CHAPTER 3
RATE-$M_{tx}$ SPACE FREQUENCY AND SPACE TIME FREQUENCY CODES

In chapter 2, we studied STF codes that achieve full diversity with rate-1 even in severely faded channel. Recently, the performance of STF codes is studied under various channel conditions and system configurations [59] over quasi-static channels [60]. However, the performance can be improved further in terms of diversity gain if we consider general block fading channels [61-62]. In block fading channels, the fading coefficients are constant over one fading block and are independent for different blocks. In [63-66], a new algebraic SF and STF code design is proposed to achieve high rate in block fading channels.

In this chapter, a rate-$M_{tx}$ full diversity STF code is presented with a different approach compared to algebraic STF codes for block fading channels. The design is motivated by the fact that there is not much $M_{tx}$ rate ST or SF or STF codes existing that are easy to design and decode for quasi-static as well as block fading channels. Various design issues for high rate SF and STF codes that are easy to design and decode are addressed. The codes behave equally well in quasi-static as well as block fading channels. Several decoding approaches like ML, SD and array processing are investigated to resolve the complexity issue. It is proved that presented STF code achieves rate-$M_{tx}$ and full-diversity of $M_{tx}M_{rx}N_bL$, where $N_b$ is number of fading blocks.

A rate-$M_{tx}$ MIMO-OFDM system is shown in Figure 3.1, which consists of $M_{tx}$ transmit and $M_{rx}$ receiving antennas. Initially, the incoming bit stream is mapped into data symbols via modulation technique like BPSK and QPSK. The block of data symbols $S$ of size $N_cM_{tx}N_b$ is split into $J$ equal size sub blocks. These sub blocks can be expressed as

$$S = [S_1^T, S_2^T, \ldots, S_J^T]^T$$

(3.1)
The total sub blocks are \( J = N_C / K \), where \( K \) is defined as \( K = 2^{\left( \log_2 M_{\text{Tx}} \right)} \). Clearly for frequency selective channels, \( L \) is always greater than 1 and \( K \) is always a power of 4. These symbols are then encoded into STF codeword matrix \( \mathbf{C} \in \mathbb{C}^{N_C \times M_{\text{Tx}} N_b} \), where codeword \( \mathbf{C} \) [66] can be written as

\[
\mathbf{C} = [c^1, c^2, \ldots, c^{N_b}]
\]

where \( N_C \times M_{\text{Tx}} \) matrix \( C^u \) is defined as \( C^u = [c^u_1, c^u_2, \ldots, c^u_{M_{\text{Tx}}}] \) for \( u = 1, 2, \ldots, N_b \). The OFDM transmitter performs an \( N_C \) point IFFT to each column of matrix \( C^u \) during the fading block \( u \). After IFFT modulation, CP is added (with length \( \geq \) channel delay spread) to remove ISI. The information is passed through MIMO channel which is characterized by Jake’s model [42] for Rayleigh frequency selective channels and is given by

\[
h_{u,i,j}(t) = \sum_{l=0}^{L-1} \alpha_{u,i,j}^l (l) \delta(t - \tau_l)
\]

Equation (3.3) represents channel impulse response (CIR) from the \( i^{th} \) transmit antenna to \( j^{th} \) receive antenna during \( u^{th} \) fading block. The \( \alpha_{u,i,j}^l (l) \)'s are zero mean complex Gaussian random variables and independent for any \((i, j, u, l)\), where \( 1 \leq i \leq \)
\( M_{\text{TX}}, 1 \leq j \leq M_{\text{RX}}, 1 \leq u \leq N_{b} \) and \( 1 \leq l \leq L-1 \). It is assumed that all path gains follow the same power delay profile i.e. \( \mathbb{E}[\alpha_{i,j}^{u}(l)^{2}] = \delta_{i}^{2} > 0 \) for any given \((i,j,u,l)\). The powers of L-paths are normalized as \( \sum_{l=0}^{L-1} \delta_{i}^{2} = 1 \). The MIMO channel experiences frequency selective fading and block fading simultaneously through \( L \) independent paths between each pair of transmitting and receiving antenna. It is assumed that the path gains are constant over one fading block and independent for different blocks.

At the receiver, the received signals are assumed to be perfectly synchronized. After removing the CP and applying FFT on frequency tones, the received signal at \( j^{th} \) receive antenna during \( u^{th} \) fading block is given by

\[
Y_{j}^{u} = \sum_{i=1}^{M_{\text{TX}}} \text{diag}(c_{i,j}^{u}) H_{i,j}^{u}
\]

where \( Y_{j}^{u} \) is defined as, \( Y_{j}^{u} = [y_{j}^{u}(0), y_{j}^{u}(1), \ldots, y_{j}^{u}(N_{c} - 1)]^{T} \). The power normalization and noise terms are neglected for simplification. The channel frequency response [66] is given by

\[
H_{i,j}^{u} = F h_{i,j}^{u}
\]

where \( h_{i,j}^{u} = [H_{i,j}^{u}(0), H_{i,j}^{u}(1), \ldots, H_{i,j}^{u}(N_{c} - 1)]^{T}, h_{i,j}^{u} = [\alpha_{i,j}^{u}(0), \alpha_{i,j}^{u}(1), \ldots, \alpha_{i,j}^{u}(L-1)]^{T} \) and \( F = [f_{0}, f_{1}, \ldots, f_{L-1}] \). The column vector \( f_{l} \) is defined as \( f_{l} = [1, \omega_{l}, \omega_{l}^{2}, \ldots, \omega_{l}^{N_{c}-1}]^{T} \) where \( \omega_{l} = \exp(-j2\pi \frac{\tau_{l}}{T_{s}}) \) and \( D_{l} = \text{diag} (f_{l}) \), which means \( D_{l}c_{i,j}^{u} = \text{diag}(c_{i,j}^{u})f_{l} \). Therefore (3.4), can be written as

\[
Y_{j}^{u} = \sum_{i=1}^{M_{\text{TX}}} \begin{bmatrix} D_{0}c_{i,j}^{u} & D_{1}c_{i,j}^{u} & \cdots & D_{L-1}c_{i,j}^{u} \end{bmatrix} h_{i,j}^{u}
\]

By putting \( \hat{h}_{i,j}^{u} = [\alpha_{i,j}^{u}(1), \alpha_{i,j}^{u}(2), \ldots, \alpha_{i,j}^{u}(L)]^{T} \) for \( l=0, 1, \ldots, L-1 \), (3.4) can be written as

\[
Y_{j}^{u} = \sum_{i=0}^{L-1} \begin{bmatrix} D_{l}c_{i,j}^{u} \end{bmatrix} \hat{h}_{i,j}^{u}
\]
\[ = \sum_{l=0}^{L-1} D_l C^u \hat{h}_{l,j}^u \]  

(3.7)

Using

\[ X^u = [D_0 C^u, D_1 C^u, \ldots, D_{L-1} C^u] \]  

(3.8)

and

\[ h_j^u = \left[ \left( \hat{h}_{0,j}^u \right)^T, \left( \hat{h}_{1,j}^u \right)^T, \ldots, \left( \hat{h}_{L-1,j}^u \right)^T \right]^T \]  

(3.9)

In (3.6), we get \( Y_j^u = X^u h_j^u \) for \( u = 1, 2, \ldots, N_b \) and \( j = 1, 2, \ldots, M_{Rx} \). We can further generalize \( Y, h \) and \( X \) as

\[ Y = \begin{bmatrix} (Y_1^1)^T & \ldots & (Y_1^{N_b})^T & \ldots & (Y_{M_{Rx}}^1)^T & \ldots & (Y_{M_{Rx}}^{N_b})^T \end{bmatrix}^T \]  

(3.10)

\[ h = \begin{bmatrix} (h_1^1)^T & \ldots & (h_1^{N_b})^T & \ldots & (h_{M_{Rx}}^1)^T & \ldots & (h_{M_{Rx}}^{N_b})^T \end{bmatrix} \]  

(3.11)

\[ X = I_{M_b} \otimes \text{diag}(X^1, X^2, \ldots, X^{N_b}) \]  

(3.12)

Thus we obtain

\[ Y = \sqrt{\frac{\tilde{\gamma}}{M_{Tx}}} X h + N \]  

(3.13)

where the size of \( Y, X, h \) and \( N \) is \( N_b M_{Rx}, N_b M_{Rx} \times M_{Tx} M_{Rx} N_b L, M_{Tx} M_{Rx} N_b L \) and \( N_b M_{Rx} \) respectively. The factor \( \sqrt{\frac{\tilde{\gamma}}{M_T}} \) is the power normalization factor, where \( \tilde{\gamma} \) is average SNR.

3.1 RATE-M_{Tx} STF CODE PERFORMANCE DESIGN CRITERIA

Assume that \( \mathbf{C} \) and \( \hat{\mathbf{C}} \) are two STF codewords of size \( N_C \times M_{Tx} N_b \) related to \( S \) and \( \hat{S} \) respectively, the PEP between \( \mathbf{C} \) and \( \hat{\mathbf{C}} \) can be upper bounded [33] as
\[ P_e(\mathbf{X} \rightarrow \hat{\mathbf{X}}) \leq \left( \frac{2r_x - 1}{r_x} \right) \left( \prod_{i=1}^{r_x} \lambda_i \right)^{-r_x} \left( \frac{\gamma}{M_T} \right)^{-r_x} \]  \hspace{1cm} (3.14)

where \( r_x \) is the rank of \((\mathbf{X} - \hat{\mathbf{X}})^H \mathcal{R} (\mathbf{X} - \hat{\mathbf{X}})\) and \( \lambda_1, \ldots, \lambda_{rx} \) are the non-zero eigen values of \((\mathbf{X} - \hat{\mathbf{X}})^H \mathcal{R} (\mathbf{X} - \hat{\mathbf{X}})\) and \( \mathcal{R} = \mathbb{E}[\mathbf{h}\mathbf{h}^H] \) is the correlation matrix of \( \mathbf{h} \). The codewords \( \mathbf{X} \) and \( \hat{\mathbf{X}} \) are related to \( \mathbf{C} \) and \( \hat{\mathbf{C}} \) as shown in (3.8). Based upon PEP criteria two general STF performance criteria can be defined as follows.

**Diversity Criterion:** It is also called rank criterion. It states that minimum rank of \((\mathbf{X} - \hat{\mathbf{X}})^H \mathcal{R} (\mathbf{X} - \hat{\mathbf{X}})^H \) over all pairs of codewords \( \mathbf{C} \) and \( \hat{\mathbf{C}} \) should be as large as possible.

**Product Criterion:** It states that minimum value of the product \( \prod_{i=1}^{r_x} \lambda_i \) over all pairs of different codewords \( \mathbf{C} \) and \( \hat{\mathbf{C}} \) should be maximized.

In spatially uncorrelated MIMO channels, the channel taps \( \alpha_{i,j}(l) \) for \( l \in [0 \ldots L-1] \) between each pair of transmit antenna \( i \) and receive antenna \( j \) are independent of each other. Thus, correlation matrix \( \mathcal{R} = \mathbb{E}[\mathbf{h}\mathbf{h}^H] \) can be written as

\[ \mathcal{R} = \mathbf{I}_{MRx} \otimes \mathbf{I}_{Nb} \otimes \text{diag}(\delta_0^2, \delta_1^2, \ldots, \delta_{L-1}^2) \otimes \mathbf{I}_{MTx} \]  \hspace{1cm} (3.15)

Factorizing \( \mathcal{R} \) as \( \mathcal{R} = (\mathcal{R}^{1/2})(\mathcal{R}^{1/2})^H \), we get \( \mathcal{R}^{1/2} = \mathbf{I}_{MRx} \otimes \mathbf{I}_{Nb} \otimes \text{diag}(\delta_0, \delta_1, \ldots, \delta_{L-1})^{1/2} \) \( \otimes \mathbf{I}_{MT} \). Thus from (3.12) and (3.15), we have

\[ (\mathbf{X} - \hat{\mathbf{X}})^{1/2} = \mathbf{I}_{MRx} \otimes \mathbf{G} \]  \hspace{1cm} (3.16)

Block diagonal matrix \( \mathbf{G} \) can be further written as

\[ \mathbf{G} = \text{diag}(\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_{Nb}) \]  \hspace{1cm} (3.17)
and the $N_c \times M_{Tx} L$ matrix $G_u$ can be represented as

$$G_u = (X_u - \hat{X}_u) \text{diag}(\delta_0^2, \delta_1^2, \ldots, \delta_{L-1}^2)^{1/2}$$

for $u = 1, 2, \ldots, N_b$. Let $r_G$ and $r_{G_u}$ be the rank of $G$ and $G_u$ respectively, where $r_G$ is given by $r_G = \sum_{u=1}^{N_b} r_{G_u}$. Let $\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_{r_G}$ and $\hat{\lambda}_{u,1}, \hat{\lambda}_{u,2}, \ldots, \hat{\lambda}_{u,r_{G_u}}$ are non-zero eigenvalues of $G$ and $G_u$. Thus we have

$$\prod_{i=1}^{r_G} \hat{\lambda}_i = \prod_{u=1}^{N_b} \left( \hat{\lambda}_{u,1}, \hat{\lambda}_{u,2}, \ldots, \hat{\lambda}_{u,r_{G_u}} \right).$$

Further, $(X - \hat{X}) \Re (X - \hat{X})^H$ can be simplified to

$$(X - \hat{X}) \Re (X - \hat{X})^H = I_{M_{Rx}} \otimes (GG^H)$$

(3.18)

The rank of $I_{M_{Rx}} \otimes (GG^H)$ is defined as $r_s = r_G M_{Rx}$. Thus, the performance criteria of STF codes can be modified for frequency selective block fading as follows:

**Diversity Criterion for Block Fading:** It is also called sum of ranks criterion, which states that maximum transmit diversity gain is given by

$$r_G = \sum_{u=1}^{N_b} r_{G_u}$$

(3.19)

For all pairs of distinct codewords $C$ and $\hat{C}$.

**Product Criterion for Block Fading:** Maximize the product value of for all pairs of codewords $C$ and $\hat{C}$. The maximum value is called coding gain.

$$\prod_{i=1}^{r_G} \hat{\lambda}_i = \prod_{u=1}^{N_b} \left( \hat{\lambda}_{u,1}, \hat{\lambda}_{u,2}, \ldots, \hat{\lambda}_{u,r_{G_u}} \right)$$

(3.20)

The MIMO channels will experience frequency selective fading if $L > 1$. Also, if $N_b = 1$, the design rules of STF code are same as that of SF codes in quasi-static fading channels. The main requirement is to construct high rate codes with full diversity. Full diversity is directly related to the rank of the STF codes in MIMO frequency selective block fading channels. The rank is given by
\[ r_s = r_0 M_{Rs} \leq \min(M_{Rx}N_bN_c, M_{Rx}N_bM_{Tx}L) \] (3.21)

If we consider \( N_C \geq M_{Tx}L \) then rank \( r_s \) can be approximated as \( r_s \leq M_{Rx}N_bM_{Tx}L \) to achieve the full diversity. The matrix \( G \) should also be full rank for every distinct pair of codewords \( C \) and \( \hat{C} \).

### 3.2 RATE-M_{Tx} STF CODE DESIGN STRUCTURE

The coding algorithm provides different steps to design rate-M_{Tx} SF and STF codes in quasi-static and block fading channels. Although some work exits in [64], but the design was not generalized for STF codes. In the proposed design the algorithm work for STF codes with SF codes as a special case. Initially, the algorithm processes block wise data and precodes it by multiplying with unitary matrix, and subsequently with Hadamard matrix of order 2\( \times \)2 or 4\( \times \)4. The order of Hadamard matrix depends upon the number of transmitting antennas used. The processed block symbols are then concatenated to form complete codeword, which are then transmitted by \( M_{Tx} \) antennas. The design can be generalized for any number of transmitting antennas.

#### 3.2.1 Code Structure

The rate-M_{Tx} STF code scheme is shown in Figure 3.2. Initially, the block of data symbols \( S \) of size \( N_C \times M_{Tx} \times N_b \) is split into \( J \) equal size sub blocks.

![Figure 3.2: Details of rate-M_{Tx} STF encoder](image)

The total sub blocks are \( J = N_C / K \), where \( K = 2^{\log_2 M_{Tx}L} \). Afterwards, each sub block data symbols \( S_i \) is sent through STF encoder. The generalized STF encoder is same for every input sub block \( S_i \). The Input block \( S_i \) is linearly precoded [67] with a unitary matrix \( \Theta \). The algebraic construction of unitary matrix \( \Theta \) [62] per block is given by
\[
\Theta^u = \frac{1}{\sqrt{KM_{Tx} N_b}} \begin{bmatrix}
1 & \theta^{(u)}_{1(u)} & \theta^{(u)}_{2(u)} & \cdots & \theta^{K_{M_{Tx}}-1(u)}_{1(u)} \\
1 & \theta^{(u)}_{2(u)} & \theta^{(u)}_{3(u)} & \cdots & \theta^{K_{M_{Tx}}-1(u)}_{2(u)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & \theta^{(u)}_{K_{M_{Tx}}(u)} & \theta^{(u)}_{K_{M_{Tx}}(u)+1} & \cdots & \theta^{K_{M_{Tx}}-1(u)}_{K_{M_{Tx}}(u)}
\end{bmatrix}
\] (3.22)

where \( \theta_k = e^{\frac{\pi(4k-3)}{2K_{M_{Tx}}}} \), \( k=1,2,\ldots,K_{M_{Tx}} \) and \( u=1,2,\ldots,N_b \). The precoded matrix \( \tilde{X}_i \) can expressed as

\[
\tilde{X}_i = \Theta S_i
\] (3.23)

The size of \( \tilde{X}_i \), \( \Theta \) and \( S_i \) is \( K \times M_{Tx} \times N_b \). The resultant matrix [64] is reshaped and then some matrix manipulations are performed on it as shown below

\[
B_i = \tilde{X}_i \circ (\hat{H}_{m \times M_{Tx}} \otimes 1_{n \times 1})
\] (3.24)

where \( \hat{H}_{m \times M_{Tx}} \) is the first \( M_{Tx} \) column of \( m \times m \) Hadamard matrix \( \hat{H}_{m \times m} \) with \( m = 2^\lceil \log_2 M_{Tx} \rceil \) and \( n=K/m \). The values of \( m, n \) and \( K \) corresponding to \( M_{Tx} \) and \( L \) are given in Table 3.1, 3.2 and 3.3.

**Table 3.1:** Different values of \( L, K \) and \( n \) with \( M_{Tx}=2 \) and \( m=2 \)

<table>
<thead>
<tr>
<th>L</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>n</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

**Table 3.2:** Different values of \( L, K \) and \( n \) with \( M_{Tx}=3 \) and \( m=4 \)

<table>
<thead>
<tr>
<th>L</th>
<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>8</td>
<td>16</td>
<td>16</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>n</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
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</tbody>
</table>

**Table 3.3:** Different values of \( L, K \) and \( n \) with \( M_{Tx}=4 \) and \( m=4 \)

<table>
<thead>
<tr>
<th>L</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>8</td>
<td>16</td>
<td>16</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>n</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
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</tr>
</tbody>
</table>

The Hadamard matrices of order 2 and 4 are given by
\[ \hat{H}_{2\times2} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \text{ and } \hat{H}_{4\times4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \] (3.25)

Finally, \( B_i \) matrices are concatenated to form codeword matrix \( C \) of size \( N_c \times M_{Tx} N_b \) i.e.

\[ C = \begin{bmatrix} B_1^T, B_2^T, \ldots, B_i^T \end{bmatrix}^T \] (3.26)

### 3.2.2 Simulated Examples of STF Code Design

The coding strategy is same for every sub block \( B_i^T \) but with different variables and therefore formulation of only one sub block is considered.

**A. \( M_{Tx}=2 \) with \( N_b=1 \) and \( N_b=2 \)**

Consider \( N_c = 64 \) and \( L=2 \). When \( N_b = 1 \), STF codes resembles SF codes. When \( M_{Tx} = 2 \), the value of \( K, m \) and \( n \) are 4, 2 and 2 respectively as per Table 3.1. STF code corresponding to these parameters is a \( 4 \times 2 \) matrix as given by

\[ \begin{bmatrix} x_1 & x_5 \\ x_2 & x_6 \\ x_3 & -x_7 \\ x_4 & -x_8 \end{bmatrix} \] (3.27)

where \( x_i \)'s are elements of matrix \( \hat{X}_i \). It can be easily observed that 8 symbols are transmitted in 4 time slots that results in code rate of 2, which is same as that of \( M_{Tx} \). Above code structure can be extended to more fading blocks by choosing \( N_b=2 \) or more. The STF code with above parameters is given by

\[ \begin{bmatrix} x_1 & x_5 \\ x_2 & x_6 \\ x_3 & -x_7 \\ x_4 & -x_8 \end{bmatrix} \begin{bmatrix} x_9 & x_{13} \\ x_{10} & x_{14} \\ x_{11} & -x_{15} \\ x_{12} & -x_{16} \end{bmatrix} \] (3.28)
Comparing the codes in (3.27) and (3.28), the numerator component in (3.28) exactly resembles that are given in (3.27) with same code rate.

**B. $M_{tx}=3$ with $N_b = 1$ and $N_b=2$**

In this case, rate 3 STF code is constructed with $N_b=1$ and $N_b=2$. When $M_{tx}=3$, The values of $K$, $m$ and $n$ is 8, 4 and 2 respectively as per Table 3.2. STF code corresponding to these parameters is an $8 \times 3$ matrix given by

$$
\begin{bmatrix}
  x_1 & x_9 & x_{17} \\
  x_2 & x_{10} & x_{18} \\
  x_3 & -x_{11} & x_{19} \\
  x_4 & -x_{12} & -x_{20} \\
  x_5 & x_{13} & -x_{21} \\
  x_6 & x_{14} & -x_{22} \\
  x_7 & -x_{15} & -x_{23} \\
  x_8 & -x_{16} & -x_{24}
\end{bmatrix}
$$

(3.29)

Above code structure shows code rate of 3. This can be extended to more fading blocks, e.g., $N_b=2$. The code structure corresponding to $N_b = 2$ is given by

$$
\begin{bmatrix}
  x_1 & x_9 & x_{17} \\
  x_2 & x_{10} & x_{18} \\
  x_3 & -x_{11} & x_{19} \\
  x_4 & -x_{12} & -x_{20} \\
  x_5 & x_{13} & -x_{21} \\
  x_6 & x_{14} & -x_{22} \\
  x_7 & -x_{15} & -x_{23} \\
  x_8 & -x_{16} & -x_{24}
\end{bmatrix}
\begin{bmatrix}
  x_{25} & x_{33} & x_{41} \\
  x_{26} & x_{34} & x_{42} \\
  x_{27} & -x_{35} & x_{43} \\
  x_{28} & -x_{36} & -x_{44} \\
  x_{29} & x_{37} & -x_{45} \\
  x_{30} & x_{38} & -x_{46} \\
  x_{31} & -x_{39} & -x_{47} \\
  x_{32} & -x_{40} & -x_{48}
\end{bmatrix}
$$

(3.30)

where numerator matrix is for fading block 1 and denominator matrix is for fading block 2 with same code rate.

**C. $M_{tx}=4$ with $N_b = 1$ and $N_b=2$**

The parameter corresponds to $M_{tx}=4$ are $K=8$, $m=4$ and $n=2$ respectively as per Table 3.3. The STF code structure corresponds to above parameters is an $8 \times 4$ matrix and is given by
\[
\begin{bmatrix}
  x_1 & x_9 & x_{17} & x_{25} \\
  x_2 & x_{10} & x_{18} & x_{26} \\
  x_3 & -x_{11} & x_{19} & -x_{27} \\
  x_4 & -x_{12} & x_{20} & -x_{28} \\
  x_5 & x_{13} & -x_{21} & -x_{29} \\
  x_6 & x_{14} & -x_{22} & -x_{30} \\
  x_7 & -x_{15} & -x_{23} & x_{31} \\
  x_8 & -x_{16} & -x_{24} & x_{32}
\end{bmatrix}
\]

(3.31)

The above code structure can be extended to two fading blocks with same parameters and same code rate and is given by:

\[
\begin{bmatrix}
  x_1 & x_9 & x_{17} & x_{25} \\
  x_2 & x_{10} & x_{18} & x_{26} \\
  x_3 & -x_{11} & x_{19} & -x_{27} \\
  x_4 & -x_{12} & x_{20} & -x_{28} \\
  x_5 & x_{13} & -x_{21} & -x_{29} \\
  x_6 & x_{14} & -x_{22} & -x_{30} \\
  x_7 & -x_{15} & -x_{23} & x_{31} \\
  x_8 & -x_{16} & -x_{24} & x_{32}
\end{bmatrix}
\begin{bmatrix}
  x_{33} & x_{41} & x_{49} & x_{57} \\
  x_{34} & x_{42} & x_{50} & x_{58} \\
  x_{35} & -x_{43} & x_{51} & -x_{59} \\
  x_{36} & -x_{44} & x_{52} & -x_{60} \\
  x_{37} & x_{45} & -x_{53} & -x_{61} \\
  x_{38} & x_{46} & -x_{54} & -x_{62} \\
  x_{39} & -x_{47} & -x_{55} & x_{63} \\
  x_{40} & -x_{48} & -x_{56} & x_{64}
\end{bmatrix}
\]

(3.32)

3.3 BER EXPRESSION OF RATE-\(M_{Tx}\) STF CODED MIMO-OFDM SYSTEMS

In this section, BER expressions for STF block coded MIMO-OFDM systems are derived and evaluated for frequency selective block fading channels. On the receiver side of MIMO-OFDM systems, data can be extracted and detected through (3.13). The BER expression [67] can be written as

\[
\text{BER} = \frac{1}{N_c} \sum_{n=0}^{N_c-1} \text{BER}(n)
\]

(3.33)

Considering MPSK modulation with gray bit mapping for each subcarrier and ignoring degradation due to CP, instantaneous BER expression for the \(n^{th}\) subcarrier [68] can be represented as

90
BER_{MPSK} (n) = \frac{1}{\beta} \text{erfc}\left(\sqrt{\gamma_s [H^2 (n)]} \sin \left(\frac{\pi}{2^\beta}\right)\right) \tag{3.34}

The exponential approximation [69] of above expression is given as

BER_{MPSK} (n) = 0.2 \exp \left(-\frac{7\gamma_s [H(n)]^2}{2^{1.9\beta} + 1}\right) \tag{3.35}

Thus, the BER expression in (3.33) can be rewritten as

BER_{MPSK} = \frac{1}{N_C \beta} \sum_{n=0}^{N_C-1} \text{erfc}\left(\sqrt{\gamma_s [H^2 (n)]} \sin \left(\frac{\pi}{2^\beta}\right)\right) \tag{3.36}

and can be exponentially approximated as

BER_{MPSK} (n) = \frac{1}{N_C} \sum_{n=0}^{N_C-1} 0.2 \exp \left(-\frac{7\gamma_s [H(n)]^2}{2^{1.9\beta} + 1}\right) \tag{3.37}

Now the average BER can be obtained by integrating BER_{MPSK} over infinite interval and is given by.

BER_{MPSK}^{\text{avg}} = \int_0^\infty BER_{MPSK} P(\gamma) \, d\gamma \tag{3.38}

where \( \gamma = [H(n)]^2 \gamma_s \). Since \( H(n) \) is Rayleigh distributed with variance 1, \( H(n)^2 \) will be chi-square pdf with two degree of freedom. Consequently, by putting (3.37) in (3.38) we get

BER_{MPSK}^{\text{avg}} = 0.2 \left(1 + \frac{7\gamma_s}{2^{1.9\beta} + 1}\right)^{-1} \tag{3.39}

Equation (3.39) gives BER expression for uncoded OFDM system. It can be extended to STF coded MIMO-OFDM systems with \( M_{Tx} \) transmitting antenna, \( M_{Rx} \) receiving antennas and \( N_b \) fading blocks. The normalized instantaneous SNR [70] in MIMO-OFDM is given by
\[
\gamma = \frac{1}{M_{Tx}N_{b}R_{C}} \sum_{i=1}^{M_{Rx}} \sum_{j=1}^{M_{Rx}} \sum_{l=0}^{N_{b}-1} \left[ H_{i,j}^{u}(n,l) \right] \gamma_{s}^{2} 
\]

(3.40)

Using above expression, the BER of MPSK-STFBC-MIMO-OFDM over frequency selective block fading channels can be expressed as

\[
\text{BER}_{\text{MPSK}} = \frac{1}{N_{C}^{2}} \sum_{n=0}^{N_{C}-1} \text{erfc} \left( \sqrt{\frac{M_{Tx}M_{Rx}N_{b}L-1}{R_{C}M_{Tx}N_{b}} \frac{1}{\sin \left( \frac{\pi}{2^{\beta}} \right)}} \right) 
\]

(3.41)

Equation (3.41) can be exponentially approximated as

\[
\text{BER}_{\text{MPSK}}(n) = \frac{0.2}{N_{C}} \sum_{n=0}^{N_{C}-1} \exp \left( -\frac{7\gamma_{s}^{2} M_{Tx}M_{Rx}N_{b}L-1}{R_{C}M_{Tx}N_{b}(2^{1.9\beta} + 1)} \right) 
\]

(3.42)

Average BER in (3.38) can be extended to STFBC-MIMO-OFDM as

\[
\text{BER}_{\text{MPSK}}^{\text{avg}} = \int_{0}^{\infty} \left[ P(\gamma_{i,j}^{u}(l))d\gamma_{i,j}^{u} \right] \left[ P(\gamma_{M_{Tx},M_{Rx}}^{l}(l))d\gamma_{M_{Tx},M_{Rx}}^{l} \right] 
\]

(3.43)

with the knowledge that \( H_{i,j}^{u}(n,l) \) is an i.i.d (independent and identically distributed) Rayleigh channel with variance 1. The pdf \( P(\gamma_{i,j}^{u}(l)) \) is given by

\[
P(\gamma_{i,j}^{u}(l)) = \frac{1}{\gamma_{i,j}^{u}(l)} \exp \left( -\frac{\gamma_{i,j}^{u}(l)}{\gamma_{i,j}^{u}(l)} \right) 
\]

(3.44)

where \( \gamma_{i,j}^{u} \geq 0 \). Substituting (3.42) and (3.44) in (3.43) we get
\[ \text{BER}_{\text{MPSK}}^{\text{avg}} = 0.2 \left( 1 + \frac{7 \gamma_s}{R_C M_{1x} N_b (2^{1.96} + 1)} \right)^{-M_{\text{rec}} M_{1x} N_b} \]  

(3.45)

### 3.4 Decoding of High Rate SF and STFBC MIMO-OFDM

ISI caused by multipath MIMO channels distorts the MIMO-OFDM transmitted signal and this produces bit errors at the receiver. To minimize these errors, equalization or proper decoding is needed. In this section, we discuss various equalizers or decoders like ML, SD and array processing, also their performance evaluation is done in terms of BER and complexity.

#### 3.4.1 Maximum Likelihood (ML) Decoder

ML decoder [54] is already discussed in section 2.4.5. In this section, the results are generalized for block fading channels. The minimization problem for the fading block to find codeword \( \hat{x}^u \) can be written as

\[
\hat{x}^u(n) = \arg \min_{x^u(n)} \sum_{u=1}^{N_b} \sum_{n=0}^{N_c-1} \|Y^u(n) - c^u(n)H^u(n)\|_F^2
\]  

(3.46)

The channel is assumed to be constant in one fading block. (3.46) can be expanded using Frobenius norm as follows:

\[
\hat{x}^u(n) = \arg \min_{x^u(n)} \sum_{u=1}^{N_b} \sum_{n=0}^{N_c-1} \text{Trace}[(Y^u(n) - x^u(n)H^u(n))H^u(n) - x^u(n)H^u(n)]
\]  

(3.47)

\[
\hat{x}^u(n) = \arg \min_{x^u(n)} \text{Trace} 
\begin{bmatrix}
(x^u(n))^H (Y^u(n) - x^u(n)H^u(n))H^u(n) - x^u(n)H^u(n)
\end{bmatrix}
\]  

(3.48)

If \((Y^u)^H Y^u\) is independent of the transmitted codeword, (3.48) can be written as

\[
\hat{x}^u(n) = \arg \min_{x^u(n)} \sum_{u=1}^{N_b} \sum_{n=0}^{N_c-1} \text{Trace} 
\begin{bmatrix}
(H^u(n))^H (x^u(n))^H x^u(n)H^u(n)
\end{bmatrix}
\]  

(3.49)

For multiple receivers (3.49) becomes
\[
\hat{x}^u(n) = \arg \min_{x^u(n)} \left[ \sum_{j=1}^{M_R} (H_j^u(n))^H (x^u(n))^H (x^u(n)H_j^u(n)) + 2 \Re(\sum_{j=1}^{M_R} (H_j^u(n))^H (x^u(n))^H Y_j^u(n)) \right]
\]

(3.50)

In case of ST coding, the above metric can be decomposed into two separate parts for detecting each individual symbol, i.e., ML decoding becomes SML. In SF and STF coding, joint ML decoder (JML) is preferred that detects two symbols jointly. Similarly in STF coding, two symbols can be detected jointly in one fading block which increases decoding complexity.

### 3.4.2 Sphere Decoder (SD)

As discussed in section 2.4.6, a full search for finding the optimal codeword in ML requires lot of computations which increases further with increase in constellation size, typically proportional to \(2^{RM} \). Therefore in SD [56], instead of searching all possible vectors for finding optimal codeword, a hyper-sphere of radius \( R_{SD} \) centered on the received signal vector is searched in \( u^{th} \) fading block as shown below.

\[
\hat{X}^u = \arg \min_{X} \| Y^u - X^uH^u \|_F^2 \leq R_{SD}^2
\]

(3.51)

After optimizing \( \hat{X}^u \), the radius of the search sphere is reduced and procedure described above is repeated till no point lie inside the search sphere. It is implemented in two steps namely, pre-processing and search step. In first step, the solution of optimization problem mentioned in (3.51) becomes \( Z_s^u = (H^u)^+ Y^u \). Equation (3.51) can be written as

\[
\min_{X^{u} \in \Lambda} \left( X^u - Z_s^u \right)^H (H^u)^H (H^u)(X^u - Z_s^u)
\]

(3.52)

where \( \Lambda \) is a complex lattice in the sense that each coordinates of \( X \) is chosen from the defined complex constellation. Unlike QR decomposition used in section 2.4.6, Cholskey decomposition is performed on \((H^u)^H(H^u)\) matrix to get upper triangular matrix as \( U = \left\{ u_{k_1,1} \mid u_{k_2,k_s} \in R_{SD} > 0 \right\} \) such that \((H^u)^H(H^u) = (U^u)^H(U^u)\). The modified optimization problem becomes
\[ \left\| U^u (Z^u_s - X^u) \right\|^2 \leq R_{SD}^2 \] (3.53)

After finding unconstrained solution \( Z^u_s \) and forming upper triangular matrix, a matrix \( Q^u \) [46] is formed and is given by

\[
Q^u = \begin{bmatrix}
q_{k_s,k_s}^u &= \left( u_{k_s,k_s}^u \right)^2 \\
q_{k_s,1}^u &= u_{k_s,1}^u / u_{k_s,k_s}^u \text{ for } k_s < 1
\end{bmatrix}
\] (3.54)

In search step, the points inside the sphere are examined to locate the optimal codeword. Thus (3.54) can be modified in terms of matrix \( Q \) as follows

\[
\sum_{i=0}^{K_s} \left| q_{i,1}^u \left( X_{i}^u - Z_{S_i}^u \right) + \sum_{j=i+1}^{K_s} q_{i,j}^u \left( X_{j}^u + Z_{S_j}^u \right) \right|^2 \leq R_{SD}^2
\] (3.55)

To find optimal codeword we start searching with \( k_s = K_s \) and find the distance between \( X_{k_s}^u \) and the center of the \( K_s \)-dimensional sphere as

\[
d_{k_s}^2 = \sum_{i=0}^{K_s} \left[ \sum_{j=i}^{K_s} q_{i,j}^u \left( X_{i}^u - Z_{S_i}^u \right) \right]^2
\] (3.56)

Let a variable \( S_{K_s}^u \) is defined as

\[
S_{K_s}^u = Z_{S_{K_s}}^u - \sum_{i=K_s+1}^{K_s} q_{i,k_s}^u \left( X_{i}^u - Z_{S_{i}}^u \right)
\] (3.57)

The condition for optimal codeword being inside the search sphere is given by

\[
d_{k_s}^2 = d_{K_s+1}^2 + q_{K_s,K_s}^u \left| X_{K_s}^u - S_{K_s}^u \right|^2 \leq R_{SD}^2
\] (3.58)

Hence, a search space for \( S_{K_s}^u \) can be specified as

\[
\left| C_{K_s}^u - S_{K_s}^u \right|^2 \leq \frac{R_{SD}^2 - d_{k_s}^2 + 1}{q_{k_s,k_s}^u}
\] (3.59)

when \( K_s \) become 1, which means a valid codeword is found. If the distance between center and the searched point is less than the radius of the hyper sphere [56], this distance becomes new radius. The procedure is then repeated, and if at any moment \( d_{k_s}^2 \) is greater than the radius of the sphere, the procedure is terminated.
3.4.3 Array Processing Decoder

In ML decoding, transmitted symbols are detected in pairs which increase decoding complexity. This complexity increases further with modulation level and with antennas employed, which in turn increases transmission delay. To overcome this problem, the array processing [71] decoding algorithm is used. In this approach, signals which are transmitted via different antennas are separated by null space. The null space decomposes received symbols into several independent parts that are decoded separately and linearly. The transmitted signals can be divided into two parts with one part transmitted by antenna group 1 that includes 1\textsuperscript{st} and 2\textsuperscript{nd} transmitting antenna and other part by antenna group 2 which includes 3\textsuperscript{rd} and 4\textsuperscript{th} transmitting antenna. After this division, MIMO channel per fading block for $4\times M_{Rx}$ system can be written as $H^u = \begin{bmatrix} H_1^u & H_2^u \end{bmatrix}$, where $H_1^u$ and $H_2^u$ [72] are given by

$$
H_1^u = F \begin{bmatrix}
    h_{1,1}^u & h_{2,1}^u \\
    h_{1,2}^u & h_{2,2}^u \\
    \vdots & \vdots \\
    h_{1,M_{Rx}}^u & h_{2,M_{Rx}}^u 
\end{bmatrix}
$$

(3.60)

In (3.60), $F = [f_0, f_1, \ldots, f_{L-1}]$. The column vector $f_i$ is defined as $f_i = [1, \omega_i, \omega_i^2, \ldots, \omega_i^{N_c-1}]$ with $\omega_i = \exp(-j2\pi \frac{\tau_i}{T_s})$ and $T_s$ is the effective duration of the OFDM symbol.

$$
H_2^u = F \begin{bmatrix}
    h_{3,1}^u & h_{4,1}^u \\
    h_{3,2}^u & h_{4,2}^u \\
    \vdots & \vdots \\
    h_{3,M_{Rx}}^u & h_{4,M_{Rx}}^u 
\end{bmatrix}
$$

(3.61)
The null space of a matrix $A$ is the subspace of vectors $x$ for which $Ax=0$. There should be more than two antennas at the receiver to ensure the existence of null space. Let $\Psi_1^u$ and $\Psi_2^u$ denote the null space of $H_1^u$ and $H_2^u$ respectively. Therefore by definition

$$\Psi_1^u (H_1^u)^T = 0$$  \hspace{1cm} (3.62)

and

$$\Psi_2^u (H_2^u)^T = 0$$  \hspace{1cm} (3.63)

Multiplying (3.13) with $\Psi_1^u$ and $\Psi_2^u$, respectively, we get

$$(\Psi_1^u)^T Y^u = (\Psi_1^u)^T H^u X^u + (\Psi_1^u)^T n^u$$  \hspace{1cm} (3.64)

and

$$(\Psi_2^u)^T Y^u = (\Psi_2^u)^T H^u X^u + (\Psi_2^u)^T n^u$$  \hspace{1cm} (3.65)

Using the definition of null matrix [72] we have

$$(\Psi_1^u)^T H^u X^u = \begin{bmatrix} 0 & (\Psi_1^u)^T H_2^u \\ (\Psi_2^u)^T H_1^u & 0 \end{bmatrix} X^u$$  \hspace{1cm} (3.66)

$$(\Psi_2^u)^T H^u X^u = \begin{bmatrix} (\Psi_2^u)^T H_1^u \\ 0 \end{bmatrix} X^u$$  \hspace{1cm} (3.67)

where $[X^u]$ is the rate-$M_{Tx}$ STF code described in section 3.2 for different number of fading blocks. During decoding process, the channel matrix is repeated for all sub blocks. Hence the STF code with 4 transmit antennas can be decoded in two parallel steps. To conclude, the decoding complexity can be considerably reduced compared to traditional ML decoding. Decoding complexity is calculated in terms of number of complex valued additions, subtractions and multiplications that are performed to decode one block of information. While one complex multiplication is considered to be equivalent to 4 real multiplications and 2 real additions, one complex addition is considered as 2 real additions. Further, the multiplication of a real valued quantity by a factor 2, like the term on right hand side of equation (3.50) is implemented using one real valued addition. In case of ML decoding, we have to compare single symbol decodable ML for ST codes and jointly decodable ML for SF and STF codes. In the first case, we need to compute $2^B$
metrics for each of the two transmitted symbols. In joint ML, we need $2^{2\beta}$ metrics to determine symbols which jointly minimizes (3.50). The number of necessary complex valued additions, subtractions and multiplications are summarized in Table 3.4. We can compute exact amount with different modulation schemes like BPSK, QPSK and 16-QAM as shown in Table 3.5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SML</th>
<th>JML</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of additions/subtractions</td>
<td>$4 + 3 \times 2^\beta$</td>
<td>$4 + 3.5 \times 2^\beta$</td>
</tr>
<tr>
<td>Number of complex multiplications</td>
<td>$8 + 2.5 \times 2^\beta$</td>
<td>$10 + 4 \times 2^\beta$</td>
</tr>
</tbody>
</table>

**Table 3.5:** Number of complex valued operations in SML, JML and array processing decoder

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BPSK($\beta=1$)</th>
<th>QPSK($\beta=2$)</th>
<th>16-QAM($\beta=4$)</th>
<th>Array Processing decoder[73]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of additions/subtractions</td>
<td>SML</td>
<td>JML</td>
<td>SML</td>
<td>JML</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>10 18</td>
<td>16</td>
<td>60</td>
<td>52</td>
<td>900</td>
</tr>
<tr>
<td>Number of complex multiplications</td>
<td>13</td>
<td>26</td>
<td>18</td>
<td>74</td>
</tr>
</tbody>
</table>

From the Table 3.5, it can be concluded that complexity increases with increase in constellation size and among all decoders array processing decoder exhibits least complexity. In SD, complexity is measured in terms of FLOPS. Average FLOPS per block in case of SD comes out to be 10(approx.) for BPSK and increased up to 200 for 16-QAM. Thus, the FLOPS are considerably less than real multiplications and additions in SML and JML but more than that of array processing decoder. Therefore the array processing decoder is significantly less complex compared to SML, JML and SD. The total system complexity also includes complexity of functional blocks like IFFT and FFT at transmitter and receiver. The complex multiplications required for an $N_C$ point FFT is $N_C \log_2 N_C$ and for $N_C=64$, complex multiplications becomes 384. Although, decoding complexity in array processing decoder is proportional to $\sqrt{M}$ compared to $M^2$ in ML decoder but for faster decoding it requires more power. This decoding scheme can be used even for more transmit antennas.
3.5 SIMULATION RESULTS

In this section, simulation results are presented for rate-$M_{Tx}$ full diversity SF and STF codes. Our method provides high rate SF and STF codes that are easy to design and decode. We will justify that our designed codes behave equally well in quasi-static as well as block fading channels. Further the proposed STF code achieves rate-$M_{Tx}$ with full diversity of $M_{Tx}M_{Rx}N_bL$. The simulation parameters used for simulating rate-$M_{Tx}$ MIMO-OFDM transceiver model mentioned in Figure 3.1 are given in Table 3.6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total bandwidth</td>
<td>20MHz</td>
</tr>
<tr>
<td>Number of transmitting antenna</td>
<td>2 and 4</td>
</tr>
<tr>
<td>Number of receiving antenna</td>
<td>2 and 4</td>
</tr>
<tr>
<td>Number of data subcarriers</td>
<td>64</td>
</tr>
<tr>
<td>Number of pilot-subcarriers</td>
<td>None</td>
</tr>
<tr>
<td>IFFT size</td>
<td>64</td>
</tr>
<tr>
<td>Guard period type</td>
<td>Cyclic extension</td>
</tr>
<tr>
<td>Cyclic prefix length</td>
<td>16</td>
</tr>
<tr>
<td>Carrier modulation used</td>
<td>BPSK-rate-4 codes</td>
</tr>
<tr>
<td></td>
<td>4-QAM-rate-2 codes</td>
</tr>
<tr>
<td>Channel model</td>
<td>Two-ray equal power delay profile model</td>
</tr>
<tr>
<td>Delay spread</td>
<td>0.2μsec</td>
</tr>
<tr>
<td>Transmission rate</td>
<td>4/bits/sec/Hz</td>
</tr>
<tr>
<td>Maximum Doppler spread</td>
<td>200 Hz</td>
</tr>
<tr>
<td>Maximum Doppler shift</td>
<td>$2\pi f_m=1.256\times10^7$</td>
</tr>
<tr>
<td>Direction of mobile Travel (θ)</td>
<td>In the direction of base station</td>
</tr>
<tr>
<td>Window type</td>
<td>Rectangular pulse</td>
</tr>
</tbody>
</table>

Simulation is done in two phases. In phase 1, results are plotted for two transmit and two receiving antennas, and in phase 2, four transmit and four receiving antennas are considered. Also the results are compared with existing codes for similar code rate and modulations. The channel is MIMO frequency selective block fading channel derived from simple two-ray equal power delay profile Jake’s model. The effects of Doppler shift and Doppler spread due to relative motion between transmitter and receiver is also considered.
Figure 3.3 shows performance comparison of rate-2 STF codes with rate-2 SF codes both implemented with ML, SD and array processing decoders. Rate-2 STF results have large slope than SF due to higher diversity order. Among the decoders, STF with SD clearly outperforms the other combinations because in array processing, there is error in calculating null matrix with 2 receivers. In ML, the size of search space for selecting optimum code is large. It can be observed that the results using the closed form expression (CFE) are very close to the simulated results.

![BER comparison for rate-2 STFC and SFC for 2x2 MIMO-OFDM system with various equalizers using 4-QAM](image)

**Figure 3.3:** BER comparison for rate-2 STFC and SFC for 2x2 MIMO-OFDM system with various equalizers using 4-QAM

Figure 3.4 compares our STF codes with existing rate-2 STF in [66] and rate-1 STF code described in [38] and [66] with ML and SD detectors. Also our rate-2 SF codes are compared with existing rate-2 SF in [65] and rate-1 SF codes in [36]. To fix the transmission rate at 4/bits/sec/Hz, 4-QAM modulation is used for rate-2 codes and 16-QAM for rate-1 codes respectively. Figure 3.4 shows that proposed STF and SF codes dominates in lower SNR region and STF code in [66] and SF code in [65] dominates in higher SNR region. Also rate-2 STF results has higher slope than SF due to higher
diversity order of 16 instead of 8. Comparison results for our rate-2 STF and SF codes are summarized in Table 3.7.

![BER comparison for our rate-2 STF and SF codes with other codes for 2x2 MIMO-OFDM](image)

**Figure 3.4:** BER comparison for our rate-2 STF and SF codes with other codes for 2x2 MIMO-OFDM

**Table 3.7:** Result comparison for our rate-2 STF and SF codes with other codes for 2x2 MIMO-OFDM system

<table>
<thead>
<tr>
<th>Code</th>
<th>BER=$10^{-4}$ at SNR of</th>
<th>Diversity Order</th>
<th>Decoder Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate-2 STF-SD [ours]</td>
<td>15.2dB</td>
<td>Maximum</td>
<td>Moderate</td>
</tr>
<tr>
<td>Rate-2 STF-SD [66]</td>
<td>15dB</td>
<td>High</td>
<td>Moderate</td>
</tr>
<tr>
<td>Rate-2 SF-SD [ours]</td>
<td>18.5dB</td>
<td>Moderate</td>
<td>Low</td>
</tr>
<tr>
<td>Rate-2 SF-SD [65]</td>
<td>18dB</td>
<td>Low</td>
<td>Moderate</td>
</tr>
<tr>
<td>Rate-1 STF-SD [66]</td>
<td>19dB</td>
<td>Low</td>
<td>Moderate</td>
</tr>
<tr>
<td>Rate-1 STF-ML [38]</td>
<td>18.8dB</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Rate-1 SF-[36]</td>
<td>&gt; 20dB</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>
Figure 3.5 compares proposed rate-4 STF codes with rate-4 SF codes both implemented with ML, SD and array processing decoders. This shows that rate-4 STF codes with array processing decoders have better performance than all other combinations due to advantage in calculating error free null matrix with 4 receiving antennas. Further, it is observed that the results using the closed form expression are very close to the simulation results. Figure 3.6 compares proposed rate-4 STF code with rate-4 STF code given in [66] and proposed rate-4 SF with existing rate-4 SF code in [64]. To fix the transmission rate at 4/bits/sec/Hz, BPSK modulation technique is used for both rate-4 STF and SF codes. Figure 3.6 shows that presented STF and SF codes have better performance than existing rate-4 STF and SF codes due to reduction in decoder complexity. BER performance can be further improved by considering higher delay spread. The comparison results for our rate-4 STF and SF codes with other rate-4 codes are summarized in Table 3.8.

![Figure 3.5: BER comparison for rate-4 STF and SF codes for 4x4 MIMO-OFDM system with various equalizers using BPSK](image-url)
Figure 3.6: BER comparison for our rate-4 STF and SF codes with other rate-4 codes for $2\times2$ MIMO-OFDM system

Table 3.8: Result comparison for our rate-4 STF and SF codes with other rate-4 codes for $2\times2$ MIMO-OFDM system

<table>
<thead>
<tr>
<th>Code</th>
<th>BER=$10^{-4}$ at SNR of</th>
<th>Diversity Order</th>
<th>Decoder Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate-4 STF-Array Processing [ours]</td>
<td>11dB</td>
<td>Maximum</td>
<td>Low</td>
</tr>
<tr>
<td>Rate-4 STF-SD [66]</td>
<td>11.8dB</td>
<td>High</td>
<td>Moderate</td>
</tr>
<tr>
<td>Rate-4 SF-Array Processing [ours]</td>
<td>15dB</td>
<td>Moderate</td>
<td>Low</td>
</tr>
<tr>
<td>Rate-4 SF-SD [64]</td>
<td>17dB</td>
<td>Low</td>
<td>Moderate</td>
</tr>
</tbody>
</table>
3.6 CONCLUSION

In this chapter, a rate $M_{TX}$ STF code with full diversity of $M_T M_R N_b L$ is investigated with a design approach different from existing algebraic STF codes in block fading channels. This chapter addresses the issue of designing high rate SF and STF codes that are easy to design and decode. The decoder complexity is resolved by using ML, SD and array processing techniques. It is also proved that the proposed code for 4×4 MIMO system along with array processing decoder achieves goals of lower complexity. These results are verified by simulation plots. The performance of STF code is also compared with other existing STF codes in terms of BER and decoder complexity. Further, closed form expressions for BER performance of STFBC MIMO-OFDM systems are derived and evaluated for frequency selective block fading channels with MPSK constellations. It is concluded that these expressions are very close to simulation results.