_f) \). Also, the loss in coding gain can be characterized in a similar way as given in section 4.5.3 but with some factors corresponds to non zero Eigen values of \( R_T \) and \( R_F \).

### 4.6 PEP PERFORMANCE ANALYSES FOR LDPC CODES WITH MIMO-OFDM

In the previous section, the performance analysis of channel codes under various propagation environments has been investigated. In this section, we integrate the analysis with OFDM systems. For OFDM system with \( N_c \) subcarriers, the received signal in \( p^{th} \) time slot at \( j^{th} \) \((j=1, \ldots, M_{\text{RX}})\) receiving antenna using \( m^{th} \) subcarrier can be represented as

\[ y_j[p,m] = \sum_{i=1}^{M_{\text{TX}}} H_{ij}[p,m] c_i[p,m] + n_j[p,m] \] (4.60)

where \( m=0, \ldots, N_c-1 \), \( p = 1, \ldots, P \), and \( H[p,m] \in C^{M_{\text{TX}} \times M_{\text{RX}}} \) is the channel matrix of size \( M_{\text{TX}} \times M_{\text{RX}} \). \( c_i[p,m] \in C^{M_{\text{TX}}} \) is \( M_{\text{TX}} \) dimensional codeword matrix, \( y_j[p,m] \in C^{M_{\text{RX}}} \) is \( M_{\text{RX}} \) dimensional received vector and \( n_j[p,m] \in C^{M_{\text{RX}}} \) is \( M_{\text{RX}} \) dimensional additive white Gaussian noise matrix with unit variance. It is assumed that the channel experiences frequency selective quasi-static fading, i.e. the channel remains constant in one codeword and varies independently from one codeword to another. With the assumption of perfect channel estimation at the receiver (3.13) can be rewritten as

\[ \hat{c} = \arg \min_{\hat{c}} \sum_{j=1}^{M_{\text{RX}}} \sum_{m=0}^{N_c-1} \left[ y_j[p,m] - \sum_{i=1}^{M_{\text{TX}}} H_{ij}[p,m] c_i[p,m] \right]^2 \] (4.61)

The error patterns \( f \) considered so far are of singular values, but in this case it can have multiple values. Therefore, the construction of \( f \) is complex compared to previous section. In an \( M \)-ary constellation the well known result for pairwise conditional
probability of error with all zero transmitted codeword and received codeword \( e \) is given by

\[
P_e(0 \rightarrow e / f, \gamma) = Q \left( \sqrt{2 \sum_{w=1}^{W} \sum_{i=1}^{W} \gamma_{w,i} \alpha^w} \right)
\]  

(4.62)

The distance metric \( \alpha^w \) for each block pattern can be defined as

\[
\alpha^w = \sum_{M} v_w d^2(c, e)
\]

(4.63)

where \( v_w \) are multiplications of symbols in block pattern and \( d^2(c, e) \) is the Euclidean distance between codewords \( c \) and \( e \). \( d^2(c, e) \) is evaluated in such a way that both codewords have same error patterns and belong to one block indexed by \( w \). The distance \( d \) between two codewords of constellation \( M \) is calculated as

\[
d^2(c, e) = \sum_{j=1}^{N_w} \sum_{m=0}^{N_{C-1}} \sum_{p=0}^{M_{Tx}} \sum_{i=1}^{M_{Tx}} H_{ij} [p, m] \in_i [p, m]
\]

(4.64)

where \( \in_i [p, m] = c_i[p, m] - e_i[p, m] \) and \( \in [p, m] = [\in_1 [p, m] \ldots \in_{M_{Tx}} [p, m]] \in_{M_{Tx} \times 1} \). If we assume the transmission of all zero codeword, (4.64) can be modified accordingly. Averaging (4.62) over \( \gamma \) we get

\[
P_e(0 \rightarrow e / f, \gamma) = \frac{1}{\pi \sqrt{\prod_{w=1}^{W} W}} \left[ M_\gamma \left( \frac{-\alpha^w}{2 \sin^2 \theta} \right) \right]^{\gamma_w} d\theta
\]

(4.65)

Substituting the value of \( \alpha^w \) in (4.65),

\[
P_e(0 \rightarrow e / f, \gamma) = \frac{1}{\pi \sqrt{\prod_{w=1}^{W} W}} \left[ M_\gamma \left( \frac{-\sum_{M} v_w d^2(c, e)}{2 \sin^2 \theta} \right) \right]^{\gamma_w} d\theta
\]

(4.66)

The error bound (4.66) can be modified for uncorrelated and spatially correlated fading by substituting their corresponding MGF directly, i.e. putting (4.43) and (4.50) in (4.66). In case of temporally and frequency correlated channels, MGF cannot be
substituted directly because in such cases SNR is not a function of H only. The effective or equivalent SNR can be written as

\[ \gamma = \prod_{m=0}^{N_C-1} \prod_{i=1}^{d_H} \prod_{j=1}^{M_{TX}} \prod_{l=0}^{M_{RX}} \left[ h_{i,j,l} \right]^2 \]  \hspace{1cm} (4.67) \]

Equation (4.67) can be rewritten as

\[ \gamma = \prod_{m=0}^{N_C-1} \prod_{i=1}^{d_H} \prod_{j=1}^{M_{TX}} \prod_{l=1}^{M_{RX}} \left[ \hat{h}_{i,j,l} \right]^2 \frac{\tilde{\lambda}_m \tilde{\lambda}_j \lambda_i \lambda_j}{\lambda_m \lambda_j \lambda_i \lambda_j} \]  \hspace{1cm} (4.68) \]

where \( \tilde{\lambda}_m \) and \( \tilde{\lambda}_j \) are the eigen values of \( \text{diag}(d_1, \ldots, d_d) \) and \( \text{diag}(d_1, \ldots, d_d) \).

Replacing \( \lambda^F_m, \lambda^T_j \) with \( \tilde{\lambda}_m, \tilde{\lambda}_j \) in the definition of MGF, the PEP for MIMO-OFDM system with temporal and frequency correlation fading can be obtained by substituting (4.54) and (4.57) in (4.66). Further, the diversity loss and coding gain loss can be quantified by applying Chernoff bound [96] on resultant equations.

4.7 BER ANALYSIS OF LDPC CODES WITH MIMO-OFDM UNDER VARIOUS CONSTELLATIONS

The PEP bound analysis of LDPC codes with MIMO-OFDM is discussed in the previous sections. These bounds can be utilized to develop BER bounds. Each pairwise error probability event is weighted with the number of information bits in error. Using these derived MGF expressions and BER bounds, we can derive BER expressions for all kinds of channel conditions with varying modulation schemes.

4.7.1 BER with M-PAM and M-QAM Constellation

According to [111], the probability that \( k \)-th bit is in error for pulse amplitude modulation (PAM) constellation is given by

\[ P_b(k) = \frac{1}{M} \sum_{i=0}^{1-2^{-k}} \left(-1\right)^{i} \left[ \frac{1}{2} \right] \left(2^{-1} - \left[ \frac{1.2^{k-1}}{M} + \frac{1}{2} \right] \right) \times Q \left( \left(2i+1\right) \sqrt{\frac{3 \log_2 M \gamma}{(M^2-1)}} \right) \]  \hspace{1cm} (4.69)
where \( [x] \) is integer part of \( x \). If we assume \( B_{i,k} = (-1)^{\left\lfloor \frac{i-2^{k-1}}{M} \right\rfloor} \left( 2^{k-1} - \left\lfloor \frac{i-2^{k-1}+1}{2} \right\rfloor \right) \) and

\[
G_i = (2i+1) \sqrt{\frac{3 \log_2 M}{(M^2-1)}}.
\]

(4.69) reduces to

\[
P_b(k) = \frac{1}{M} \sum_{i=0}^{\left\lfloor \frac{1-2^{-k}}{M-1} \right\rfloor} B_{i,k} \cdot Q(G_i \sqrt{\gamma})
\]

Using Craig’s definition [114] for \( Q \) function, (4.70) becomes

\[
P_b(k) = \frac{1}{\sqrt{\pi}} \sum_{i=0}^{\left\lfloor \frac{1-2^{-k}}{M-1} \right\rfloor} B_{i,k} \cdot \int_0^{\frac{\pi}{2}} \exp \left( \frac{-G_i^2 \gamma}{\sin^2 \theta} \right) d\theta
\]

(4.71)

In order to obtain BER expression for various fading conditions, we take expectation of (4.71) with respect to \( H \), and substitute the values of equivalent SNR (\( \gamma \)) and its corresponding MGF (\( M_\gamma(t) \)). After doing so, we get (4.71) as linear combinations of finite known integrals that can be expressed in closed form using (77) of [115]. In case of uncorrelated and spatially correlated fading, MGF \( M_\gamma(t) \) is given by (4.43) and (4.50), and hence we have

\[
P_b^{M-PAM}(k, \gamma) = \frac{1}{M} \sum_{i=0}^{\left\lfloor \frac{1-2^{-k}}{M-1} \right\rfloor} B_{i,k} \left( \sum_j \sum_{l=1}^{M_{Rs}} \left( \lambda_i \lambda_j \gamma \right)^{n_l} \right) \left[ 1 - \sqrt{\frac{G_i^2 \gamma}{2 + G_i^2 \gamma}} \sum_k \frac{(2k)^n}{k} \left( 1 + \frac{G_i^2 \gamma}{2} \right)^{-k} \right]
\]

(4.72)

In uncorrelated fading, the correlation matrices \( R_{Tx} \) and \( R_{Rx} \) are of full rank. In case of joint spatial and temporal correlation as well as joint spatial, temporal and frequency correlation the BER can be expressed as

\[
P_b^{M-PAM}(k, \gamma) = \frac{1}{M} \sum_{i=0}^{\left\lfloor \frac{1-2^{-k}}{M-1} \right\rfloor} \sum_{m=0}^{N_t-1} \sum_{l=1}^{M_{Tx}} \sum_{j=1}^{M_{Rs}} \left( \lambda_i \lambda_j \gamma \right)^{n_l} \left[ 1 - \sqrt{\frac{G_i^2 \gamma}{2 + G_i^2 \gamma}} \sum_k \frac{(2k)^n}{k} \left( 1 + \frac{G_i^2 \gamma}{2} \right)^{-k} \right]
\]

(4.73)
Finally, the exact value of average BER for M-PAM under various fading conditions can be obtained by averaging (4.72) and (4.73) and is given by

\[ P_b^{M\text{-PAM}}(\gamma) = \frac{1}{\log_2 M} \sum_{k=1}^{\log_2 M} P_b^{M\text{-PAM}}(k, \gamma) \]  

(4.74)

Above equations can be extended to two-dimensional constellation i.e. quadrature amplitude modulation (QAM) constellation. Here two independent PAM constellations are considered, namely I-ary PAM for in-phase component and J-ary PAM for quadrature component. Then \( M = I \times J \), and the BER for \( P_b^{I\text{-PAM}}(k, \gamma) \) under various fading conditions is given by (4.73) by replacing \( M \) by \( I \) and \( G \) with \( \hat{G}_1 = (2i + 1) \frac{3 \log_2(I, J)}{I^2 + J^2 - 1} \). Similarly BER for \( P_b^{J\text{-PAM}}(k, \gamma) \) under various fading conditions is given by (4.73) but with \( M \) replaced by \( J \) and \( G \) with \( \hat{G}_1 \). Finally, the exact average BER for an arbitrary M-QAM modulation is given by

\[ P_b^{M\text{-QAM}}(\gamma) = \frac{1}{\log_2(I, J)} \left[ \sum_{k=1}^{\log_2 I} P_b^{I\text{-QAM}}(k, \gamma) + \sum_{k=1}^{\log_2 J} P_b^{J\text{-QAM}}(k, \gamma) \right] \]  

(4.75)

### 4.7.2 BER with M-PSK Constellation

According to [112], the general tight bound approximate expression for the BER of coherent M-ary PSK in AWGN channels is given by

\[ P_b^{M\text{-PSK}}(\gamma) \leq \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max(M/4 - 1)} Q\left( \sqrt{2 \gamma \log_2 M \sin^2 \left( \frac{2i - 1}{M} \pi \right)} \right) \]  

(4.76)

Using Craig’s definition [114] of Q function and substituting values of equivalent SNR (\( \gamma \)) for different fading conditions we get BER expressions for different fading conditions. In case of uncorrelated and spatially correlated fading (4.76) can be rewritten as

\[ P_b^{M\text{-PSK}}(\gamma) \leq \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max(M/4 - 1)} \sum_{j=1}^{M_{TX}} \sum_{s=1}^{M_{RX}} (\lambda_{si}^T \lambda_j^R)^{-1} \left[ 1 - \frac{7}{2 + 7} \sum_{k=0}^{2k} \binom{2k}{k} \right] \left( 1 + \frac{\gamma}{2} \right)^{-k} \]  

(4.77)
After substituting $\gamma$ and $M_r(t)$, (4.77) becomes linear combination of finite known integrals which can be expressed in closed form using (77) of [115]. BER for joint spatial and temporal correlation as well as joint spatial, temporal and frequency correlation can be approximated as

$$P_b^{M-PSK}(\gamma) \equiv \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max(M/2-1,N/2)} \sum_{m=0}^{d_H M_t} \sum_{l=1}^{M_{RX}} \sum_{j=1}^{M_{RX}} \left( \lambda_m \lambda_j \gamma \right)^{-1}_{i} \left[ 1 - \sqrt{\frac{\gamma}{2 + \gamma}} \sum_{k=0}^{\infty} \frac{2^k}{k+1} \left( 1 + 2 \gamma \right)^{-\frac{1}{2}} \right]$$

(4.78)

### 4.8 SIMULATION RESULTS

This section shows the simulation results for LDPC coded Alamouti MIMO-OFDM system under various fading conditions along with theoretical results. The various simulation parameters used for mentioned concatenation scheme are given in Table 4.3.

**Table 4.3:** Simulation parameters for various fading conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total bandwidth</td>
<td>8MHz</td>
</tr>
<tr>
<td>$M_{TX}$</td>
<td>2</td>
</tr>
<tr>
<td>$M_{RX}$</td>
<td>2</td>
</tr>
<tr>
<td>FFT size</td>
<td>4×1024</td>
</tr>
<tr>
<td>Carrier modulation used</td>
<td>BPSK, QPSK, 16-QAM, 32-QAM</td>
</tr>
<tr>
<td>Cyclic prefix length</td>
<td>1024</td>
</tr>
<tr>
<td>LDPC code rate</td>
<td>1/2</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>3000</td>
</tr>
<tr>
<td>Number of 1's per column/per row</td>
<td>3/6</td>
</tr>
<tr>
<td>Channel model used [87]</td>
<td>Rayleigh quasi-static DVB-NGH Outdoor</td>
</tr>
<tr>
<td>Channel fading</td>
<td>Spatially independent, spatially correlated, spatial time correlated, spatial time frequency correlated</td>
</tr>
<tr>
<td>Number of channel taps</td>
<td>8</td>
</tr>
<tr>
<td>Path delays (in $\mu$sec)</td>
<td>0.02, 0.1094, 0.2188, 0.6094, 1.109, 2.109, 4.109, 8.109</td>
</tr>
<tr>
<td>Interleaver type</td>
<td>Uniform</td>
</tr>
<tr>
<td>Doppler shift</td>
<td>33.3Hz</td>
</tr>
</tbody>
</table>
To support and validate the derived bounds, BER performance is plotted with variation in SNR values under various fading conditions. Figure 4.9 shows BER performance of LDPC coded MIMO-OFDM system with various modulation techniques like BPSK, QPSK, 16-QAM and 32-QAM under spatially independent fading conditions. Figure 4.9 shows that the best BER performance is achieved with BPSK modulation where BER of about $10^{-5}$ is achieved at SNR of around 6.5dB, 7.5dB and 8.5dB for BPSK, QPSK, 16-QAM and 32-QAM respectively. Figure 4.10 shows BER performance under spatially correlation channel with transmitter side correlation of $\rho_t = 0.7$. To evaluate theoretical expressions, IOWEF of codes is calculated [74]. Figure 4.10 clearly indicates that BER performance degrades compared to the one plotted in Figure 4.9 due to rank deficient channel matrix., to achieve BER value of $10^{-5}$, the SNR required for BPSK, QPSK, 16-QAM and 32-QAM are 7.5dB, 8dB, 8.5dB, and 9.5dB respectively. The results of Figure 4.9 and Figure 4.10 shows almost same diversity order but Figure 4.10 suffers coding gain loss of the order of 1dB in all modulation schemes.

![Figure 4.9: BER comparison for LDPC coded Alamouti MIMO-OFDM system under spatially independent fading for various modulation schemes](image-url)
Figure 4.10: BER comparison for LDPC coded Alamouti MIMO-OFDM system with spatial correlation ($\rho_s = 0.7$) for various modulation schemes.

Figure 4.11 shows BER performance under spatially and time correlated channel. The effect of time correlation is modeled using Bessel functions mentioned in [116] with $f_d T_s = 0.1$. Similar to Figure 4.9 and 4.10, we achieved BER of $10^{-5}$ at SNR of around 10dB, 11dB, 12dB and 14dB for BPSK, QPSK, 16-QAM and 32-QAM respectively. Figure 4.11 shows the effect of spatial and time correlation is severe as the constellation size increases. Figure 4.12 shows the BER performance in extremely degraded channel consists of combinations of spatial, time and frequency correlation. The effect of frequency correlation is modeled using [105]. Figure 4.12 indicates that BER performance degrades further compared to Figures 4.9-4.11. The BER value of $10^{-5}$ achieved at SNR of 11dB, 12dB, 14dB and 17dB for BPSK, QPSK, 16-QAM and 32-QAM respectively. From Figures 4.9-4.12, it can be observed that the simulation results are very close to results obtained from derived equations. Further, the simulation results look identical to the analytical results.
Figure 4.11: BER comparison for LDPC coded Alamouti MIMO-OFDM system with spatial time correlation for various modulation schemes

Figure 4.12: BER comparison for LDPC coded Alamouti MIMO-OFDM system with spatial time frequency correlation for various modulation schemes
Figure 4.13: BER comparison for LDPC coded Alamouti MIMO-OFDM system with other codes

Figure 4.13 shows BER comparison of proposed system with existing works in literature under various fading conditions like spatially independent, spatially correlated, spatial time correlated and spatial time frequency correlated. From the results of Figure 4.13 it can be concluded that our concatenation scheme gives better performance compared to other codes in literature for all fading conditions. This is happened due to the fact that our concatenation scheme of LDPC with Alamouti MIMO-OFDM systems ensures (i) good error correction capability due to LDPC code, (ii) mitigation of frequency selective fading due to the OFDM modulation.
**Table 4.4**: Result comparison for LDPC coded Alamouti MIMO-OFDM system with other codes

<table>
<thead>
<tr>
<th>Code</th>
<th>BER=10^-4 at SNR of</th>
<th>Diversity Order</th>
<th>Decoder Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.I.D[ours]</td>
<td>5.5dB</td>
<td>Maximum</td>
<td>Low</td>
</tr>
<tr>
<td>I.I.D[74]</td>
<td>6dB</td>
<td>High</td>
<td>Moderate</td>
</tr>
<tr>
<td>I.I.D[83]</td>
<td>11dB</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>S.Corr [ours]</td>
<td>6.2dB</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>S.Corr [74]</td>
<td>6.7dB</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>S.T.Corr [ours]</td>
<td>8.5dB</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>S.T.Corr [74]</td>
<td>10dB</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>S.T.F.Corr [ours]</td>
<td>9.5dB</td>
<td>Low</td>
<td>Moderate</td>
</tr>
<tr>
<td>S.T.F.Corr [105]</td>
<td>&gt;20dB</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

### 4.9 CONCLUSION

In this chapter, we considered the performance analysis of serially concatenated regular and irregular LDPC codes with Alamouti STBC and SFBC MIMO-OFDM systems for high data rate wireless transmission. The DE tools based on pdf approach have been used for analyzing and optimizing LDPC codes. It is also used to compute probability of error and minimum operational SNR required for DVB-NGH standard. We have evaluated BER results of irregular and regular LDPC codes with Alamouti STBC and SFBC under MIMO and MIMO-OFDM system configurations. From simulation results, we showed that optimized irregular LDPC codes concatenated with Alamouti SFBC in 2×2 MIMO-OFDM systems achieved best BER results. Comparing the results with existing results, it can be concluded that above mentioned combination will outperform the existing schemes in terms of complexity and minimum operational SNR. Further the simulation results and results obtained from DE analysis are almost identical. Further, we used computationally inexpensive MGF approach rather than PDF approach to analyze LDPC coded Alamouti MIMO-OFDM systems under various fading conditions. The fading conditions include independent and correlative Rayleigh quasi-static fading. Due to the correlation structure there is loss of diversity and coding gain. These losses are quantified in spatial, spatial temporal and spatial time frequency correlation models. The concatenation scheme is assembled in such a way that it takes advantage of all individual blocks. Result section shows that the simulation results almost match with the theoretical results.